Sudakov dynamics in SIDIS at small x

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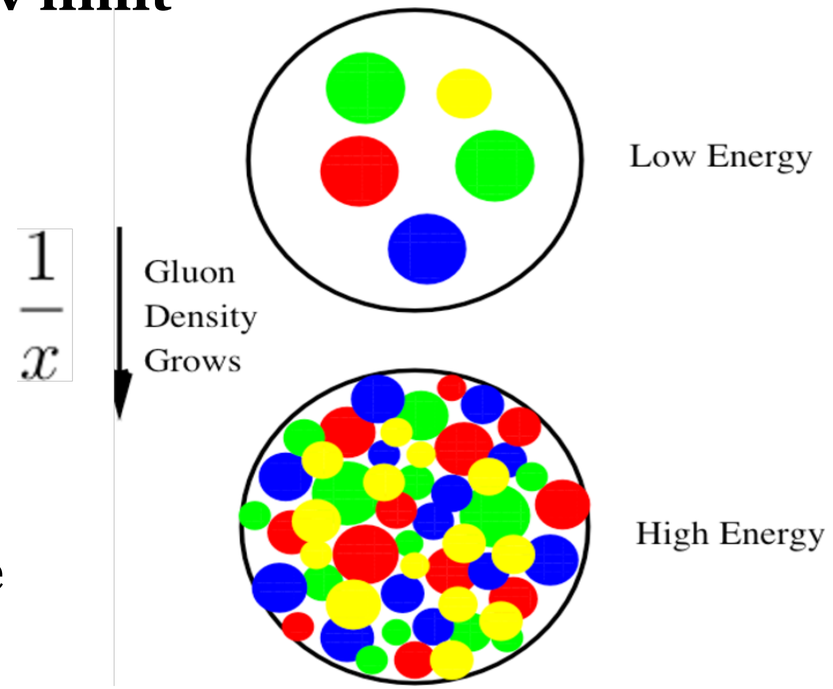
Santiago de Compostela, Spain, October 19-23, 2026

Based on:
arXiv:2406.08277, arXiv:2505.04557
and in progress: Altinoluk, JJM, Marquet, Shi

Resolving the nucleus/hadron: Regge-Gribov limit

Gribov-Levin-Ryskin

Q^2 fixed and $\sqrt{S} \to \infty \quad (x \equiv \frac{Q^2}{S} \to 0)$



gluons are radiated into a fixed resolved area

number of gluons increases due to increased longitudinal phase space

hadron/nucleus becomes a dense state of gluons (CGC) $\frac{\alpha_s}{Q^2}\frac{xG(x,Q^2)}{S_\perp} \sim 1 \qquad Q_s^2(x,A,b_\perp) \sim A^{1/3}\left(\frac{1}{x}\right)^{0.3}$

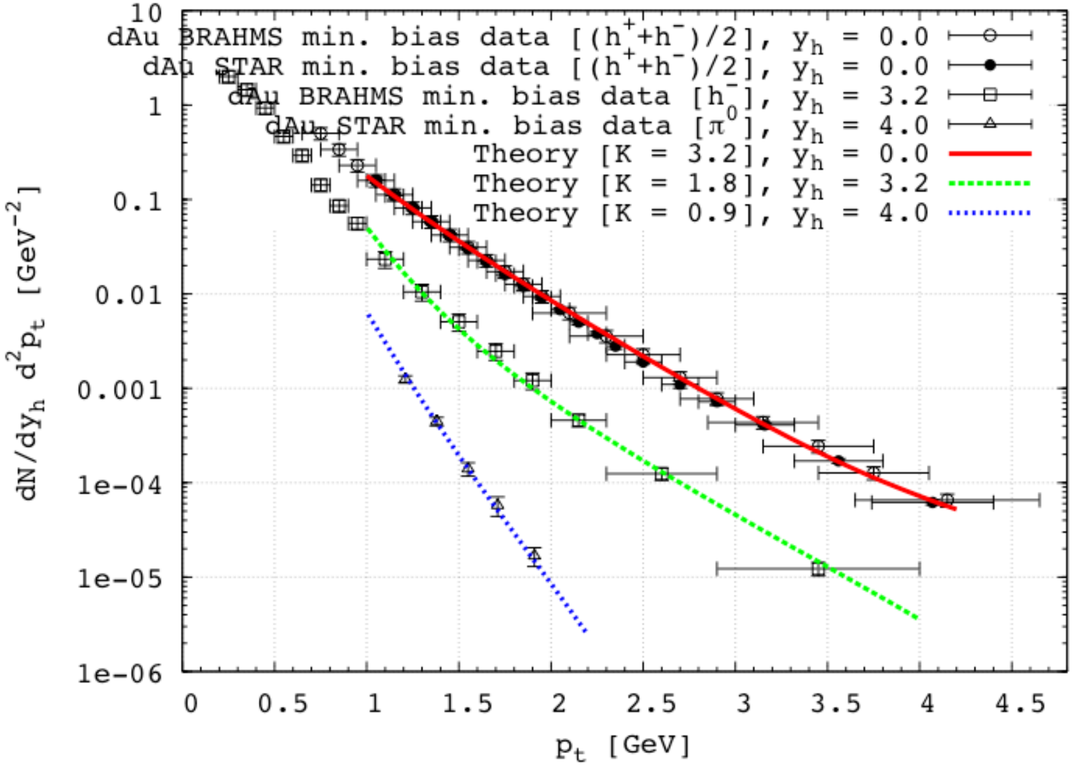
strong color fields

possible universal properties of QCD observables ?

