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Twist-three quark TMDs from back-to-back dijet production in DIS

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From the eikonal approximation to the quark background

Eikonal approximation

Energetic partons propagate through the dominant A^- field of the target. Multiple interactions are resummed into Wilson lines:

$$U_R(\boldsymbol{b}) = \mathcal{P}_+ \exp\left[ig \int dx^+ \, T_R^a A_a^-(x^+, \boldsymbol{b})\right].$$

Beyond eikonal accuracy

Corrections account for finite target width, suppressed field components and target dynamics.

Our focus: interactions with the quark background of the target.

In DIS, these contribute to both $\gamma^* \to qg$ and $\gamma^* \to q\bar{q}$.

Mukherjee, Skokov, Tarasov, Tiwari, Yao (2026), | Altinoluk, Armesto, Beuf (2023)

Quark-background contributions at next-to-eikonal accuracy

At fixed transverse scales, quark exchange is suppressed in the high-energy expansion.

Relative energy counting: the overall eikonal normalization is factored out,

$$\mathcal{A}_{\rm eik} \sim s^0, \qquad s \equiv W^2 = (q + P_{\rm tar})^2 \simeq 2q^+ P^-_{\rm tar}.$$

Channel	**Amplitude**	**Contribution to the cross section**
$\gamma^* \to qg$	Single quark exchange $\mathcal{A}_q \sim s^{-1/2}$	Square of the single-exchange amplitude $\lvert\mathcal{A}_q\rvert^2 \sim s^{-1}$
$\gamma^* \to q\bar{q}$	Double quark exchange $\mathcal{A}_{q\bar{q}} \sim s^{-1}$	Interference with the eikonal amplitude $2\,\mathrm{Re}(\mathcal{A}^*_{\rm eik}\mathcal{A}_{q\bar{q}}) \sim s^{-1}$

Both channels probe the quark background at NEik accuracy
through different amplitude structures.

Back-to-back kinematics and the correlation limit

$$z = \frac{k_1^+}{k_1^+ + k_2^+}, \qquad \underbrace{\boldsymbol{k} = \boldsymbol{k}_1 + \boldsymbol{k}_2}_{\text{dijet imbalance}}, \qquad \underbrace{\boldsymbol{P} = (1-z)\boldsymbol{k}_1 - z\boldsymbol{k}_2}_{\text{hard relative momentum}}$$

$$|\boldsymbol{k}| \ll |\boldsymbol{P}| \quad \text{(back-to-back correlation limit)}$$

Conjugate transverse coordinates

$$\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \qquad \boldsymbol{b} = z\boldsymbol{x}_1 + (1-z)\boldsymbol{x}_2$$

$$\boldsymbol{k}_1 \cdot \boldsymbol{x}_1 + \boldsymbol{k}_2 \cdot \boldsymbol{x}_2 = \boldsymbol{k} \cdot \boldsymbol{b} + \boldsymbol{P} \cdot \boldsymbol{r}$$

Expand the target operators around $\boldsymbol{r} = 0$.

We keep NEik accuracy in energy and NLP accuracy in the transverse-size expansion.

Dominguez, Marquet, Xiao, Yuan (2011)

Leading power and twist-three quark correlations

Known leading-power qg result: $\langle \bar{\Psi}\, U\, \gamma^{-} \Psi \rangle \;\longrightarrow\; f_1.$

Kinematic NLP

Corrections to the hard coefficient multiplying the two-body correlator.

Genuine twist three

Three-body correlators with a field-strength insertion: $\langle \bar{\Psi}\, U\, g F_i{}^{-}\, U\, \Psi \rangle$.

The next term in the correlation-limit expansion resolves three-body quark correlations.

LP DIS result: Altinoluk, Armesto, Beuf (2023) | Rodini, Vladimirov (2022)

Two-body quark operator and f_1

The two-body operator entering the qg cross section

$$\mathcal{T}_{11}[\Gamma](\boldsymbol{k}) = \frac{1}{2P^-_{\rm tar}} \int d^2\Delta\boldsymbol{b}\, e^{i\boldsymbol{k}\cdot\Delta\boldsymbol{b}} \int d\Delta z^+ \times \Big\langle P_{\rm tar}\Big|\bar\Psi(\Delta z^+,\Delta\boldsymbol{b}) U_F^\dagger(+\infty,\Delta z^+;\Delta\boldsymbol{b})\,\Gamma\, U_F(+\infty,0;\boldsymbol{0})\Psi(0,\boldsymbol{0})\Big|P_{\rm tar}\Big\rangle.$$

