Deep inelastic scattering as a photon shockwave

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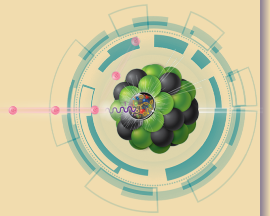
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Outline

Work in progress with Patricia Gimeno-Estivill, Xinyu Qin, Mikko Seppälä

This talk

- Shockwave scattering, CGC
- Motivation: single inclusive hadron production in pA
- DIS as a photon shockwave
  - Parton level
  - Leading order quark channel F_2
  - Next to leading order quark channel
  - Gluon channel, SIDIS



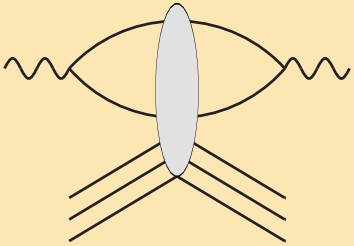
See also:

- T.L., R. Paatelainen and M. Seppälä, "Two-loop DGLAP splitting functions from light cone perturbation theory," JHEP **05** (2026), 156 [arXiv:2601.10374 [hep-ph]].
- M. Bertilsson, T. L., H. Mäntysaari and X. B. Tong, "DIS dipole picture cross section at finite energy," Phys. Rev. D **114** (2026) no.5, 5 [arXiv:2601.07302 [hep-ph]].

Shockwaves everywhere

Inclusive DIS in dipole picture

Dipole frame: γ^* moving fast

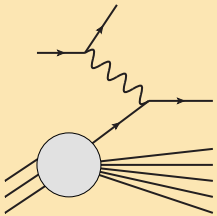


- γ^* consists of partons
- Partons struck by target gluon field:
  interaction time short $\Longrightarrow$ structure of γ^* frozen
- LO: γ^* is only $q\bar{q}$ dipole
- NLO corrections: also $\gamma^* \to q\bar{q}g$

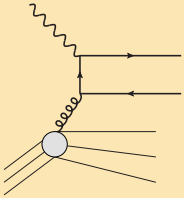
Nonperturbative physics: q/g+p scattering amplitude

DIS in usual parton picture

Infinite momentum frame: proton moving fast



- Proton consists of partons
- Struck by γ^*: interaction time short
  $\Longrightarrow$ partonic interaction factorizes
- LO $\gamma^* + q \to q$ counts quarks
  $\Longrightarrow$ PDF's
- NLO: also gluon-initiated

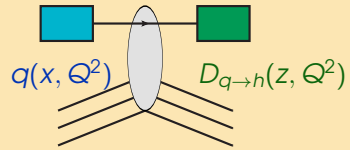


Nonperturbative physics parametrized by PDF's: count quarks/gluons

Forward hadrons in pA

"Hybrid formalism"; Dumitru, Jalilian-Marian 2002

- q/g from collinear pdf, large p^+
- Collinear fragmentation function



Shockwave = CGC picture

Interaction with target gluon field: Wilson line

$$V(\mathbf{x}) = P \exp\left\{ ig \int dx^- A^+_{\rm cov}(\mathbf{x}, x^-) \right\} \quad \in \quad \mathrm{SU}(3)$$

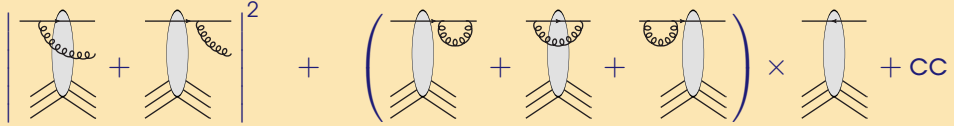
- Coordinate-space amplitude
- Includes multiple gluon exchanges = saturation
- Eikonal: no p^+ transfer
- Infinite energy, "as much p^- as needed" to put final state on shell

Forward hadron production in pA at NLO

Forward hadron production at NLO

Chirilli, Xiao, Yuan [arXiv:1112.1061 [hep-ph]] "CXY"

- Parton from high-x PDF, with large p^+
- Emit gluons, pass through target gluon shockwave



Decompose Wilson line color structures (quark channel) $\Longrightarrow$ 2 separate terms:

- $\propto C_F$: collinear divergences $\mathbf{k} \to 0$
  $\Longrightarrow$ DGLAP evolution of pdf and fragmentation function
- $\propto N_c$: soft divergences $k^+ \to 0$
  $\Longrightarrow$ Large $\log 1/x$, absorb into BK/JIMWLK evolution of target

Motivation for this work

C_F-part, DGLAP

- Calculate in $2 - 2\varepsilon$ dimensions, subtract collinear $1/\varepsilon$. Result fine, but:
- Having to subtract collinear $1/\varepsilon$ hassle
- Natural pdf operator definition: **off-shell** incoming parton

  T.L., R. Paatelainen and M. Seppälä [arXiv:2601.10374 [hep-ph]]
- Idea: off-shell $p^2 = -\mu_0^2$ incoming partons, from pdf in off-shell-scheme

N_c-part, BK

- CXY: subtract at (probe) $k_g^+ = 0 \implies$ oversubtract $\implies$ (famously) $\sigma < 0$
- Reason is known: BK evolution should be in **target** p^- momentum fraction
- Same problem in NLO DIS in dipole picture

  B. Ducloué, H. Hänninen, T.L., Y. Zhu, [arXiv:1708.07328 [hep-ph]]
- Idea: reconstruct p^- from final state, then target rapidity BK evolution

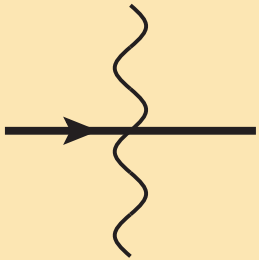
DIS as a photon shockwave: leading order

Photon shockwave

Simpler case study

Gluon shockwave $\Longrightarrow$ γ^* shockwave

- Charge current: $j^-(x^+,0,\mathbf{x}) = e\delta(x^+)\delta(\mathbf{x})$
- Field $A^-(x^+,0,\mathbf{x}) = -e\,\delta(x^+)\,\frac{\delta^{(2)}(\mathbf{x})}{\boldsymbol{\nabla}^2}$
- Wilson line
  $V_q(\mathbf{x}) = \mathrm{P}\exp\left(ie\int \mathrm{d}x^+ A^-\right) = \exp\left(-ie_f e^2\,\frac{\delta^{(2)}(\mathbf{x})}{\boldsymbol{\nabla}^2}\right)$
- Scattering amplitude for quark at LO in $\alpha_{\mathrm{e.m.}}$
  $$1 - V_q(\mathbf{x}) \approx -ie^2 e_f \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\,\frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{\mathbf{q}^2}$$



Only one photon polarization, eikonal scattering $\Longrightarrow$ we get $y = 0$ limit:

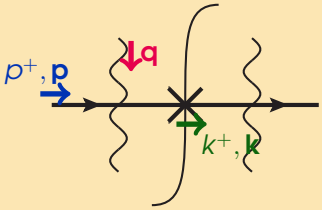
$$\frac{\mathrm{d}^2\sigma^{\mathrm{DIS}}}{\mathrm{d}x_{Bj}\,\mathrm{d}Q^2} = \frac{4\pi\alpha_{\mathrm{e.m.}}^2}{x_{Bj}Q^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x_{Bj},Q^2) - \frac{y^2}{2}F_L(x_{Bj},Q^2)\right] \Longrightarrow \frac{4\pi\alpha_{\mathrm{e.m.}}^2}{x_{Bj}Q^4}F_2(x_{Bj},Q^2)$$

$\Longrightarrow$ Using small y to understand small x. (In DIS $Q^2 = xys$)

Parton level cross section

- Scattering amplitude:

$$ {}^{\text{out}}_{\text{D}}\langle b|\ \overbrace{\hat{S}-1}^{\text{shockwave}}\ |a\rangle^{\text{in}}_{\text{D}} = 2p^+(2\pi)\delta\Big(p^+ - \sum_{i\in b} k_i^+\Big)\, i\mathcal{M}^{a\to b}, $$



