Non-linear DGLAP/BFKL duality

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Linear DGLAP/BFKL duality

High-energy (Regge) limit of QCD

Let us introduce the problem of High-Energy (Regge) limit of QCD on a particular example of a process – *production of Mueller-Navelet dijets* at hadron collider (X =anything):

$$p(P_1) + p(P_2) \to jet(k_{J1}) + X + jet(k_{J2}),$$

where $M_{JJ}^2 = (k_{J1} + k_{J2})^2 \gg |\boldsymbol{k}_{J1}|^2$ or $|\boldsymbol{k}_{J2}|^2 \gg \Lambda_{\rm QCD}^2$ (**bold**–$\perp$-vectors!).

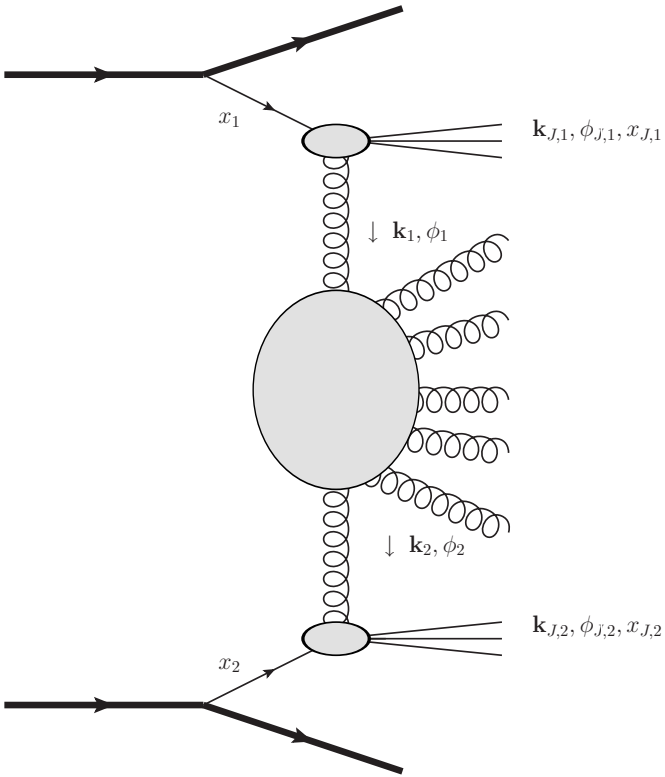
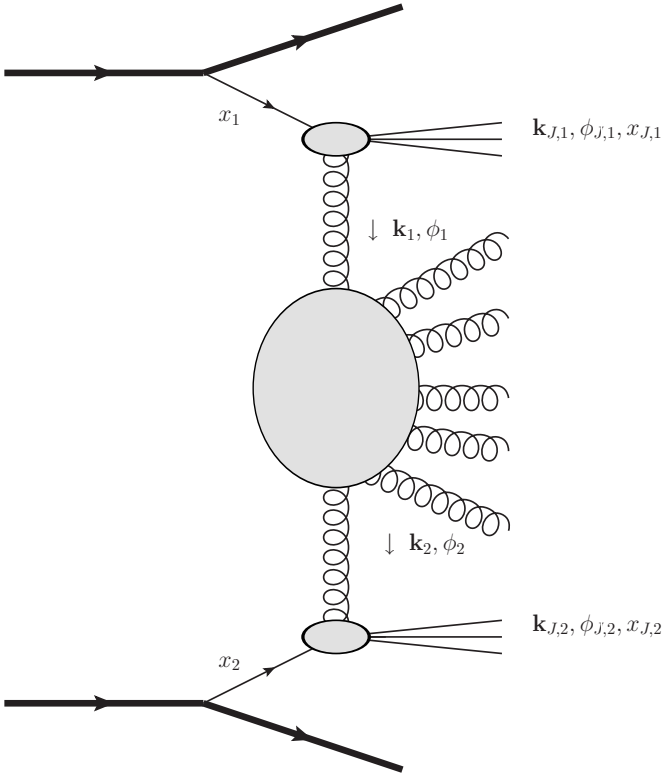


Figure from [hep-ph/1302.7012] .

- $d\sigma = \underbrace{\mathrm{PDF}_i \otimes \mathrm{PDF}_j}_{\text{“non-perturbative”}} \otimes \underbrace{d\hat{\sigma}_{ij}}_{\text{perturbative}}$,
- Fixed-order: $d\hat{\sigma}$ known up to NNLO in α_s,
- Regge limit: $Y = \ln \frac{M_{JJ}}{\sqrt{|\boldsymbol{k}_{J1}||\boldsymbol{k}_{J2}|}} \gg 1$, the $d\hat{\sigma}$ recieves corrections $\sim (\alpha_s Y)^n$, which should be resummed.
- The *Balitsky-Fadin-Kuraev-Lipatov (BFKL)* equation allows to rigirously resum them in the LLA ($\sum_n (\alpha_s Y)^n$) and NLLA ($\sum_n \alpha_s (\alpha_s Y)^n$).