Relation to the two-body TMD correlator

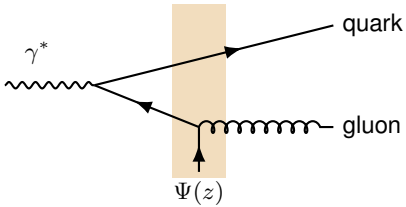
$$\mathcal{T}_{11}[\Gamma](\boldsymbol{k}) = \frac{1}{P^-_{\rm tar}} \int d^2\Delta\boldsymbol{b}\, e^{i\boldsymbol{k}\cdot\Delta\boldsymbol{b}} \int d\Delta z^+\, \widetilde\Phi^{[\Gamma]}_{11}(\Delta z^+,0;\Delta\boldsymbol{b}) = \frac{2\pi}{P^-_{\rm tar}} \int d^2\Delta\boldsymbol{b}\, e^{i\boldsymbol{k}\cdot\Delta\boldsymbol{b}} \Phi^{[\Gamma]}_{11}(0;\Delta\boldsymbol{b}).$$

f_1: unpolarized twist-two quark TMD $\quad \mathcal{T}_{11}[\gamma^-](\boldsymbol{k}) = \dfrac{(2\pi)^3}{P^-_{\rm tar}} f_1(0;\boldsymbol{k}).$

Unrestricted longitudinal integration selects the formal zero-momentum-fraction projection.

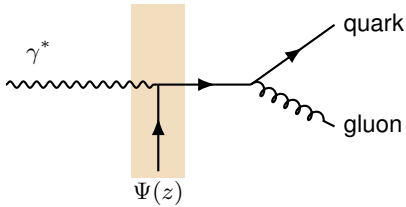
qg production: before and inside contributions

Before contribution



Photon splitting before the target.

Inside contribution



Photon interaction inside; gluon emission after.

$$\mathcal{M}_{qg} = \mathcal{M}_{\text{bef}} + \mathcal{M}_{\text{in}}, \qquad \mathcal{M}_{\text{in},L} = 0$$

Before: expand the two transverse positions around $\boldsymbol{b}$.

Inside: a single transverse position; contributes for transverse photons.

Through NLP: $|\mathcal{M}|^2 = |\mathcal{M}_{\text{LP}}|^2 + 2\,\text{Re}(\mathcal{M}_{\text{LP}}^\dagger \mathcal{M}_{\text{NLP}}) + \cdots$

Altinoluk, Armesto, Beuf (2023)

qg: the operator expansion separates the two NLP terms

Expand to first order in r and integrate by parts in b:

$\mathcal{U}_{\rm bef}$ contains the quark-background insertion dressed by Wilson lines.

$$\mathcal{U}_{\rm bef}(\boldsymbol{z}) \equiv \int_{z^+} U_A(+\infty, z^+; \boldsymbol{z})_{a_2 b}\, U_F^\dagger(z^+, -\infty; \boldsymbol{z}) t^b \Psi(z^+, \boldsymbol{z})$$

$$\mathcal{O}_{\rm bef} = \int_{\boldsymbol{b},\boldsymbol{r}} e^{-i\boldsymbol{k}\cdot\boldsymbol{b} - i\boldsymbol{P}\cdot\boldsymbol{r}} f(\boldsymbol{r}) \Big\{ \underbrace{[1 - iz\,\boldsymbol{r}\cdot\boldsymbol{k}] U_F(\boldsymbol{b}) \mathcal{U}_{\rm bef}(\boldsymbol{b})}_{\text{LP + kinematic NLP}} + \underbrace{r^i (\partial_i U_F(\boldsymbol{b})) \mathcal{U}_{\rm bef}(\boldsymbol{b})}_{\text{genuine twist three}} \Big\} + \cdots .$$

$$\partial_i U_F(\boldsymbol{b}) = \int dx^+\, U_F(+\infty, x^+; \boldsymbol{b}) \big[- igF_i{}^-(x^+, \boldsymbol{b}) \big] U_F(x^+, -\infty; \boldsymbol{b})$$

Interference with the LP amplitude generates

$\bar{\Psi}\, U\, gF_i{}^-\, U\, \Psi$ with future-pointing gauge links.

