A Global Analysis of the Thrust Distribution in e^+e^- Annihilation: Determination of α_S and Non-Perturbative QCD Parameters

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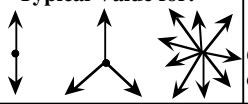
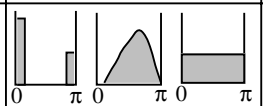




REF26 - 19/10/26

Event Shape Distributions

- Event Shape Distributions continuously encode a geometrical property of the event in $e^+e^- \to hadrons$.
- Powerful tool to determine α_S from experimental data.
- Must be free from InfraRed (IR) singularities to be computable in pQCD⇒ IR safety.

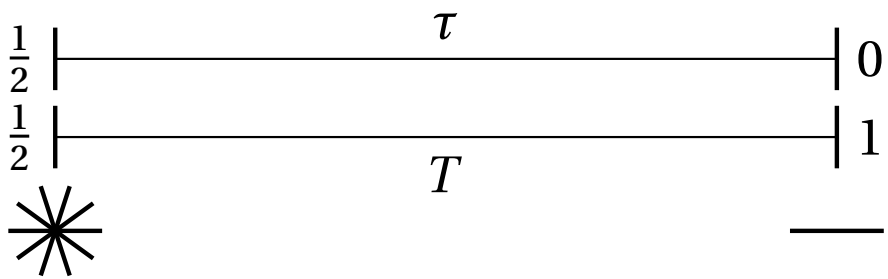
Name of Observable	Definition	Typical Value for: 			QCD calculation
Thrust	$T = \max_{\vec{n}} \left(\frac{\Sigma_i \lvert \vec{p}_i \vec{n} \rvert}{\Sigma_i \lvert \vec{p}_i \rvert} \right)$	1	≥2/3	≥1/2	(resummed) $O(\alpha_s^2)$
Thrust major	Like T, however T_{maj} and $\vec{n}_{maj}$ in plane $\perp \vec{n}_T$	0	≤1/3	≤1/√2	$O(\alpha_s^2)$
Thrust minor	Like T, however T_{min} and $\vec{n}_{min}$ in direction $\perp$ to $\vec{n}_T$ and $\vec{n}_{maj}$	0	0	≤1/2	$O(\alpha_s^2)$
Oblateness	$O = T_{maj} - T_{min}$	0	≤1/3	0	$O(\alpha_s^2)$
Sphericity	$S = 1.5\,(Q_1 + Q_2)$; $Q_1 \le ... \le Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\Sigma_i p_i^\alpha p_i^\beta}{\Sigma_i p_i^2}$	0	≤3/4	≤1	none (not infrared safe)
Aplanarity	$A = 1.5\, Q_1$	0	0	≤1/2	none (not infrared safe)
Jet (Hemisphere) masses	$M^2_{\pm} = \left(\Sigma^i E_i^2 - \Sigma_i \vec{p}_i^{\,2}\right)_{i \in S_\pm}$ ($S_\pm$: Hemispheres $\perp$ to $\vec{n}_T$) $M_H^2 = \max(M_+^2, M_-^2)$ $M_D^2 = \lvert M_+^2 - M_-^2 \rvert$	 0 0	 ≤1/3 ≤1/3	 ≤1/2 0	(resummed) $O(\alpha_s^2)$
Jet broadening	$B_\pm = \frac{\Sigma_{i \in S_\pm} \lvert \vec{p}_i \times \vec{n}_T \rvert}{2\,\Sigma_i \lvert \vec{p}_i \rvert}$; $B_T = B_+ + B_-$ $B_w = \max(B_+, B_-)$	0 0	≤1/(2√3) ≤1/(2√3)	≤1/(2√2) ≤1/(2√3)	(resummed) $O(\alpha_s^2)$
Energy-Energy Correlations	$EEC(\chi) = \sum_{events} \sum_{i,j} \frac{E_i E_j}{E_{vis}^2} \int_{\chi + \frac{\Delta\chi}{2}}^{\chi - \frac{\Delta\chi}{2}} \delta(\chi - \chi_{ij})$				(resummed) $O(\alpha_s^2)$
Asymmetry of EEC	$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$				$O(\alpha_s^2)$
Differential 2-jet rate	$D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$				(resummed) $O(\alpha_s^2)$

Thrust

- In this work we consider the thrust:

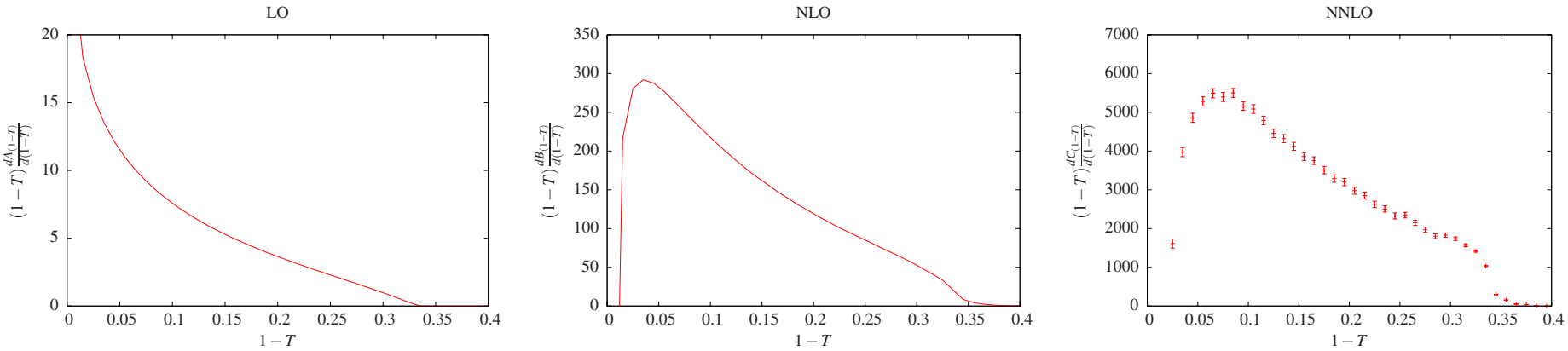
$$T \hat{=} \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} = 1 - \tau \ .$$

With $\vec{p}_i$: i-th particle final state tri-momentum, $\vec{n}$: Euclidean versor.



- Thrust encodes the jet-like nature of the event:
  - $T = \tau = \frac{1}{2}$: isotropic.
  - $T = 1, \tau = 0$: pencil-like.
- The real upper limit τ_{Max} depends on the number of final state particles N. $\tau_{Max} = \frac{1}{2}$ is only realized as $N \to \infty$.

Fixed order computations



Thrust at fixed order in pQCD from [Weinzierl ('09)]

- The thrust event shape distribution is computable in pQCD:

$$R(\tau) = \int_0^\tau d\tau' \frac{1}{\sigma_{tot}} \frac{d\sigma}{d\tau'}$$

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{d\tau} = \delta(\tau) + \frac{\alpha_S}{\pi} \frac{d\mathscr{A}}{d\tau} + \left(\frac{\alpha_S}{\pi}\right)^2 \frac{d\mathscr{B}}{d\tau} + \left(\frac{\alpha_S}{\pi}\right)^3 \frac{d\mathscr{C}}{d\tau} + \mathscr{O}(\alpha_S^4)$$

- $\mathscr{A}$ is known analytically, $\mathscr{B}, \mathscr{C}$ numerically [Ridder et al. ('07)][Weinzierl ('09)][Duca et al. ('16)].

Breakdown of perturbative expansion

- Enhanced infrared logarithmic terms appear as $\tau \to 0$ in the perturbative series:

$$L = \ln\left(\frac{1}{\tau}\right) \gtrsim 1, \quad \tau \to 0.$$

- Caused by unbalanced cancellation of constrained real and unconstrained virtual emissions.
- The cumulant $R(\tau)$ at low τ has the form:

$$R(\tau) \underset{\tau\to 0}{\sim} \sum_{n=1}^{\infty} \alpha_S^n \sum_{k=0}^{2n} R_{nk} L^k$$

- Predictivity lost at fixed order n as $\alpha_S L^2 \gtrsim 1, \tau \to 0$: large logs (Sudakov) resummation is necessary.