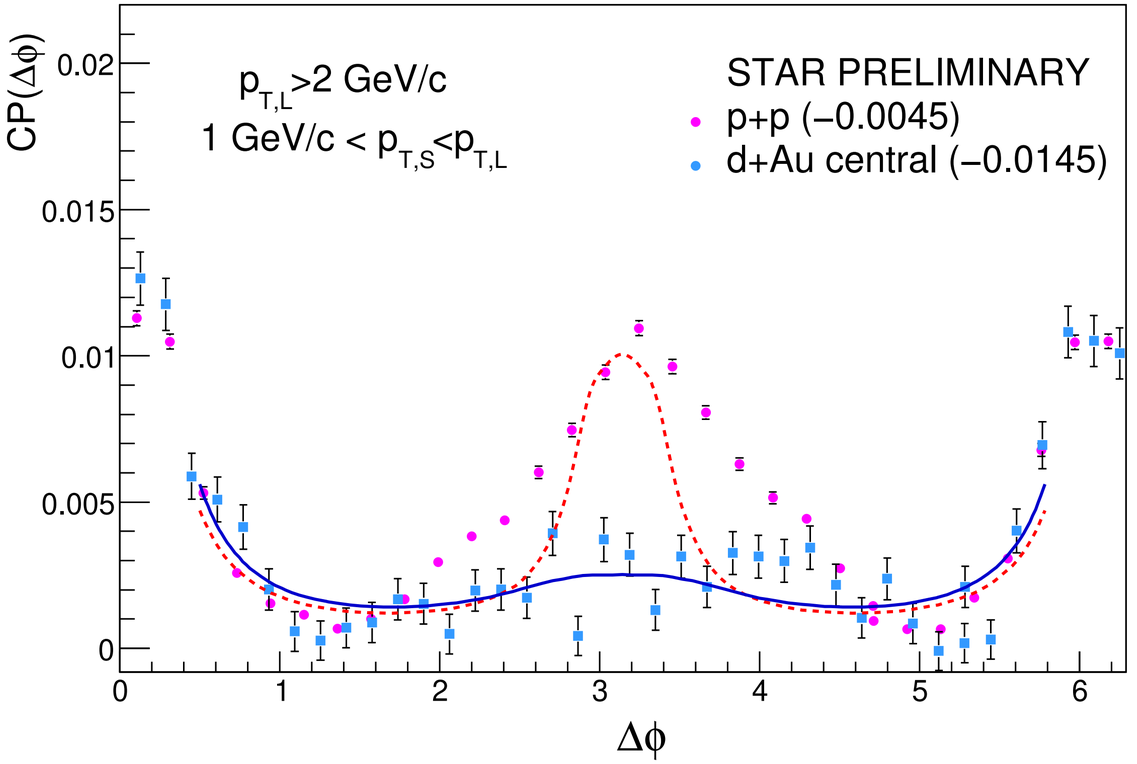
EIC

Some early “hints” of CGC at RHIC

Single and double inclusive hadron production

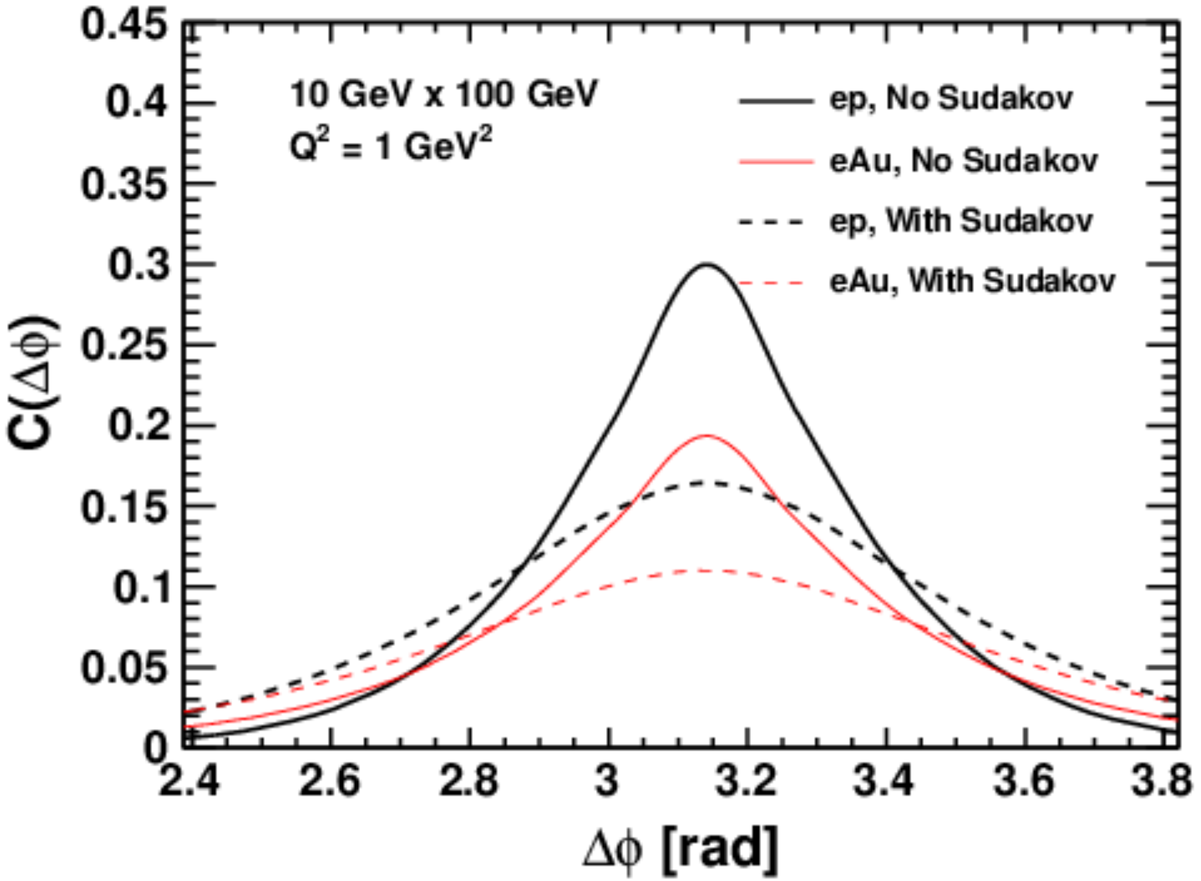