Three-body quark TMD correlators

qg: field strength on the antiquark side

$$\widetilde{\Phi}^{[\Gamma]}_{i,21}(z_1^+, z_2^+, z_3^+; \Delta\boldsymbol{b}) = \Big\langle T\Big\{ \bar{\Psi}(z_1^+, \Delta\boldsymbol{b}) U_F(z_1^+, z_2^+; \Delta\boldsymbol{b}) g F_i^{\ -}(z_2^+, \Delta\boldsymbol{b}) \\ \times \frac{\Gamma}{2} U_F(z_2^+, +\infty; \Delta\boldsymbol{b}) U_F(+\infty, z_3^+; \boldsymbol{0}) \Psi(z_3^+, \boldsymbol{0}) \Big\} \Big\rangle_{P_{\rm tar}}.$$

$\Phi^{[\Gamma]}_{i,21}(x_1, x_2, x_3; \boldsymbol{k})$ is the Fourier transform of $\widetilde{\Phi}^{[\Gamma]}_{i,21}$, with $x_1 + x_2 + x_3 = 0$.

For $\Phi_{i,12}$, the field strength lies on the **quark side**.

$$\Phi^{[\Gamma]}_{i,\oplus}(x_1, x_2, x_3; \boldsymbol{k}) = \tfrac{1}{2}\big[\Phi^{[\Gamma]}_{i,21}(x_1, x_2, x_3; \boldsymbol{k}) + \Phi^{[\Gamma]}_{i,12}(-x_3, -x_2, -x_1; \boldsymbol{k})\big],$$

$$\Phi^{[\Gamma]}_{i,\ominus}(x_1, x_2, x_3; \boldsymbol{k}) = \tfrac{i}{2}\big[\Phi^{[\Gamma]}_{i,21}(x_1, x_2, x_3; \boldsymbol{k}) - \Phi^{[\Gamma]}_{i,12}(-x_3, -x_2, -x_1; \boldsymbol{k})\big].$$

Rodini, Vladimirov (2022) - with our conventions

Twist-three quark TMDs: definitions

Unpolarized target

Vector projection: f distributions Axial-vector projection: g distributions

Below, $X = (x_1, x_2, x_3)$ with $x_1 + x_2 + x_3 = 0$; all correlators are in transverse-momentum space.

qg channel

$$\Phi^{[\gamma^-]}_{i,\oplus}(X;\boldsymbol{k}) = k^i f^{\perp}_{\oplus}(X;\boldsymbol{k}^2)$$

$$\epsilon^{ij}\Phi^{[\gamma^-\gamma_5]}_{i,\ominus}(X;\boldsymbol{k}) = k^j g^{\perp}_{\ominus}(X;\boldsymbol{k}^2)$$

The field strength shares a transverse position with one quark field.

$q\bar{q}$ channel

$$\Phi'^{[\gamma^-]}_{i,\ominus}(X;\boldsymbol{k}) = k^i f'^{\perp}_{\ominus}(X;\boldsymbol{k}^2)$$

$$\Phi'^{[\gamma^-\gamma_5]}_{i,\ominus}(X;\boldsymbol{k}) = \epsilon^{ij}k^j g'^{\perp}_{\ominus}(X;\boldsymbol{k}^2)$$

The quark pair shares a transverse position, opposite the field strength.

Prime $\rightarrow$ different transverse operator arrangement.

$\oplus, \ominus \rightarrow$ symmetric and antisymmetric combinations of companion correlators.

qg: longitudinal cross section through NLP

$$\mathcal{N}_{qg} = \frac{4\alpha_{\rm em}\alpha_s e_f^2}{2z(1-z)} \underbrace{\frac{1}{2q^+ P^-_{\rm tar}}}_{\simeq 1/W^2}$$

$$\frac{1}{\mathcal{N}_{qg}} \frac{d\sigma_L^{qg}}{d^2\boldsymbol{k}\, dz\, d^2\boldsymbol{P}} = \mathcal{C}_L f_1(0;\boldsymbol{k}) + 2\pi\, \mathcal{C}_L^i k^i f_\oplus^\perp(0,0,0;\boldsymbol{k}) + \mathcal{O}(\text{NNLP})$$

$$\mathcal{C}_L = \frac{4z^3(1-z)^2 Q^2 C_F}{[\boldsymbol{P}^2 + \bar{Q}^2]^2} \left[1 - 4z \frac{\boldsymbol{P}\cdot\boldsymbol{k}}{\boldsymbol{P}^2 + \bar{Q}^2}\right]$$

$$\mathcal{C}_L^i = 16z^3(1-z)^2 Q^2 \left(\frac{C_A}{2} - C_F\right) \frac{P^i}{[\boldsymbol{P}^2 + \bar{Q}^2]^3}, \qquad \bar{Q}^2 = m^2 + z(1-z)Q^2$$

Kinematic NLP: correction in $\mathcal{C}_L$. **Genuine twist three:** $\mathcal{C}_L^i f_\oplus^\perp$.

