Unified Resummation of Soft Gluon Radiation for Heavy Meson Pair correlations:

probing saturation effects in γA and forward pA collisions

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- Cyrille Marquet, YS, Bo-wen Xiao, arXiv: 2510.18949
- Zhan Gao, Cyrille Marquet, YS, Bo-wen Xiao, arXiv: 2605.01527

Resummation, Evolution and Factorization REF 2026

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Outline

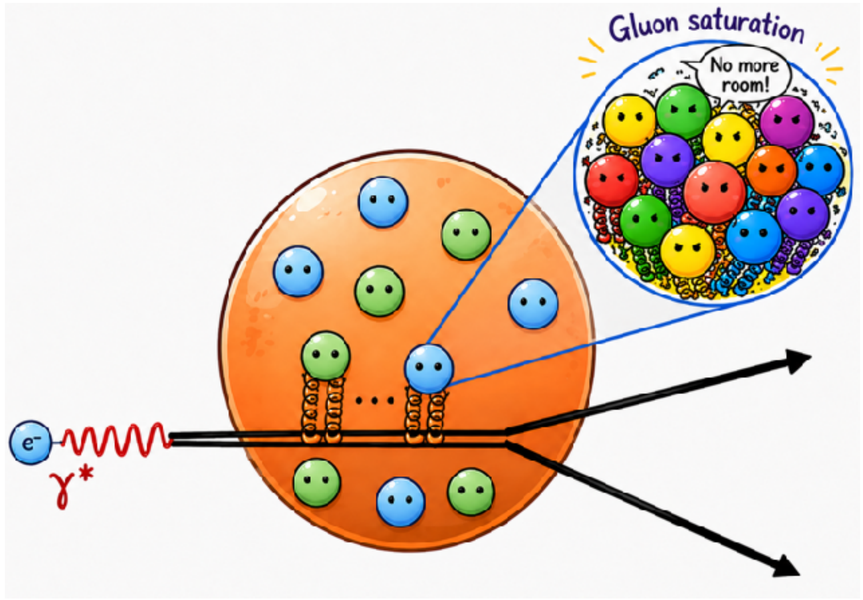
1) Introduction

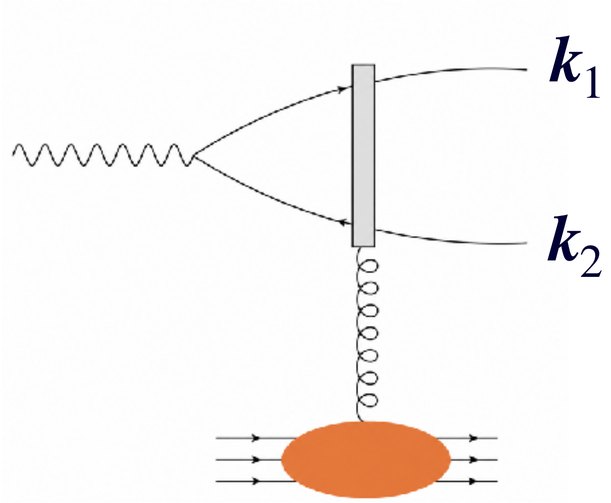
2) Unified resummation of soft gluon radiation for heavy meson pair production

3) Comparisons with experimental data from both ep and foward pp/pA collisions

4) Probing saturation effects in γA and foward pA

Probing Saturation via two-particle correlations at small-x





[Dominguez, Marquet, Xiao, Yuan, Phys.Rev.D 83 (2011) 105005]

In the correlation limit ($\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \ll \boldsymbol{P} = \dfrac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{2}$), CGC can be factorized as hard factor ⊗ gluon TMDs

- WW gluon TMD encodes the gluon saturation information. $\langle \boldsymbol{q} \rangle \sim Q_s$

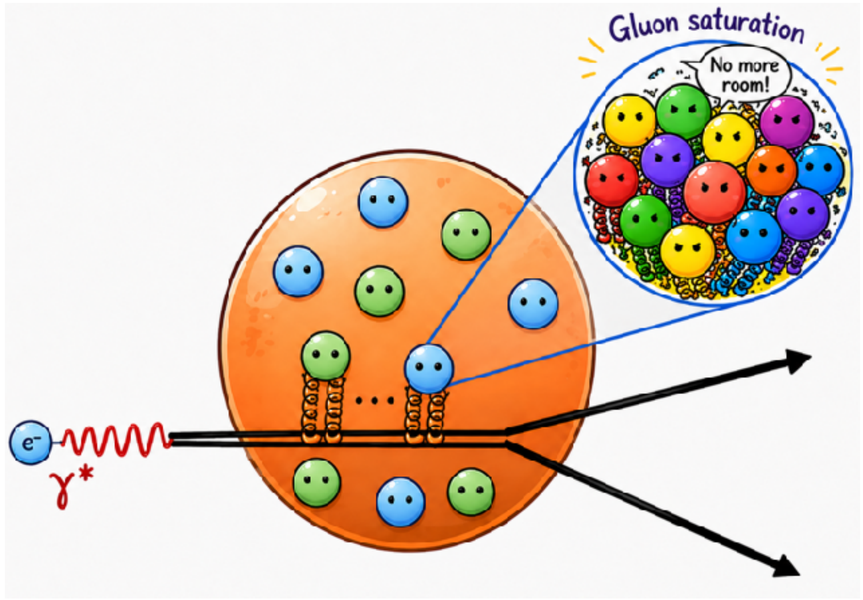
$$xG^{ij}_{WW}(x_g, \boldsymbol{q}) = -\frac{4}{g_s^2}\int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})} \langle \mathrm{Tr}\left[(\partial^i U_x)U_y^\dagger (\partial^j U_y)U_x^\dagger\right]\rangle$$

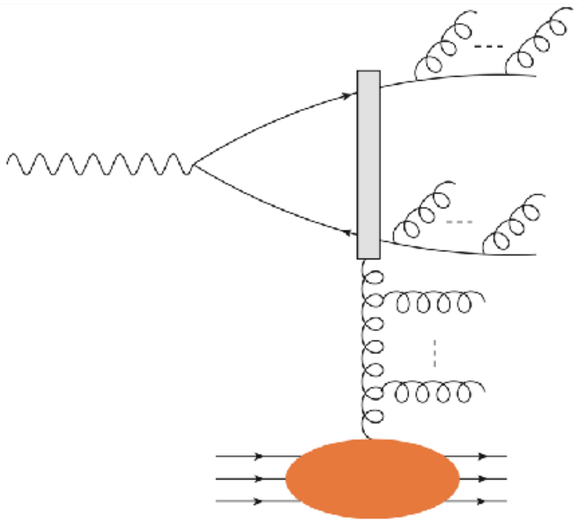
un-polarized & linearly-polarized

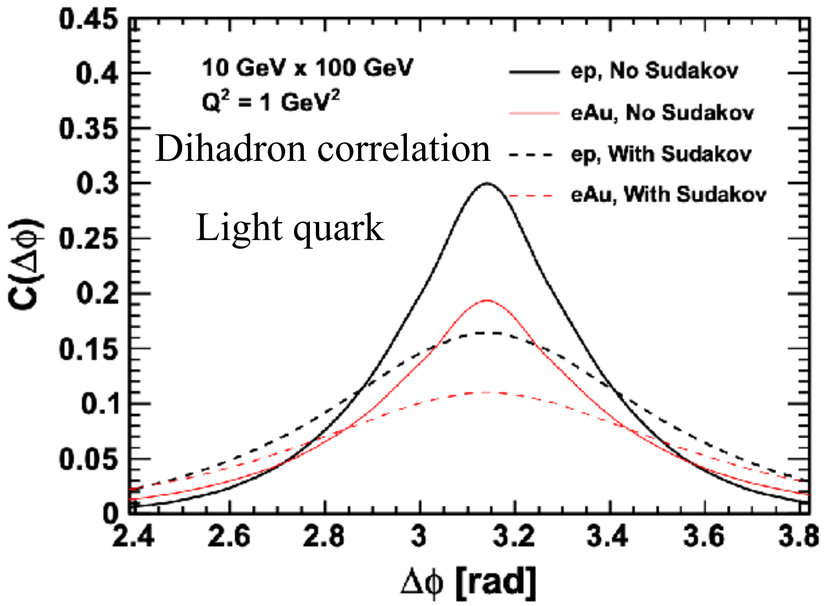
$$\frac{d\sigma^{\gamma A \to q\bar{q}X}}{d^2\boldsymbol{P} d^2\boldsymbol{q}} = \int H^{ij}_{\mathrm{TMD}}(\boldsymbol{P}) \otimes xG^{WW}_{ij}(x_g, \boldsymbol{q}')\delta^{(2)}\left(\boldsymbol{q} - \boldsymbol{q}'\right)$$

[Metz, Zhou, Phys.Rev.D 84 (2011) 051503 ; Dominguez, Qiu, Xiao, Yuan, Phys.Rev.D 85 (2012) 045003]

Probing Saturation via two-particle correlations at small-x







In the correlation limit ($\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \ll \boldsymbol{P} = \dfrac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{2}$), CGC can be factorized as hard factor $\otimes$ gluon TMDs

- gluon TMD encodes the gluon saturation information. $\langle \boldsymbol{q} \rangle \sim Q_s$
- Sudakov effect are also important, and compete with/hide the saturation effect for light quarks.

$$\frac{d\sigma^{\gamma A \to q\bar{q}X}}{d^2\boldsymbol{P} d^2\boldsymbol{q}} = \int H^{ij}_{\mathrm{TMD}}(\boldsymbol{P}) \otimes xG^{WW}_{ij}(x_g, \boldsymbol{q}') \otimes \mathcal{S}_r(\boldsymbol{k}_g)\delta^{(2)}\left(\boldsymbol{q} - \boldsymbol{k}_g - \boldsymbol{q}'\right)$$

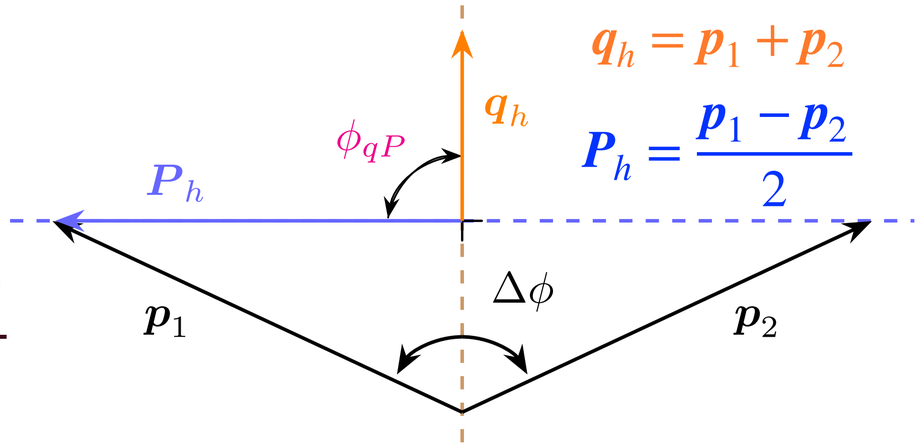
How about heavy-quark pairs?