Dumitru, Hayashigaki, JJM, NPA770 (2006) 57



Albacete, Marquet, PRL105 (2010) 162301

Dihadrons at EIC



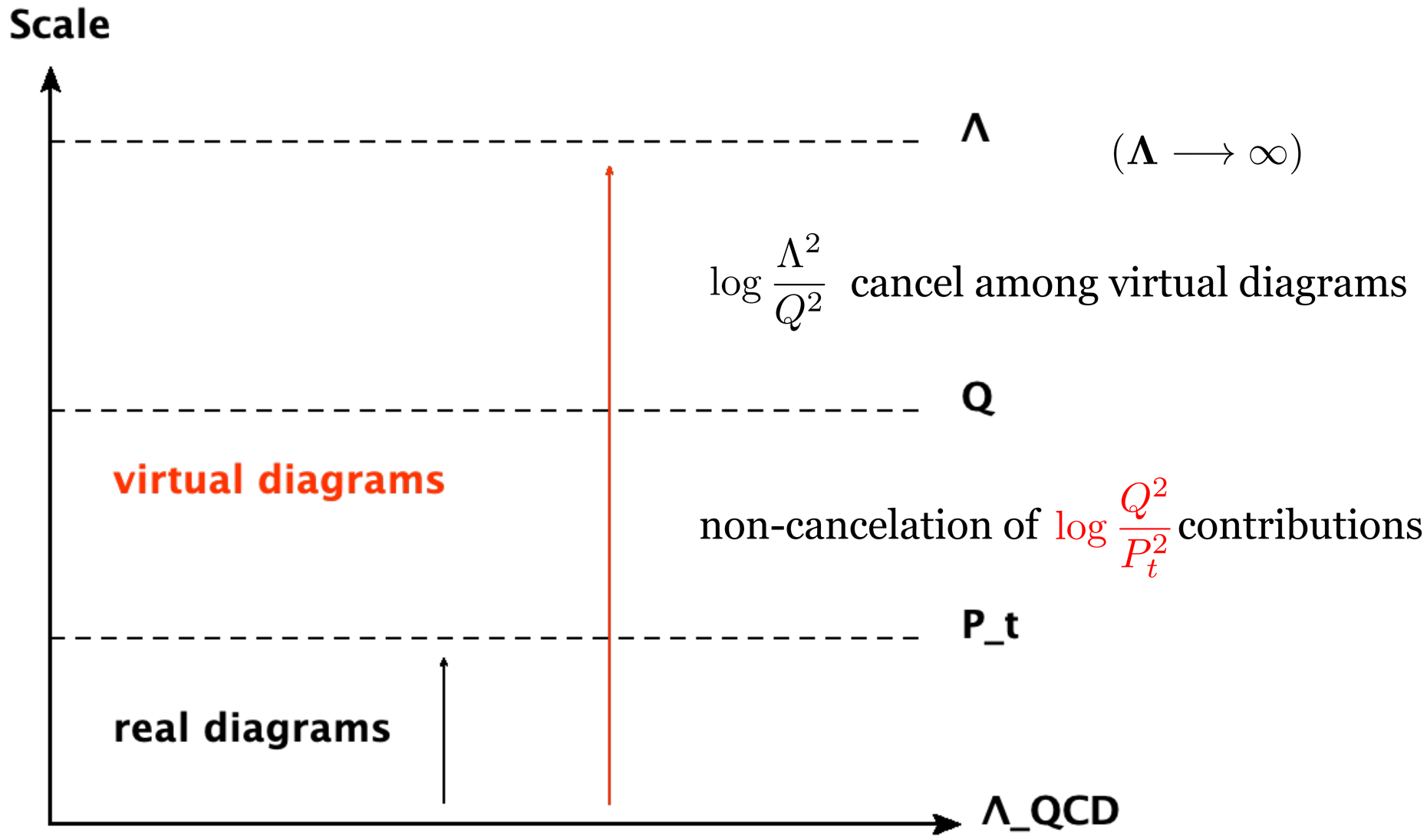
Zheng, Aschenauer,Lee,Xiao, arXiv:1403.2413