BFKL equation



Gluons in t-channel are **Reggeised**.

- Up to NLLA, the partonic cross-section is:

$$d\hat{\sigma}_{ij} = \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2\, C_i(\boldsymbol{k}_1) G(Y, \boldsymbol{k}_1, \boldsymbol{k}_2) C_j(\boldsymbol{k}_2),$$

where $C_i(\boldsymbol{k})$ – *impact-factors*, G – *BFKL Green's functon*. From now on: $\int d^2\boldsymbol{k} \to \int_{\boldsymbol{k}}$.

- The latter satisfies the BFKL equation:

$$\frac{\partial}{\partial Y} G(Y, \boldsymbol{k}_1, \boldsymbol{k}_2) = \int_{\boldsymbol{q}} K(\boldsymbol{k}_1, \boldsymbol{q}) G(Y, \boldsymbol{q}, \boldsymbol{k}_2),$$

The BFKL kernel is computed perturbatively:

$$K(\boldsymbol{k}, \boldsymbol{q}) = \frac{\alpha_s(\mu) N_c}{\pi} \left(K_0(\boldsymbol{k}, \boldsymbol{q}) + a_s(\mu) K_1(\boldsymbol{k}, \boldsymbol{q}) + \dots \right),$$

where $a_s = \alpha_s/(4\pi)$. The K_0 is known since [BFKL '76, '78] , the K_1 is also known in QCD [Fadin, Lipatov, '98] and $\mathcal{N} = 4$ SYM [Kotikov, Lipatov, 2000] . The K_2 is known in $\mathcal{N} = 4$ SYM [Caron-Huot, Herranen, 2016] and is being computed now in QCD [Gardi, Del Duca, Fadin, Papa, Caola, Falcioni, Byrne,Fucilla,...]

("Generalised") Characteristic functon

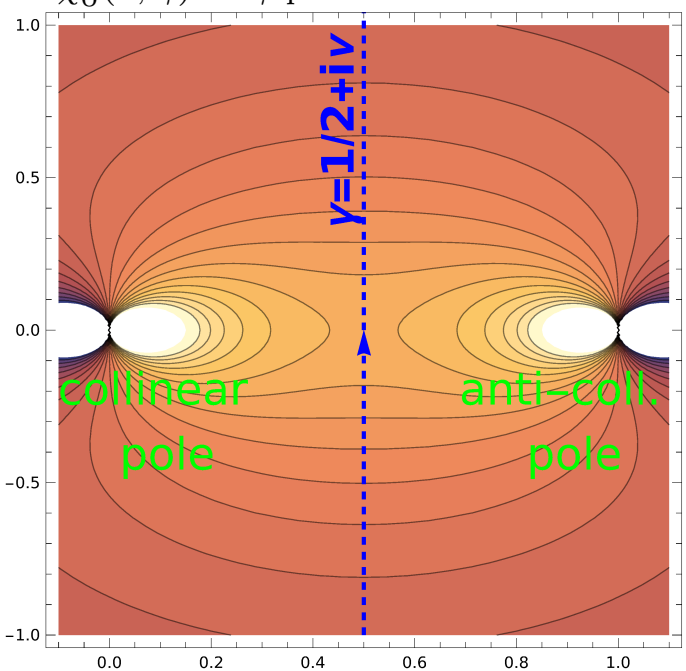
A useful mathematical tool is the *BFKL characteristic function*:

$$\int\limits_{\boldsymbol{q}} K(\boldsymbol{k},\boldsymbol{q})(\boldsymbol{q}^2)^\gamma e^{in\phi_{\boldsymbol{q}}} = \frac{\alpha_s(\boldsymbol{k}^2)N_c}{\pi}\chi(n,\gamma)(\boldsymbol{k}^2)^\gamma e^{in\phi_{\boldsymbol{k}}},$$

where $\chi(n,\gamma) = \chi_0(n,\gamma) + a_s(\boldsymbol{k}^2)\chi_1(n,\gamma) + \ldots$

Then $G(Y,\boldsymbol{k}_1,\boldsymbol{k}_2) \propto \sum\limits_n \int\limits_\gamma \exp\left[Y\frac{\alpha_s N_c}{\pi}\chi(n,\gamma)\right]\left(\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_2^2}\right)^\gamma e^{in\phi}.$

Re $\chi_0(0,\gamma)$ in γ-plane:



$\chi(0,1/2+i\nu)$, dashed line – LO, dash-dotted line – NLO, solid line – a version of resummation for leading collinear poles:

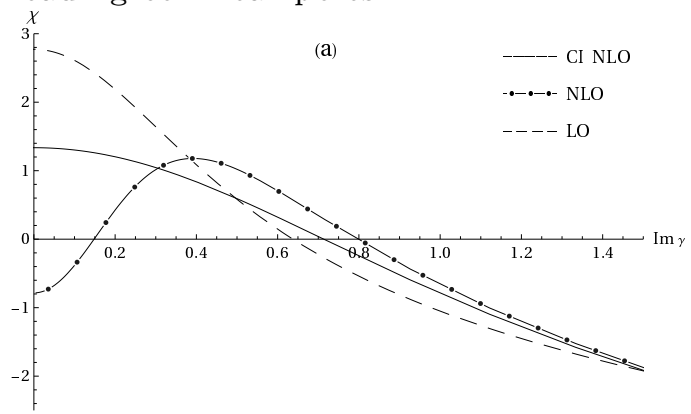
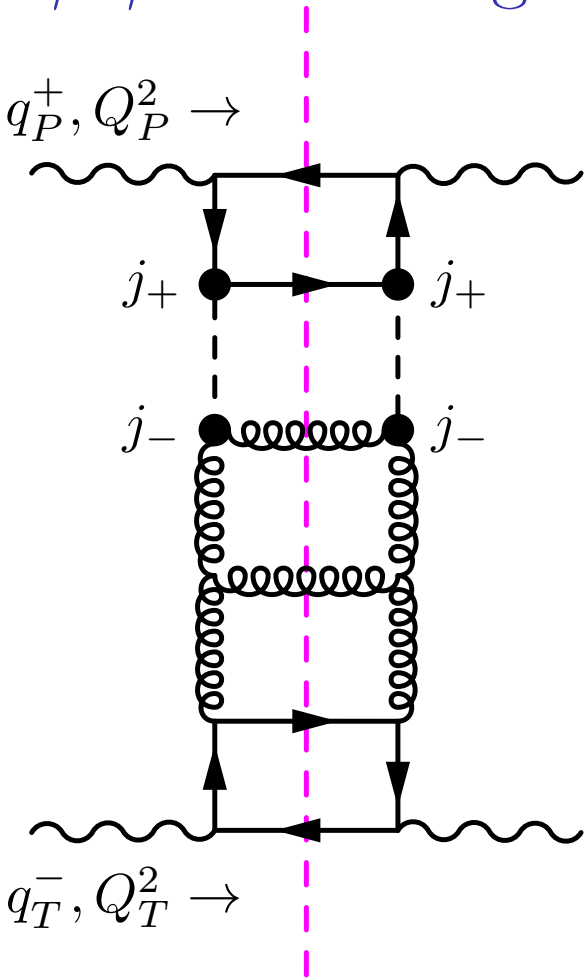


Figure from [Chernyshev,MN,Saleev, 2025]

Corrections to χ are large and mostly come from poles at $\gamma = 0, 1$!

$\gamma^*\gamma^*$-scattering and DGLAP



- **LLA BFKL** ($x_B^{(P/T)} = Q_{P/T}^2/s$):

$$\mathrm{Im}\,\mathcal{A}^{(i,j)} \propto \int \frac{d\omega}{2\pi i} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i} \left(x_B^{(T)}\right)^{-\omega} \left(\frac{Q_P^2}{Q_T^2}\right)^{\gamma-1} \times \frac{\phi_0^{(i)}(1-\gamma)\phi_0^{(j)}(\gamma)}{\omega - \hat{\alpha}_s \chi_0(0,\gamma)}.$$

- For $Q_T \gg Q_P$ (**anti-collinear limit**) the pole at $\gamma = 1 - \gamma^{(T)}(\omega)$ dominates [Jaroszewicz, '82] :

$$\mathrm{Im}\,\mathcal{A}^{(i,j)} \sim \left(\frac{Q_T^2}{Q_P^2}\right)^{\gamma^{(T)}(\omega)}.$$

$$\boxed{\frac{\alpha_s N_c}{\pi} \chi_0(0, 1-\gamma^{(T)}(\omega)) = \omega.}$$

$$\gamma^{(T)}(\omega) = \frac{\hat{\alpha}_s}{\omega} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{\omega^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{\omega^6} + \ldots$$