Zero-fraction arguments denote formal longitudinal projections.

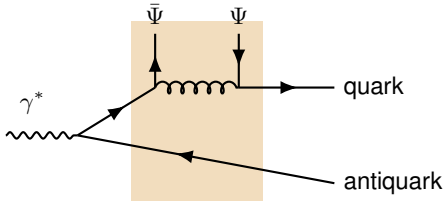
qg: what changes for transverse photons?

$$\frac{1}{\mathcal{N}_{qg}}\frac{d\sigma_T^{qg}}{d^2\boldsymbol{k}\,dz\,d^2\boldsymbol{P}} = \mathcal{C}_T f_1(0;\boldsymbol{k}) + 2\pi\,\mathcal{C}_T^{\text{unpol.},i} k^i f_{\oplus}^{\perp}(0,0,0;\boldsymbol{k}) + 2\pi\,\mathcal{C}_T^{\text{h.pol.},i} k^i g_{\ominus}^{\perp}(0,0,0;\boldsymbol{k}) + \text{mass-dependent tensor term} + \mathcal{O}(\text{NNLP}).$$

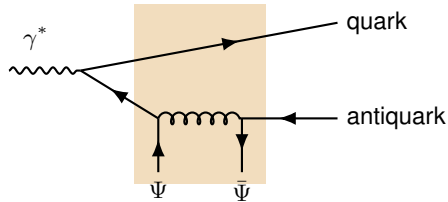
Structure	Longitudinal photon	Transverse photon
f_1 with kinematic NLP	✓	✓
$f_{\oplus}^{\perp}$	✓	✓
$g_{\ominus}^{\perp}$	✗	✓
Mass-dependent helicity flip	✗	✓

$q\bar{q}$: double quark exchange on either fermion line

Insertions on the quark line



Insertions on the antiquark line



$$d\sigma^{q\bar{q}}\big|_{\Psi\bar{\Psi}} \;\propto\; 2\,\mathrm{Re}\Big[\mathcal{M}^{\dagger}_{\mathrm{eik}}\big(\mathcal{M}^{(q)}_{\Psi\bar{\Psi}}+\mathcal{M}^{(\bar{q})}_{\Psi\bar{\Psi}}\big)\Big]$$

Eikonal amplitude: its leading small-$\boldsymbol{r}$ term supplies $F_i{}^{-}$.

Quark-background amplitude: evaluate the target operator at $\boldsymbol{r}=0$.

Light-cone ordering of the insertions: $\theta(v'^{+}-v^{+})$ on the quark line, $\theta(v^{+}-v'^{+})$ on the antiquark line.

The two channels probe different operator arrangements

qg: 21 correlator

F shares a transverse position with $\bar{\Psi}$.



$$\widetilde{\Phi}^{[\Gamma]}_{i,21}(z_1^+, z_2^+, z_3^+; \Delta\boldsymbol{b}) = \Big\langle T\Big\{ \bar{\Psi}(z_1^+, \Delta\boldsymbol{b}) U_F(z_1^+, z_2^+; \Delta\boldsymbol{b}) gF_i^-(z_2^+, \Delta\boldsymbol{b}) \times \frac{\Gamma}{2} U_F(z_2^+, +\infty; \Delta\boldsymbol{b}) U_F(+\infty, z_3^+; \mathbf{0}) \Psi(z_3^+, \mathbf{0}) \Big\} \Big\rangle_{P_{\rm tar}}.$$

$q\bar{q}$: $2'1'$ correlator

The quark pair sits opposite F.