Sudakov effect can not be avoided in the back to back kinematics

SIDIS:

larger kinematic phase space than dihadrons

Sudakov effect can be avoided (in a limited kinematic range)


Scale
Λ
$(\Lambda \longrightarrow \infty)$
$\log \frac{\Lambda^2}{Q^2}$ cancel among virtual diagrams
Q
virtual diagrams
non-cancelation of $\log \frac{Q^2}{P_t^2}$ contributions
P_t
real diagrams
Λ_QCD

SIDIS at small x: toward precision

Bergabo, JJM, JHEP 01 (2023) 095, and arXiv:2401.06259

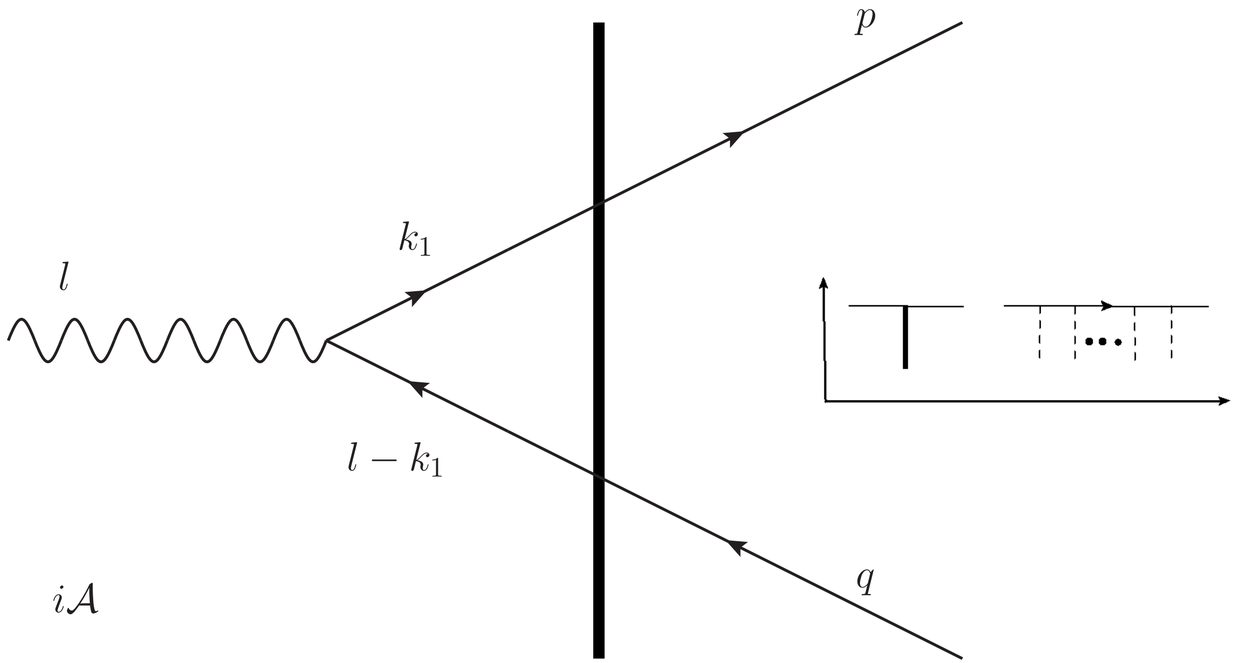
Caucal, Ferrand, Salazar, arXiv:2401.01934,.....

Altinoluk, JJM, Marquet, Shi, arXiv 2505.04557

Forward rapidity

LO:

integrate over final state antiquark



$$\frac{\mathrm{d}\sigma^{\gamma^* p/A\to q(\mathbf{p},y_1)X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2Q^2N_c}{(2\pi)^5}\int \mathrm{d}z_2\,\delta(1-z_1-z_2)\,(z_1^2\,z_2)\int \mathrm{d}^6\mathbf{x}\,[S_{11'}-S_{12}-S_{1'2}+1]\,e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{4z_1z_2K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2}|Q_1)+(z_1^2+z_2^2)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)\right\}$$

with $Q_1^2 \equiv z_1(1-z_1)Q^2$

high Q^2 limit: toward Sudakov double logs

Altinoluk, JJM, Marquet
arXiv:2406.08277

problem at end points: use a trick introduced by Marquet, Xiao, Yuan, PLB682 (2009) 207

$$Q_1^{2n}\, K_{(0,1)}(|\mathbf{x}_{12}|Q_1)\, K_{(0,1)}(|\mathbf{x}_{1'2}|Q_1) = \int_0^{Q^2/4} d\bar{Q}^2\, (\bar{Q}^2)^n K_{(0,1)}(|\mathbf{x}_{12}|\bar{Q}) K_{(0,1)}(|\mathbf{x}_{1'2}|\bar{Q})\, \delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right]$$