Recovering predictivity: Resummation

An all-order resummation is performed at fixed logarithmic accuracy:

	DL	NDL	N^2DL	N^3DL
α_S	L^2	L		
α_S^2	L^4	L^3	L^2	L
α_S^3	L^6	L^5	...	...
α_S^4	L^8	L^7	...	...

All Orders ↓

Fixed Order →

The cumulant cross section $R(\tau)$ is [Catani et al. ('91)][Catani et al. ('93)](CTTW):

$$R(\tau, \alpha_S) = C(\alpha_S)\Sigma(\tau, \alpha_S) + D(\tau, \alpha_S)$$

$$C(\alpha_S) = \sum_{n=0}^{\infty} C_n \alpha_S^n, \; D(\tau, \alpha_S) = \sum_{n=0}^{\infty} D_n(\tau)\alpha_S^n$$

With $C(\alpha_S)$ a hard-virtual factor, $D(\alpha_S)$ a remainder function vanishing as $\tau \to 0$ and $\Sigma(\tau, \alpha_S)$ a long-distance form factor resumming the large Sudakov logarithms.

Thrust Resummation Formula [CTTW]

- The thrust form factor has exponential structure in Laplace space:

$$\ln \tilde{\Sigma}_N(\alpha_S) = \int_0^1 \frac{du}{u}(e^{-uN}-1)\left[\int_{u^2Q^2}^{uQ^2}\frac{dq^2}{q^2}A(\alpha_S(q^2)) + \frac{1}{2}B(\alpha_S(uQ^2))\right]$$

$$A(\alpha_S) = \sum_{j=1}^{+\infty}\left(\frac{\alpha_S}{\pi}\right)^j A_j \quad B(\alpha_S) = \sum_{j=1}^{+\infty}\left(\frac{\alpha_S}{\pi}\right)^j B_j$$

- Using $\exp(-uN)-1 \simeq -\Theta(u-u_0),\ u_0 = (e^{\gamma_E}N)^{-1}$ (+ corrections):

$$\ln \tilde{\Sigma}_N(\alpha_S) \widehat{=} \mathcal{G}_N(\alpha_S) = Lg_1(\lambda) + g_2(\lambda) + \alpha_S g_3(\lambda)\ldots,$$
$$\lambda \widehat{=} \alpha_S\beta_0 L,\ L \widehat{=} \ln(N)$$

- g_1 resums the LL contributions ($\sim L(\alpha_S L)^n$), g_2 the NLL ($\sim(\alpha_S L)^n$) g_3 the N^{2}LL ($\sim \alpha_S(\alpha_S L)^n$)... with $\alpha_S L \sim 1$.

Laplace inversion

- Laplace inversion is required to return to τ-space and compare with data:

$$\Sigma(\tau, \alpha_S) = \frac{1}{2\pi i} \int_{\mathscr{C}} \frac{dN}{N} e^{N\tau} \tilde{\Sigma}_N(\alpha_S), \quad \tilde{\Sigma}_N(\alpha_S) = \exp(\mathscr{G}_N(\alpha_S))$$

Two possible approaches:

- Taylor-expand $\ln(N)$ in $\tilde{\Sigma}_N$ around $\ln \frac{1}{\tau}$.
- The result has an analytic expression.
- Approach from [CTTW].
- Relies on *inexact* correspondence $N \rightsquigarrow \frac{1}{\tau}$.
- Landau pole prescription is implicit.

- Numerically evaluate the inversion integral.
- Requires an explicit prescription on the integration contour $\mathscr{C}$ to avoid the Landau pole.
- Minimal prescription (MP): no NP power correction added[Catani et al. ('96)].

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Matching with fixed order

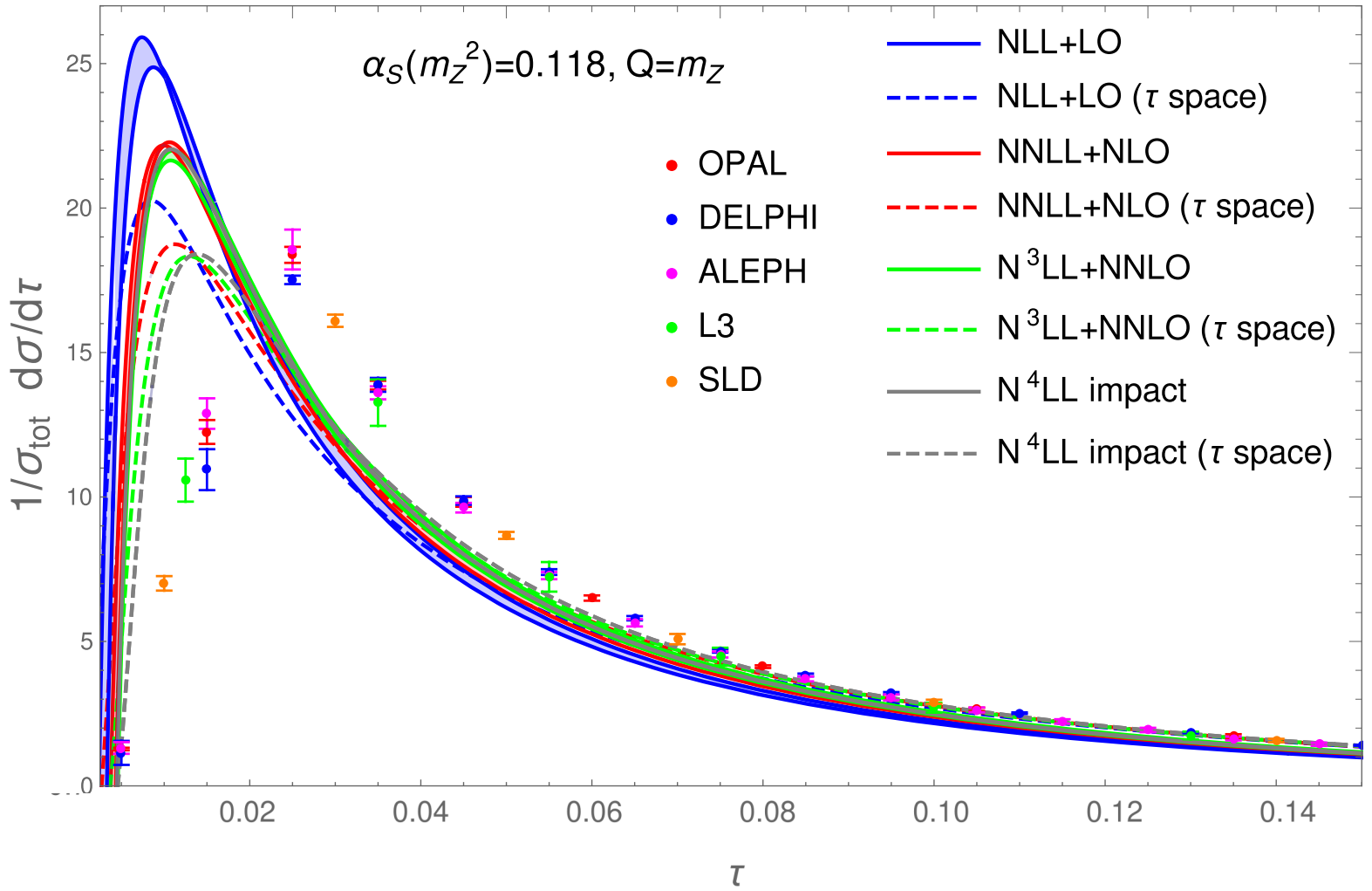
- The resummed predictions are reliable in the Sudakov region $(0 \lesssim \tau \ll 1)$.
- As $\tau \to \tau_{Max}$, matching with fixed order is necessary:

$$R_{FO}(\tau, \alpha_S)|_n = C(\alpha_S)\Sigma(\tau, \alpha_S)|_n + D(\tau, \alpha_S)|_n ,$$
$$D(\tau, \alpha_S)|_n = R_{FO}(\tau, \alpha_S)|_n - C(\alpha_S)\Sigma(\tau, \alpha_S)|_n .$$

  With "$|_n$" meaning "expanded at order α_S^n".
- Even after matching, unitarity of cross section is violated.
- Impose the physical unitarity constraint in N-space[Aglietti et al. ('25)]:

$$L \widehat{=} \ln(N) \rightsquigarrow \tilde{L} \widehat{=} \ln(N + N_C) \underset{N \gg 1}{\sim} L + \mathcal{O}(N^{-1})$$
$$R(\tau = \tau_{Max}, \alpha_S)|_{L=\tilde{L}} = 1$$

Is resummation enough?