$$\widetilde{\Phi}^{[\Gamma]}_{i,2'1'}(z_1^+, z_2^+, z_3^+; \Delta\boldsymbol{b}) = \Big\langle T\Big\{ \bar{\Psi}(z_1^+, \Delta\boldsymbol{b}) U_F^\dagger(+\infty, z_1^+; \Delta\boldsymbol{b}) t^c \frac{\Gamma}{2} U_F(+\infty, z_2^+; \Delta\boldsymbol{b}) \Psi(z_2^+, \Delta\boldsymbol{b}) \times [U_A(+\infty, z_3^+; \mathbf{0})]_{ca} gF_{i,a}^-(z_3^+, \mathbf{0}) \Big\} \Big\rangle_{P_{\rm tar}}.$$

Longitudinal projections: unrestricted and ordered integrals

qg: unrestricted integration

$$\int d\Delta z^+ \, e^{ixP^-_{\rm tar}\Delta z^+} = \frac{2\pi}{P^-_{\rm tar}}\delta(x)$$

Formal zero-fraction projection.

$q\bar{q}$: longitudinal ordering

$$\int_{v^+}^{\infty} dv'^+ \, e^{ixP^-_{\rm tar}(v'^+ - v^+)} = \frac{\pi\delta(x) + i\,\mathrm{PV}(1/x)}{P^-_{\rm tar}}$$

probes the $(x, -x, 0)$ projection.

$$(2\pi)^5\, 2z(1-z)\, \frac{d\sigma^{q\bar{q}}_{\lambda}}{d^2\boldsymbol{k}\, dz\, d^2\boldsymbol{P}}\bigg|_{\Psi\bar{\Psi}}$$
$$= \frac{e^2 e_f^2 g^2}{2q^+P^-_{\rm tar}}\left(C_F - \frac{C_A}{2}\right)\Bigg\{ -2(2\pi)^3 \mathcal{C}^j_{\lambda,u} k^j \int_{-1}^{1} dx\, \mathrm{PV}\frac{1}{x}\, f'^{\perp}_{\ominus}(x,-x,0;\boldsymbol{k}^2)$$
$$+ (2\pi)^4 (1-2z)\mathcal{C}^i_{\lambda,h} k^i g'^{\perp}_{\ominus}(0,0,0;\boldsymbol{k}^2)\Bigg\} + \mathcal{O}(\mathrm{NNLP}).$$

$\lambda = L, T, \quad 1/(2q^+P^-_{\rm tar}) \simeq 1/W^2, \quad \mathcal{C}^i_{\lambda,u/h} \equiv \mathcal{C}^{[g,q\bar{q}],i}_{\lambda;\,\mathrm{unpol./h.pol.}}$.

Both polarizations: PV integral of $f'^{\perp}_{\ominus}$. **Transverse only:** $g'^{\perp}_{\ominus}$.

Summary and outlook

- The quark-background sector of NEik DIS dijets connects to twist-three quark correlations in the back-to-back limit.
- In qg production, NLP includes kinematic corrections to f_1 and genuine three-body correlators.
- In $q\bar{q}$ production, interference probes a different transverse arrangement; longitudinal ordering generates principal-value projections.

The two channels give complementary access to three-body quark correlations.

Open question: How do these small-x results connect to the renormalization and evolution of twist-three quark TMDs?

Backup: relation to established twist-three quark TMDs

Schematic field content: longitudinal arguments and Wilson lines suppressed.

Quark–gluon channel

$$\Phi_{21}: \quad [\bar{\Psi} F](\Delta \boldsymbol{b})\ \Psi(\mathbf{0}),$$
$$\Phi_{12}: \quad \bar{\Psi}(\Delta \boldsymbol{b})\ [F\Psi](\mathbf{0}).$$

The gluon shares a transverse position with one fermion.

Connection to the twist-three quark TMD correlators of Rodini and Vladimirov.

Quark–antiquark channel

$$\Phi_{2'1'}: \quad [\bar{\Psi} t^a \Gamma \Psi](\Delta \boldsymbol{b})\ F^a(\mathbf{0}),$$
$$\Phi_{1'2'}: \quad [\bar{\Psi} t^a \Gamma \Psi](\mathbf{0})\ F^a(\Delta \boldsymbol{b}).$$

Both fermions share a transverse position, separated from the gluon.

Distinct correlators, with scalar distributions defined by analogy.

The transverse grouping of the fields is part of the operator definition.

The primed correlators lie outside the quark-TMD basis studied in Rodini and Vladimirov.