and

$$\delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right] = \frac{\delta(z_1 - z_+)}{Q^2|1-z_+|} + \frac{\delta(z_1 - z_-)}{Q^2|1-z_-|} \quad \text{with} \quad z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\bar{Q}^2/Q^2}\right)$$

then

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\, \mathrm{d}y_h}\right|_{T,LP} = \frac{1}{Q^2}\frac{e^2 N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\,[S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\, K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\, \mathrm{d}y_h}\right|_{L,LP} = \frac{1}{Q^2}\frac{e^2 N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\,[S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{4}{Q^2}\int_0^\infty d\bar{Q}^2\bar{Q}^4 K_0(|\mathbf{x}_{12}|\bar{Q})K_0(|\mathbf{x}_{1'2}|\bar{Q})$$

contribution of longitudinal photons is suppressed

Quark TMD (LO)

Marquet, Xiao, Yuan 0906.1454

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{T,LP} = \frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}xq(x,\mathbf{p}_h/z_h)$$

$$xq(x,\mathbf{p}) = \frac{2N_c}{(2\pi)^6}\int \mathrm{d}^6\mathbf{x}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\left[S_{11'}-S_{12}-S_{1'2}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

$$xq(x,\mathbf{p}) = \int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\; x\tilde{q}(x,\mathbf{x}_{11'})$$

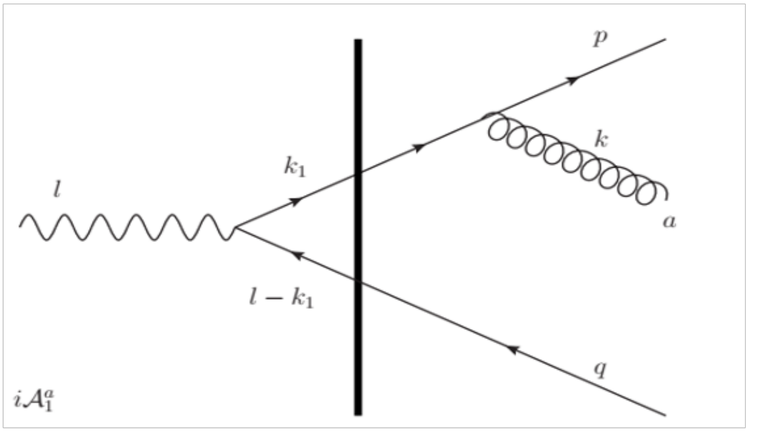
can be rewritten as

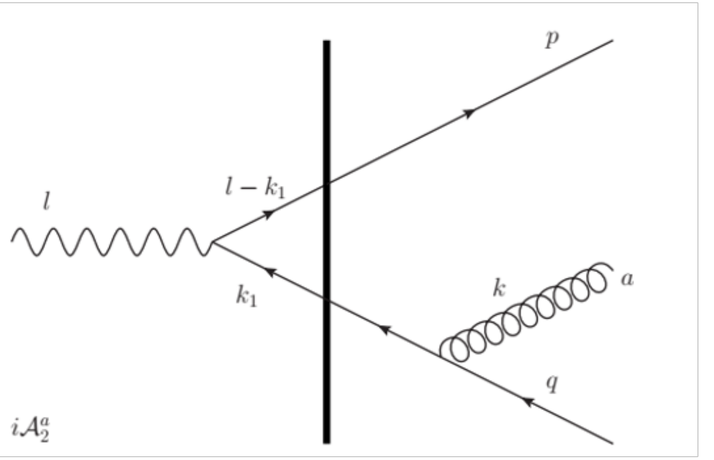
$$xq(x,\mathbf{p}) = \int \mathrm{d}^2\mathbf{k}_g\left(\underbrace{\frac{N_c}{8\pi^4}\frac{\mathbf{k}_g^2}{\alpha_s}\int \mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_{1'}\; S_{11'}\,e^{-i\mathbf{k}_g\cdot\mathbf{x}_{11'}}}\right)\left(\underbrace{\frac{1}{4\pi^2}\frac{\alpha_s}{\mathbf{k}_g^2}\int_0^\infty \mathrm{d}\bar{Q}^2\left|\frac{\mathbf{k}_g-\mathbf{p}}{\bar{Q}^2+(\mathbf{k}_g-\mathbf{p})^2}+\frac{\mathbf{p}}{\bar{Q}^2+\mathbf{p}^2}\right|^2}\right)$$