ABF conjecture

In the approach of [Altarelli, Ball, Forte, 2000] the duality equation

$$\chi_-(0,\gamma_{gg}(\omega)) = \frac{\pi\omega}{\alpha_s N_c} = \chi_+(0,1-\gamma_{gg}(\omega)),$$

is assumed to hold **exactly** at fixed coupling in gluodynamics($n_F = 0$). We also know that $\gamma_{gg}(1) = 0$ for $n_F = 0$ exactly, due to momentum sum-rule, so one predicts:

$$\chi_+(0,1) = \frac{\pi}{\alpha_s N_c}.$$

From our **local** resummation we have [Kovner, Lublinsky, MN, Skokov 2026] :

$$\chi_+^{(\text{anti-coll, LLA})}(0,\gamma\to 1) = \frac{\pi}{\alpha_s N_c}\begin{cases} 4/3 & \text{for } n_F \neq 0, \\ 12/11 & \text{for } n_F = 0. \end{cases}$$

Also if one takes the $\gamma_{gg}^{(LO)} = 2C_A\left(\frac{1}{\omega} - \frac{1}{\omega+1} + \frac{1}{\omega+2} - \frac{1}{\omega+3} - H_{\omega+1}\right) + \frac{\beta_0}{2}$ then one predicts the series:

$\chi_+(\gamma) = \frac{1}{1-\gamma} + c_1\frac{a_s N_c}{(1-\gamma)^2} + c_2\frac{(a_s N_c)^2}{(1-\gamma)^3} + c_3\frac{(a_s N_c)^3}{(1-\gamma)^4} + \ldots$

c_i	Duality	**local** resummation ($n_F = 0$)
c_1	$-11/3$	$-11/3$
c_2	$389/9 - \mathbf{16\zeta(2)}$	$121/9$
c_3	$-4493/9 + 176\zeta(2) + 64\zeta(3)$	$-1331/27$

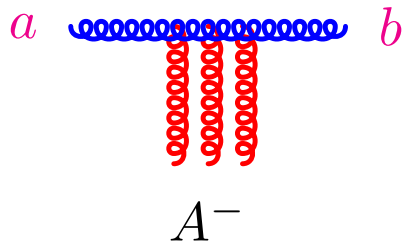
Anti-collinear resummation in JIMWLK: Non-linear duality

Let's "rise the sea level" *

Scattering amplitude of a gluon on a dense target (e.g. a nucleus A):

$$g_a(k) + A \to g_b(k') + A,$$

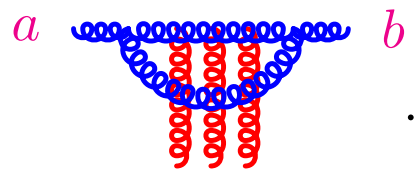
in the Regge limit ($k^+ \to \infty$) is described in the eikonal approximation, the S-matrix is:



$$= S_{ab}(\boldsymbol{x}) = \exp\left[ig \int_{-\infty}^{+\infty} dx^+ T^c A_c^-(x^+, 0^-, \boldsymbol{x})\right]_{ab}.$$

If the projectile gluon is boosted ($k^+ \to e^{\delta Y} k^+$) then the radiative correction includes:

$$\alpha_s \delta Y \times$$



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Which leads to the JIMWLK-evolution for the projectile $\mathcal{O}$.

* A. Grothendieck, "Recoltes et Semailles"

JIMWLK equation

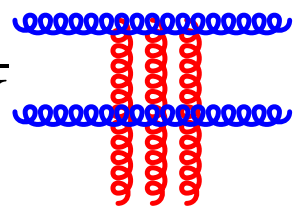
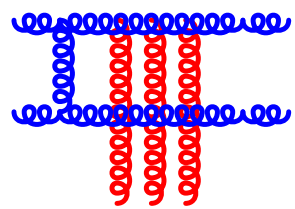
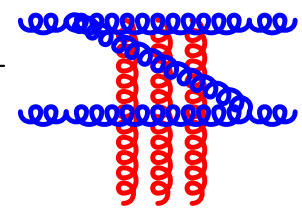
The operator of the projectile evolves with rapidity as:

$$\frac{\partial}{\partial Y}\mathcal{O} = \hat{H}^{(\mathrm{LO})}\mathcal{O},$$

where:

$$\hat{H}^{(\mathrm{LO})} = \frac{\alpha_s(\mu^2)}{2\pi^2}\int_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} K(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\left[J_L^a(\boldsymbol{x})J_L^a(\boldsymbol{y}) + J_R^a(\boldsymbol{x})J_R^a(\boldsymbol{y}) - S^{ab}(\boldsymbol{z})\left(J_L^a(\boldsymbol{x})J_R^b(\boldsymbol{y}) + J_L^a(\boldsymbol{y})J_R^b(\boldsymbol{x})\right)\right],$$

with $K(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = (\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{y}-\boldsymbol{z})/[(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2]$