[Dominguez, Marquet, Xiao, Yuan, Phys.Rev.D 83 (2011) 105005;Muller, Xiao, Yuan, PRD88, 114010 (2013); Zheng, Aschenauer, Lee, Xiao, PRD, 14]

Two key classes of observables for probing saturation effects

1. Azimuthal symmetric distribution: $\frac{d\sigma}{d\Delta\phi}$, TEEC,…

2. Azimuthal asymmetries $\langle \cos 2n\phi_{qP} \rangle = \frac{\int d\mathscr{P} \,.\, \mathcal{S} \,.\, \cos(2n\phi_{qP}) \frac{d\sigma}{d\mathscr{P} \,.\, \mathcal{S}\,.}}{\int d\mathscr{P} \,.\, \mathcal{S} \,.\, \frac{d\sigma}{d\mathscr{P} \,.\, \mathcal{S}\,.}}$



The Sudakov resummation of the heavy quark pair (c&b)/di-lepton

$\langle \cos 2\phi_{qP} \rangle \frac{d\sigma}{d\Delta\phi}$

$\gamma\gamma \to l^+l^-$ [Klein, Mueller, Xiao, Yuan, PRL.122, 132301 (2019); PRD 102, 094013 (2020); Hatta, Xiao, Yuan, Zhou, PRD, 21; Shao, Zhang, Zhou, Zhou, PRD(2023), PRD(2023); YS, Chen, Wei, Xiao, PLB 862, 139317 (2025)]

$e^+e^- \to Q\bar{Q}$ [Aglietti, Ferrera, Eur.Phys.J.C 85 (2025) 3, 272;…] TEEC

$gg \to Q\bar{Q}$ [Catani, Grazzini, Torre, Nucl.Phys.B 890 (2014) 518-538; Catani, Grazzini , Sargsyan, JHEP 06 (2017) 017] $\langle \cos 2\phi_{qP} \rangle$

[Castillo,Echevarria,Makris,Scimemi, JHEP03 (2022) 047; Dai, Liu, Shao, JHEP 07(2026)227;…]

$\gamma g \to Q\bar{Q}$ [Muller, Xiao, Yuan, PRD 88, 114010 (2013); Zhu, Sun, Yuan, PLB 727(2013) 474-479; Castillo, Echevarria, Makris, Scimemi, JHEP 01 (2021) 088;…]

Apologies for omitted references.

We also study the Massive Sudakov effects on the $\Delta\phi$ distribution in the diffractive heavy quark pair production

[Shao, YS, Zhang, Zhou, Zhou, arXiv: 2606.02230]

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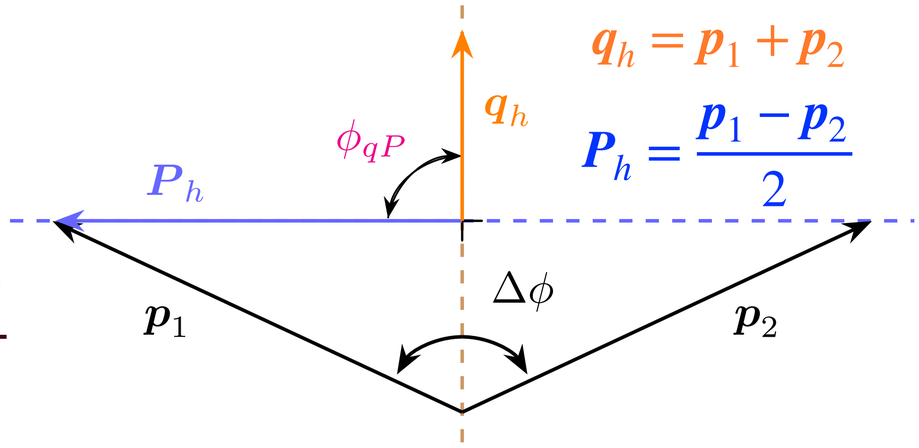

The Sudakov resummation of the heavy quark pair (c&b)/di-lepton

$\gamma\gamma \to l^+l^-$ [Hatta, Xiao, Yuan, Zhou, PRD, 21; Shao, Zhang, Zhou, Zhou, PRD(2023), PRD (2023)] $\langle \cos 2\phi_{qP} \rangle$

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Two key classes of observables for probing saturation effects

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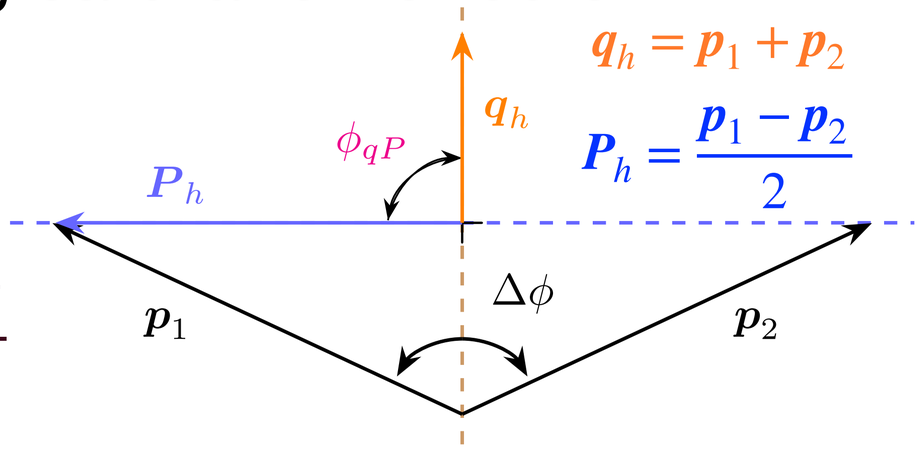

The Sudakov resummation of the heavy quark pair (c&b)/di-lepton

At leading-logarithmic accuracy

Massive Sudakov factors are ad hoc and observable-dependent

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1. Azimuthal symmetric distribution: $\frac{d\sigma}{d\Delta\phi}$, TEEC,…

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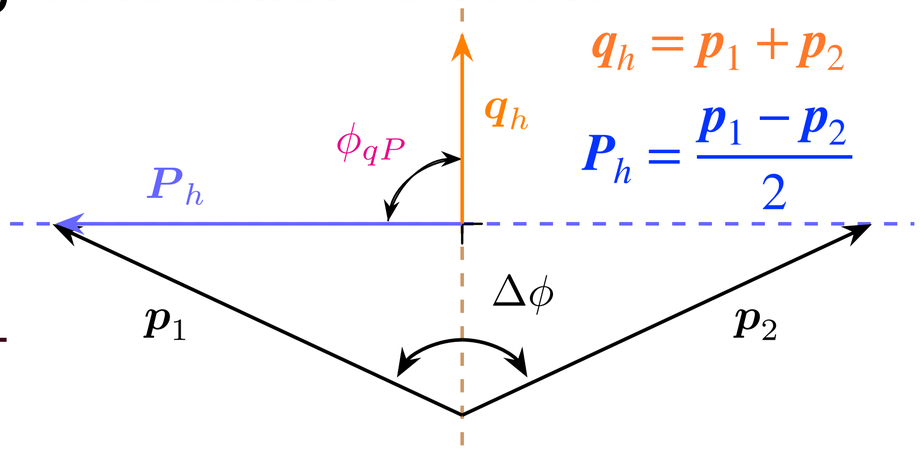


The Sudakov resummation of the heavy quark pair (c&b)/di-lepton

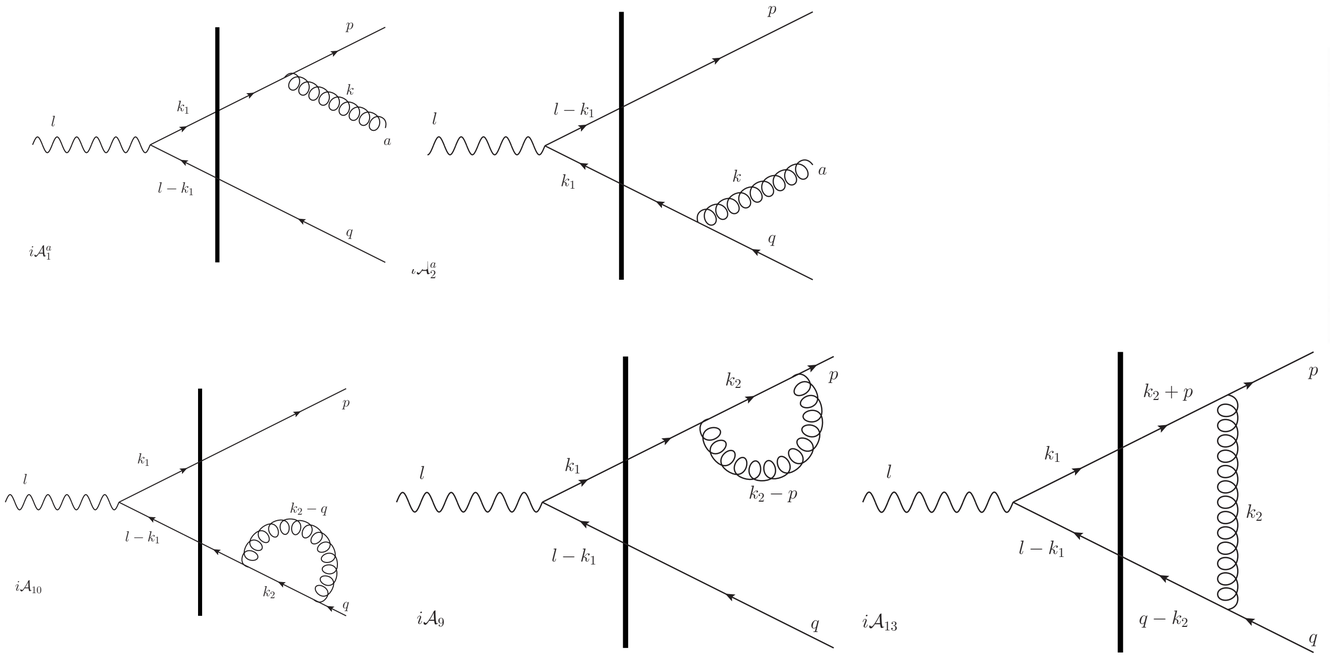
At leading-logarithmic accuracy

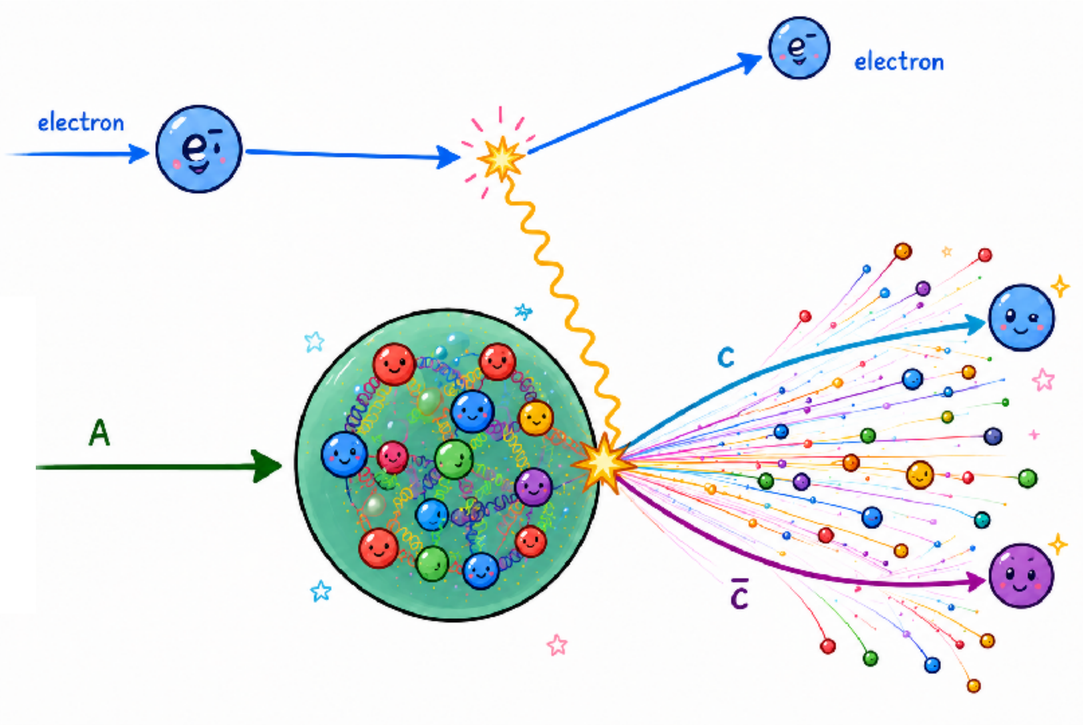
Massive Sudakov factors are ad hoc and observable-dependent

We find an unified resummation for both observables

[Marquet, YS, Xiao, arXiv: 2510.18949]

2) Unified Resummation of Soft Gluon Radiation for Heavy quark Pair Photoproduction