Thrust distribution at $Q = 91.1876\,\text{GeV}$ in pQCD[Aglietti et al. ('25)]: Numerical (solid) and analytical (dashed) inversion.

Where does this discrepancy originate?

- Thrust resummation formula only describes logarithmic terms.
- Non perturbative power suppressed terms: not included.
- The resummed $\mathcal{G}_N(\alpha_S)$ has a Landau-pole singularity $\Rightarrow$ NP physics:

$$\mathcal{G}_N(\alpha_S) = L g_1(\lambda) + g_2(\lambda) + \alpha_S g_3(\lambda) \ldots \underset{\lambda \to \frac{1}{2}}{\to} \infty$$

$$\lambda = \alpha_S \beta_0 \ln N = \frac{1}{2} \Longleftrightarrow N = N_L \widehat{=} \exp[(2\alpha_S \beta_0)^{-1}]$$

- The singularity comes from the running coupling in the resummation formula.

SOLUTION:

Include a NP model directly in the resummation formula to include NP effects and remove Landau pole divergence.

Regularized coupling models

IDEA: Introduce an IR effective coupling $\tilde{\alpha}_S$ in place of α_S.

Two possible approaches:

- Use an explicit IR-regular $\tilde{\alpha}_S$ in the resummation formula:

$$A, B(\alpha_S(q^2)) \rightsquigarrow A, B(\tilde{\alpha}_S(q^2))$$

- No prescription on $\mathscr{C}$ needed.
- Model is not data driven.

Analytic coupling model [Shirkov et al. ('96)]

- Reconstruct the IR coupling with its moments $\alpha_{l,m}$:

$$\alpha_{l,m}(\mu_I) = \frac{1}{\mu_I^{l+1}} \int_0^{\mu_I} dq\, q^l\, \tilde{\alpha}_S^m \,.$$

- $\Lambda_{QCD} < \mu_I \ll Q$.
- $\alpha_{l,m}$ fitted on data.

Dokshitzer-Marchesini-Webber model[Dokshitzer et al. ('96)]

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Consider the leading-log form factor:

$$\mathcal{G}_{LL}(N,\alpha_S) = \frac{2A_1}{\pi}\int_0^{Q^2}\frac{dq^2}{q^2}\alpha_S(q^2)\int_{\frac{q^2}{Q^2}}^{\frac{q}{Q}}\frac{du}{u}(e^{-uN}-1)$$

Introduce the effective IR-regulated coupling $\tilde{\alpha}_S(q^2)$ in the IR region $q \in [0,\mu_I]$:

$$\delta\mathcal{G}_{LL}^{NP} = \frac{2A_1}{\pi}\int_0^{\mu_I^2}\frac{dq^2}{q^2}\tilde{\alpha}_S(q^2)\int_{\frac{q^2}{Q^2}}^{\frac{q}{Q}}\frac{du}{u}(e^{-uN}-1) \approx$$

$$\approx \underbrace{-\frac{2A_1}{\pi}N\frac{\mu_I}{Q}\alpha_{0,1}(\mu_I)}_{\frac{\mu_I}{Q}} + \underbrace{\frac{2A_1}{\pi}N\frac{\mu_I^2}{Q^2}\alpha_{1,1}(\mu_I) + \frac{A_1}{2\pi}N^2\frac{\mu_I^2}{Q^2}\alpha_{1,1}(\mu_I)}_{\frac{\mu_I^2}{Q^2}\ \Rightarrow\ \text{NEW!}} + \mathcal{O}\left(\frac{\mu_I^3}{Q^3}\right) \widehat{=}$$

$$\widehat{=}\ \Omega_1^{NP}N + \Omega_2^{NP}N^2$$

Applying the same procedure to the expanded pQCD coupling:

$$-\delta\mathcal{G}_{LL}^{P} = -\frac{2A_1}{\pi}N\frac{\mu_I}{Q}\alpha_S(Q) + \frac{A_1}{\pi}N\frac{\mu_I^2}{Q^2}\alpha_S(Q) + \frac{A_1}{4\pi}N^2\frac{\mu_I^2}{Q^2}\alpha_S(Q) \widehat{=} \Omega_1^P N + \Omega_2^P N^2$$

Dokshitzer-Marchesini-Webber Model

The result generalized to any resummation order N^{k}LL is:

$$\Omega_1^{NP} = 2\sum_{i=1}^{k+1}\frac{A_i}{\pi^i}\left[\frac{\mu_I^2}{Q^2}\alpha_{1,i} - \frac{\mu_I}{Q}\alpha_{0,i}\right] - \sum_{i=1}^{k}\frac{B_i}{\pi^i}\frac{\mu_I^2}{Q^2}\alpha_{1,i}$$

$$\Omega_2^{NP} = 2\sum_{i=1}^{k+1}\frac{A_i}{\pi^i}\frac{\mu_I^2}{4Q^2}\alpha_{1,i}$$

The perturbative coefficients Ω_i^P are obtained by integrating the expanded RGE solution at the required order. The final correction is:

$$\mathcal{G}_{DMW} = \mathcal{G}_{N^kLL} + \delta\mathcal{G}^P_{N^kLL} + \delta\mathcal{G}^{NP}_{N^kLL} \widehat{=} \mathcal{G}_{N^kLL} + \delta\mathcal{G}^{DMW}_{N^kLL}, \quad \delta\mathcal{G}^{DMW}_{N^kLL} = \Omega_1 N + \Omega_2 N^2$$

The additional terms correspond to a Gaussian convolution in τ-space:

$$\exp(\delta\mathcal{G}^{DMW}_{N^kLL}) = \exp(\Omega_1 N + \Omega_2 N^2) \xrightarrow{\mathscr{L}^{-1}} \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_{NP}}\exp\left[-\frac{(\delta_{NP}-\tau)^2}{2\sigma_{NP}^2}\right]$$

$$\sigma_{NP} \widehat{=} 2\sqrt{\Omega_2}, \qquad \delta_{NP} \widehat{=} 2|\Omega_1|$$

