# Recent results on entropy production in DIS




Krzysztof Kutak

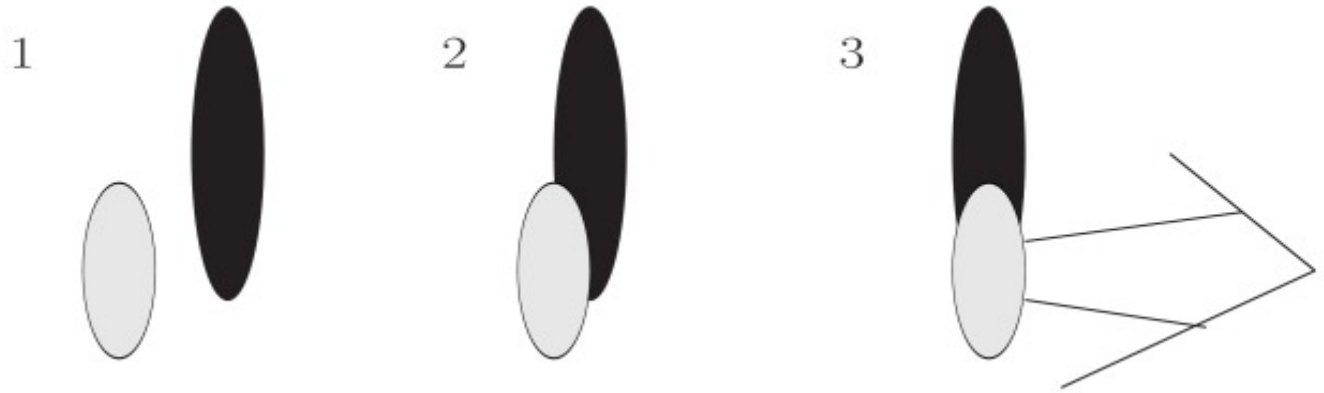
Presented results are obtained in collaboration with:

M. Hentschinski; D. Kharzeev; H. Jung; W. Płaczek;  M. Rohrmoser, R. Straka; K. Tu

# Unruh inspired particle production

## Stages of collision

Kharzeev, Tuchin '05




Hadron of mass m after interaction with external field transforms into hadrons of mass M

$$P(M \leftarrow m) = 2\pi |\mathcal{T}(M \leftarrow m)|^2 \, \rho(M),$$

$$\int dM P(M \leftarrow m)$$

probability for transition to final state

should be finite

density of states determined by typical momentum. Qs

Accelerated observer in its rest frame feels thermal radiation or Bose-Einstein distribution with temperature

$$|\mathcal{T}(M \leftarrow m)|^2 \sim \exp(-2\pi M/a)$$

transition amplitude

$$T = \frac{|a|}{2\pi}$$

$$T = \frac{Q_s(x)}{2\pi}$$

# Saturation and entropy

Kutak 1103.3654.v1 and v2

The relation $T = \frac{Q_s(x)}{2\pi}$ Can be understood in a generalized sense i.e. that saturation scale defines some temperature.

In GBW model can be shown that the saturation line has an interpretation of a Characteristics (in a partial diff equation sens) i.e. line along which the gluon density has a constant value.

$$dE = TdS$$

$$dE = dM$$

$$dM = TdS$$

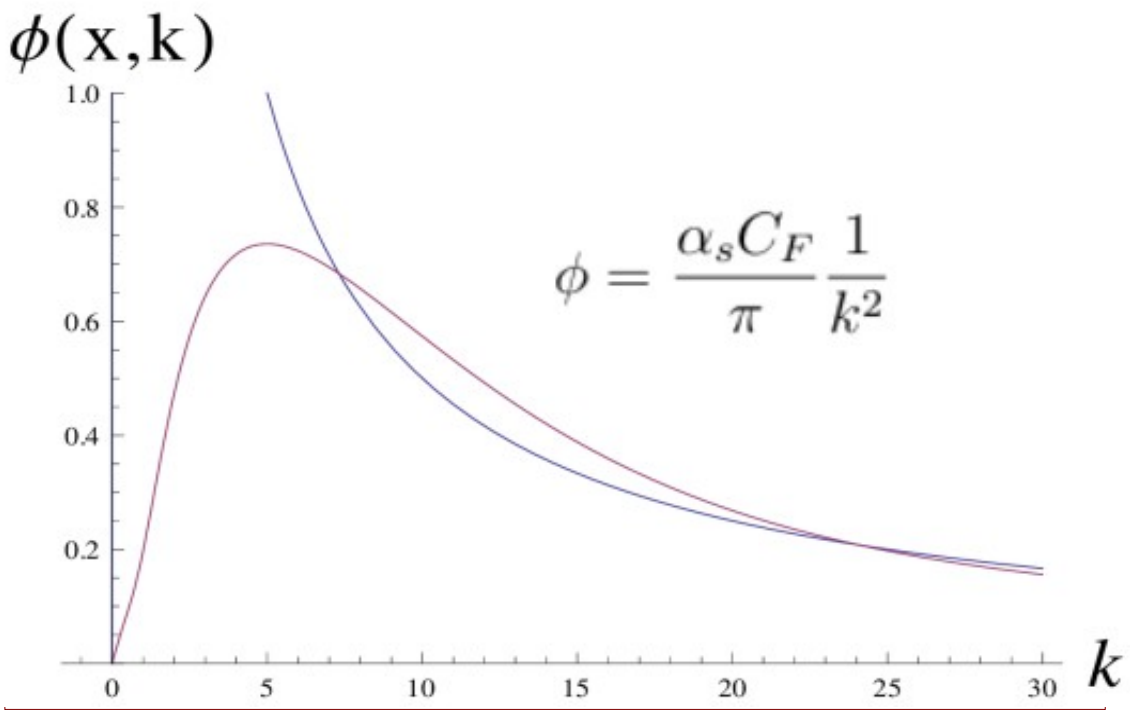
$$dM = dQ_s(x)$$

$$\frac{dQ_s(x)}{Q_s(x)} = \frac{dS}{2\pi}$$

mass of system of gluons

# Gluon density and entropy

Kutak 1103.3654.v1 and v2




$$\Delta S = \pi \lambda \Delta y$$

$$\Delta y = \ln(x_0/x)$$

$$Q_s^2 = Q_0^2 (x_0/x)^{\lambda}$$

Many-body interactions

Medium generated mass of gluon.

$$\Delta S \approx \pi \ln\left(\frac{\alpha_s \pi^2}{4 A_\perp^2 C_F}\frac{d\sigma}{dy}\right)$$

Similarly in QED. Cut on photon`s kt
Is equivalent to introducing effective mass.

Reviewed in Kutak '23

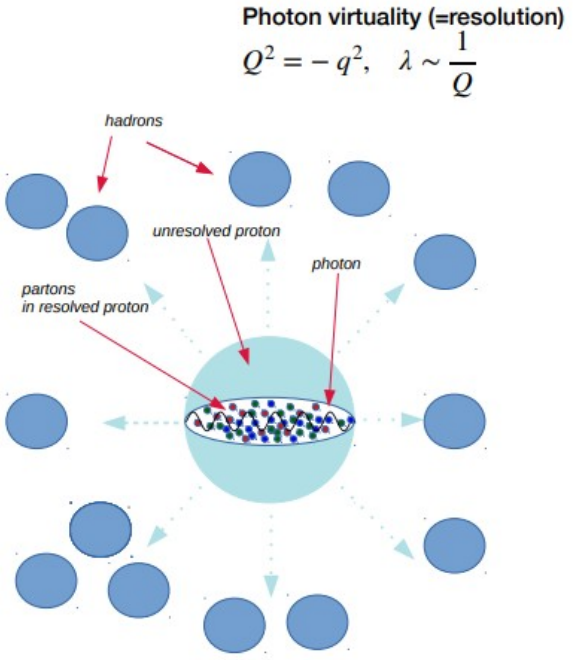
In presented approach mass is not fixed it is x dependent

# Entanglement and entropy

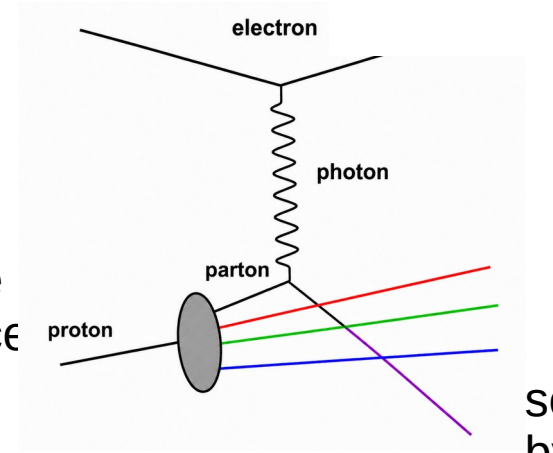
electron + proton → electron +radiation of hadrons

- collision of color singlets
- final state is color singlet
- color can not be isolated
- quark causally disconnected over period 1/Q from rest of proton. Hadronisation over period $1/\Lambda_{QCD}$ not enough time to exchange information information about color neutralization
- quark and remaining quarks and gluons form a color singlet. Strongly correlated by causaly disconnected
- DIS at large photon virtuality $Q^2$: a partial observation of the entangled quantum state
- Entanglement entropy is the measure of the incomplete information. Possible way to acce it is to measure multiplicity and calculate hadronic entropy:

$$S_h = -\Sigma_N P_N \ln P_N$$

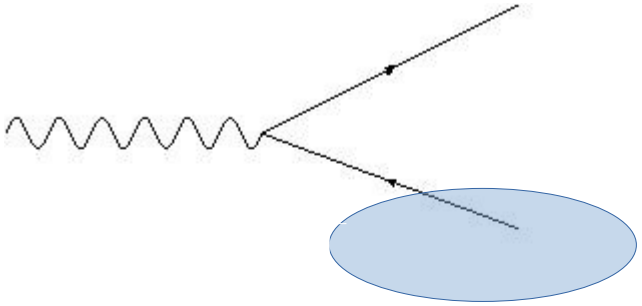



Kharzeev-Levin ‘17

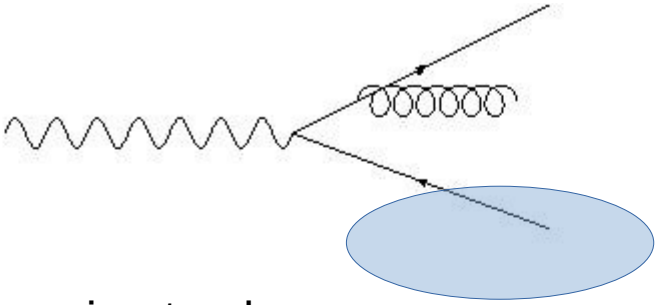



see also recent paper by Kharzeev, Rajagopal ‘26

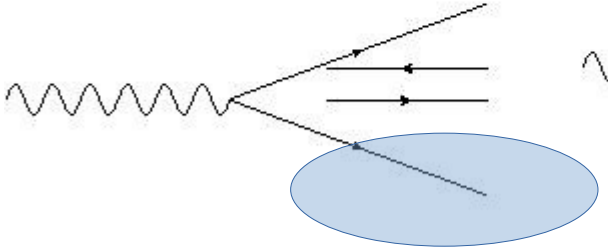
# Cascade of dipoles and entanglement entropy