Soft gluon radiation of heavy quark pair correlation

In the Sudakov region

$$\frac{d\sigma}{d^2\boldsymbol{P}d^2\boldsymbol{q}} = H^{ij}_{\rm TMD}(\boldsymbol{P})\int d^2\boldsymbol{r}d^2\boldsymbol{k}_g x G^{WW}_{ij}(x_g,\boldsymbol{b})e^{i\boldsymbol{b}\cdot\boldsymbol{q}}\,\mathcal{S}_r(\boldsymbol{k}_g)\left[e^{i\boldsymbol{b}\cdot\boldsymbol{k}_g}-\theta(M^2_{\rm in}-\boldsymbol{k}_g^2)\right]$$

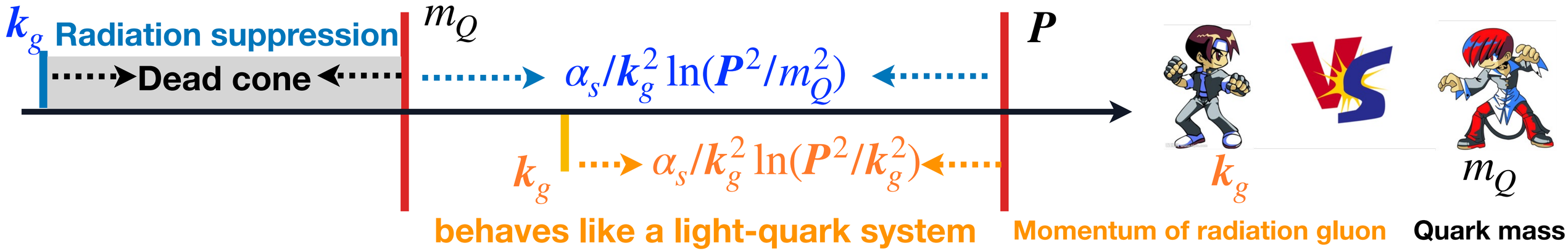
The large double logarithms are given by

$$\mathcal{S}_r(\boldsymbol{k}_g)=\frac{\alpha_s}{2\pi^2}\frac{1}{\boldsymbol{k}_g^2}\begin{cases}\left[N_c\ln\frac{\boldsymbol{P}^2}{\boldsymbol{k}_g^2}+2C_F\left(\ln\frac{M^2_{\rm in}}{m_Q^2}+2\sum_{n=1}\ln\frac{M^2_{\rm in}}{m_Q^2}\cos\left(2n\phi_{k_gP}\right)\right)+\cdots\right], & \boldsymbol{k}_g^2\le m_Q^2\ll\boldsymbol{P}^2,\ \textbf{Mass as IR cutoff}\\ \left[N_c\ln\frac{\boldsymbol{P}^2}{\boldsymbol{k}_g^2}+2C_F\left(\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}+2\sum_{n=1}\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}\cos\left(2n\phi_{k_gP}\right)\right)+\cdots\right], & \boldsymbol{k}_g^2\gg m_Q^2.\end{cases}$$

Massless limit

Collinear $\frac{1}{\epsilon}\to$ ->> FFs

The final Sudakov factor is controlled by the hierarchy among $\boldsymbol{k}_g$ and m_Q.



[Marquet, YS, Xiao, arXiv: 2510.18949]

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In the Sudakov region

$$\frac{d\sigma}{d^2\boldsymbol{P}d^2\boldsymbol{q}} = H^{ij}_{\rm TMD}(\boldsymbol{P})\int d^2\boldsymbol{r}d^2\boldsymbol{k}_g x G^{WW}_{ij}(x_g,\boldsymbol{b})e^{i\boldsymbol{b}\cdot\boldsymbol{q}}\,\mathcal{S}_r(\boldsymbol{k}_g)\left[e^{i\boldsymbol{b}\cdot\boldsymbol{k}_g}-\theta(M^2_{\rm in}-\boldsymbol{k}_g^2)\right]$$

The large double logarithms are given by

$$\mathcal{S}_r(\boldsymbol{k}_g) = \frac{\alpha_s}{2\pi^2}\frac{1}{\boldsymbol{k}_g^2}\begin{cases}\left[N_c\ln\frac{\boldsymbol{P}^2}{\boldsymbol{k}_g^2}+2C_F\left(\ln\frac{M^2_{\rm in}}{m_Q^2}+2\sum_{n=1}\ln\frac{M^2_{\rm in}}{m_Q^2}\cos\left(2n\phi_{k_gP}\right)\right)+\cdots\right], & \boldsymbol{k}_g^2\le m_Q^2\ll\boldsymbol{P}^2,\\ \left[N_c\ln\frac{\boldsymbol{P}^2}{\boldsymbol{k}_g^2}+2C_F\left(\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}+2\sum_{n=1}\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}\cos\left(2n\phi_{k_gP}\right)\right)+\cdots\right], & \boldsymbol{k}_g^2\gg m_Q^2\,.\end{cases}$$

Mass as IR cutoff

Massless limit

Collinear $\frac{1}{\epsilon}\to$ ->> FFs

Performing the Fourier transformation $\boldsymbol{k}_g\to\boldsymbol{b}$

$$\mathrm{Sud}_{\rm f}^{\rm Q} = \int_0^\infty\frac{d^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2}\left(e^{i\boldsymbol{b}\cdot\boldsymbol{k}_g}-\theta(M^2_{\rm in}-\boldsymbol{k}_g^2)\right)\left(\ln\frac{M^2_{\rm in}}{m_Q^2}\theta(m_Q^2-\boldsymbol{k}_g^2)+\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}\theta(\boldsymbol{k}_g^2-m_Q^2)\right)$$

Large logs from real and virtual digram

$$C_{2n}(\mu_b) = 2n\int_0^\infty\frac{d\boldsymbol{k}_g}{\boldsymbol{k}_g}J_{2n}(|b||\boldsymbol{k}_g|)\left(\ln\frac{M^2_{\rm in}}{m_Q^2}\theta(m_Q^2-\boldsymbol{k}_g^2)+\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}\theta(\boldsymbol{k}_g^2-m_Q^2)+\ldots\right)$$

finite parts only from real digram

[Marquet, YS, Xiao, arXiv: 2510.18949]

Unified Resummation of Soft Gluon Radiation in Heavy Quark Pair Photoproduction

- After resumming the all-order soft-gluon, we get unified resummation framework

$$\frac{d\sigma}{d\Omega} = \int d^2\boldsymbol{b} e^{i\boldsymbol{q}\cdot\boldsymbol{b}} H_{ij}^{\mathrm{TMD}}(z,\boldsymbol{P}) x G^{ij}(x_g,\boldsymbol{b}) e^{-\mathrm{Sud}(\boldsymbol{b})} \left[1 + \sum_{n=1} (-1)^n \frac{2C_F \alpha_s(\mu_b) C_{2n}(\mu_b)}{n\pi} \cos(2n\phi_{bP})\right]$$

- the Sudakov factor at DL accuracy likes, $\mathrm{Sud} = \mathrm{Sud}_{\mathrm{per}} + \mathrm{Sud}_{\mathrm{NP}}$

$$\mathrm{Sud}_{\mathrm{per}} = \int_{\mu_b^2}^{M_{\mathrm{in}}^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu)}{2\pi} \left\{ N_c \ln \frac{\boldsymbol{P}^2}{\mu^2} + 2C_F \left[\underbrace{\ln \frac{M_{\mathrm{in}}^2}{m_Q^2} \theta(m_Q^2 - \mu_b^2)}_{\text{massive part}} + \underbrace{\theta(\mu_b^2 - m_Q^2) \ln \frac{M_{\mathrm{in}}^2}{\mu^2}}_{\text{massless part}} + \dots \right] \right\}$$

Identical to Sudakov factor of the symmetrical observables in $\gamma\gamma \to l^+l^-$ and $e^+e^- \to Q\bar{Q}$.

[Klein, Mueller, Xiao, Yuan, PRL.122, 132301 (2019); Aglietti, Ferrera, Eur.Phys.J.C 85 (2025) 3, 27]

- the harmonic coefficients

$$C_{2n}(\mu_b) = \underbrace{\ln \frac{M_{\mathrm{in}}^2}{m_Q^2} \theta(m_Q^2 - \mu_{b,2n}^2)}_{\text{massive part}} + \underbrace{\ln \frac{M_{\mathrm{in}}^2}{\mu_{b,2n}^2} \theta(\mu_{b,2n}^2 - m_Q^2)}_{\text{massless part}} + \dots$$

New terms

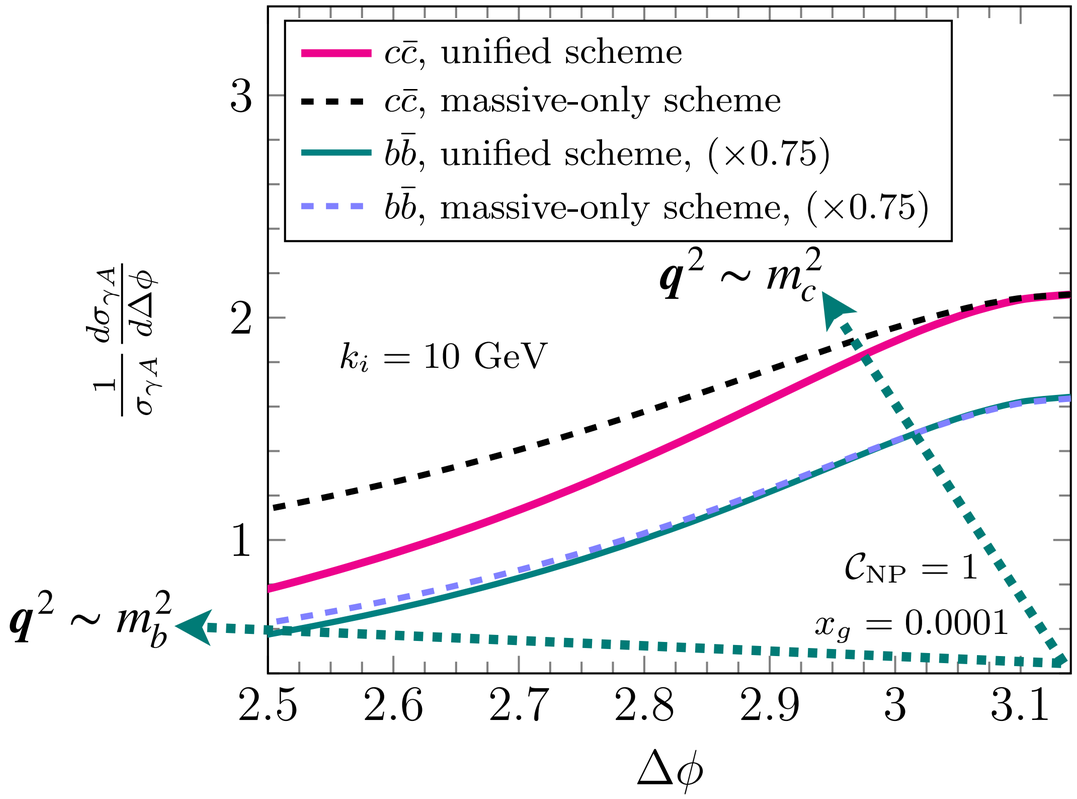
additional contributions beyond earlier $\langle \cos 2\phi_{qP} \rangle$ results