Dokshitzer-Marchesini-Webber Model

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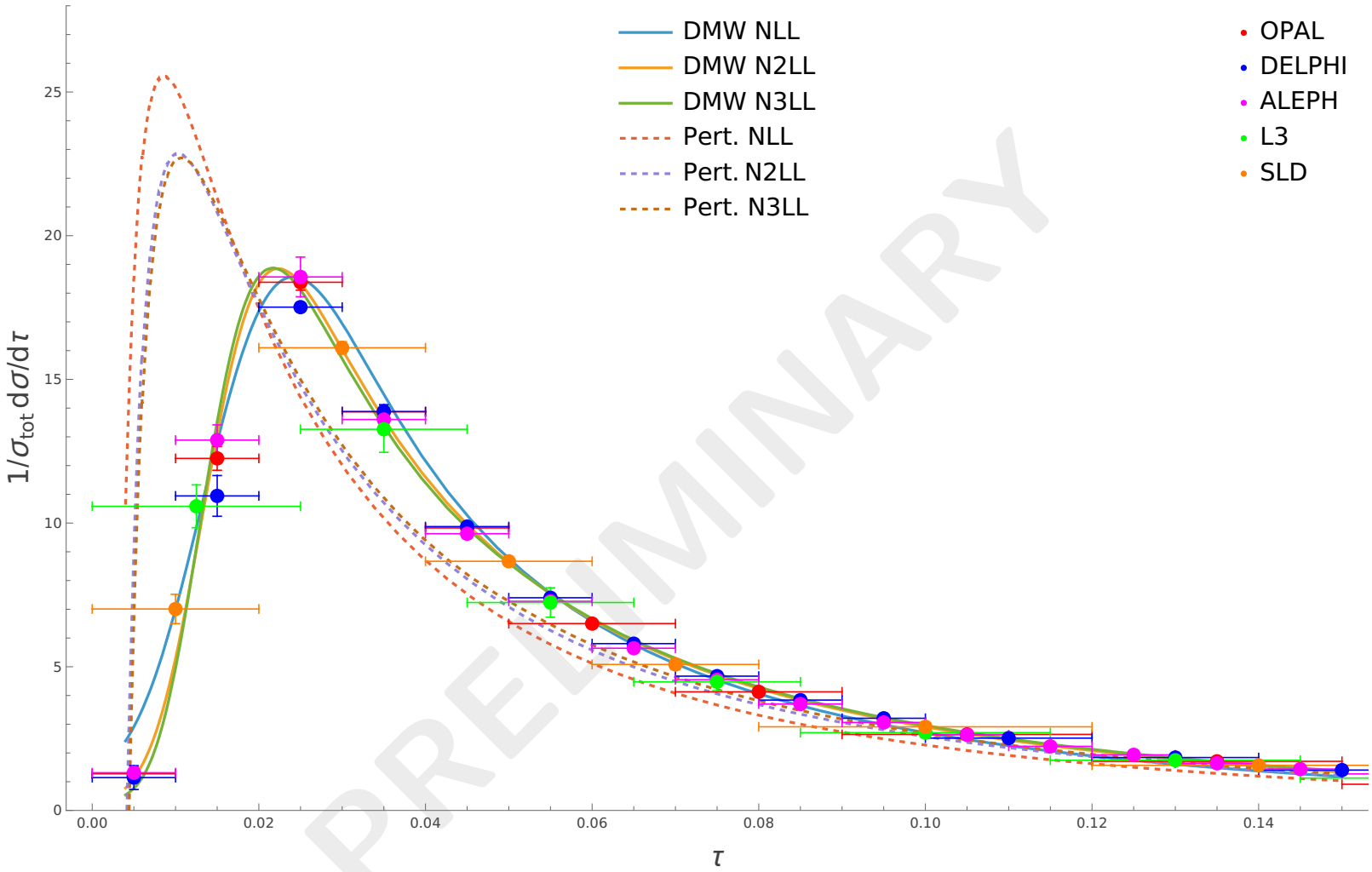
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$$\sigma_{NP} \widehat{=} 2\sqrt{\Omega_2}, \qquad \delta_{NP} \widehat{=} 2|\Omega_1|$$

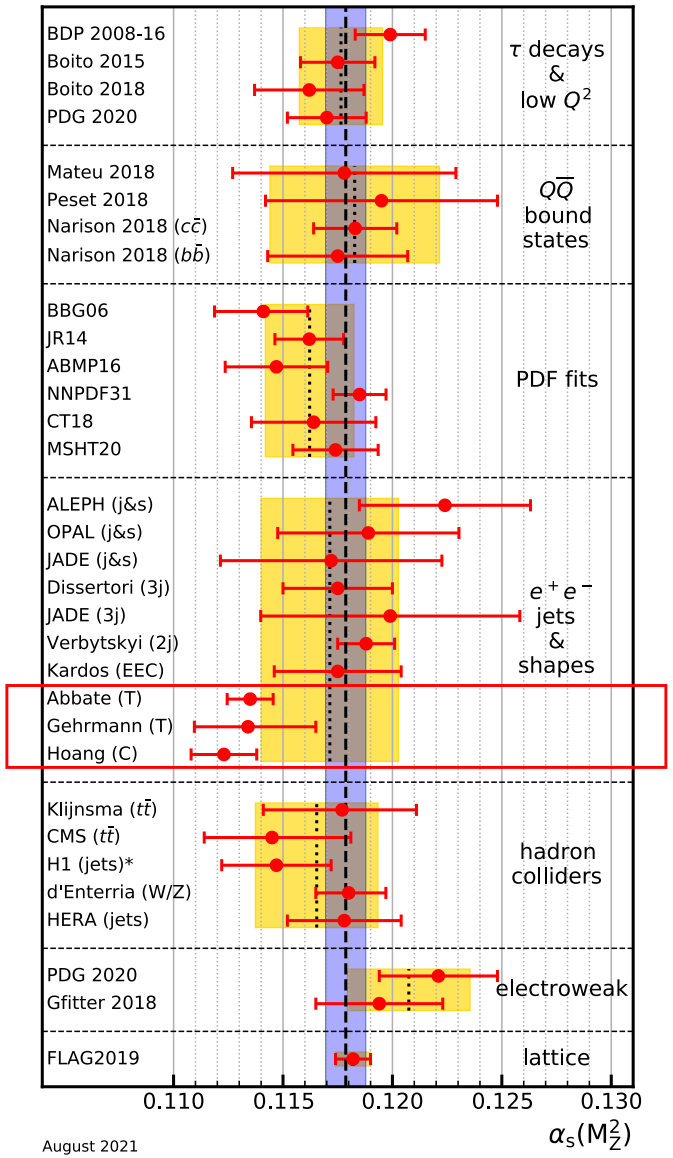
Does it work?



Thrust distribution at $Q = 91.1876\,\text{GeV}$ with (solid) and without (dashed) NP model. Numerical inversion.

α_S fit

- α_S is known with the least accuracy among the other SM couplings.
- Determinations from ESDs have produced strong tensions with global average[PDG ('21)].
- The extraction of α_S from thrust is conceptually straightforward (explicit depence in $R(\tau, \alpha_S)$).
- In practice: simultaneously fit NP parameters Ω_1, Ω_2 $(\delta_{NP}, \sigma_{NP})$ and α_S.