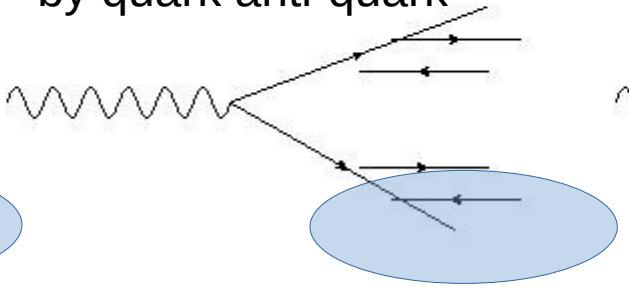


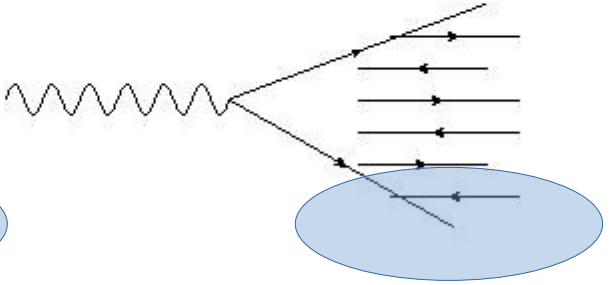
Mueller 95,
Lublinsky, Levin ‘03, ...



approximate gluon
by quark anti-quark







BFKL dipole cascade

set of partons is described by set of dipoles with fixed sizes , Y is rapidity and is related to energy

Mueller ‘94, ‘95
Mueller, Patel ‘94
Mueller, Salam ‘96

Existing Monte Carlo solutions:
Salam ‘96,
Flensburg, Gustafson, Lonnblad ‘11

# State and evolution equation

Hentschinski, Kutak, Płaczek, Rohrmoser'26

$$|\mathrm{DIS}\rangle = \sum_{n=1}^{\infty} \prod_{m=1}^{n} \int_0^1 \frac{dz_m}{z_m} \int d^2 r_m \sum_{i_1 \ldots i_n} \sum_{j_1 \ldots j_n} a_{j_1 \ldots j_n}^{i_1 \ldots i_n} \left( \{z_l, r_l\} \right) |\{z_l, r_l\}\rangle \, |i_1 \ldots i_n\rangle \, |j_1 \ldots j_n\rangle$$

$$\rho = |\mathrm{DIS}\rangle\langle \mathrm{DIS}|$$

See also:
Liu, Nowak, Zhed '22 (Mueller model approach)
Armesto, Dominguez, Kovner, Lublinsky, Skokov '19 (CGC approach)
Dumitru, Kolbusz '22

$$p_n = \prod_{m=1}^{n} \int_0^1 \frac{dz_m}{z_m} \int d^2 r_m \sum_{i_1 \ldots i_n} \sum_{j_1 \ldots j_n} \left| a_{j_1 \ldots j_n}^{i_1 \ldots i_n} \left( \{z_l, r_l\} \right) \right|^2$$

hadrons are color neutral

↓

color average

$$P_n(Y; r_1, r_2, \ldots, r_n) = \int_x^1 \frac{dz_n}{z_n} \ldots \int_{z_2}^1 \frac{dz_1}{z_1} \sum_{i_1 \ldots i_n} \sum_{j_1 \ldots j_n} \left| a_{j_1 \ldots j_n}^{i_1 \ldots i_n} \left( \{z_l, r_l\} \right) \right|^2$$

$$\frac{dP_n(Y; r_1, \ldots, r_n)}{dY} = -\sum_{i=1}^{n} \sigma(r_i) \, P_n(Y; r_1, \ldots, r_n) + \sum_{i=1}^{n-1} K(r_i, r_n) \, P_{n-1}(Y; r_1, \ldots, r_i + r_n, \ldots, r_{n-1})$$

dipole kernel

$$K(r_1, r_2) = \frac{\alpha_s N_c}{2\pi^2} \frac{(r_1 + r_2)^2}{r_1^2 r_2^2} \Theta(|r_1| - \rho) \Theta(|r_2| - \rho) \qquad \sigma(\boldsymbol{r}_i) = \int d^2 \boldsymbol{r}' K(\boldsymbol{r}_i - \boldsymbol{r}', \boldsymbol{r}') \approx \overline{\alpha}_s \ln \frac{\boldsymbol{r}_i^2}{\rho^2}$$

Levin-Lublinsky '03

# Explicit solution in 1d

$$\frac{dp_n(y)}{dy} = -\lambda\, n\, p_n(y) + (n-1)\,\lambda\, p_{n-1}(y) \qquad p_n(y) = e^{-\lambda y}(1-e^{-\lambda y})^{n-1}$$

$$\langle n\rangle = \sum_{n=1} n\, p_n(y) = e^{\lambda y} = \left(\frac{1}{x}\right) = xg(x)$$

$$S = -\sum_n p_n \ln p_n = \ln\langle n\rangle + (1-\langle n\rangle)\ln\frac{\langle n\rangle - 1}{\langle n\rangle}$$

In the large y limit we have $\langle n\rangle >> 1$

Entropy expressed in terms of gluon momentum density

$$S = \ln\langle n\rangle \qquad p_n = 1/\langle n\rangle$$

$$S = \lambda\, y = \ln\left(\frac{1}{x}\right)^{\lambda}$$

$$S = \ln xg(x)$$

$$S(x,Q) = \ln(xg(x,Q))$$

Conjectured in
Kharzeev, Levin 21

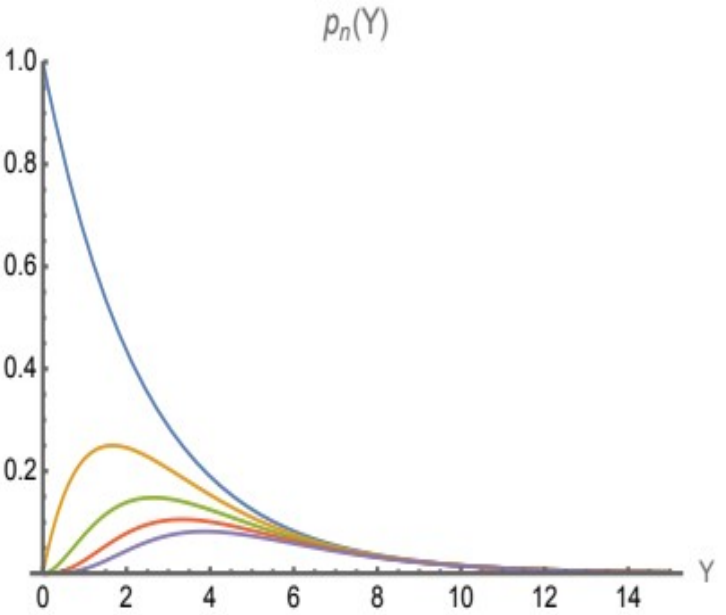
In agreement with calculations based on Unruh effect and saturation applied to p-p collisions
K. Kutak
1103.3654v1

see also
K. Kutak 2310.18510

# Cascade of dipoles – multiplicity and entropy

$$\langle n \rangle = \sum_{n=1} n\, p_n(y) = e^{\lambda y} = \left(\frac{1}{x}\right)^{\lambda} = xg(x)$$

$$S = -\sum_n p_n \ln p_n = \ln\langle n \rangle + (1 - \langle n \rangle) \ln$$




In the large y limit we have $\langle n \rangle >> 1$

$S = \ln\langle n \rangle$ $\quad p_n = 1/\langle n \rangle$

$$S = \lambda\, y = \ln\left(\frac{1}{x}\right)^{\lambda}$$

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Kharzeev, Levin 21

In agreement with calculations based on Unruh effect and saturation applied to p-p collisions
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1103.3654v1

see also
K. Kutak 2310.18510

# KL entropy formula - interpretation

$$p_n(y) = e^{-\lambda y}(1 - e^{-\lambda y})^{n-1}$$

At low x partonic microstates have equal probabilities.

In this equipartitioned state the entropy is maximal – the partonic state at small *x* is maximally entangled.

In terms of information theory as Shanon entropy:

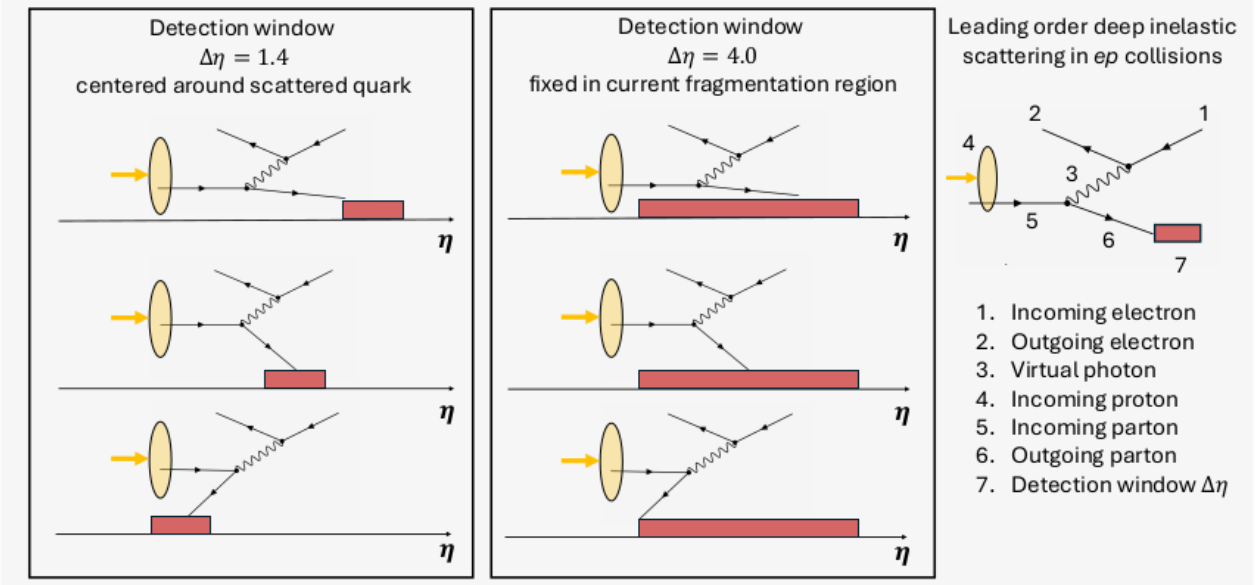
- equipartitioning in the maximally entangled state means that all “signals” with different number of partons are equally likely

- it is impossible to predict how many partons will be detected in a give event.