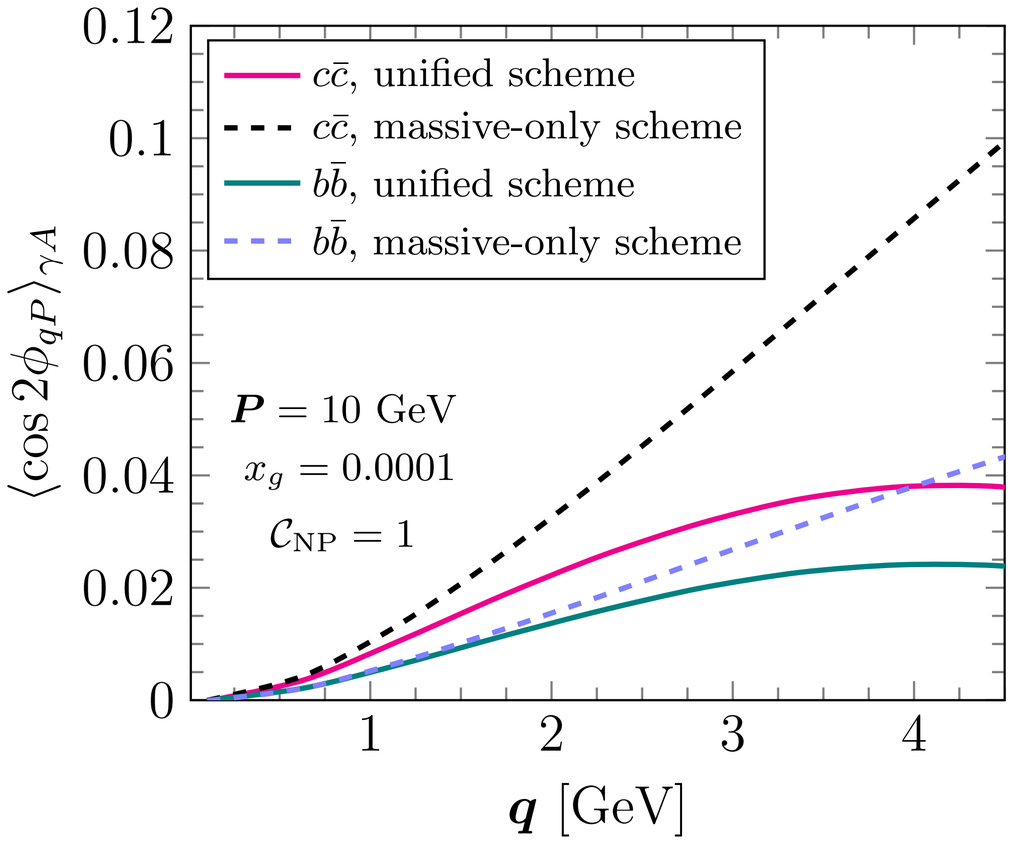
[Hatta, Xiao, Yuan, Zhou, PRD, 21; Shao, Zhang, Zhou, Zhou, PRD,23)]

It consistently describes both the symmetrical distribution and azimuthal asymmetries.

[Marquet, YS, Xiao, arXiv: 2510.18949]

Comparison of unified and massive resummation schemes





- These results show that the unified and massive-only schemes are consistent for $\boldsymbol{q}^2 < m_Q^2$, and differ at $m_Q^2 < \boldsymbol{q}^2$.
- In addition, the deviation between the two schemes sets in earlier (at smaller $\boldsymbol{q}$) for higher harmonics.

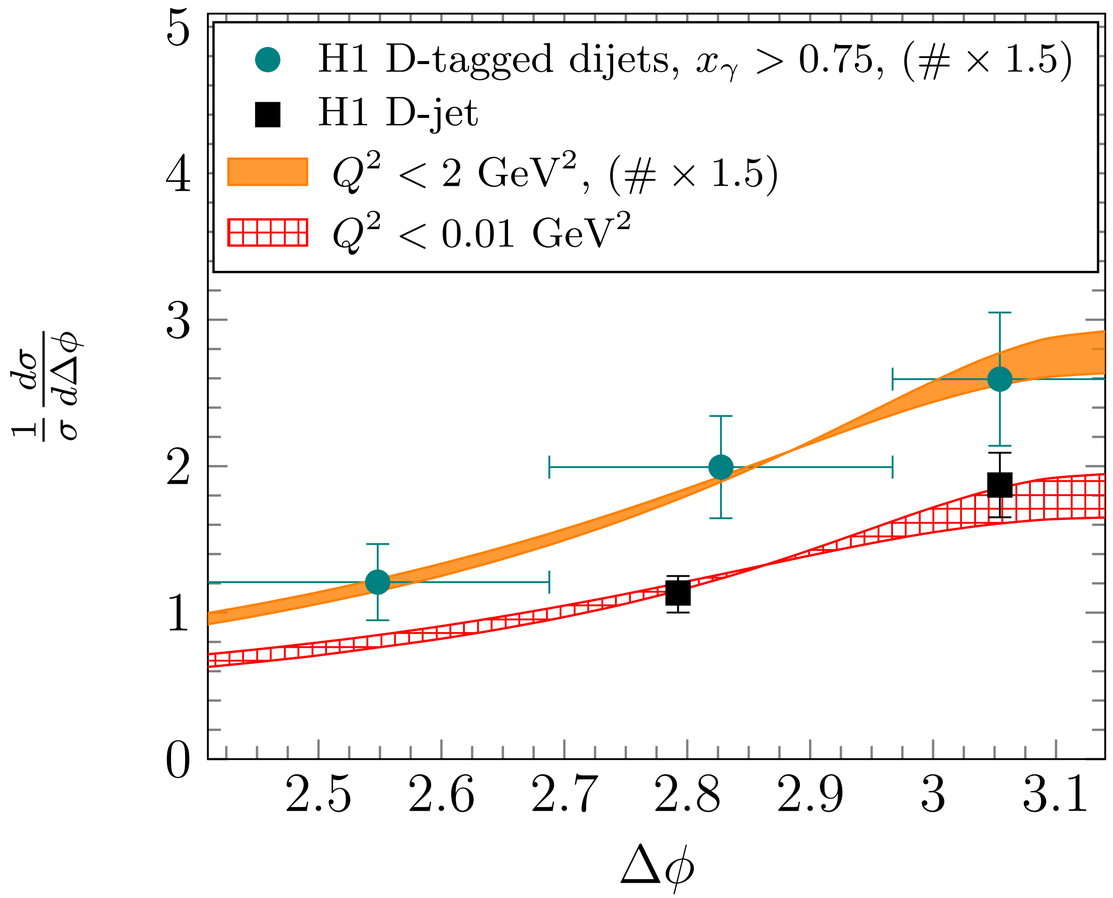
[Marquet, YS, Xiao, arXiv: 2510.18949]

3) Comparisons with experimental data from both ep and foward pp/pA collisions

Comparison with HERA ep data

[Marquet, YS, Xiao, arXiv: 2510.18949]

First description of HERA data

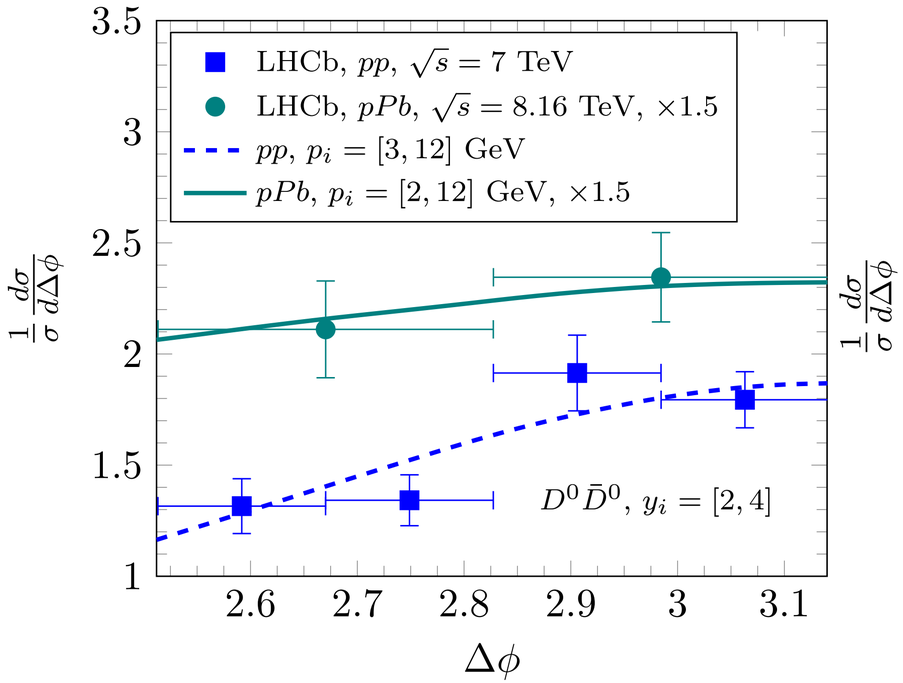


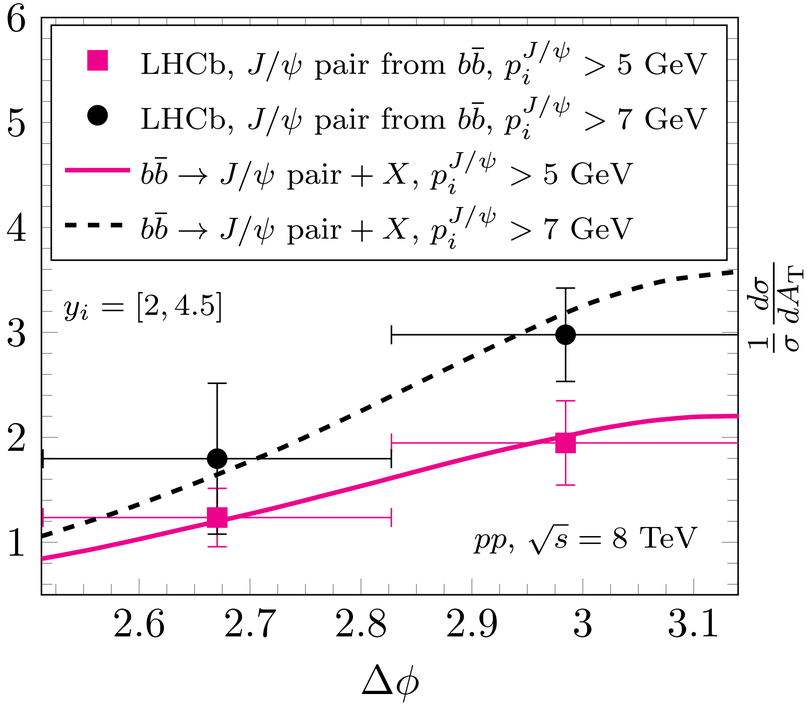
[Eur. Phys. J. C 50, 251 (2007), Eur. Phys. J.C 72 (2012)]

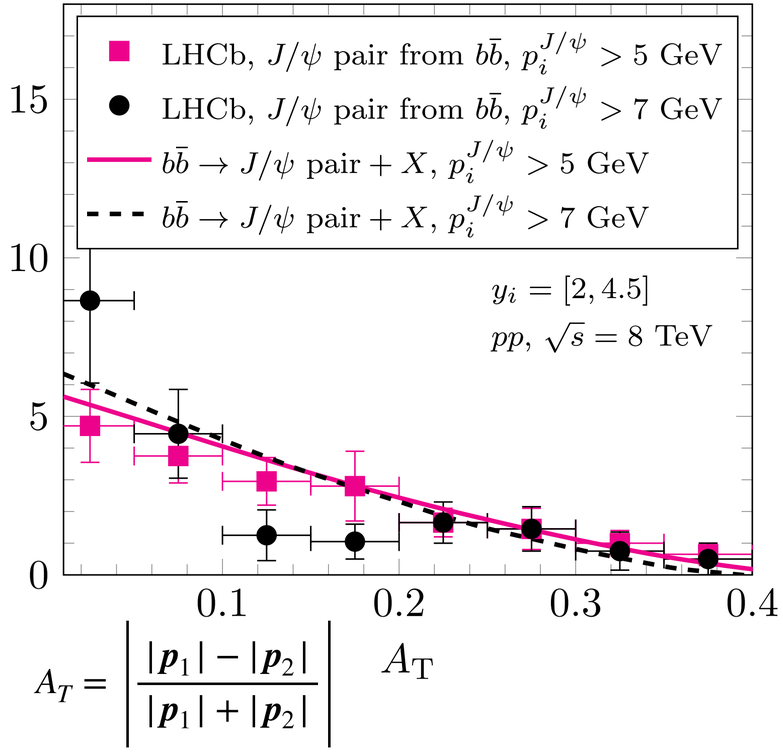
Comparison with LHC forward pp/pA data

[Gao, Marquet, YS, Xiao, arXiv: 2605.01527]