Global fit strategy

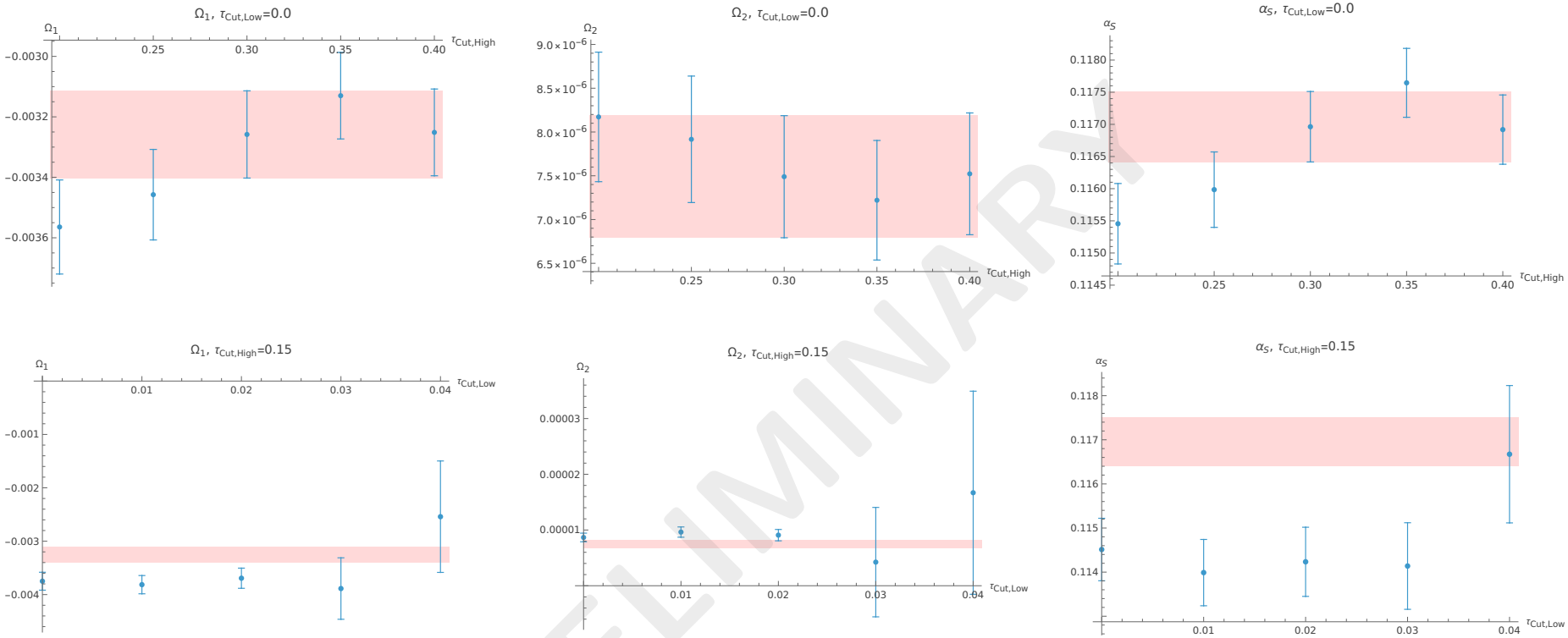
- Simplest strategy: α_S fitted at a single energy scale $Q \Rightarrow$ "local" fit.
- Thrust data exists at different values of Q, i.e. incoming beam energy.
- Utilize data across different $Q \Rightarrow$ "global" fit.

Global fit strategy:

- Choose a reference scale Q_0.
- Fit coefficients: $\theta(Q_0) = \{\alpha_S(Q_0), \Omega_i(Q_0)\}$.
- For each dataset with $Q = Q'$:
  - Compute $\theta(Q')$ from $\theta(Q_0)$ using evolution equations.
  - Evaluate $\chi^2(Q')$.
- $\chi^2_{Glob} = \sum_{Q'} \chi^2(Q')$.
- Optimize χ^2_{Glob} in $\theta(Q_0)$

Input: $\theta(Q_0) = \{\alpha_S(Q_0), \Omega_i(Q_0)\}$
$\chi^2_{Glob} \leftarrow 0$;
foreach Q' *in Energies* **do**
  $\theta(Q') \leftarrow \text{EvolPars}(Q', \theta(Q_0))$;
  $\chi^2(Q') \leftarrow \text{EvalChi}(\theta(Q'))$;
  $\chi^2_{Glob} \leftarrow \chi^2_{Glob} + \chi^2(Q')$;
end
Output: χ^2_{Glob}

Fit region choice



PRELIMINARY

Optimal parameters dependence on fit region bounds. Red band: optimal values with fit region $\tau \in [0.0, 0.30]$

- The output NP parameters heavily depend on the fit region.
- Maximum stability achieved considering data with $\tau \in [0.0, 0.30]$.

Fit details

- We used multiple datasets with energies ranging from $35\,\text{GeV}$ to $207\,\text{GeV}$.
- Theoretical error σ_{Th} estimated as the maximum envelope width of theoretical curves under joint sweep of resummation and renormalization scales.
- Total experimental error σ_{Ex} includes both systematic and statistical errors.
- Total data error for the fit given by:

$$\sigma_{Tot} = \sqrt{\sigma_{Th}^2 + \sigma_{Ex}^2}$$

- Errors on the output parameters estimated from the χ^2 Hessian.
- $\Delta\chi^2 = 1$ contours consistently produced lower error uncertainties.

Fit results: NP parameters and α_S values

Order	δ_{NP}	σ_{NP}	α_S	χ^2_{Red}
NLL	-0.004 ± 0.001	0.005 ± 0.001	0.132 ± 0.002	0.68
N^2LL	-0.0064 ± 0.0004	0.0051 ± 0.0004	0.120 ± 0.001	0.89
N^3LL	-0.0065 ± 0.0003	0.0055 ± 0.0003	0.117 ± 0.001	1.27

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N^3LL	-0.0066 ± 0.0003	0.0061 ± 0.0001	0.117 ± 0.001	1.09

Optimal Gaussian parameters value at $Q = m_Z$ from fit at $\sqrt{s} = m_Z$ and global fit ($35\,\text{GeV} \leq \sqrt{s} \leq 207\,\text{GeV}$).

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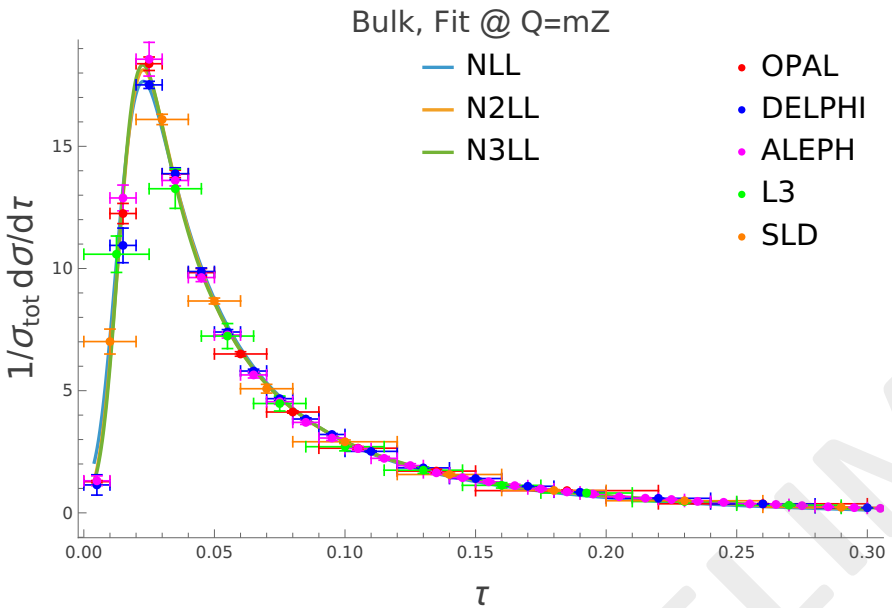
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N^{3}LL	-0.0066 ± 0.0003	0.0061 ± 0.0001	0.117 ± 0.001	1.09

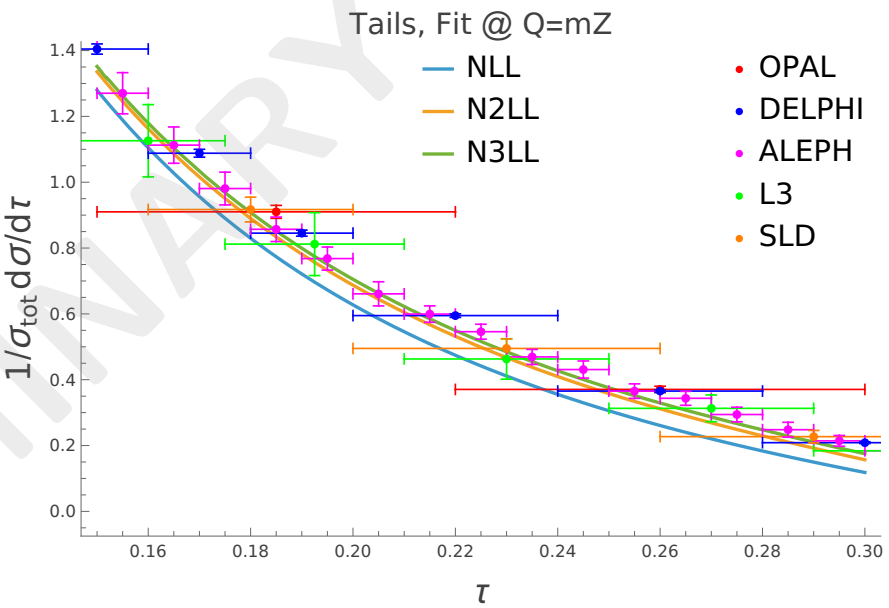
Optimal Gaussian parameters value at $Q = m_Z$ from fit at $\sqrt{s} = m_Z$ and global fit ($35\,\text{GeV} \leq \sqrt{s} \leq 207\,\text{GeV}$).