- structure function at small x should become universal for all hadrons.

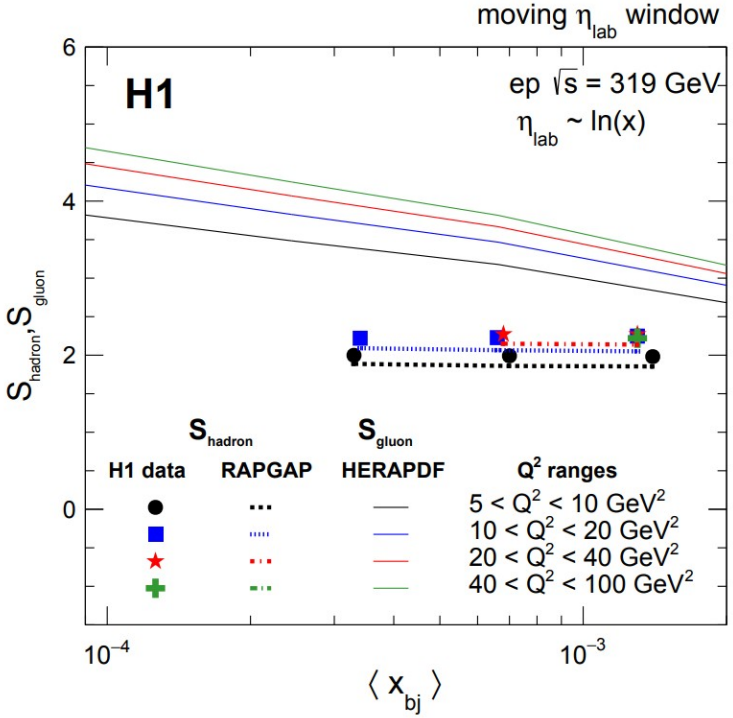
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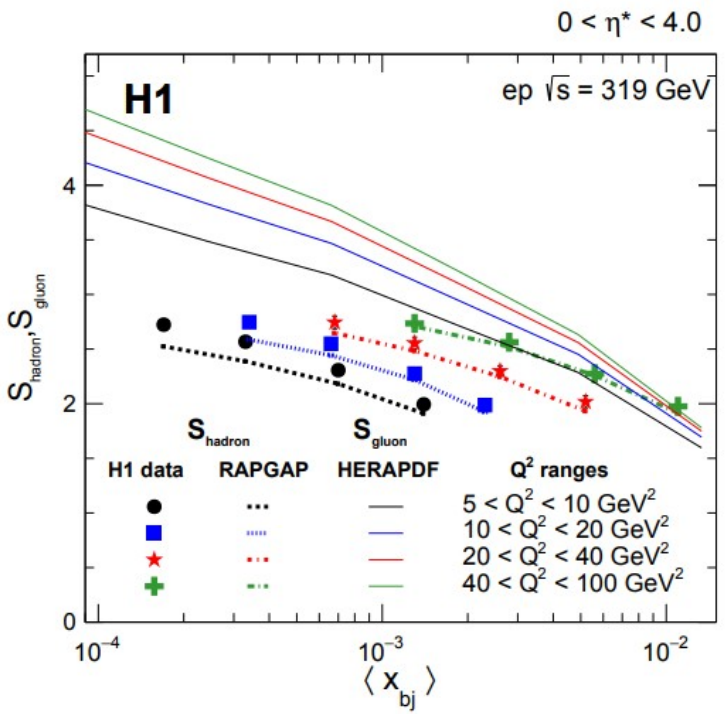
# Entropy and measurements




KL proposal

$S_{parton}=S_{hadron}$

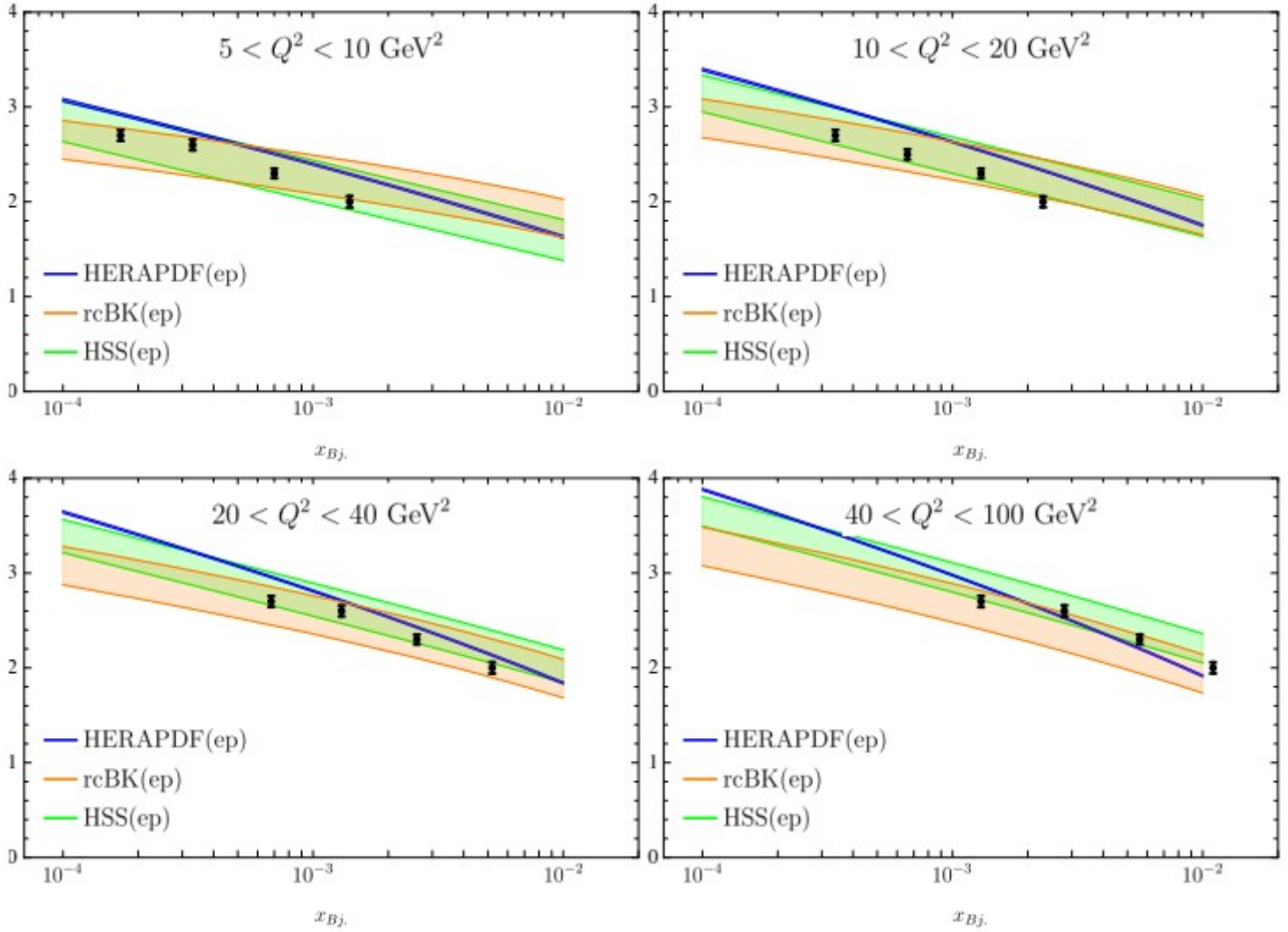



Problem with H1 escription of data...

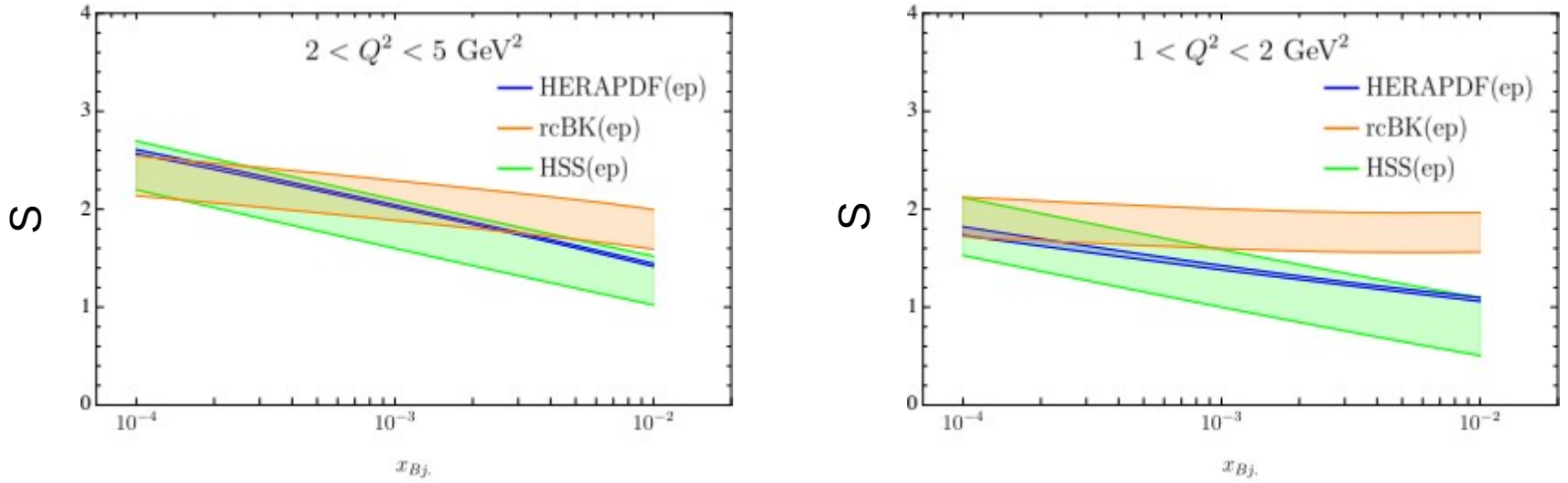
# Results – fixed rapidity window

Kutak, Hentschinski ‘22
Kutak, Hentschinski, Straka ‘23




Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured.