$\frac{\partial}{\partial Y}$  $=$  $+$  $+\ldots$

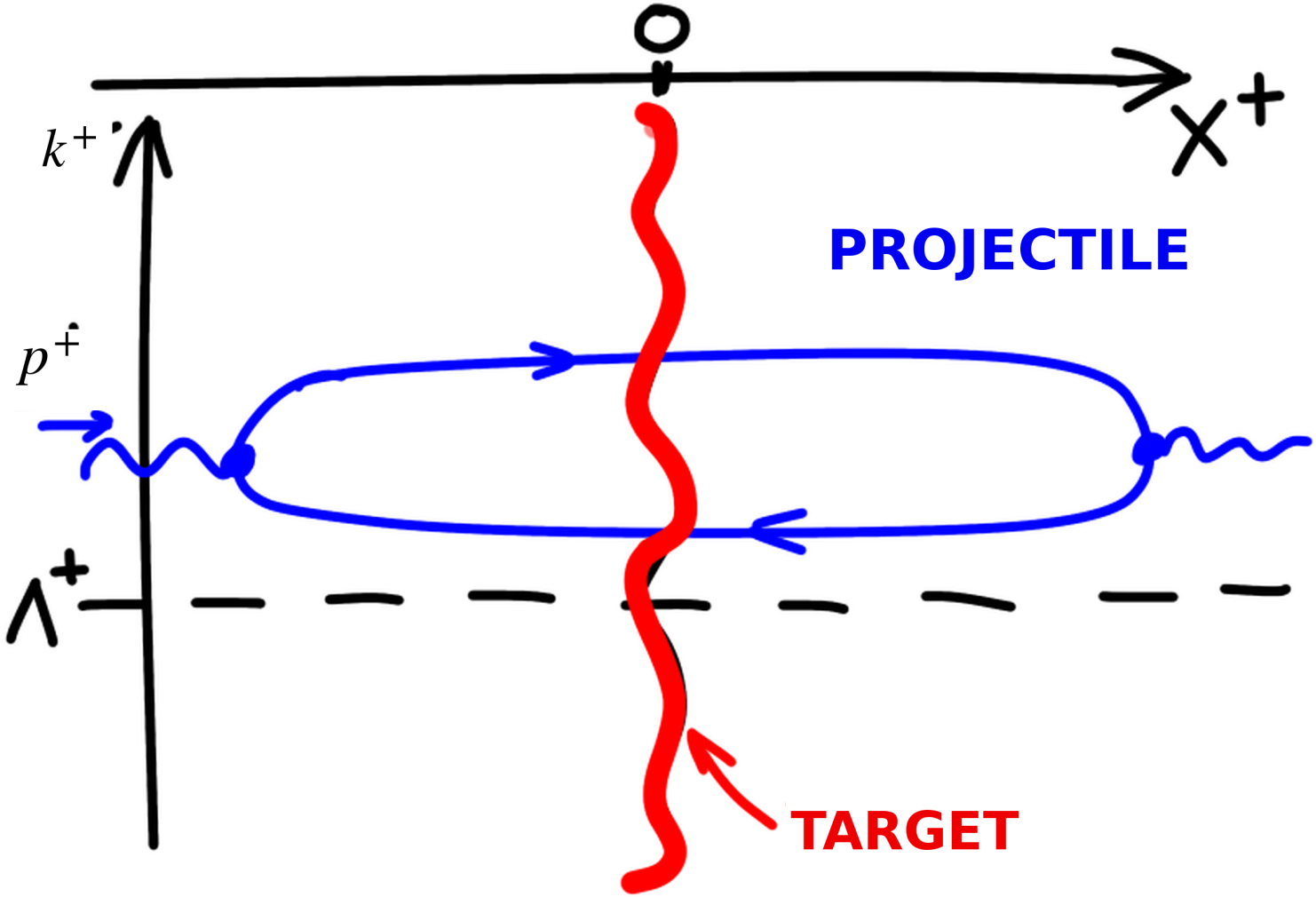
If the target is **not** dense, the JIMWLK picture reduces to BFKL and Wilson lines S^{ab} can be expanded in the Reggeized gluon fields $\alpha^a(\boldsymbol{x})$

[Lipatov '95;...;Kovner, Lublinsky, 2005;...;Caron-Huot 2012;...] :