First description of LHCb c & b-hadrons data



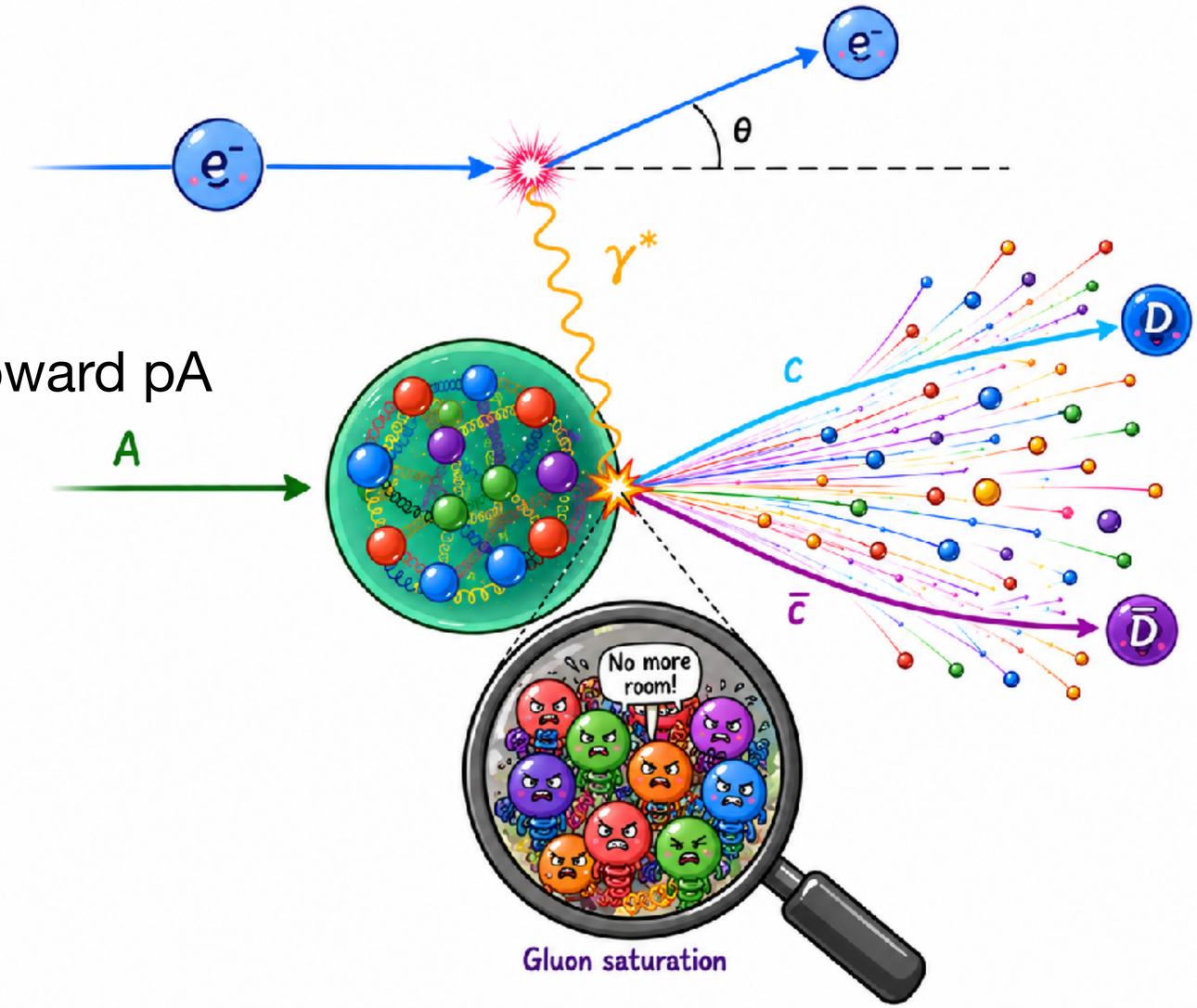




we also compare to p+Pb data in order to further validate our massive Sudakov resummation

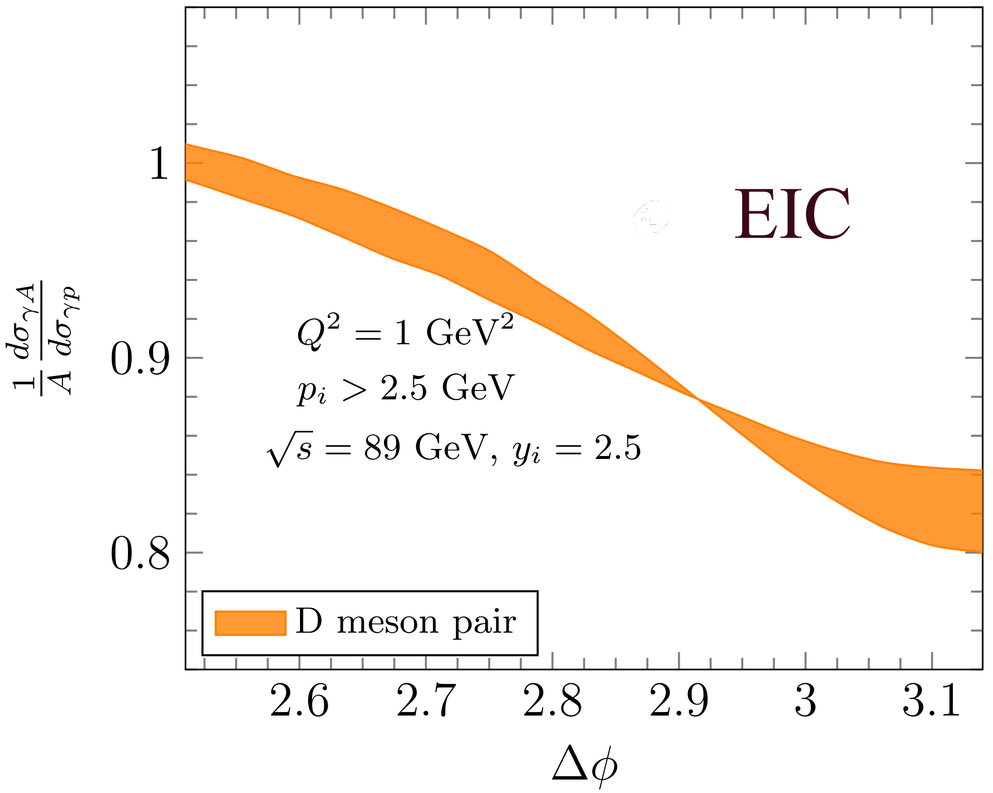
[JHEP 11, 030 (2017); JHEP 06, 141 (2012);PRL 125, 212001(2020)]

4) Probing saturation effects in γA and foward pA

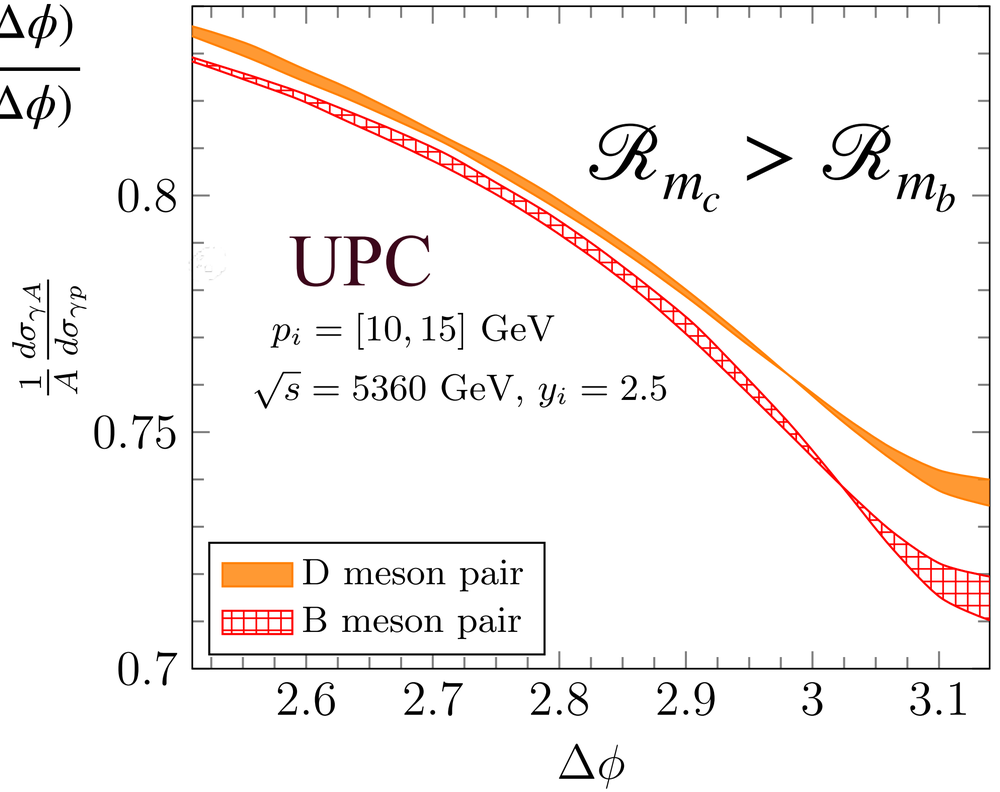


Probing non-linear effects in RpA

Saturation-induced suppression survives after Sudakov factor is included.

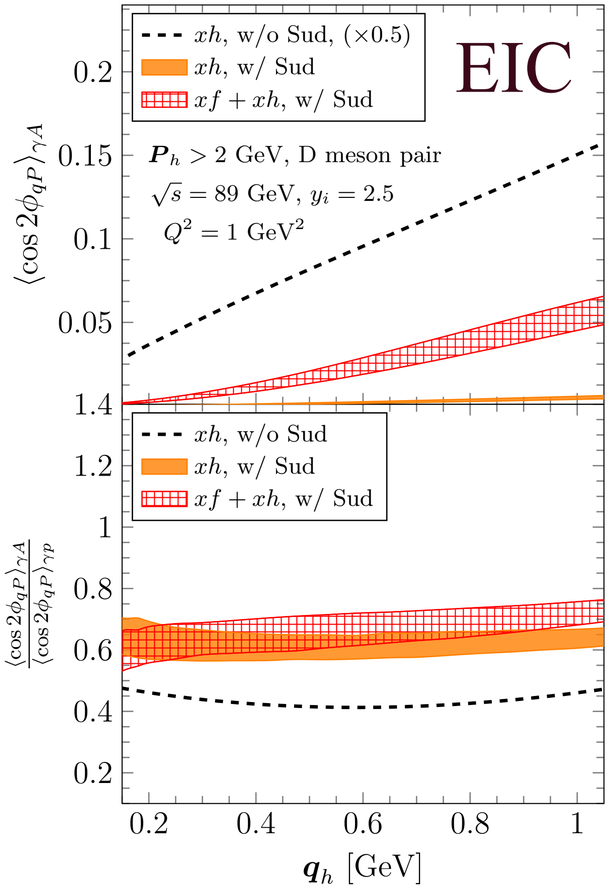


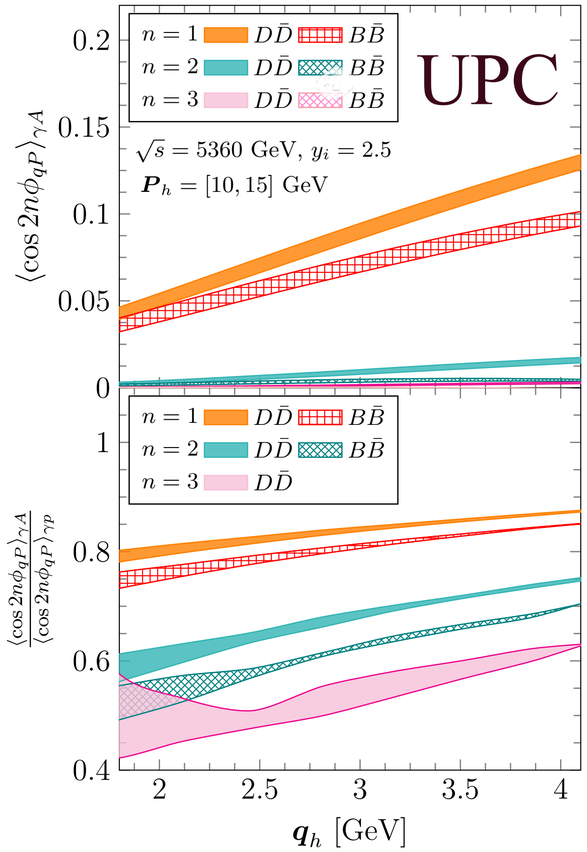
$$\mathscr{R} \equiv \frac{1}{A}\frac{d\sigma_{\gamma A}(\Delta\phi)}{d\sigma_{\gamma p}(\Delta\phi)}$$