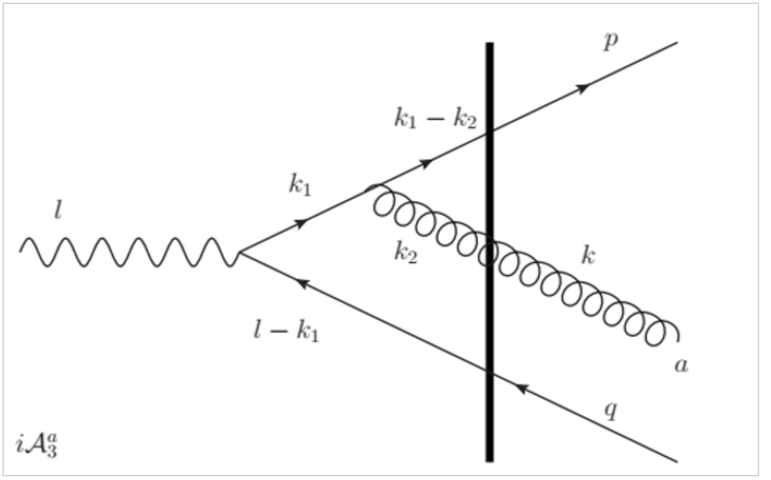
dipole gluon TMD

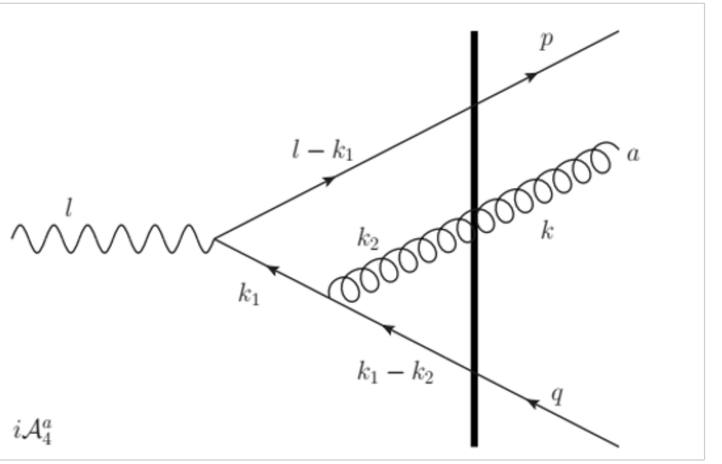
momentum dependent splitting function

One loop corrections - real diagrams





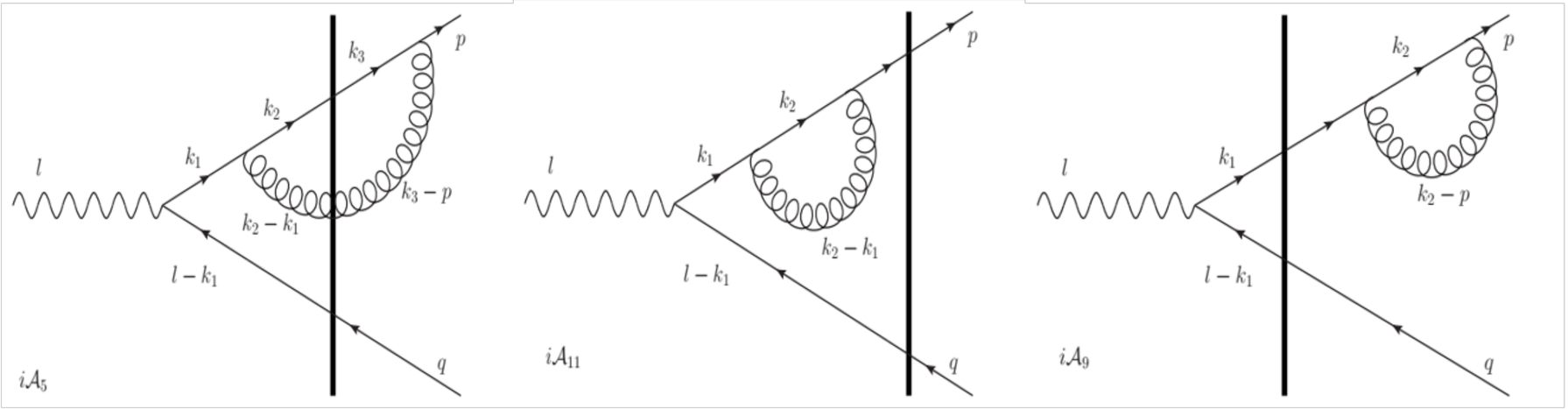


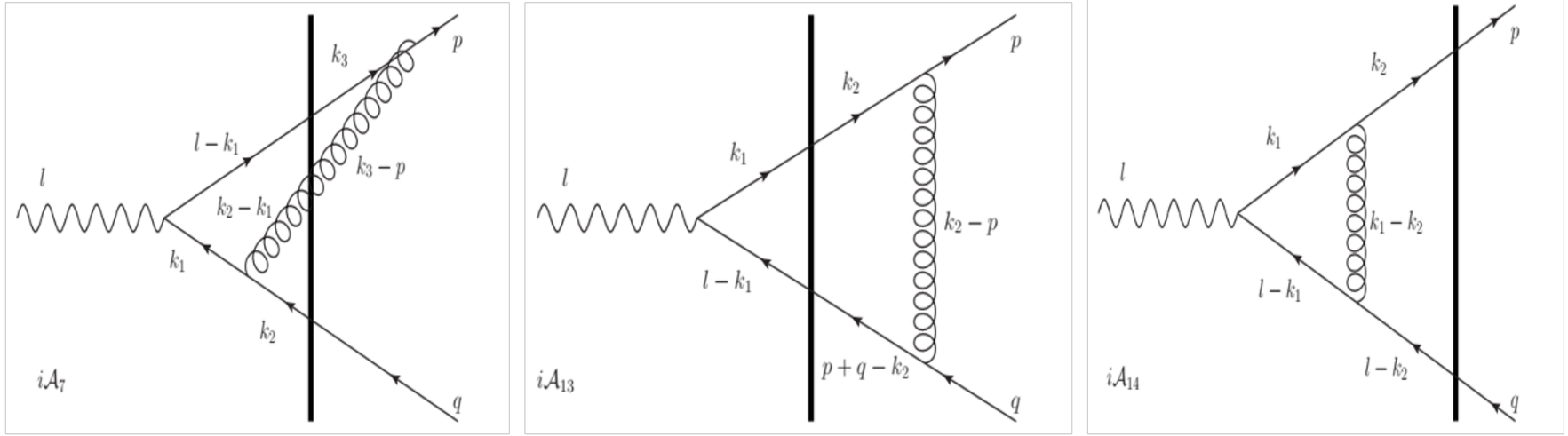


3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

Iancu, Mulian, JHEP 07 (2023) 121

One loop corrections – virtual diagrams





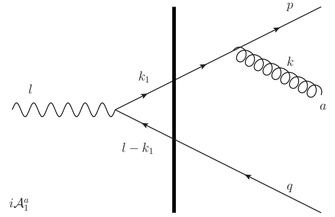
F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

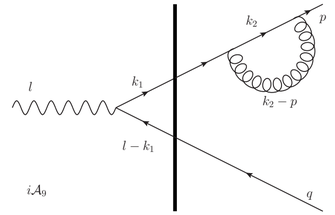
NLO corrections to SIDIS: some of the contributions are

Altinoluk, Bergabo, JJM, Marquet, Shi, arXiv:2505.04557

$$\frac{\mathrm{d}\sigma^T_{1\times 1}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2g^2Q^2}{(2\pi)^8}\int_0^{1-z_1}\frac{\mathrm{d}z}{z}\frac{(1-z-z_1)(z+z_1)}{z_1}\left[z_1^2(1-z-z_1)^2+\left(z_1^2+(1-z-z_1)^2\right)(z+z_1)^2+(z+z_1)^4\right]$$
$$\int \mathrm{d}^8\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_{1z})K_1(|\mathbf{x}_{1'2}|Q_{1z})N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,\Delta^{(3)}_{11'}\,e^{i\frac{z_1+z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}$$

$$\frac{\mathrm{d}\sigma^T_{9}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = -\frac{e^2g^2Q^2}{2(2\pi)^6}\int_0^{z_1}\frac{\mathrm{d}z}{z}(1-z_1)(z_1^2+(1-z_1)^2)\left[z_1^2+(z_1-z)^2\right]\int \mathrm{d}^6\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int\frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2}\frac{1}{\mathbf{x}_{31}^2}$$





with $\Delta^{(3)}_{ij}=\dfrac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}_{3i}^2\mathbf{x}_{3j}^2}$ and $Q_1^2\equiv z_1(1-z_1)Q^2$, $Q_{1z}^2\equiv(1-z-z_1)(z+z_1)Q^2$