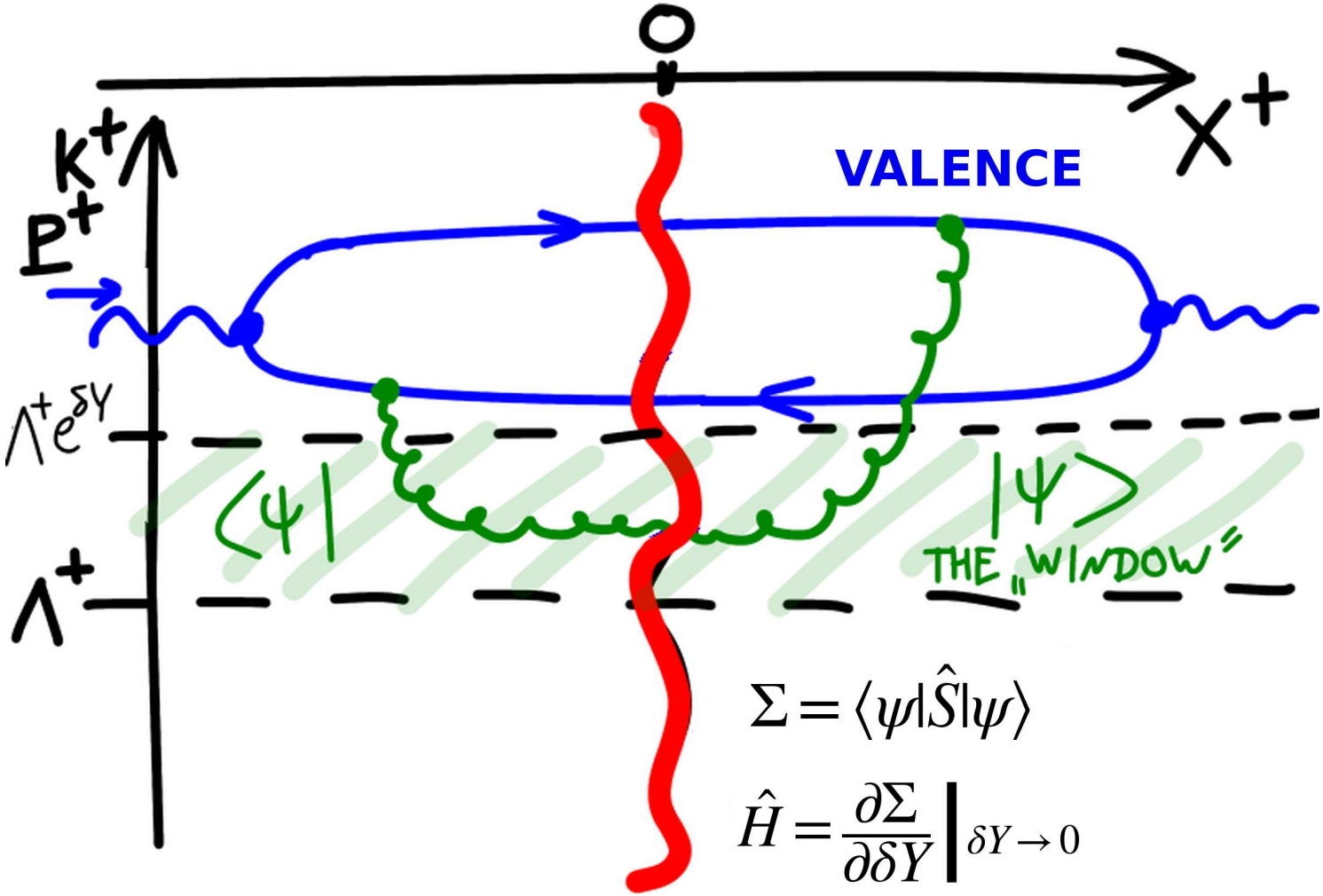
$$S^{ab}(\boldsymbol{x}) = \exp\left[igT^c\alpha_c(\boldsymbol{x})\right]^{ab},\ (ig)J^a_{(L,R)} = \left[F(\pm iT^c\alpha^c(\boldsymbol{x}))\right]^{ab}\frac{\delta}{\delta\alpha^b(\boldsymbol{x})},$$

with $F(x) = x/(1-e^{-x})$.

How to derive JIMWLK?