- Cross section

$$ \mathrm{d}\sigma = |\mathcal{M}|^2 2p^+(2\pi)\delta\Big(p^+ - \textstyle\sum_i k_i^+\Big) \prod_i \frac{\mathrm{d}k_i^+\,\mathrm{d}^2\mathbf{k}_i}{2k_i^+(2\pi)^3} $$

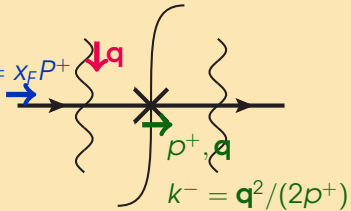
- Leading order

$$ 2k^+ \frac{\mathrm{d}\sigma}{\mathrm{d}k^+\,\mathrm{d}^2\mathbf{k}} = \frac{4\alpha_{\text{e.m.}}^2}{\mathbf{q}^4} 2k^+\delta(k^+ - p^+) \qquad \mathbf{q} = \mathbf{k} - \mathbf{p} = \mathbf{k} \quad \text{(choose } \mathbf{p} = \mathbf{0}\text{)} $$

Parton level to hadron level

- Parton level

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k^+\,\mathrm{d}^2\mathbf{q}} = \frac{4\alpha^2_{\mathrm{e.m.}}}{\mathbf{q}^4}\delta(k^+ - p^+) \qquad \mathrm{d}^2\mathbf{q} = \pi\,\mathrm{d}Q^2$$



1. Collinear factorization: quark from PDF with $x_F P^+$

$$\frac{\mathrm{d}^2\sigma^{\mathrm{DIS}}}{\mathrm{d}k^+\,\mathrm{d}Q^2} = \int_0^1 \mathrm{d}x_F\, q(x_F)\frac{\mathrm{d}\sigma(p^+ = x_F P^+)}{\mathrm{d}k^+\,\mathrm{d}Q^2}$$

2. Differential in $x_{Bj} = \frac{Q^2}{2P^+q^-}$, reconstruct **from final state** $q^- = \sum_{\mathrm{f.s.}} k_i^-$

$$\frac{\mathrm{d}^2\sigma^{\mathrm{DIS}}}{\mathrm{d}x_{Bj}\,\mathrm{d}Q^2} = \int_0^1 \mathrm{d}x_F\, q(x_F)\int \mathrm{d}k^+\delta\left(x_{Bj} - \frac{Q^2}{2P^+}\frac{2p^+}{\mathbf{q}^2}\right)\frac{\mathrm{d}\sigma(p^+ = x_F P^+)}{\mathrm{d}k^+\,\mathrm{d}Q^2} = \frac{4\pi\alpha^2_{\mathrm{e.m.}}}{x_{Bj}Q^4}q(x_{Bj})$$

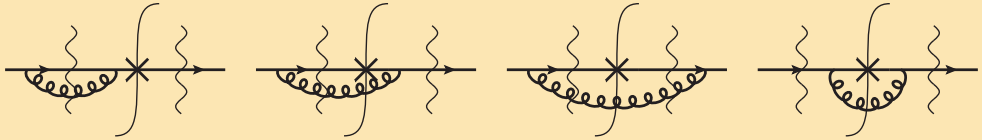
- Standard result: LO DIS in collinear factorization
- Now x_{Bj} is a **target** ($e^- \to e^-\gamma^*$) k^- **momentum fraction** (More properly $1/x_{Bj} \sim q^-$)

Quark channel F_2 at NLO

Generalize same to NLO

Here: quark-initiated channel. Can also do gluon

- Calculate radiative and virtual corrections at parton level



- From parton-level cross section to hadron level

$$\frac{\mathrm{d}\sigma^{\mathrm{DIS}}}{\mathrm{d}x_{Bj}} = \int_0^1 \mathrm{d}x_F q(x_F) \int \mathrm{d}[\mathrm{P.S.}] \frac{\mathrm{d}\sigma(q + \gamma^* \to X)}{\mathrm{d}[\mathrm{P.S.}]} \delta\left(x_{Bj} - \frac{Q^2}{2P^+(\sum_i k_i^-)}\right)$$