# Small scales - prediction

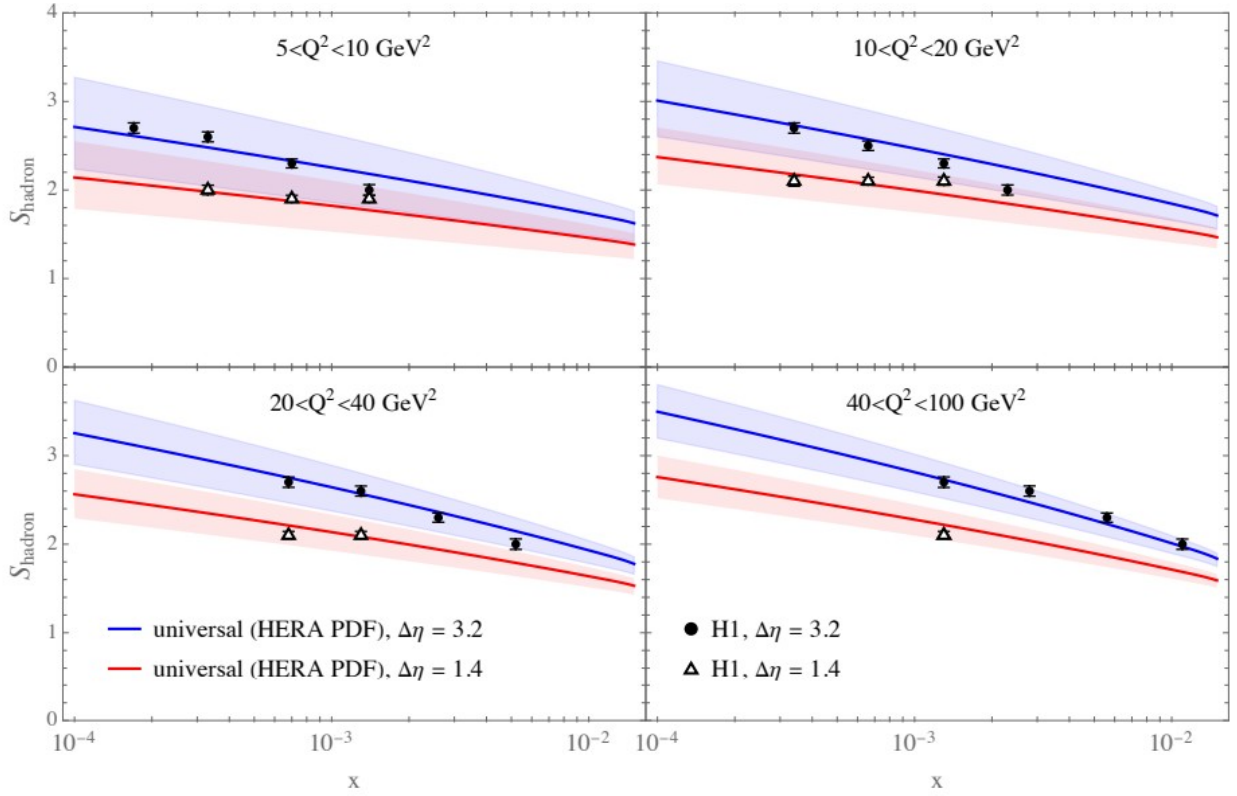



Entropy saturates as a consequence of gluon saturation.
However, the calculation is an approximation that uses that brakes down for small mean multiplicities

$$S = \ln\left(\frac{2}{3}\langle n\rangle\right) + C$$

# Results – fixed rapidity window

Hentschinski, Kharzeev, Kutak, Tu ‘24




S

blue – fixed rapidity window

moving – rapidity window

Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured.

# Solving the Levin-Lublinsky equation

Hentschinski, Kutak, Płaczek, Rohrmoser'26

$$\frac{dP_n(Y;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n)}{dY} = -\sum_{i=1}^{n}\sigma(\boldsymbol{r}_i)\,P_n(Y;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n) + \sum_{i=1}^{n-1}K(\boldsymbol{r}_i,\boldsymbol{r}_n)\,P_{n-1}(Y;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_i+\boldsymbol{r}_n,\ldots,\boldsymbol{r}_{n-1})$$

$$p_n(Y) = \prod_{m=1}^{n}\int d^2\mathbf{r}_m P_n(Y,\mathbf{r}_1,...,\mathbf{r}_n)$$

Initial condition: $P_n(Y=0;\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n) = \delta_{n1}\delta^{(2)}(\boldsymbol{r}_1-\boldsymbol{r})$

Solution with Monte Carlo programs

DIPMAR by W. Płaczek

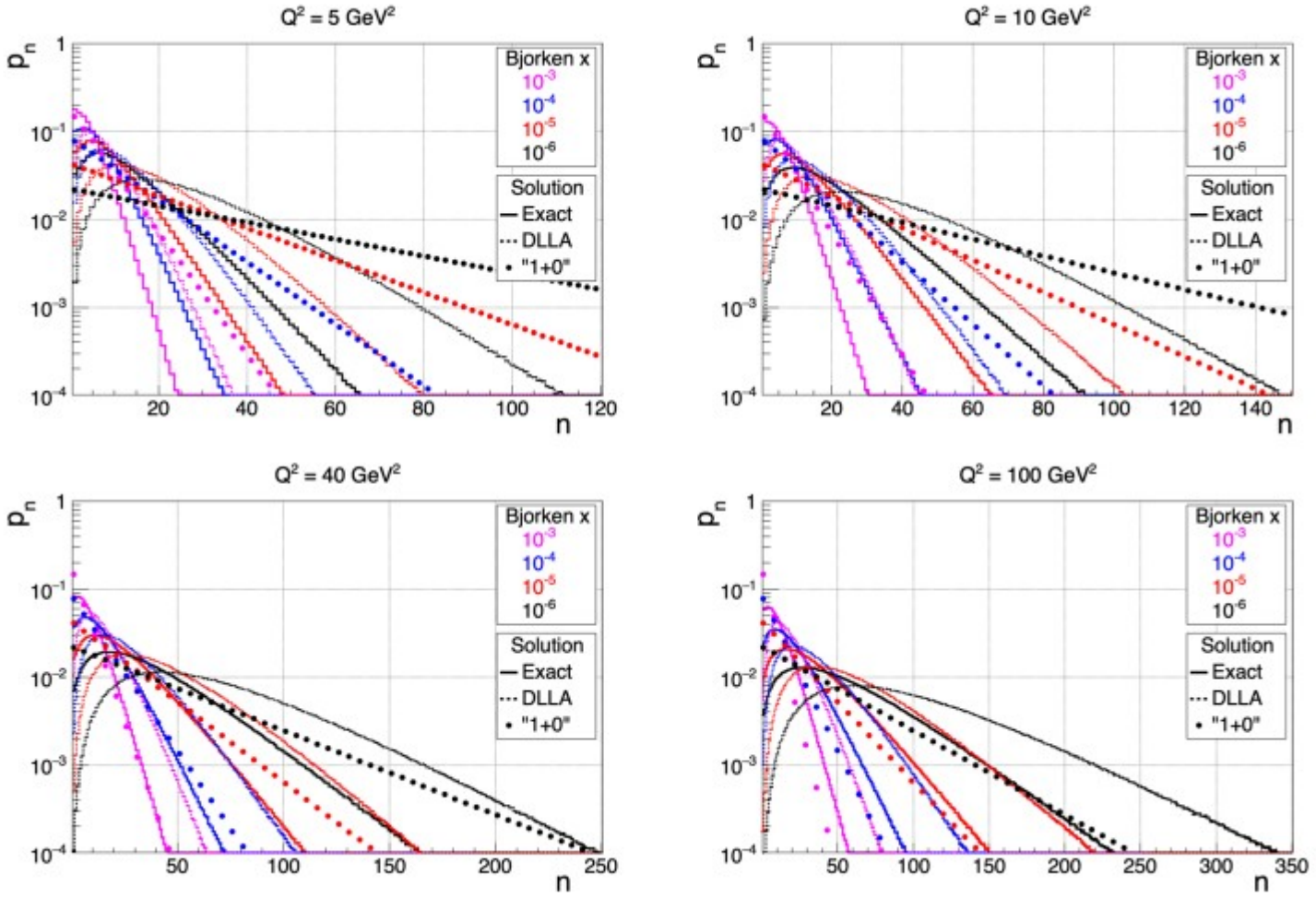
LLMC by M. Rohrmoser

$$\sigma(\boldsymbol{r}_i) = \int d^2\boldsymbol{r}' K(\boldsymbol{r}_i-\boldsymbol{r}',\boldsymbol{r}') \approx \overline{\alpha}_s \ln\frac{\boldsymbol{r}_i^2}{\rho^2}$$

The scale dependence is set by cutoff

# Results – $p_n(Y,Q^2)$

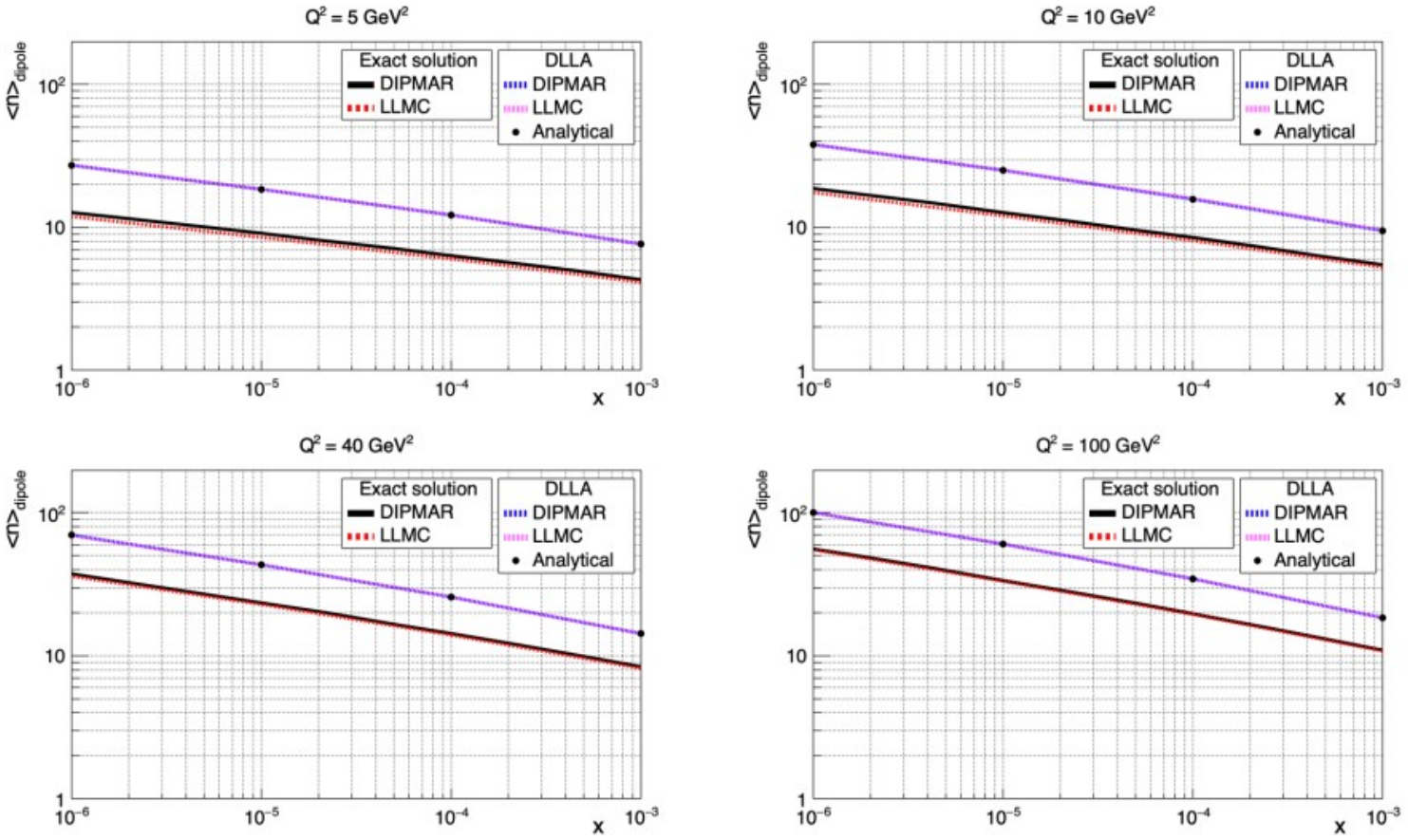
Hentschinski, Kutak, Płaczek, Rohrmoser'26