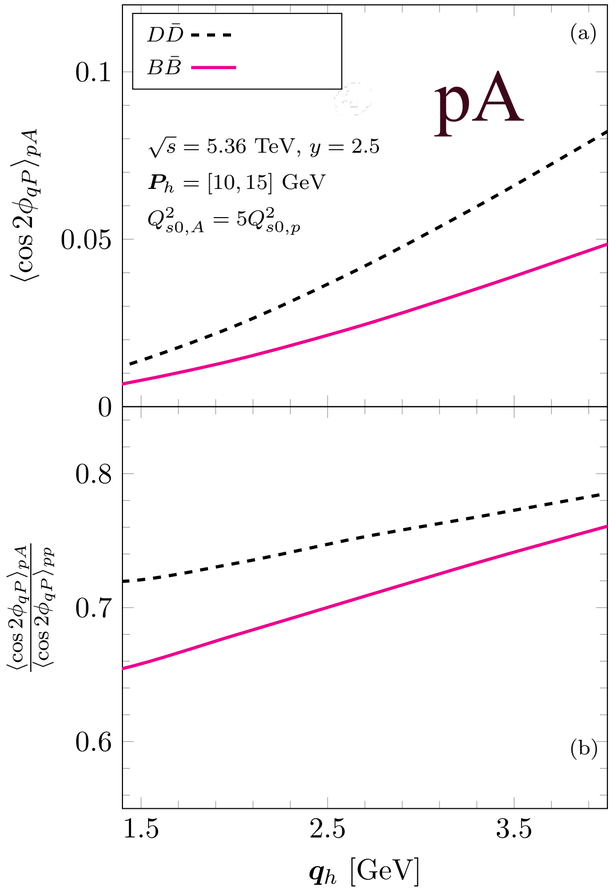


Mass hierarchy due to suppression of soft gluon emissions.

Azimuthal asymmetries $\langle \cos 2n\phi \rangle$







Higher harmonic ratios are even more sensitive to saturation effects.

$$\left.\frac{\langle \cos 2n\phi_{qP} \rangle_{\gamma A}}{\langle \cos 2n\phi_{qP} \rangle_{\gamma p}}\right|_{m_c} > \left.\frac{\langle \cos 2n\phi_{qP} \rangle_{\gamma A}}{\langle \cos 2n\phi_{qP} \rangle_{\gamma p}}\right|_{m_b}$$

Heavy meson pair correlations may serve as a good probe of saturation effects.

$$\frac{\langle \cos 6\phi_{qP} \rangle_{\gamma A}}{\langle \cos 6\phi_{qP} \rangle_{\gamma p}} < \frac{\langle \cos 4\phi_{qP} \rangle_{\gamma A}}{\langle \cos 4\phi_{qP} \rangle_{\gamma p}} < \frac{\langle \cos 2\phi_{qP} \rangle_{\gamma A}}{\langle \cos 2\phi_{qP} \rangle_{\gamma p}}$$

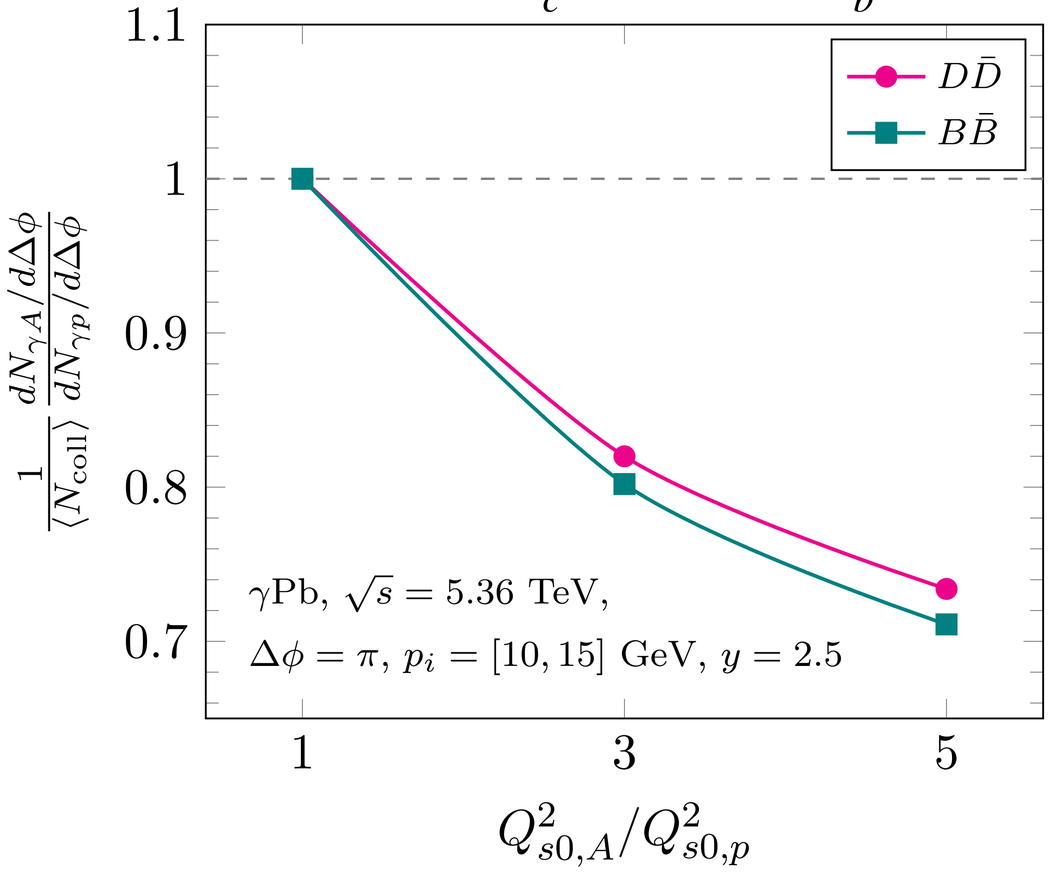
Summary

- **Unified Sudakov resummation** for heavy-quark pair correlation.
- **First pQCD resummation description** of HERA and LHCb data.
- **RpA and** $\langle \cos 2n\phi \rangle$ **ratios** are sensitive to saturation in both γA/pA.
- **Heavy-quark mass** suppresses soft radiation, enhancing saturation signals.
- **Upcoming LHC measurements and future EIC:** heavy meson pair correlations may serve as a good probe of saturation effects.

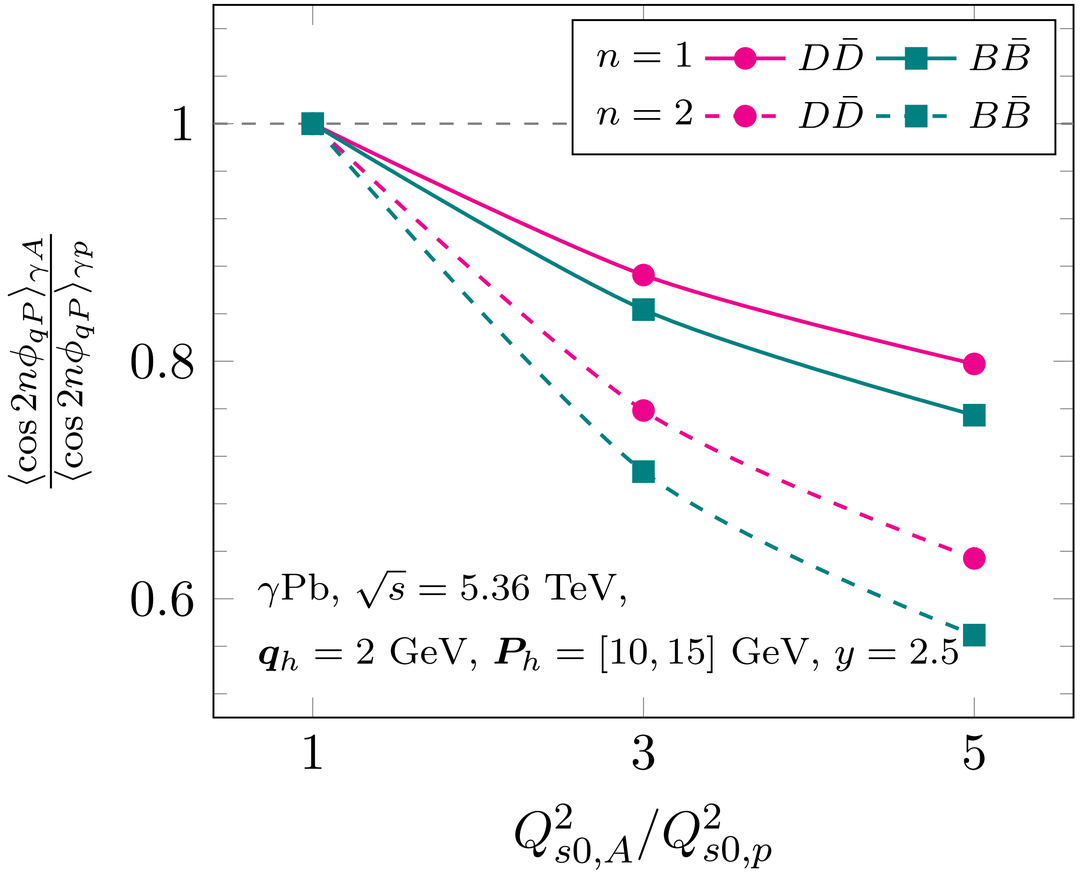
Thank you.

Dependence on the Saturation Scale $Q^2_{s0,A}$

$$R_{pA}\Big|_{m_c} > R_{pA}\Big|_{m_b}$$



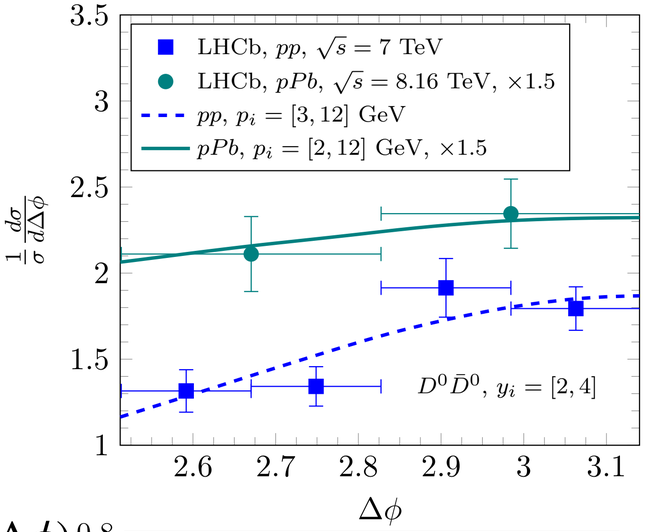
$$\frac{\langle\cos 2n\phi_{qP}\rangle_{\gamma A}}{\langle\cos 2n\phi_{qP}\rangle_{\gamma p}}\Big|_{m_c} > \frac{\langle\cos 2n\phi_{qP}\rangle_{\gamma A}}{\langle\cos 2n\phi_{qP}\rangle_{\gamma p}}\Big|_{m_b}$$

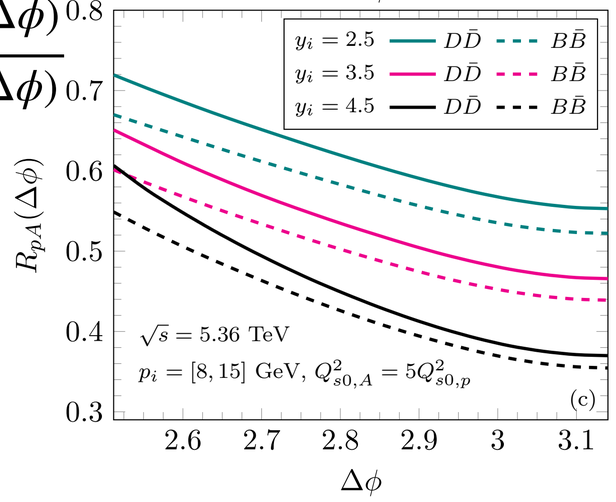


Heavy-mesons correlations in forward pA collisions

$D^0\bar{D}^0$ in both forward pp and pA

[JHEP 06, 141 (2012);PRL 125, 212001(2020)]