To be compared against the PDG world average:
$$\alpha_S^{Avg}(m_Z) = 0.1180 \pm 0.0009$$

PRELIMINARY

Fit results: Z-Boson peak

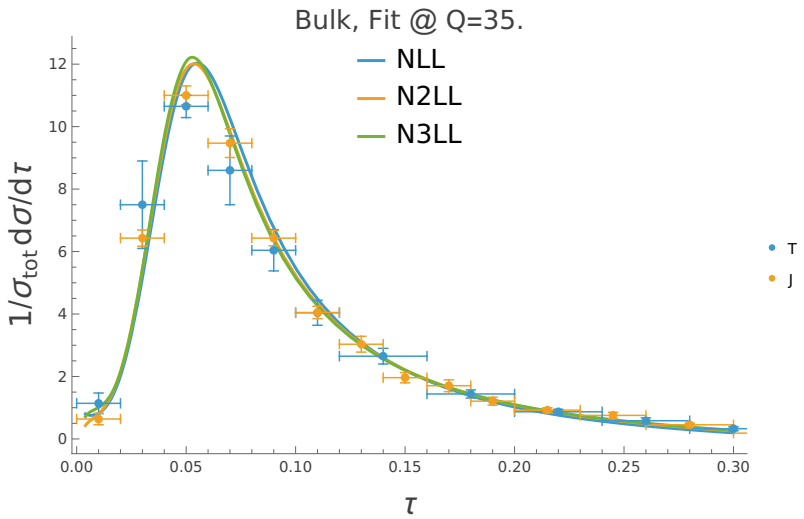


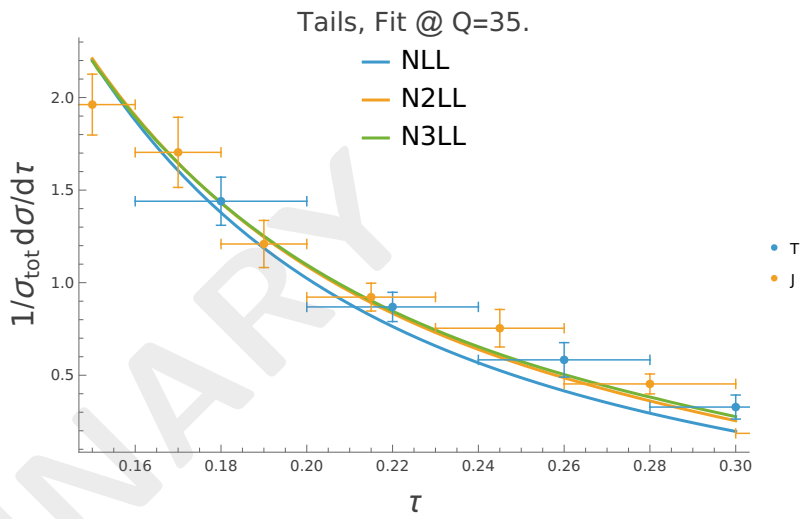


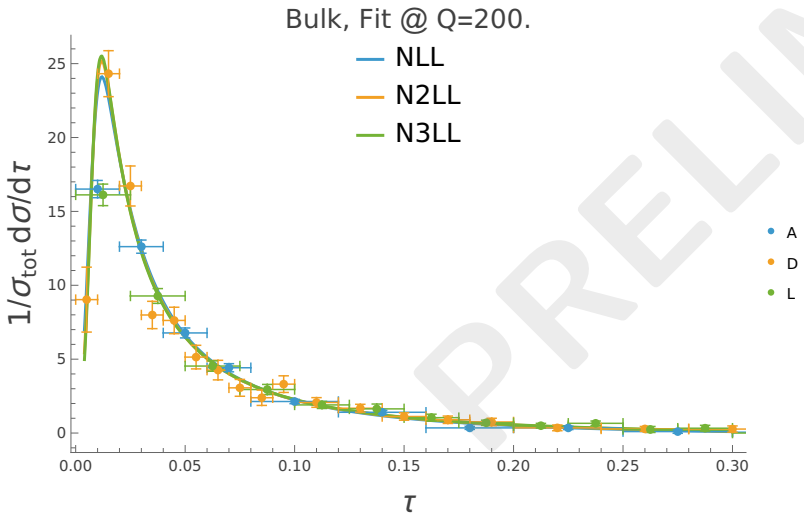
PRELIMINARY

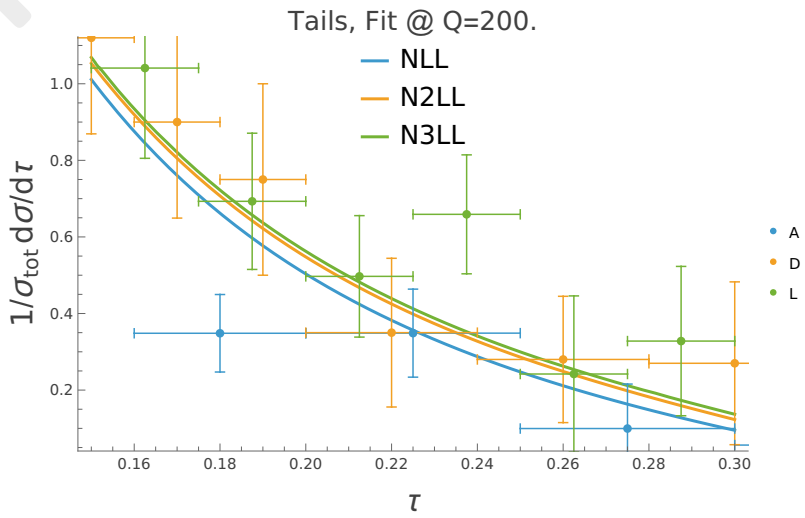
Thrust distribution at $Q = 91.1876\,\text{GeV}$ with globally fitted NP parameters and α_s. Bulk cross section (left) and tails zoom (right).

Fit results: Multiple Q plot









PRELIMINARY

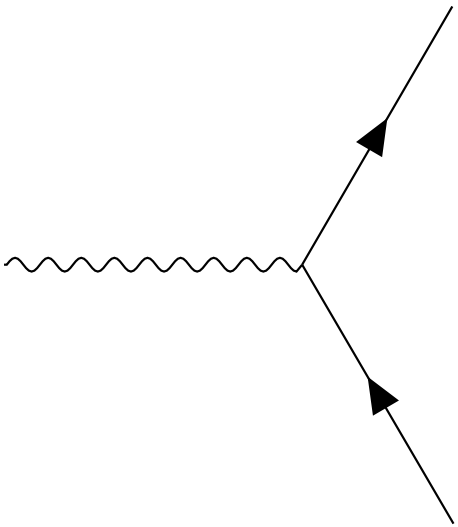
Thrust distribution at $Q = 35\,\text{GeV}, 200\,\text{GeV}$ with globally fitted NP parameters and α_s.