- 1 D solution $p_{n+1} \leq p_n$ always
- exact and double logarithmic approximation changes this: maxima of $p_n$ moves towards larger Y and ln $Q^2$ feature found also in H1 and LHC data → a more realistic distribution

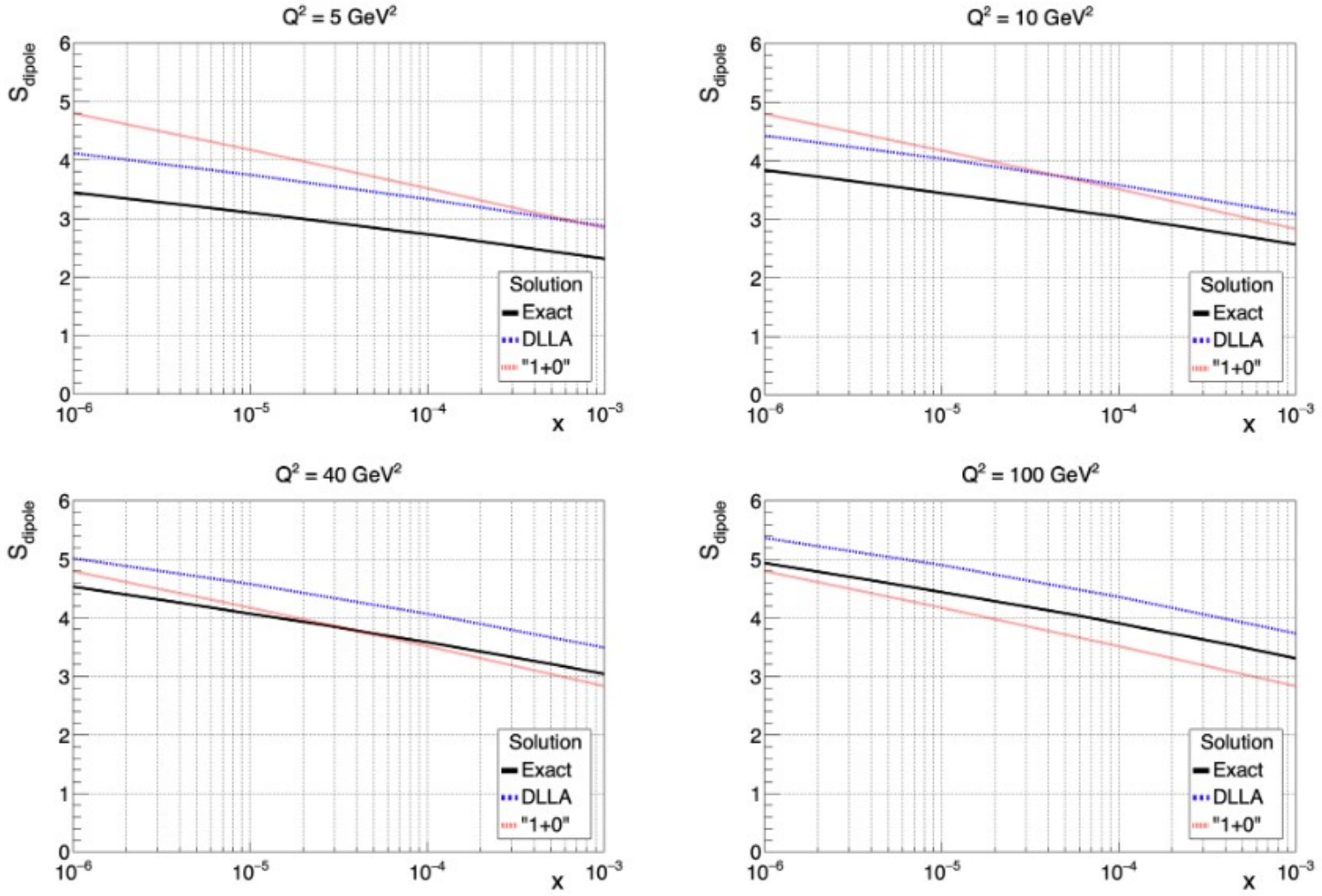
# Results – mean multiplicity

Hentschinski, Kutak, Płaczek, Rohrmoser'26

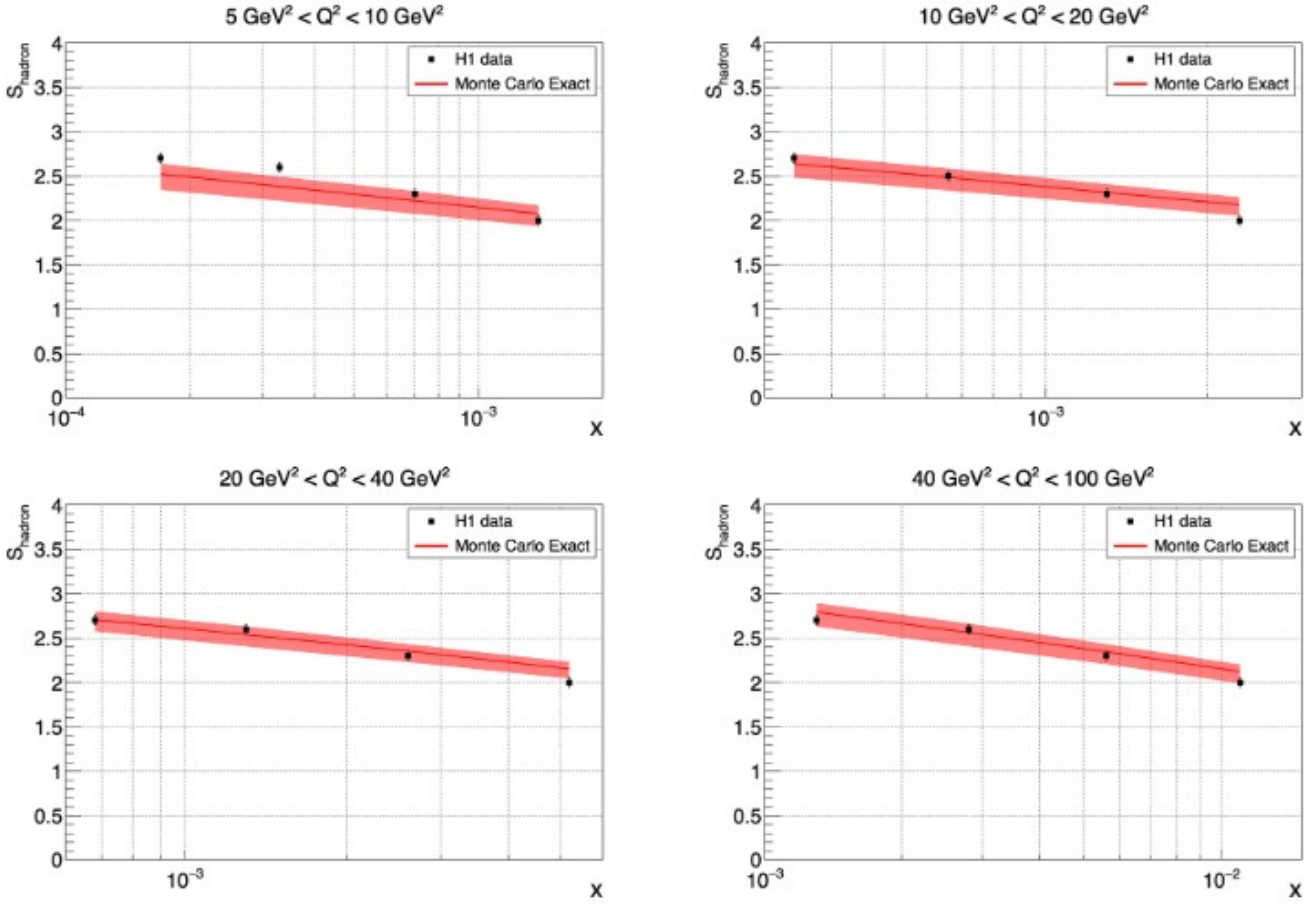



- with given initial condition, rise with 1/x is too strong; reason: unaccounted NLO corrections, we set $\bar{\alpha}_s = 0.1$
- initial dipole size r = 0.85 fm, UV regulator: ρ = 1/Q