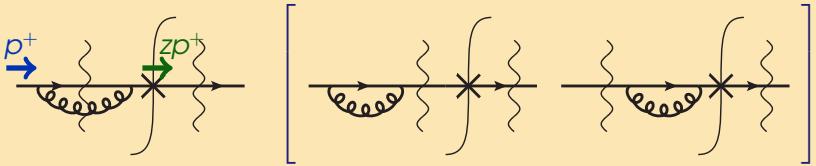
- Phase space is in terms of $k_i^+, \mathbf{k}_i$

$$\mathrm{d}[\mathrm{P.S.}] = \prod_i \frac{\mathrm{d}k_i^+ \, \mathrm{d}^2\mathbf{k}_i}{2k_i^+(2\pi)^3}$$

$\Longrightarrow$ Change variables to access target momentum $q^- = \sum_i k_i^-$

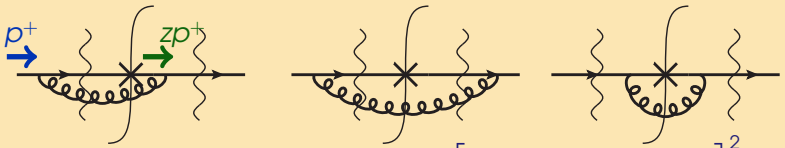
Collinear divergence

- Virtual



$$\frac{\mathrm{d}\sigma_V}{\mathrm{d}z\,\mathrm{d}Q^2} \sim -\delta(1-z)\int_0^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi}\int \mathrm{d}^2\ell \left[\frac{\ell}{\ell^2} - \frac{\ell+(1-\xi)\mathbf{q}}{(\ell+(1-\xi)\mathbf{q})^2}\right]^2$$

- Real



$$\frac{\mathrm{d}\sigma_R}{\mathrm{d}z\,\mathrm{d}Q^2} \sim \frac{1+z^2}{1-z}\int \mathrm{d}^2\ell \left[\frac{\ell}{\ell^2} - \frac{\ell+(1-z)\mathbf{q}}{(\ell+(1-z)\mathbf{q})^2}\right]^2$$

- Collinear divergences, whole cross section $\sim$ splitting function $P_{q\leftarrow q}(z)$

Have we rediscovered DGLAP? Kind of, but need to be careful.

Off-shell states

First, let us be more careful with collinear divergence

- Bare partons on shell. Partons from renormalized pdf are physically off-shell
- Operator definition of quark pdf, bare:

$$f^0_{q/h}(\mu, x) \equiv \frac{1}{4\pi x} \int \frac{\mathrm{d}^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \frac{\langle h(p^+)|\, a^\dagger_q(xp^+, \mathbf{k}) a_q(xp^+, \mathbf{k})\, |h(p^+)\rangle}{\langle h(p^+)|h(p^+)\rangle}$$

- UV-divergent (d.r. scale μ) $\Longrightarrow$ renormalize (factorization scale μ_F)
- Natural renormalization condition: time-like off shell ($p^2 = -\mu_0^2$) parton with

$$f^R_{q/q(\mu_0)}(\mu_F, x)\Big|_{\mu_F=\mu_0} = \delta(1-x) \quad \textbf{off-shell pdf scheme}$$

- Prospect: calculate cross section with $\mu_F = \mu_0$ off-shell partons,
  $\Longrightarrow$ no need to subtract
- Hard? No, we do this in dipole picture DIS all the time.
  (Light cone quantize spacelike photon $q^2 = -Q^2$) (Gauge invariance? L.C. gauge!)
- Especially for processes with 2 scales (DY, SIDIS) : good if logs of μ_F explicit

Projectile to target momentum fraction

Now back to rederiving DGLAP

- Partonic cross section

$$\frac{\mathrm{d}\sigma_R}{\mathrm{d}z\,\mathrm{d}Q^2} \sim P_{q\leftarrow q}(z)$$

- But essential: need to integrate over final state with x_{Bj} constraint

$$\frac{\mathrm{d}\sigma^{\mathrm{DIS}}}{\mathrm{d}x_{Bj}\,\mathrm{d}Q^2} \sim \int_0^1 \mathrm{d}x_F\, q(x_F) \int_0^1 \mathrm{d}z\,\mathrm{d}^2\ell\; P_{q\leftarrow q}(z)\;\left[\ldots \ell, z\right]^2\;\delta\left(x_{Bj} - x_{Bj}(z,\ell,x_F,Q^2)\right)$$