SIDIS at small x: including Sudakov double logs

Altinoluk, JJM, Marquet, arXiv:2406.08277

taking the high $\mathbf{Q^2}$: introduce a delta function,..., cross section becomes

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}_{LO+NLO}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{1}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$

$$\times\Bigg\{D_{h/q}(z_h)$$

$$+\left[\frac{\alpha_s C_F}{2\pi^2}\left(\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{1+(1-z)^2}{1-z}D_{h/q}(\frac{z_h}{1-z})-\int_0^1\frac{\mathrm{d}z}{z}[1+(1-z)^2]D_{h/q}(z_h)\right)\right]\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}$$

$$+\left[\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}\left(\int\mathrm{d}^2\mathbf{x}_3\,\Delta^{(3)}_{1'1}-\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\right]D_{h/q}(\frac{z_h}{1-z})\Bigg\}$$

the first two terms are LO (DGLAP) evolution of quark/hadron fragmentation function

in the last term introduce a factorization scale for momentum fraction z $\displaystyle\int_0^{1-z_h}\mathrm{d}z=\int_0^{z_f}\mathrm{d}z+\int_{z_f}^{1-z_h}\mathrm{d}z$

SIDIS at small x: including Sudakov double logs

to get Sudakov logs one must introduce a kinematic constraint

needed for self-consistency of evolution equations (avoid negative cross sections)

at small x we require $k^+ < z_f\, l^+$, this is sufficient at LL accuracy

at NLO accuracy we also need to impose a condition on lifetimes of fluctuations

introduce a cutoff on - component of momenta $k^- > \tilde{z}_f\, l^-$ with $z_f\, \tilde{z}_f \sim 1$