Backup: three-body Fourier conventions

Longitudinal momentum fractions

$$\widetilde{\Phi}^{[\Gamma]}_{i,21}(z_1^+, z_2^+, z_3^+; \boldsymbol{b}) = (P^-_{\rm tar})^2 \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3\, \delta(x_1+x_2+x_3) \times e^{-iP^-_{\rm tar}(x_1 z_1^+ + x_2 z_2^+ + x_3 z_3^+)} \Phi^{[\Gamma]}_{i,21}(x_1, x_2, x_3; \boldsymbol{b}).$$

Transverse momentum

$$\Phi^{[\Gamma]}_{i,21}(x_1, x_2, x_3; \boldsymbol{k}) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{k}\cdot\boldsymbol{b}} \Phi^{[\Gamma]}_{i,21}(x_1, x_2, x_3; \boldsymbol{b}).$$

Primed antisymmetric combination

$$\Phi'^{[\Gamma]}_{i,\ominus}(x_1, x_2, x_3; \boldsymbol{k}) = \tfrac{i}{2}\big[\Phi^{[\Gamma]}_{i,2'1'}(x_1, x_2, x_3; \boldsymbol{k}) - \Phi^{[\Gamma]}_{i,1'2'}(-x_2, -x_1, -x_3; \boldsymbol{k})\big].$$

Backup: transverse mass-dependent tensor contribution

The tensor correlator is kept explicit in the draft:

$$\frac{1}{\mathcal{N}_{qg}}\frac{d\sigma_T^{qg}}{d^2\boldsymbol{k}\,dz\,d^2\boldsymbol{P}}\bigg|_{\text{h.flip}} = -2\pi i m\,\mathcal{C}_T^{\text{h.flip},ij}(z,\boldsymbol{P})\int\frac{d^2\Delta\boldsymbol{b}}{(2\pi)^2}\,e^{i\boldsymbol{k}\cdot\Delta\boldsymbol{b}}\Phi_{i,\ominus}^{[\gamma^-\gamma^j]}(0,0,0;\Delta\boldsymbol{b}).$$

- This term contributes for transverse photons.
- It vanishes in the massless limit.
- The main-slide scalar distributions do not exhaust the massive transverse result.

Backup: hard coefficients in the $q\bar{q}$ channel

$$\bar{Q}^2 = m^2 + z(1-z)Q^2$$

$$\mathcal{C}^j_{L,u} = -8Q^2 z^2 (1-z)^2 \frac{P^j}{[\boldsymbol{P}^2 + \bar{Q}^2]^3}, \qquad \mathcal{C}^i_{L,h} = 0,$$

$$\mathcal{C}^j_{T,u} = \frac{P^j}{[\boldsymbol{P}^2 + \bar{Q}^2]^3} \left[\left(z^2 + (1-z)^2\right)(\bar{Q}^2 - \boldsymbol{P}^2) - 2m^2 \right],$$

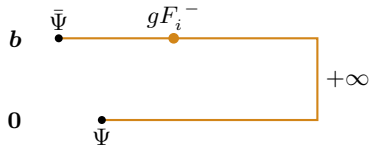
$$\mathcal{C}^i_{T,h} = (1-2z) \frac{P^i}{[\boldsymbol{P}^2 + \bar{Q}^2]^2}.$$

$$C_F - \frac{C_A}{2} = -\frac{1}{2N_c}$$

Backup: DIS three-body operators and gauge links

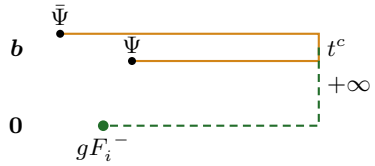
All longitudinal links run to $+\infty$. Solid: fundamental Dashed: adjoint.

qg: 21 arrangement



12: field strength on the Ψ side.

$q\bar{q}$: $2'1'$ arrangement



$1'2'$: exchange the transverse locations.

Example: the primed DIS operator

$$\widetilde{\Phi}^{[\Gamma]}_{i,2'1'} = \Big\langle T\Big\{ \bar{\Psi}(z_1^+, \boldsymbol{b}) U_F^\dagger(+\infty, z_1^+; \boldsymbol{b})\, t^c \frac{\Gamma}{2}\, U_F(+\infty, z_2^+; \boldsymbol{b}) \Psi(z_2^+, \boldsymbol{b}) \\ \times [U_A(+\infty, z_3^+; \mathbf{0})]_{ca}\, gF^-_{i,a}(z_3^+, \mathbf{0}) \Big\} \Big\rangle_{P_{\rm tar}}.$$