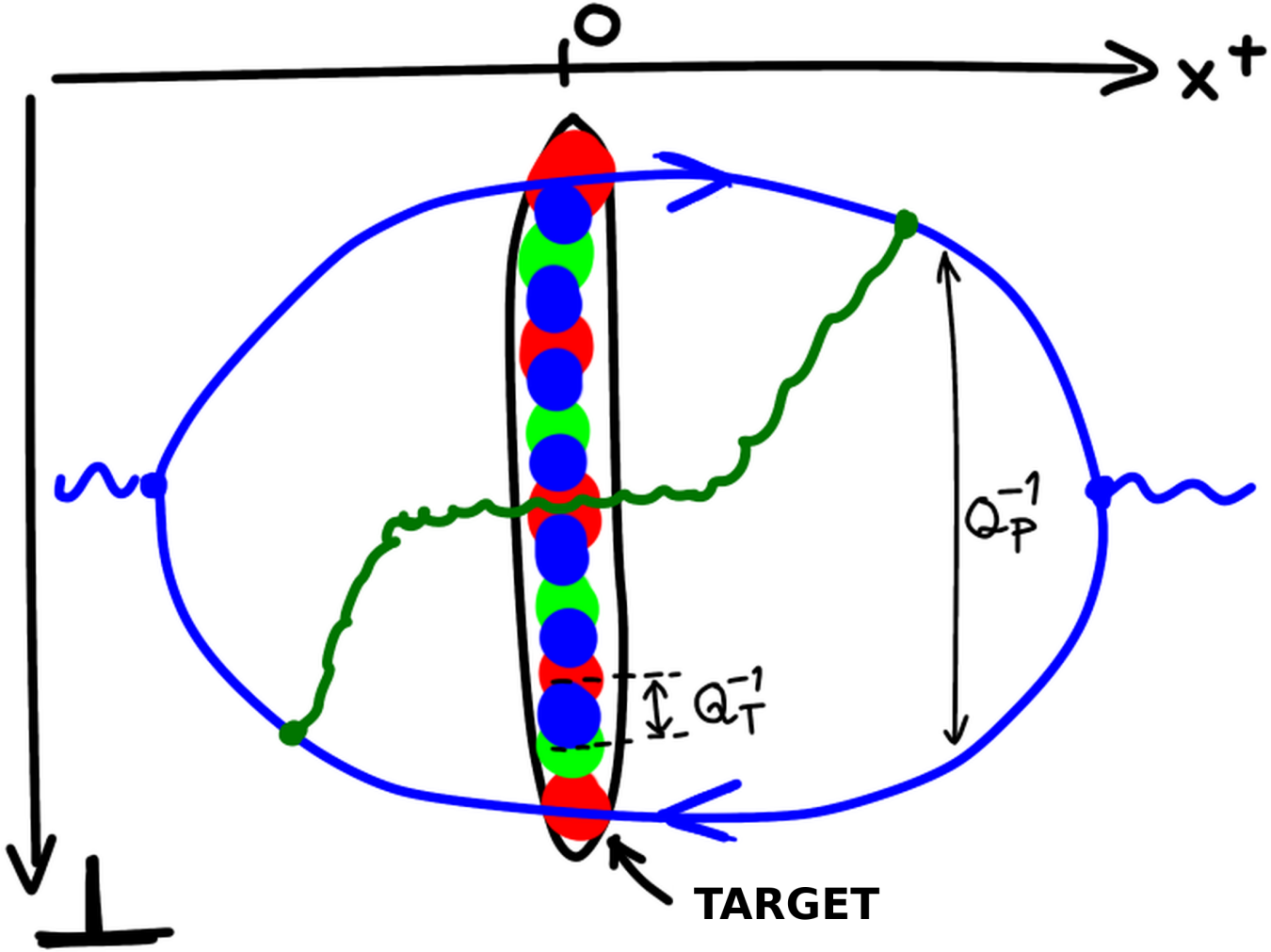
How to derive JIMWLK?