Sudakov single logs may be sensitive to the details of this cutoff

Altinoluk, JJM, Marquet, Yu, in progress

SIDIS at small x: including Sudakov double logs

longitudinal factorization $\int_0^{1-z_h} dz = \int_{z_f}^{1-z_h} dz + \int_0^{z_f} dz\,[1-\theta(kin.const.)] + \int_0^{z_f} dz\theta(kin.const.)$

where we add and subtract the kinematic constraint $\boldsymbol{\Theta}(\text{kin.const.}) = \boldsymbol{\Theta}\left(\mathbf{z_f}\dfrac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)$

$$\int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\left[\mathbf{1} - \boldsymbol{\Theta}\left(\mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)\right] = \int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\boldsymbol{\Theta}\left(\mathbf{z} - \mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}}\right) = \boldsymbol{\Theta}\left(\mathbf{Q^2} - \mathbf{k^2}\right)\ln\left(\frac{\mathbf{Q^2}}{\mathbf{k^2}}\right)$$

and dipoles satisfy kinematically-constrained BK evolution

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}(z_f)\otimes D_{h/q}(z_h,Q^2) + d\sigma_{NLO-rap-finite} +$$

$$\frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$

$$\times\left\{\frac{\alpha_s C_F}{\pi^2}\int^{Q^2}\frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left(e^{i\mathbf{k}\cdot\mathbf{x}_{1'1}}-1\right)\ln\left(\frac{Q^2}{\mathbf{k}^2}\right)\right\}$$

Sudakov double logs in SIDIS

using

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left[e^{-i\mathbf{k}\cdot\mathbf{x}_{11'}}-1\right]\ln\left(\frac{Q^2}{\mathbf{k}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau)-1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right)$$

$$= -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) + O\left(\frac{1}{\sqrt{Q|\mathbf{x}_{11'}|}}\right)$$

we get

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}_{LO+NLO}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{NLO-rap-finite}$$

$$+\frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h,Q^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\,e^{-S_{sud}(\mathbf{x}_{11'})}$$

with

$$S_{sud}(\mathbf{x}_{11'}) \equiv \frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right)$$

SIDIS at small x : can one avoid Sudakov logs?

EIC will have a reasonably large window in $\mathbf{Q^2}$ where $\mathbf{Q^2} >> \mathbf{p_{t,h}^2}$ so that $\alpha_s \log\left(\frac{\mathbf{Q^2}}{\mathbf{p_{t,h}^2}}\right) \sim \mathbf{1}$

CGC contains <u>*no*</u> Sudakov double logs in SIDIS <u>*unlike*</u> dihadron production (wrong sign)

avoid large Sudakov logs: $\mathbf{p_{t,h}^2} \sim \mathbf{Q^2}$ so that $\log\left(\frac{\mathbf{Q^2}}{\mathbf{p_{t,h}^2}}\right) \simeq \mathbf{0}$

Summary

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,…

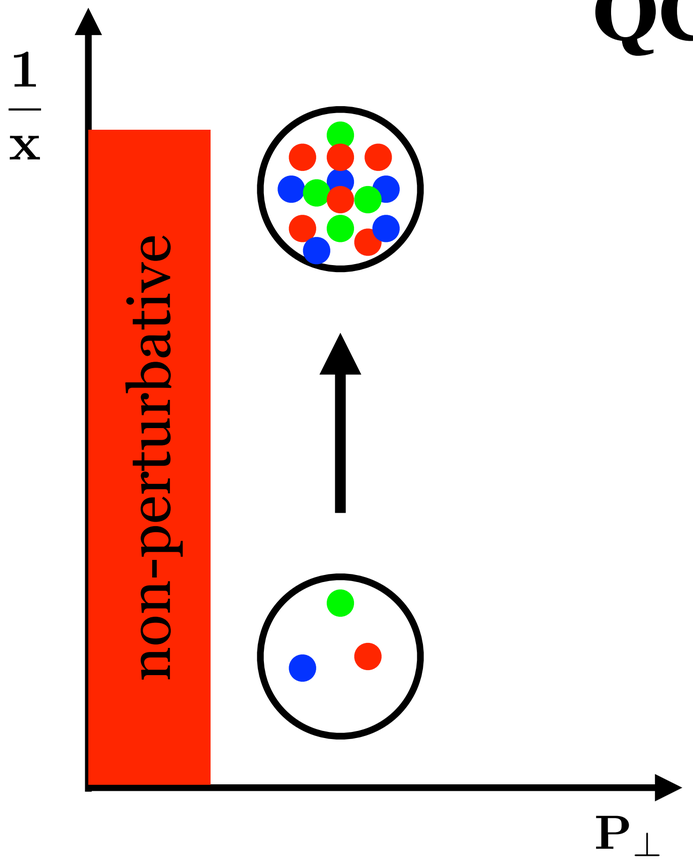
to be probed precisely at EIC

toward precision: NLO, beyond-eikonal corrections, …

deep connections to TMD, Sudakov physics, …

missing a unified approach to large and small x!

QCD at small x: gluon saturation



high gluon density: multiple eikonal scatterings

high energy: evolution in x via BK/JIMWLK

p_t broadening

suppression of single inclusive spectra/away side peak

A systematic formalism for multi-particle production in QCD

nuclear shadowing/modification factor, azimuthal correlations

long range rapidity correlations

connections to TMD, spin physics, diffraction,....

$$Q_s^2(x, A, b_\perp) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$Q_s^2 \sim 1\,\mathrm{GeV}^2$ at

$x \sim 3 \times 10^{-4}$