# Entropy: complete vs. DLL vs. 1 d



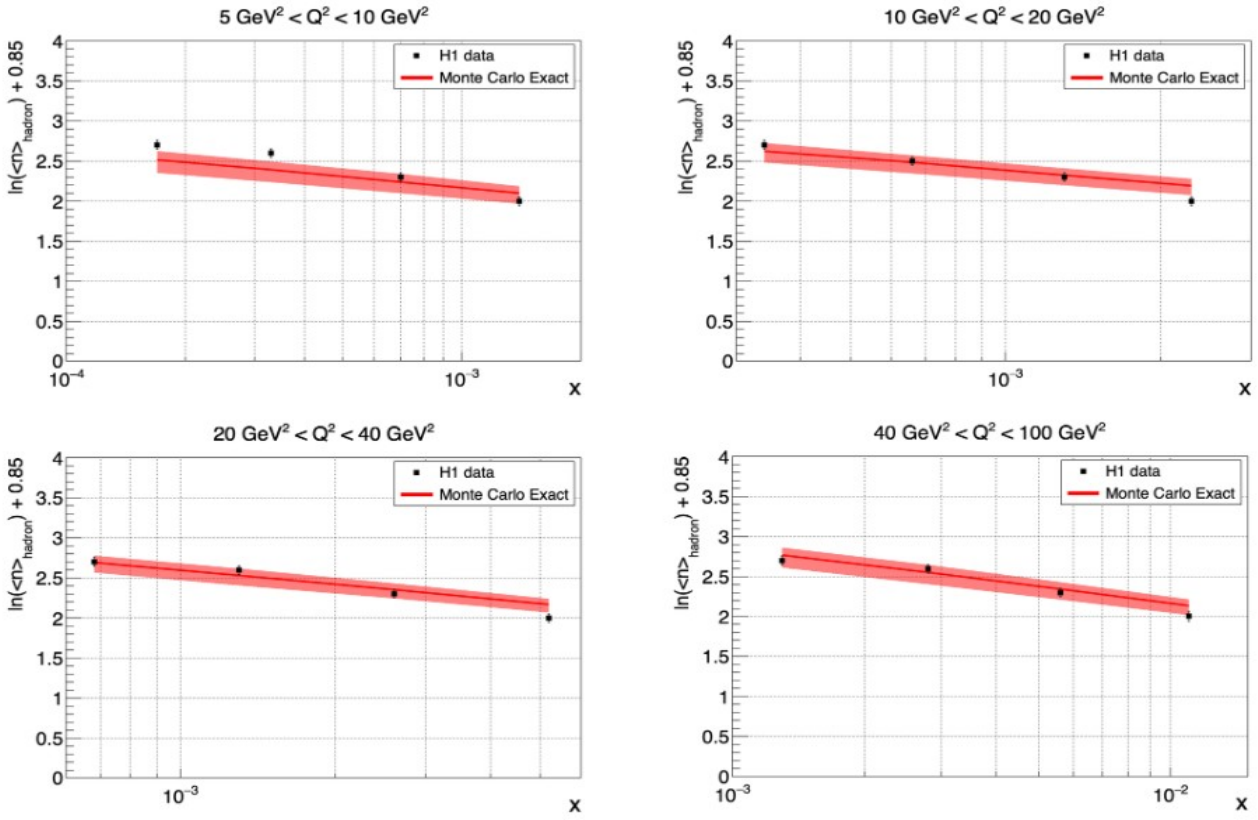
# Hadronic entropy description




Accounts for transition from partons to hadrons

$$P_N = \sum_{n=N}^{\infty} \binom{n}{N} q^N (1-q)^{n-N} P_n, \qquad q = \frac{2}{3}$$

# Maximal entanglement and Levin Lublinksy equation

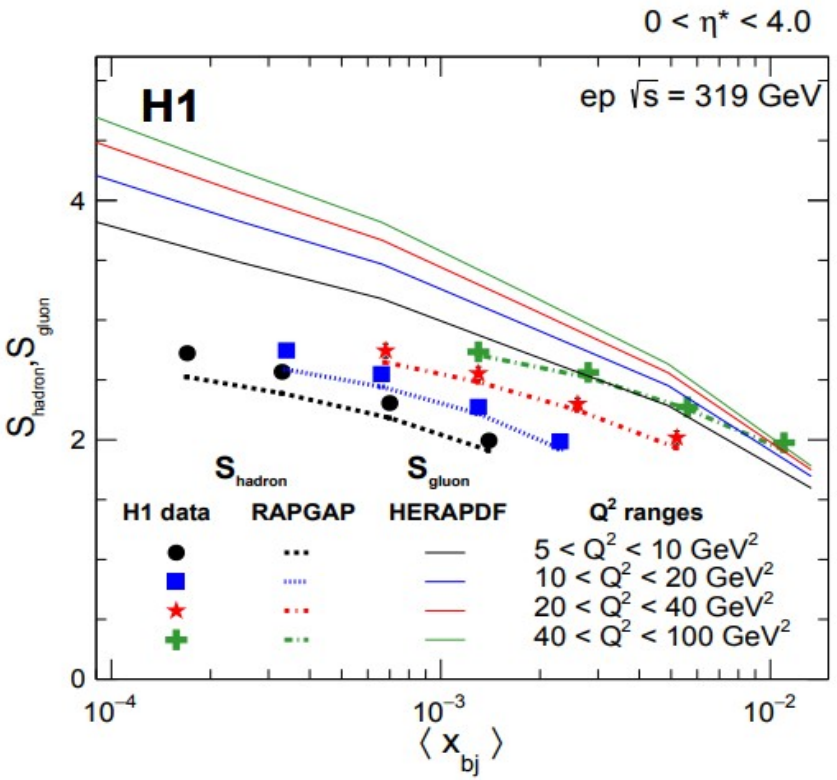


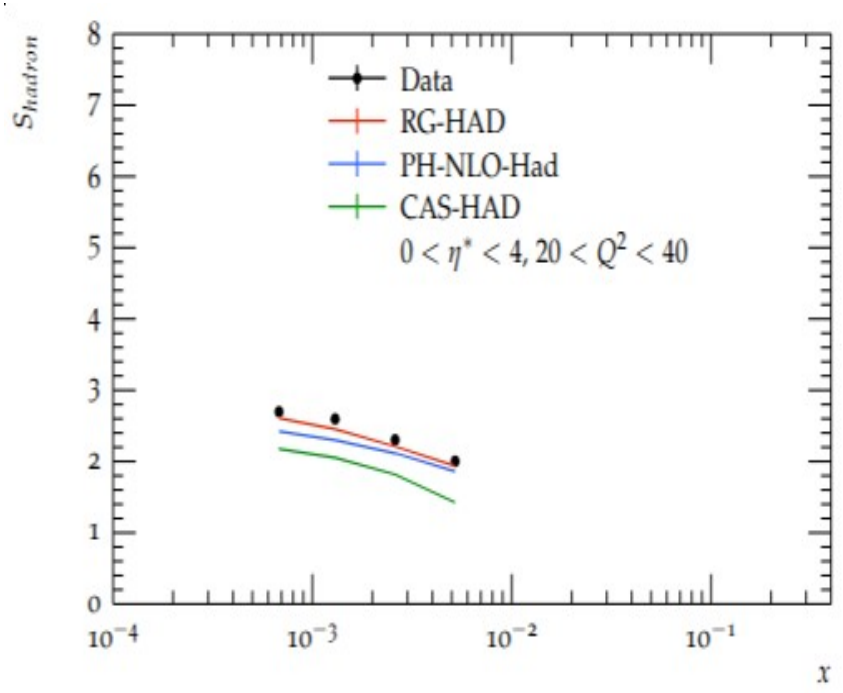

the maximal entanglement limit

$$S = \ln\left(\frac{2}{3}\langle n\rangle\right) + C$$

- a good description is possible using C ≃ 0.85
- interpretation: color and position of struck quark are maximally entangled with rest of the partons

# Entropy from H1 and CASCADE, RapGap, PowHeg




M. Hentschinski, H. Jung, K. Kutak ’25

Hadronisation accounted for

Z. Tu, D. E. Kharzeev, and T. Ullrich, Phys. Rev. Lett. 124 (2020) 062001 – statement that Monte Carlo event generators at parton level can not describe the entropy.

# Entropy from measurement and Monte Carlo at parton level

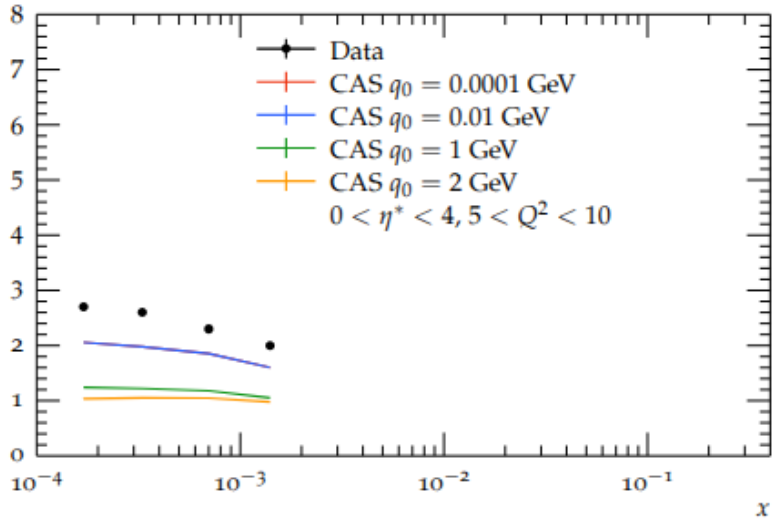
$$xf_a(x,\mu^2) = \Delta_a^S(\mu^2)\, xf_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{dq^2}{q^2} \frac{\Delta_a^S(\mu^2)}{\Delta_a^S(q^2)} \int_x^{z_M} dz\, \frac{\alpha_s}{2\pi} \hat{P}_{ab}(z) \frac{x}{z}\, f_b\left(\frac{x}{z}, q^2\right)$$

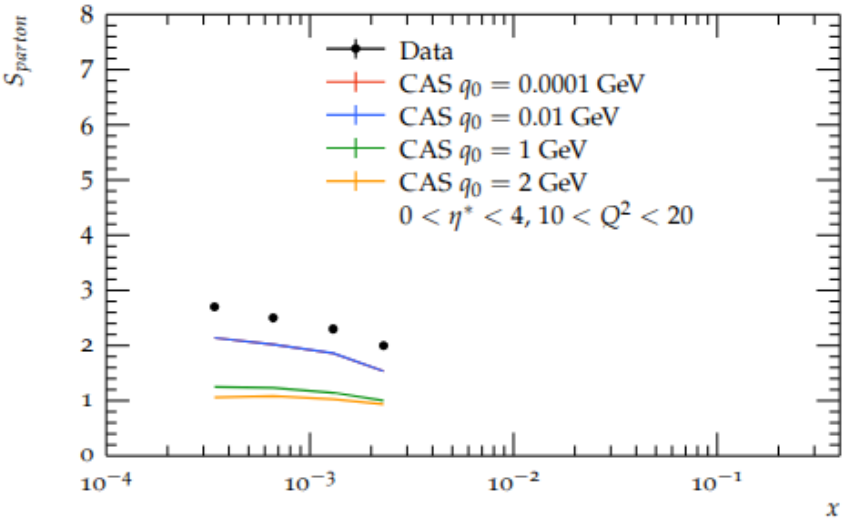
Hentschinski, Jung, Kutak ’25

$$z_M = 1 - \frac{q_0}{q}$$

$$q = \frac{q_t}{1-z}$$

minimal scale




Motivated by
Kharzeev, Tu Ullrich’19

Typically in Monte Carlo generators $q_0$=1 or 2 GeV. Now if pdf (obtained with no cut) are used in backward shower and the cut is 2 GeV one looses some multiplicity, do not obey the shower equation. This has consequences for entropy….