Conclusions

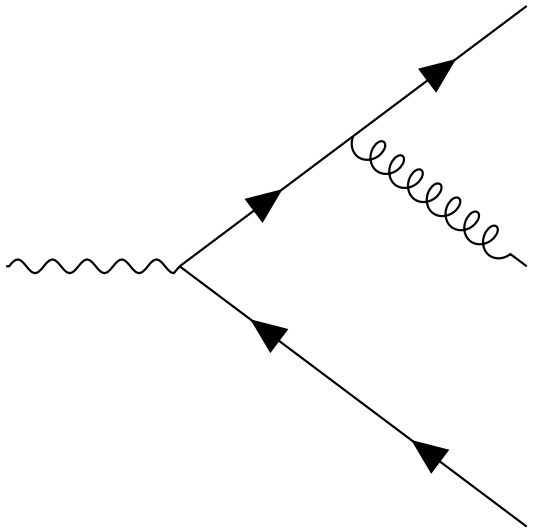
- A purely perturbative theoretical approach to the thrust is not sufficient to describe data.
- Non perturbative models can be directly implemented in the resummation formula.
- We extended the Dokshitzer-Marchesini-Webber model up to second order power corrections $\frac{\mu_I^2}{Q^2}$ (shift correction + smearing).
- We have shown the corrsepondence between this model and a NP Gaussian smearing.
- By fitting the NP parameters and α_S we managed to describe experimental data.
- We performed a global fit of NP parameters and α_S using data in a wide energy range.
- Our determination of α_S is in good agreement with the PDG world average.

Thank you for your attention.

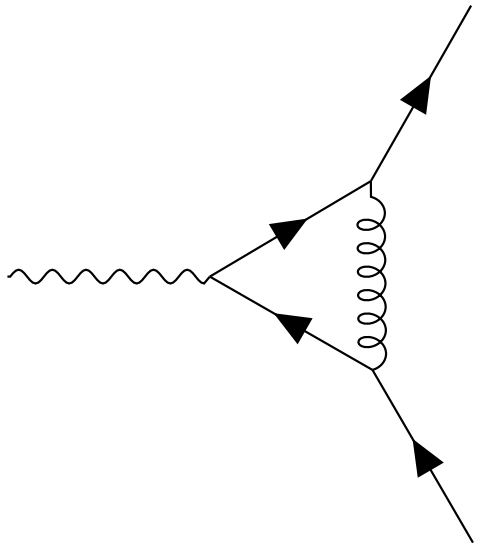
IR safety



(a) Tree Level, $\mathscr{M}_0$.



(b) Real Emissions, $\mathscr{M}_R$.



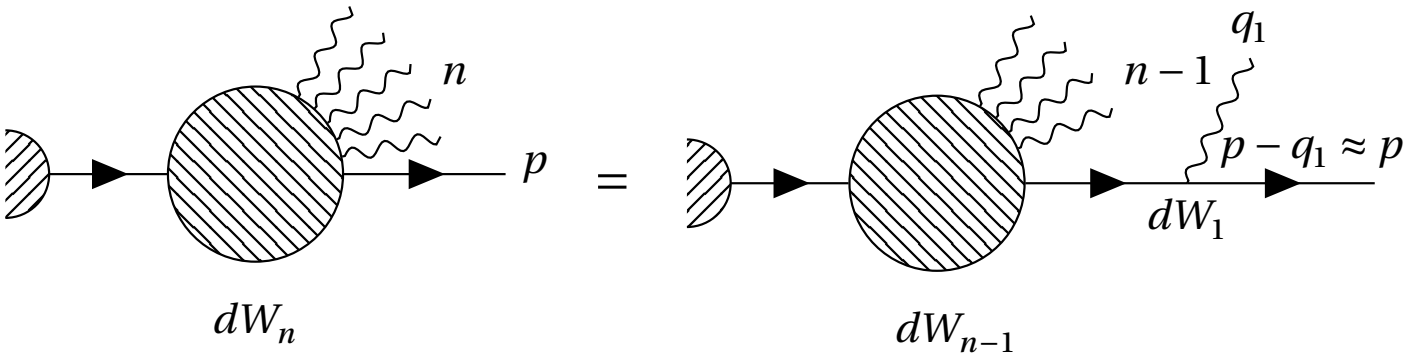
(c) Virtual Corrections, $\mathscr{M}_V$.

$$\mathscr{M}_0 = \mathscr{O}(g_S),\ \mathscr{M}_R = \mathscr{O}(g_S^2),\ \mathscr{M}_V = \mathscr{O}(g_S^3)$$

$$|\mathscr{M}_0 + \mathscr{M}_V|^2 = |\mathscr{M}_0|^2 + 2\Re\mathfrak{e}\left(\mathscr{M}_0\mathscr{M}_V^\star\right) = \mathscr{O}(g_S^2) + \mathscr{O}(g_S^4), \quad |\mathscr{M}_R|^2 = \mathscr{O}(g_S^4)\,.$$

When is exponentiation possible?

- Exponentiation happens if the multi-emission dynamics and kinematics factorizes.
- Dynamics factorization is general in QCD for soft emissions:



$$dw_n(q_1,\dots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_1(q_i)$$

- Kinematics is factorized in Laplace space for the Thrust:

$$\Theta\left(\tau - \sum_{j=1}^{n}\frac{k_j^2}{Q^2}\right) = \frac{1}{2\pi i}\int_{\mathscr{C}}\frac{d\nu}{\nu}e^{\nu\tau}\prod_{j=1}^{n}e^{-\nu\frac{k_j^2}{Q^2}}$$

τ-space Laplace inversion [CTTW]

$$R(\tau) \underset{L=\ln\frac{1}{\tau}}{=} \frac{1}{2\pi i} e^{2\mathscr{G}(L)} \int_{\mathscr{C}} du \exp[u-(1-2\gamma(L))\ln u] \cdot \cdot \left\{ 2 \sum_{m=0}^{\infty} \frac{1}{m!} \left[\sum_{n=2}^{\infty} \frac{1}{n!} \mathscr{G}^{(n)}(L) \ln^n u \right]^m \right\} .$$

With:

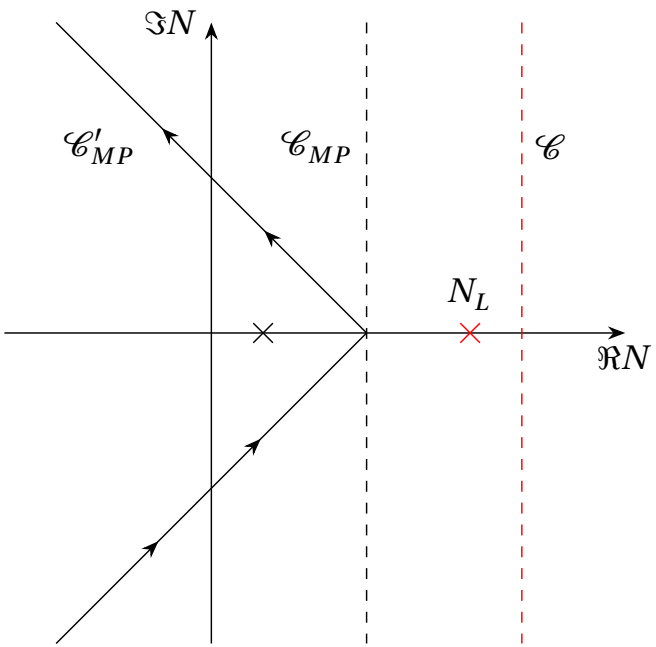
$$\mathscr{G}^{(n)}(L) \widehat{=} \frac{\partial^n}{\partial L^n} \mathscr{G}(L) ,$$
$$\gamma(L) \widehat{=} \mathscr{G}^{(1)}(L) .$$

The expression can be analytically evaluated using:

$$\frac{1}{2\pi i} \int_{\mathscr{C}} \ln^k u \exp[u-(1-2\gamma)\ln u] = \frac{d^k}{d\gamma^k} \frac{1}{\Gamma(1-2\gamma)} .$$

Minimal Prescription

[Catani et al. ('96)]



- Exact numerical inversion requires a prescription to avoid the Landau pole.
- Minimal prescription: take the contour $\mathscr{C}_{MP}$ to the right of all physical singularities and to left of the unphysical N_L.
- The result converges asymptotically to the perturbative series without introducing any NP power correction.
- Deforming the contour to ($\mathscr{C}'_{MP}$) improves numerical stability.