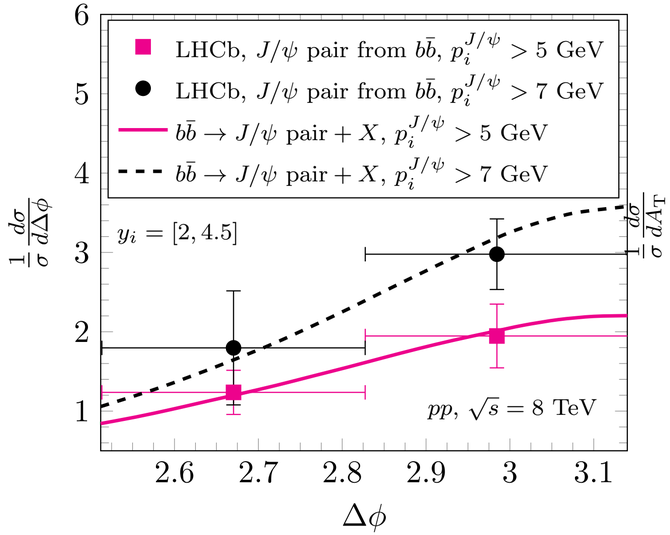


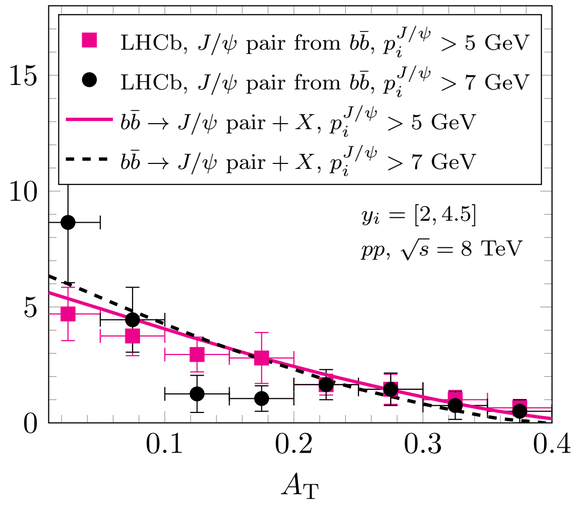


$$R_{pA} \equiv \frac{1}{A}\frac{d\sigma_{pA}(\Delta\phi)}{d\sigma_{pp}(\Delta\phi)}$$

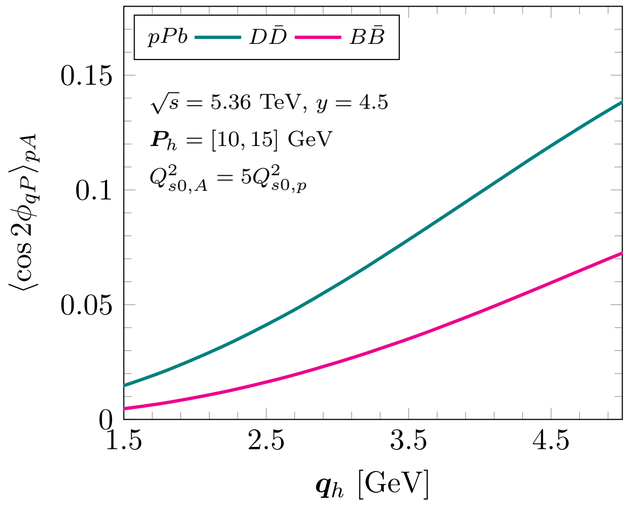
J/ψ pair from $b\bar{b}$ decay in forward pp

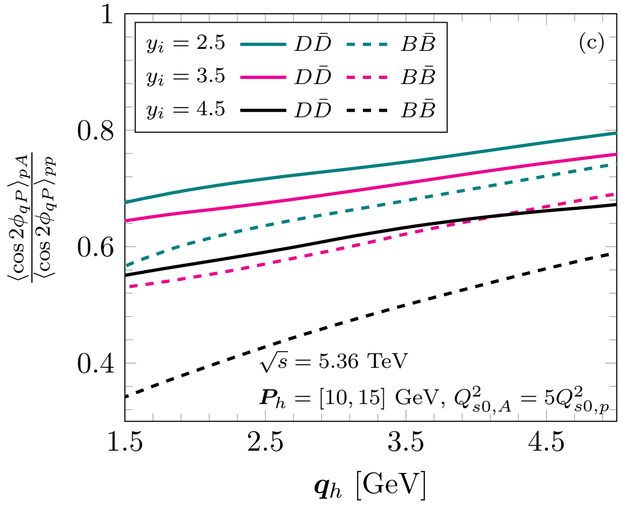
[JHEP 11, 030 (2017)]





$$A_T = \left|\frac{|\boldsymbol{p}_1| - |\boldsymbol{p}_2|}{|\boldsymbol{p}_1| + |\boldsymbol{p}_2|}\right|$$

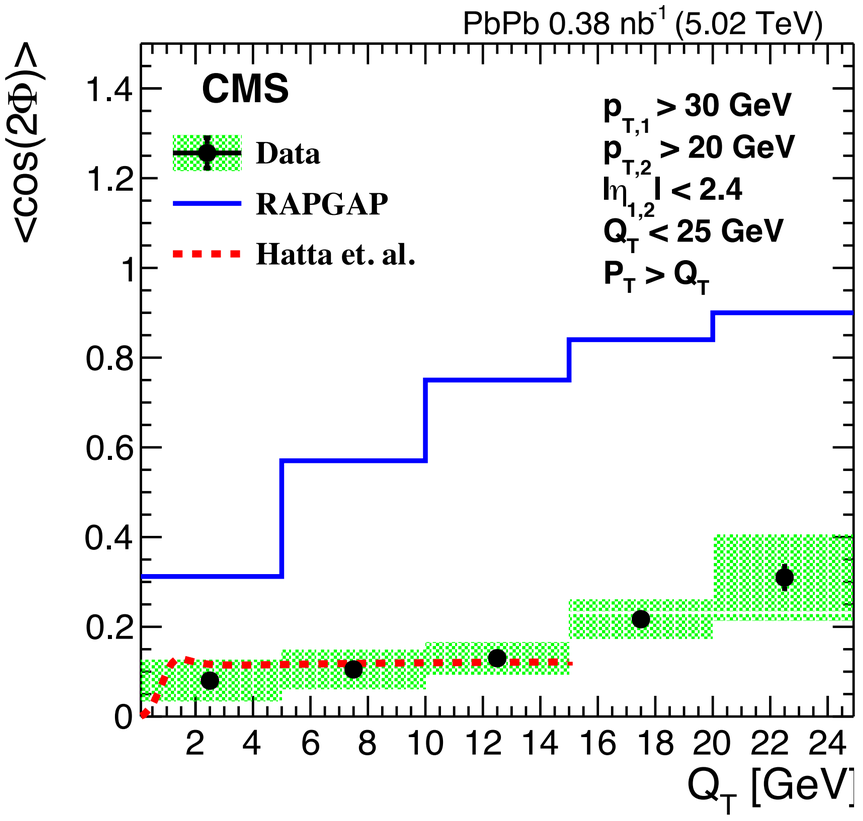




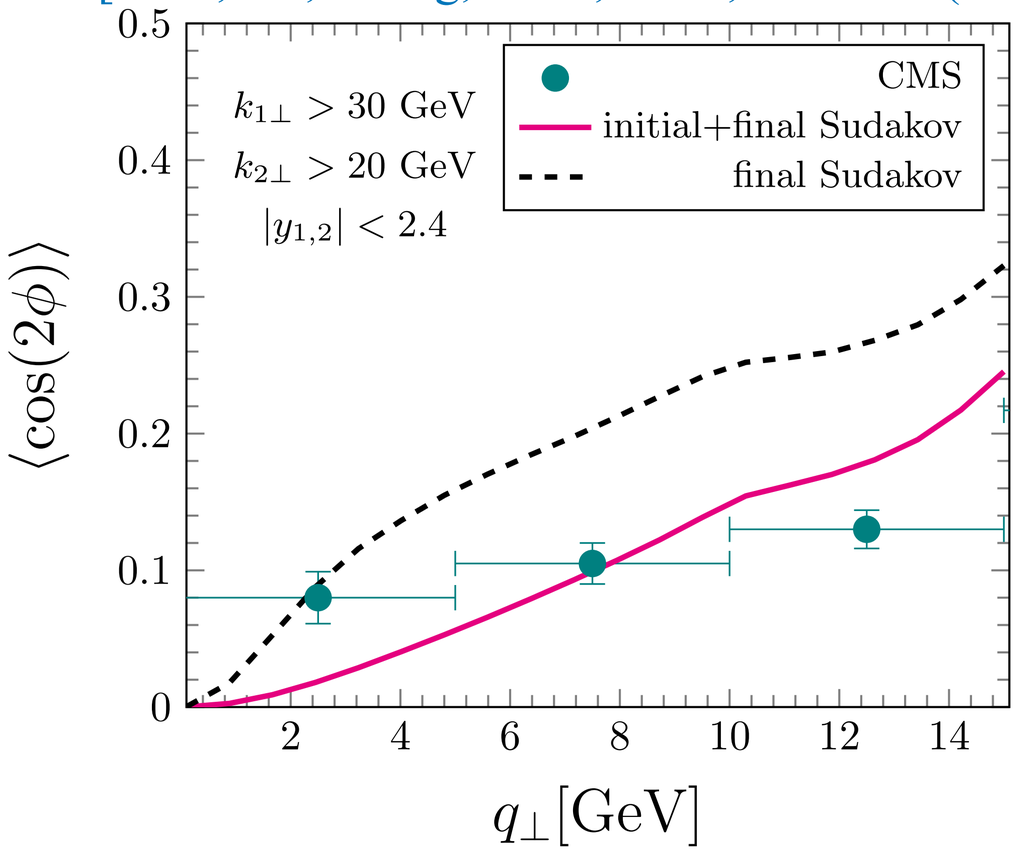
$\Delta\phi$ ratios, $\langle\cos 2n\phi_{qP}\rangle$ and its ratios show a clear mass hierarchy, which is sensitive to saturation effects.

Azimuthal asymmetries $\langle \cos 2n\phi_{qP} \rangle$ observables

previously measured in exclusive light-dijet UPCs



[Shao, YS, Zhang, Zhou, Zhou, JHEP 07 (2024) 189]



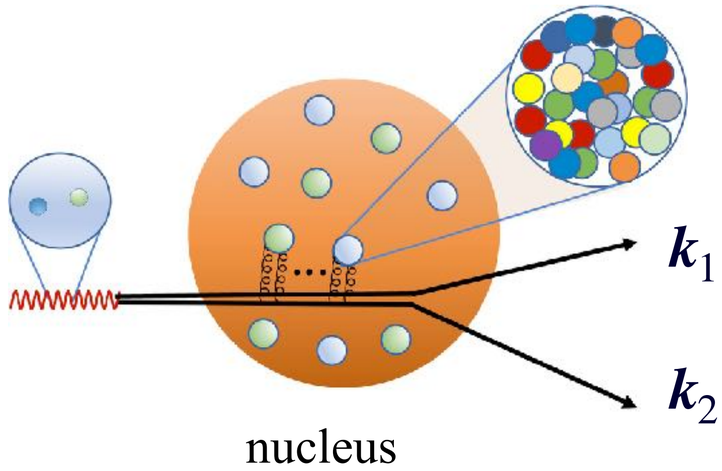
we advocate doing the same with inclusive UPCs and forward pA collisions

The heavy quark pair in UPC

In the back-to-back limit ($\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \ll \boldsymbol{P} = \frac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{2}$), it can be factorized as TMD hard factor ⊗ WW gluon dis

$$\frac{d\sigma_{\mathrm{LO}}^{\gamma A \to Q\bar{Q}X}}{d^2\boldsymbol{P} d^2\boldsymbol{q}} = x_\gamma f(x_\gamma) \otimes H_{\mathrm{TMD}}^{ij}(\boldsymbol{P}) \otimes xG_{ij}(x_g, \boldsymbol{q})$$

ultra-dense gluonic matter



Weizsacker Williams (WW) gluon distribution

$$xG^{ij}(x_g, \boldsymbol{q}) = -\frac{4}{g_s^2}\int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3} e^{-\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})} \langle \mathrm{Tr}\left[(\partial_i U_x) U_y^\dagger (\partial_i U_y) U_x^\dagger\right]\rangle$$

$$= \frac{\delta^{ij}}{2} xf(x_g, \boldsymbol{q}) + \left(\frac{\boldsymbol{q}^i \boldsymbol{q}^j}{\boldsymbol{q}^2} - \frac{\delta^{ij}}{2}\right) xh(x_g, \boldsymbol{q})$$

un-polarized linearly-polarized

- WW gluon encodes the gluon saturation information in the small-x regime.