# Conclusions and outlook and comments

- We described results of published by H1 on entropy both in sliding and fixed rapidity window
- We applied various quantum measures to  the multiplicity densities of the dipole model. New handle to look for saturation
- The Monte Carlo simulation at parton level demonstrated to give bulk contribution to entropy. This further provides evidence that most of the entropy is generated on parton level i.e. entropy is associated with pdfs. Soft gluons are crucial element
- This result might be of relevance for Monte Carlo simulations in heavy ion physics and studies of quark gluon plasma and thermalisation. Also in studies of TMD in Monte Carlo settings.
- We described entropy data using complete Mueller cascade

# Backup

# Generalization of KL to moving rapidity window

Hentschinski, Kharzeev, Kutak, Tu '24

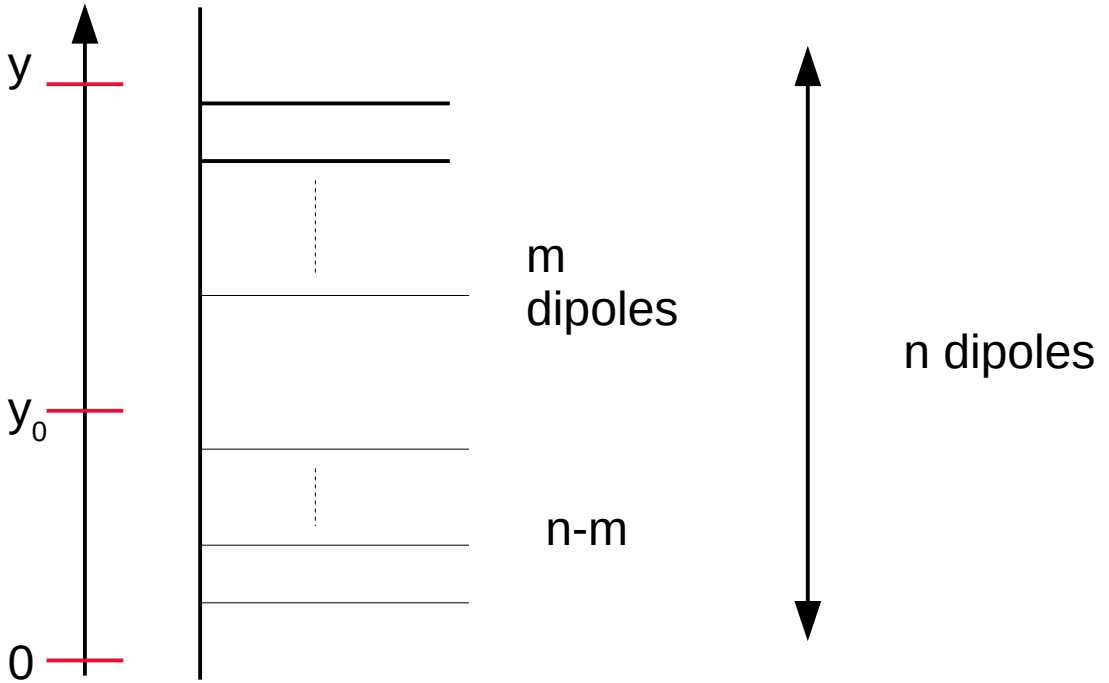
Generalize the KL formula to address measurement in rapidity window

$$p_n(y) = \sum_{m=0}^{n} p_{n-m}(y_0)\tilde{p}_m(y, y_0)$$

$$\tilde{p}_0(y, y_0) = e^{-\lambda(y-y_0)}$$

$$\tilde{p}_{n\geq 1}(y, y_0) = p_n(y) \cdot (1 - \tilde{p}_0(y, y_0))$$

$$\sum_{m=0} \tilde{p}_m(y, y_0) = 1$$




probability to have m dipoles in $[y_0,y]$

n-m dipoles in the range $[0,y_0]$

probability to have n dipoles in $[0,y]$

# Gluon production and entropy – another assumptions

Bialas; Janik; Fialkowski, Wit; Iancu, Blaizot, Peschanski,...

Kutak '11

$$\phi = \frac{\alpha_s C_F}{\pi}\frac{1}{k^2}$$

Many-body interactions

Medium generated mass of gluon.
Framework of Hard Thermal Loops.

Similarly in QED. Cut on photon`s kt
Is equivalent to introducing mass.

$$M_G(x) = Q_s(x)$$

energy dependent gluon's mass

$$M(x) = N_G(x) M_G(x)$$

mass of system of gluons

$$N_G(x) \equiv \frac{dN}{dy} = \frac{1}{S_\perp}\frac{d\sigma}{dy}$$

number of gluons

$$dE = TdS$$

$$dM = TdS$$

$$d\left[N_G(x)\, M_G(x)\right] = \frac{Q_s(x)}{2\pi} dS$$

$$S = \frac{6 C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

In presented approach mass is not fixed it is x dependent

$$[L_0, \phi(z)] = \left( z\frac{d}{dz} + h \right) \phi(z)$$ scaling

$$[L_{-1}, \phi(z)] = \frac{d}{dz}\phi(z)$$ translations

$$[L_1, \phi(z)] = \left( z^2\frac{d}{dz} + 2hz \right) \phi(z)$$ special conformal

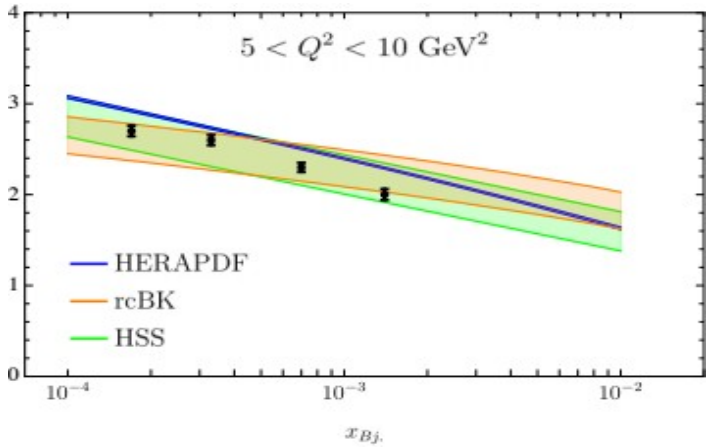
$$L_0 = \frac{1}{2}(a^\dagger a + b^\dagger b + 1), \qquad L_{-1} = a^\dagger b^\dagger, \qquad L_1 = ab$$

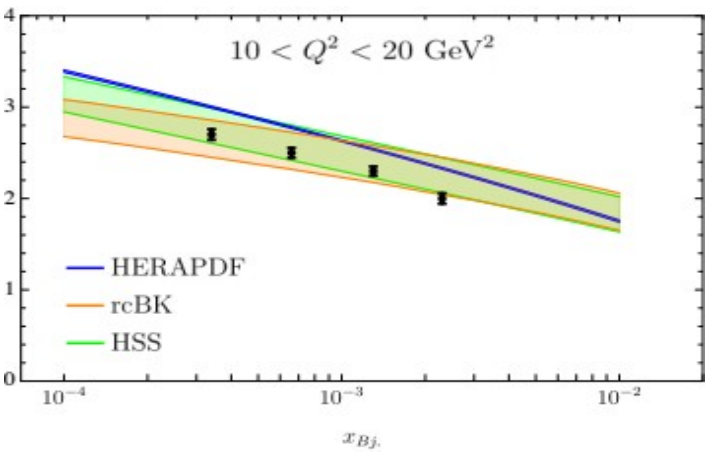
$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}, \qquad [L_1, L_{-1}] = 2L_0$$

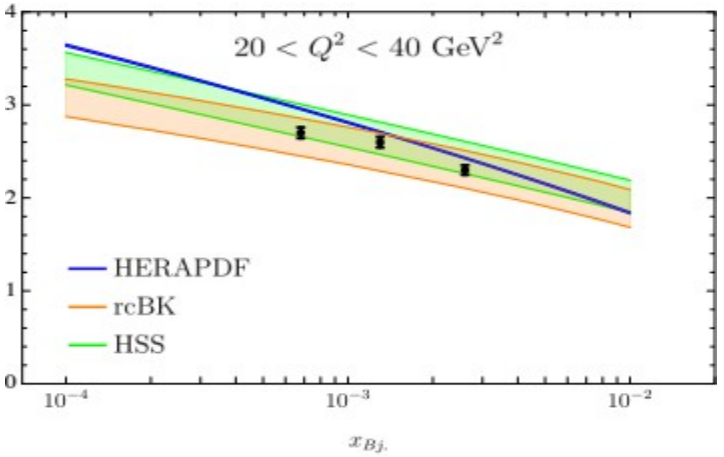
$$[a, a^\dagger] = [b, b^\dagger] = 1$$

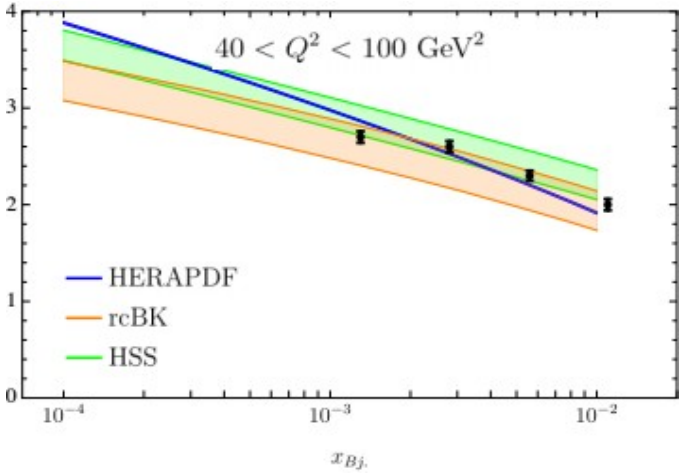
# Large scales - description

Martin Hentschinski, K. Kutak, Robert Straka ‘23



# Krylov subspace, complexity – motivation

Simple reference quantum state spreads and becomes complex in Hilbert space

Visvanath, Muller ‘63
Altman, Avdoshkin, Cao, Parker, Scaffidi ‘19
Balasubramanian, Caputa, Magan, Wu ‘22,...