Analytic Coupling

[Aglietti et al. ('07)]

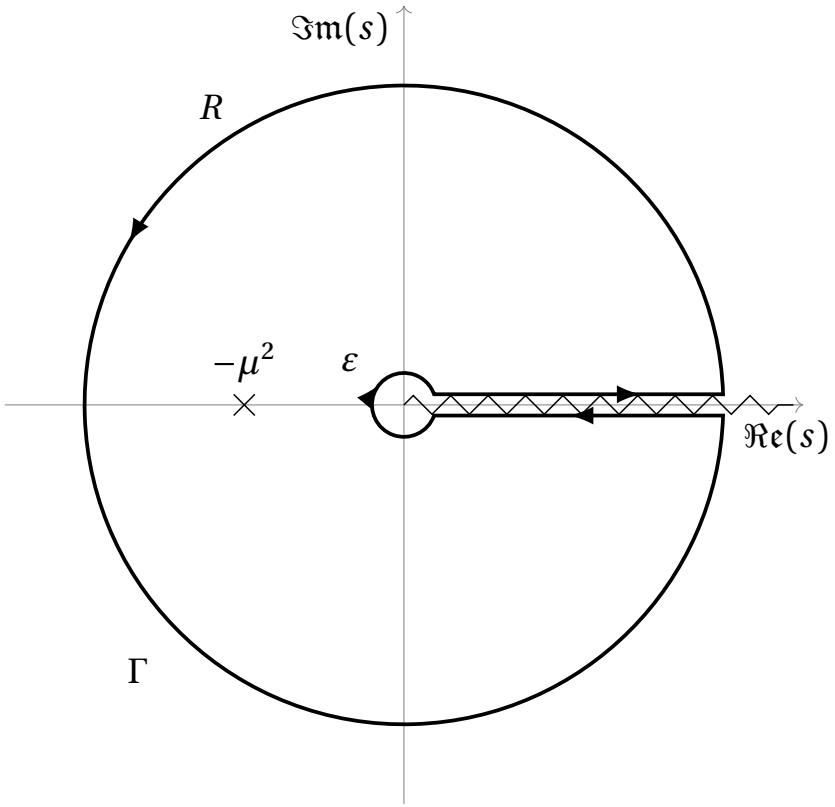
Starting from:

$$f(s) \widehat{=} \frac{\tilde{\alpha}_S(-s)}{s+\mu^2}\,,$$

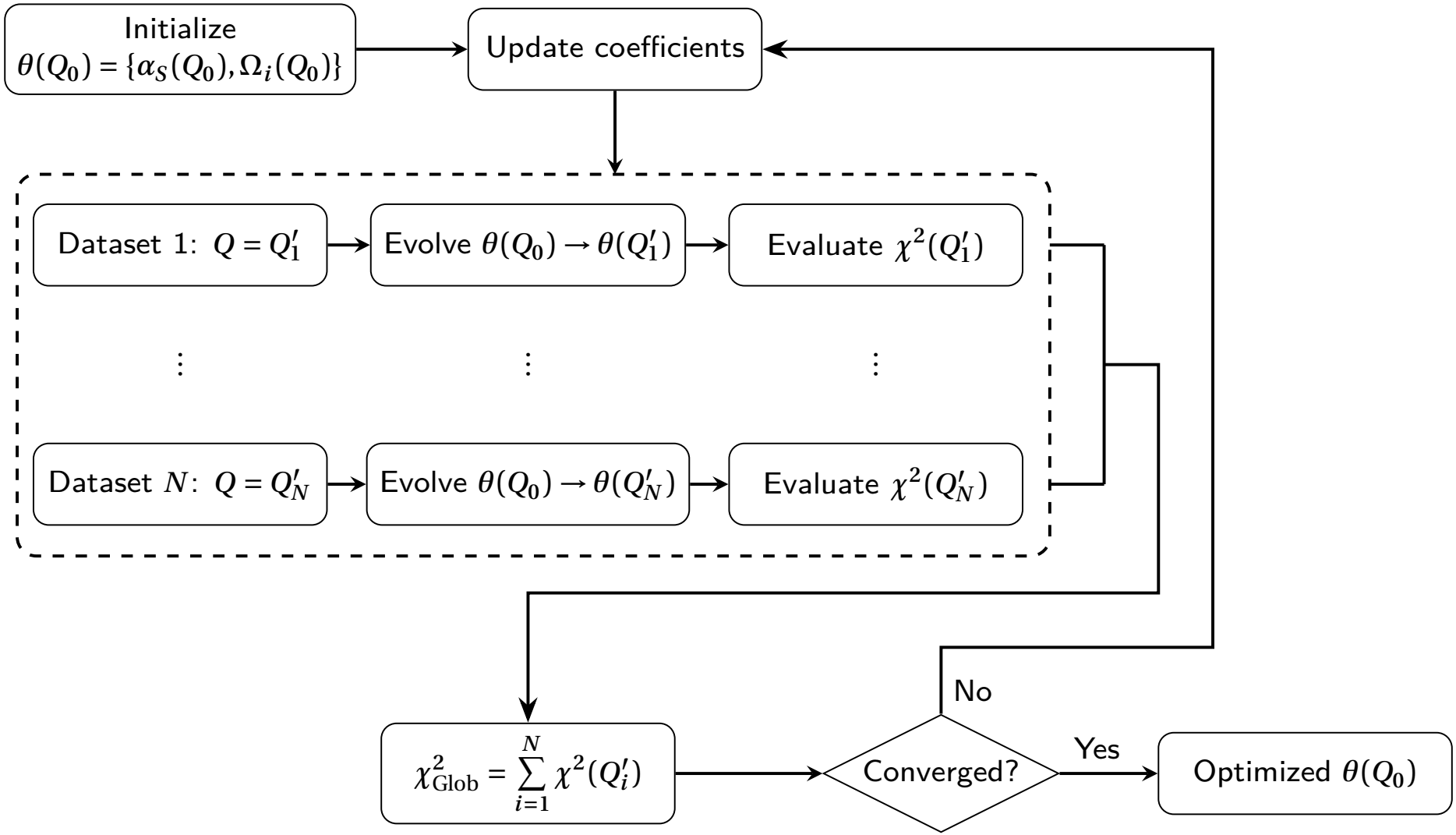
One obtains:

$$\tilde{\alpha}_S(\mu^2) = \frac{1}{2\pi i}\oint_\Gamma f(s)ds = \ldots =$$

$$= \frac{1}{2\pi i}\int_0^\infty \frac{ds}{s+\mu^2}\,\mathrm{Disc}[\alpha_S(-s)]\,.$$



Global fit algorithm



Global fit datasets

- TASSO: $Q = \{35, 44\}$ GeV
- AMY: $Q = 54.5$ GeV
- JADE: $Q = \{35, 44\}$ GeV
- SLD: $Q = 91.2$ GeV
- L3: $Q = \{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2\}$ GeV
- DELPHI: $Q = \{45, 66, 76, 89.5, 91.2, 93, 133, 161, 172, 183, 189, 192, 196, 200, 202, 205, 207\}$ GeV
- OPAL: $Q = \{91, 133, 161, 172, 183, 189, 199\}$ GeV
- ALEPH: $Q = \{91.2, 133, 161, 172, 183, 189, 200, 206\}$ GeV