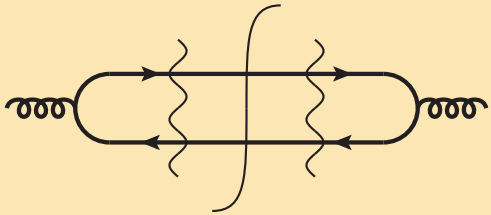
- Initial state collinear divergence at $\frac{x_{Bj}}{x_F} = z \implies \sim P_{q\leftarrow q}(x_{Bj}/x_F)\log Q^2/\mu_F^2$
- Final state collinear divergence cancels in F_2 (will be there for SIDIS)
- But: **also finite part** ($\not\propto P_{q\leftarrow q}$) , recover (correct) NLO DIS coefficient function C_q

$$F_2(x_{\mathrm{Bj}},Q^2) = \sum_{q,\bar{q}} e_q^2 \left\{ x_{Bj} q(x_{Bj}) + \frac{\alpha_s}{2\pi} x_{Bj} [C_q \otimes q](x_{Bj}) \right\},$$

(We get it in the off-shell pdf scheme, but can also convert to $\overline{\mathrm{MS}}$)

In addition to quark channel F_2

Gluon channel



- Only contributes at NLO
- Will have a collinear divergence leading to $P_{q\leftarrow g}(z)$ at parton level
- Similarly final state integration with $\delta(x_{Bj} - \dots)$ transforms this into
  - Target momentum fraction $P_{q\leftarrow g}(x_{Bj}/x_F)$ times collinear log
  - Finite part of gluon channel coefficient function

SIDIS

We can similarly calculate SIDIS

$$\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x_{Bj}\,\mathrm{d}z_H} = \sum_{i,j} \int_0^1 \mathrm{d}x_F\, f_i(x_F, \mu_F^2) \int_0^1 \mathrm{d}\xi\, D_j^H(\xi, \mu_D^2) \int \mathrm{d}[\mathrm{P.S.}] \delta(x_{Bj} - \dots)\, \delta(z_H - \dots) \frac{\mathrm{d}\sigma^{i\gamma^* \to jX}}{\mathrm{d}[\mathrm{P.S.}]}$$

Textbook definition of fragmentation momentum fraction

$$z_H = \frac{p_H \cdot P}{q \cdot P} = \frac{p_H^-}{q^-}$$

also involves a target k^- momentum $\Longrightarrow$ treatment analoguous to x_{Bj}

Conclusions

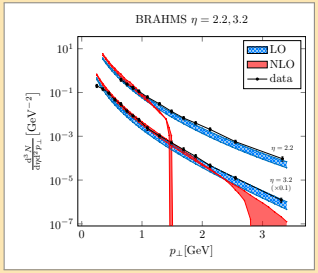
- We rederive F_2 and SIDIS cross sections at NLO in collinear factorization, treating the photon as an "abelian CGC shockwave"
- Propose to directly resum collinear logs by calculating hard cross section with off-shell incoming partons (from renormalized pdf)
- Explicitly transform from projectile to target momentum fractions:
  - Correct result for DIS
  - Expect perturbative stability in CGC calculations

  Target momentum fraction in dipole picture DIS:

  M. Bertilsson, T. L., H. Mäntysaari and X. B. Tong, [arXiv:2601.07302 [hep-ph]].

Outlook: try the same for single inclusive hadron production.

Negative cross sections



- Analytical calculation
  Chirilli, Xiao, Yuan 2012
- Numerics: Stasto, Xiao, Zaslavsky 2013 $\Longrightarrow$ cross section negative
  (large N_c; mix C_F and N_c terms)
- Kinematics? Large k_T logs?? Beuf et al 2014, Watanabe, Xiao & Zaslavsky 2015

Ducloué, T.L., Zhu 2016: q channel at finite N_c

also Kang et al 2014

- Problem is in the rapidity divergence
- Most easily identified by color factor