[Dominguez, Marquet, Xiao, Yuan, Phys.Rev.D 83 (2011) 105005; Metz, Zhou, Phys.Rev.D 84 (2011) 051503 ; Dominguez, Qiu, Xiao, Yuan, Phys.Rev.D 85 (2012) 045003]

the Sudakov resummation in heavy quark pair

- we computed the Sudakov factor for heavy quark production

- Following the method in [Hatta, Xiao, Yuan, Zhou, PRD, 21], we obtain

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{P} d^2\boldsymbol{q}} \propto \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}\cdot\boldsymbol{b}} e^{-\mathrm{Sud}(\boldsymbol{b},M_{\mathrm{in}})} \left[1 + \sum_{n=1} (-1)^n \frac{2C_F \alpha_s(\mu_b) C_{2n}(\mu_b)}{n\pi} \cos(2n\phi_{bP})\right]$$
$$\times \int d^2\boldsymbol{q}' e^{-i\boldsymbol{q}'\cdot\boldsymbol{b}} \left[x f_{x_g}(\boldsymbol{q}') + \cos(2\phi_{Pq'}) x h_{x_g}(\boldsymbol{q}')\right]$$

Soft gluon radiations also generate azimuthal asymmetry!!!

Linearly-polarized gluon generate azimuthal asymmetry.

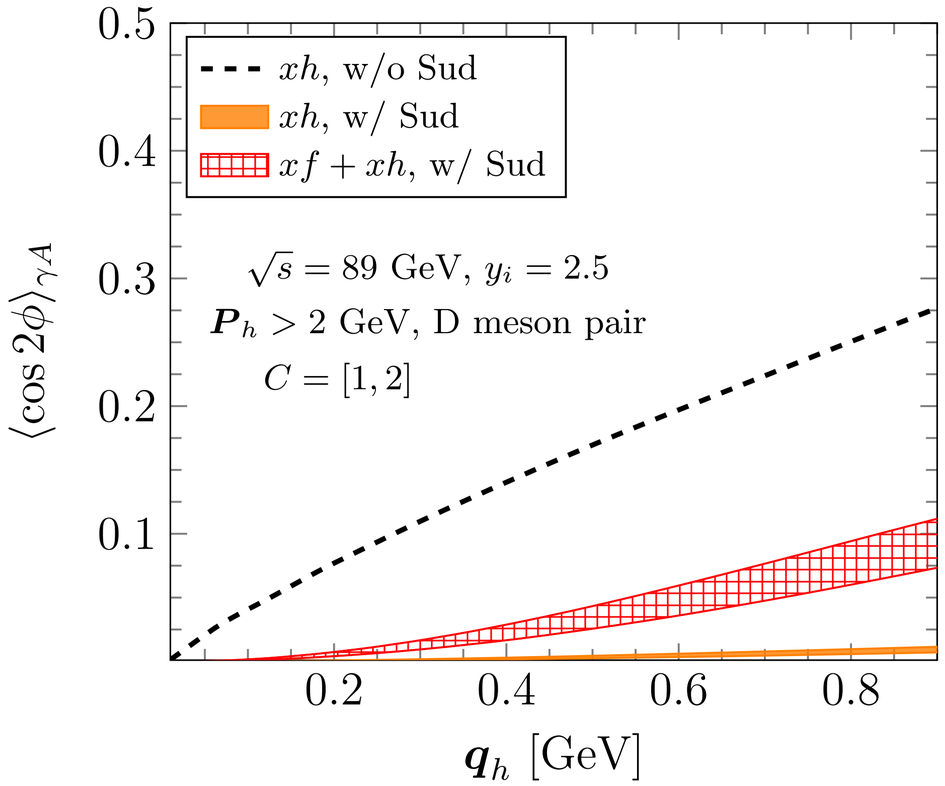
- the Sudakov factor

$$\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}}) = \mathrm{Sud}_i(\boldsymbol{b}, M_{\mathrm{in}}) + 2\mathrm{Sud}_f(\boldsymbol{b}, M_{\mathrm{in}}) + \mathrm{Sud}^i_{\mathrm{NP}}(\boldsymbol{b}, M_{\mathrm{in}})$$

Initial gluon radiation | Final heavy quarks radiation | Non-perturbative effects

Results for heavy quark pair photo-production at EIC

$$\langle \cos 2\phi \rangle = \frac{\int d\mathscr{P} \,.\, \mathcal{S} \,.\, \cos(2\phi) \frac{d\sigma}{d\mathscr{P} \,.\, \mathcal{S} \,.}}{\int d\mathscr{P} \,.\, \mathcal{S} \,.\, \frac{d\sigma}{d\mathscr{P} \,.\, \mathcal{S} \,.}}$$



- without Sudakov effect, only linearly polarised WW gluon xh generates $\langle \cos 2\phi \rangle$.
- With Sudakov effect, the linearly polarised WW has been suppressed, and the soft-gluon radiation with unpolarised gluon xf dominates.

Unified Resummation of Soft Gluon Radiation in Heavy Meson Pair Photoproduction

In the back-to-back limit ($\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \ll \boldsymbol{P} = \dfrac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{2}$), it can be factorized as TMD hard factor ⊗ WW gluon dis

$$\frac{d\sigma}{d^2\boldsymbol{P}d^2\boldsymbol{q}} = H^{ij}_{\rm TMD}(\boldsymbol{P})\int d^2\boldsymbol{r}d^2\boldsymbol{k}_g x G^{WW}_{ij}(x_g,\boldsymbol{b})e^{i\boldsymbol{b}\cdot\boldsymbol{q}}\,\mathcal{S}_r(\boldsymbol{k}_g)\left[e^{i\boldsymbol{b}\cdot\boldsymbol{k}_g} - \theta(M^2_{\rm in} - \boldsymbol{k}_g^2)\right]$$

The eikonal radiation kernel $\mathcal{S}_r(\boldsymbol{k}_g)$ is given by

$$\mathcal{S}_r(\boldsymbol{k}_g) = \frac{\alpha_s}{2\pi^2}\int dy_g\left[\frac{N_c}{2}\frac{2k_1\cdot p}{(k_1\cdot k_g)(p\cdot k_g)} + \frac{N_c}{2}\frac{2k_2\cdot p}{(k_2\cdot k_g)(p\cdot k_g)} - \frac{1}{2N_c}\frac{2k_1\cdot k_2}{(k_1\cdot k_g)(k_2\cdot k_g)}\right]$$

Following techniques developed in [Hatta, Xiao, Yuan, Zhou, PRD, 21], it can be decomposed as follows

$$\mathcal{S}_r(\boldsymbol{k}_g) = \mathcal{S}_{r0}(\boldsymbol{k}_g) + 2\sum_{n=1}^{\infty}\cos(2n\phi_{k_gP})\mathcal{S}_{r2n}(\boldsymbol{k}_g)$$

$$\mathcal{S}_r(\boldsymbol{k}_g) = \frac{\alpha_s}{2\pi^2}\frac{1}{\boldsymbol{k}_g^2}\left[N_c\ln\frac{\boldsymbol{P}^2}{\boldsymbol{k}_g^2} + 2C_F\left(\ln\frac{M^2_{\rm in}}{m_Q^2} + 2\sum_{n=1}\ln\frac{M^2_{\rm in}}{m_Q^2}\cos\left(2n\phi_{k_gP}\right)\right) + \dots\right], \qquad \boldsymbol{k}_g^2 \le m_Q^2$$

$$\mathcal{S}_r(\boldsymbol{k}_g) = \frac{\alpha_s}{2\pi^2}\frac{1}{\boldsymbol{k}_g^2}\left[N_c\ln\frac{\boldsymbol{P}^2}{\boldsymbol{k}_g^2} + 2C_F\left(\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2} + 2\sum_{n=1}\ln\frac{M^2_{\rm in}}{\boldsymbol{k}_g^2}\cos\left(2n\phi_{k_gP}\right)\right) + \dots\right], \qquad \boldsymbol{k}_g^2 \gg m_Q^2$$

Comparison of unified, massive and massless resummation schemes

$$\langle \cos 2\phi_{qP} \rangle \propto \frac{\int d\boldsymbol{b}^2 \boldsymbol{b}^2 J_2(|\boldsymbol{q}|\,|\boldsymbol{b}|) x f(x_g, \boldsymbol{b}) \alpha_s(\mu_b) e^{-\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}})} C_2(\mu_b)}{\int d\boldsymbol{b}^2 J_0(|\boldsymbol{q}|\,|\boldsymbol{b}|) x f(x_g, \boldsymbol{b}) e^{-\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}})}} \longrightarrow \sim \frac{\boldsymbol{q}^2}{\boldsymbol{P}^2} \ln \frac{M_{\mathrm{in}}^2}{m_Q^2} \text{ for } \boldsymbol{q}^2 \ll m_Q^2$$

$$\mathrm{Sud}_f^Q = \int_{\mu_b^2}^{M_{\mathrm{in}}^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu)}{2\pi} \left[\underbrace{\ln \frac{M_{\mathrm{in}}^2}{m_Q^2} \theta\left(m_Q^2 - \mu_b^2\right)}_{\text{massive part}} + \underbrace{\ln \frac{M_{\mathrm{in}}^2}{\mu_b^2} \theta\left(\mu_b^2 - m_Q^2\right)}_{\text{massless part}} \right]$$

$$\mu_b^2 \sim \boldsymbol{q}^2$$

$$C_2(\mu_b) = \underbrace{\ln \frac{M_{\mathrm{in}}^2}{m_Q^2} \theta\left(\frac{m_Q^2}{e} - \mu_b^2\right)}_{\text{massive part}} + \underbrace{\left(\ln \frac{M_{\mathrm{in}}^2}{\mu_b^2} - 1\right) \theta\left(\mu_b^2 - \frac{m_Q^2}{e}\right)}_{\text{massless part}} + f_2(\Delta y_{12})$$

These results show that the unified and massive-only schemes are consistent for $\boldsymbol{q}^2 < m_Q^2$.

In addition, the deviation between the two schemes sets in earlier (at smaller $\boldsymbol{q}$) for higher harmonics.

[Marquet, YS, Xiao, arXiv: 2510.18949]

rcBK equation and power-law fit

The impact parameter independent rcBK equation likes

$$\frac{\mathrm{d}N(\eta, r_\perp)}{\mathrm{d}\eta} = \int \mathrm{d}^2 r_{1\perp} \mathcal{K}_{\mathrm{BK}}(r_\perp, r_{1\perp}, r_{2\perp})[N(\eta, r_{1\perp}) + N(\eta, r_{2\perp}) - N(\eta, r_\perp) - N(\eta, r_{1\perp})N(\eta, r_{2\perp})]$$

We use the modified MV model as the initial condition [Albacete, Armesto, Milhano, Quiroga-Arias and Salgado, 11]

$$N(\eta = 0, r_\perp) = 1 - \exp\left[-\frac{1}{4}(r_\perp^2 Q_{s0}^2)^\gamma \ln(e + \frac{1}{|r_\perp|\Lambda_{\mathrm{BK}}})\right]$$

Taking Fourier transforms to $k_\perp$ space

$$F(\eta, k_\perp) = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} (1 - N(\eta, r_\perp))$$

At large $k_\perp$, the power-law behaviour emerge

$$F(\eta, k_\perp) \sim \frac{C(\eta, \beta)}{k_\perp^{2\beta}}$$