$$i\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle \qquad |\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle$$

Krylov basis

$$|\Psi(t)\rangle = e^{-iHt} |\Psi_0\rangle = \sum_n \phi_n(t) |K_n\rangle \qquad p_n(t) = |\phi_n(t)|^2 \qquad \sum_n |\phi_n(t)|^2 = 1$$

Comes from studies of efficient diagonalization of matrices and computation of characteristic polynomial coefficients

The complexity has simple form in Krylov basis.
It can be to used to quantify chaotic behavior of quantum systems.

$$\mathcal{C}_K(t) = \langle n \rangle = \sum_n n\, p_n(t)$$

# Krylov basis and dipole evolution

$$|\Psi(t)\rangle = e^{-iHt}\,|\Psi_0\rangle = \sum_n \phi_n(t)\,|K_n\rangle$$

P. Caputa, K. Kutak, 2404.07657

$$i\partial_t \phi_n(t) = a_n \phi_n(t) + b_n \phi_{n-1}(t) + b_{n+1}\phi_{n+1}(t)$$

The equation above can be solved for

$$a_n = 0$$

$$b_n = \alpha n$$

boost operator

$$|\psi(t)\rangle = e^{-i\alpha(L_1+L_{-1})t}\,|0\rangle \otimes |0\rangle$$

vacuum of un-observed part of proton

vacuum of observed part of proton

One can calculate reduced density matrix

$$\rho(t) = \sum_{n=0}^{\infty} p_n(t)\,|n\rangle\langle n|$$

$$p_n(t) = |\phi_n(t)|^2$$

$$\mathcal{C}_K(t) = \sum_{n=0}^{\infty} n|\phi_n(t)|^2$$

$$\mathcal{C}_K(t) = \sum_{n=0}^{\infty} n|\psi_n(t)|^2 = \sinh^2(\alpha t)$$

After expressing it in terms rapidity and probabilities rapidity variable one gets

$$p_n(Y) = \frac{\Gamma(2h+n)}{n!\Gamma(2h)}(e^{-\alpha Y})^{2h}\left(1-e^{-\alpha Y}\right)^n$$

$$\mathcal{C}_K(Y) = e^{\alpha Y} - 1 \qquad Y = \ln(1/x)$$

$$\partial_Y p_n(Y) = \alpha n\, p_{n-1}(Y) - \alpha(n+1)\, p_n(Y)$$

$$\mathcal{C}_K = x g(x)$$

# Krylov subspace – construction

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle = \sum_{n=0}^{\infty} \frac{(-it)^n}{n!}|\Psi_n\rangle \qquad |\Psi_n\rangle \equiv \{|\Psi_0\rangle, H|\Psi_0\rangle, ..., H^n|\Psi_0\rangle, ...\}$$

n consequtive application of Hamiltonian

$$|K_0\rangle = |\psi(0)\rangle = |\psi_0\rangle$$

Gram-Schmidt orthogonalization procedure. Construct with $K_2$ by subtracting the previous two vectors, $K_3$ by subtracting the previous 3 vectors, and so forth

$$|z_1\rangle = \hat{H}|K_0\rangle - a_0|K_0\rangle \qquad |K_1\rangle = \frac{|z_1\rangle}{\langle z_1|z_1\rangle}$$

$$|z_{n+1}\rangle = (\hat{H} - a_n)|K_n\rangle - b_n|K_{n-1}\rangle$$

Strength of the Lanczos algorithm. n + 1 is determined by n and n – 1. Low memory requirements Visvanath, Muller '63

$$|K_n\rangle = b_n^{-1}|z_n\rangle \qquad b_n = \langle z_n|z_n\rangle^{\frac{1}{2}} \qquad a_n = \langle K_n|\hat{H}|K_n\rangle$$

Project a high-dimensional problem onto a lower-dimensional Krylov subspace

$$H|K_n\rangle = a_n|K_n\rangle + b_{n+1}|K_{n+1}\rangle + b_n|K_{n-1}\rangle$$

$$\langle K_n|K_m\rangle = \delta_{nm}$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle = \sum_n \phi_n(t)|K_n\rangle$$

$$H_{nm} := \langle K_n|\hat{H}|K_m\rangle = \begin{pmatrix} a_1 & b_1 & 0 & 0 & \cdots \\ b_1 & a_2 & b_2 & 0 & \cdots \\ 0 & b_2 & a_3 & b_3 & \cdots \\ 0 & 0 & b_3 & a_4 & \ddots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix}$$

probability amplitudes for each vector

$$i\partial_t\phi_n(t) = a_n\phi_n(t) + b_n\phi_{n-1}(t) + b_{n+1}\phi_{n+1}(t)$$

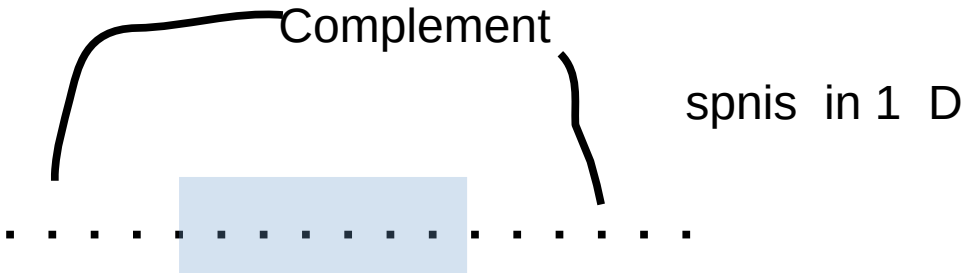
# Comments

CFT result for EE

central charge

$$S = \frac{c}{3} \ln \frac{L}{\epsilon}$$

UV cutoff




Entanglement entropy obtained from CFT
calculations as well as from gravity using Ryu-Takayanagi formula

See also
Callan, Wilczek ‘94
Calabrese, Cardy ‘04

and lectures by Headrick

Relation to Kharzeev-Levin formula

$$L = (mx)^{-1} \qquad \epsilon \equiv 1/m$$

Length of tube probed in DIS

Proton’s Compton wave length

$$S = \ln \left( \frac{1}{x} \right)^{1/3}$$

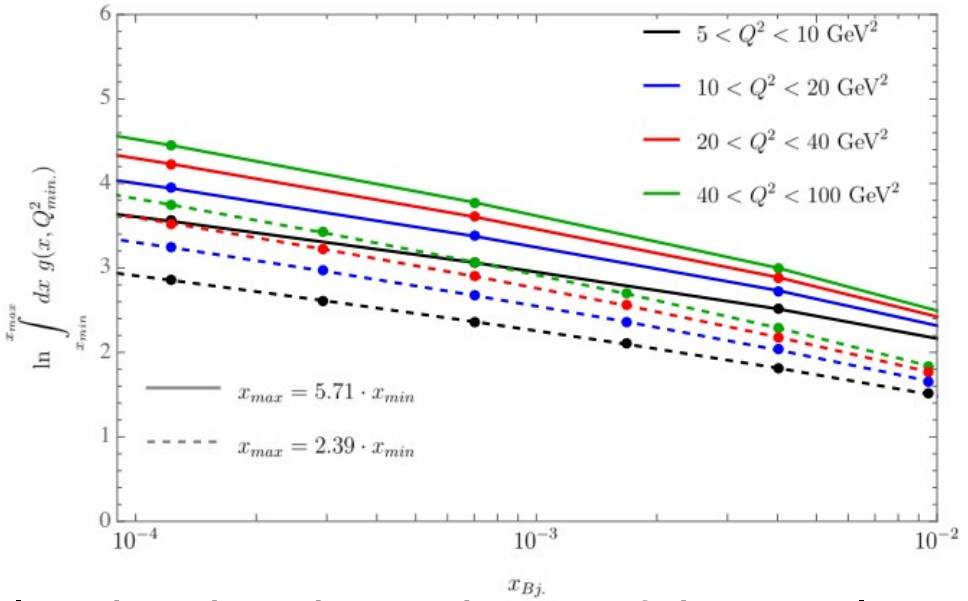
$$S(x) = \ln(xg(x))$$

Studied also in the context of 2 D QCD

Liu, Nowak, Zahed, ‘22

Casini, Huerta, Hosco ‘05

# Bining and KL formula




plot showing dependence of the result on the sie of bins if binig is naive

$$n_g(Q^2) = \int_0^1 dx g(x, Q^2)$$

Formal definition of number of gluons

$$n_g(\bar{x}) = \int_{x_{\min}}^{x_{\max}} dx g(x, Q^2) \qquad \bar{x} \in [x_{\min}, x_{\max}]$$

$$\bar{x} = \frac{\int_{x_{\min}}^{x_{\max}} dx\, xg(x, Q^2)}{\int_{x_{\min}}^{x_{\max}} dx g(x, Q^2)}$$

average *x*

Data binning takes place in rapidity

$$\bar{n}_g(\bar{x}) = \frac{1}{y_{max} - y_{min}} \int_{y_{min}}^{y_{max}} dy \frac{dn_g}{dy} = \frac{n_g(y_{max}) - n_g(y_{min})}{y_{max} - y_{min}}$$

$$y_{max,min} = \ln 1/x_{min,max}$$

for small bins

$$\bar{n}_g(x, Q^2) = \frac{dn_g}{d\ln(1/x)} = xg(x, Q^2)$$

$$\langle \bar{n}(x, Q^2) \rangle_{Q^2} = \frac{1}{Q^2_{\max} - Q^2_{\min}} \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \left[ xg(x, Q^2) + x\Sigma(x, Q^2) \right]$$

$$\langle S(x, Q^2) \rangle_{Q^2} = \ln \langle \bar{n}(x, Q^2) \rangle_{Q^2}$$

# Gluon production and entropy – another assumptions

Bialas; Janik; Fialkowski, Wit; Iancu, Blaizot, Peschanski,...

Kutak '11

$$M_G(x) = Q_s(x)$$

energy dependent gluon's mass

$$M(x) = N_G(x) M_G(x)$$

mass of system of gluons

$$\phi = \frac{\alpha_s C_F}{\pi} \frac{1}{k^2}$$

$$N_G(x) \equiv \frac{dN}{dy} = \frac{1}{S_\perp} \frac{d\sigma}{dy}$$

number of gluons

Many-body interactions

$$dE = TdS$$

Medium generated mass of gluon.
Framework of Hard Thermal Loops.

$$dM = TdS$$

$$d\left[N_G(x)\, M_G(x)\right] = \frac{Q_s(x)}{2\pi} dS$$

Similarly in QED. Cut on photon`s kt
Is equivalent to introducing mass.

$$S = \frac{6 C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

In presented approach
mass is not \fixed it is x dependent

$$S = 3\pi \left[N_G(x) + N_{G\,0}\right]$$