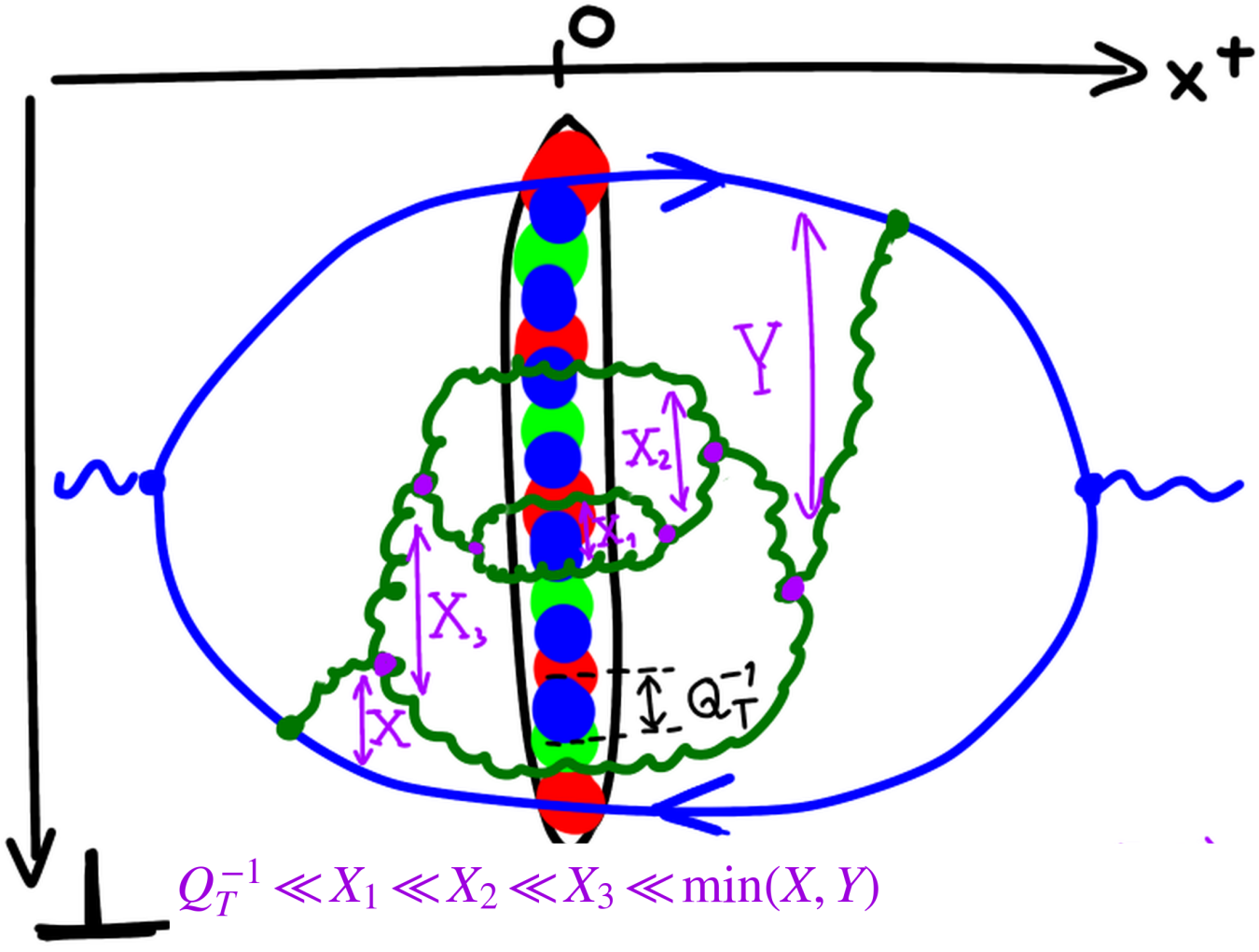
$$\Sigma = \langle \psi | \hat{S} | \psi \rangle$$

$$\hat{H} = \frac{\partial \Sigma}{\partial \delta Y} \Big|_{\delta Y \to 0}$$

Anti-collinear regime



Anti-collinear regime



Each splitting contributes to $\Sigma_\perp \sim \alpha_s \int \frac{d^2 \boldsymbol{X}_i}{\boldsymbol{X}_i^2}$.

Anti-collinear resummation in JIMWLK (gluons)

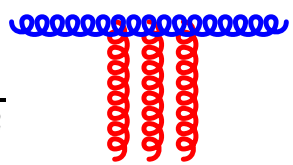
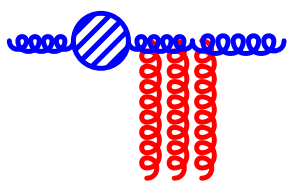
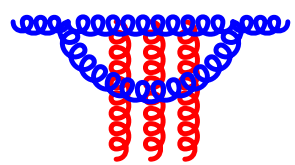
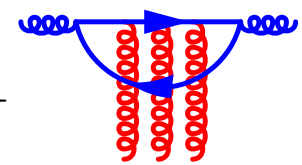
In [Kovner,Lublinsky,Skokov 2023] the method to resum large corrections to JIMWLK Hamiltonian, arising when $Q_T \gg Q_P$ was derived. One evolves the "bare" Wilson lines (S^{ab}) at the scale Q_T down to the scale Q_P ($\Rightarrow \mathbb{S}^{ab}_{Q_P}$):

$$\frac{\partial}{\partial \ln Q^2}\mathbb{S}_{ab}(\boldsymbol{x},Q) = -a\mathbb{S}_{ab}(\boldsymbol{x},Q)$$

$$-\frac{a}{\beta_0}\int\limits_0^1 d\xi \int\limits_0^{2\pi}\frac{d\phi}{2\pi}\Big[2N_c p_{gg}(\xi)\mathbb{D}_{ab}\left(\boldsymbol{x}+(1-\xi)Q^{-1}\boldsymbol{n}_\phi, \boldsymbol{x}-\xi Q^{-1}\boldsymbol{n}_\phi, Q\right)$$

$$+2T_F n_F p_{qg}(\xi)\mathbb{D}^{(F)}_{ab}\left(\boldsymbol{x}+(1-\xi)Q^{-1}\boldsymbol{n}_\phi, \boldsymbol{x}-\xi Q^{-1}\boldsymbol{n}_\phi, Q\right)\Big],$$

where $a = \alpha_s\beta_0/(4\pi)$, $\mathbb{D}_{ab}(\boldsymbol{x},\boldsymbol{y},Q) = \mathrm{tr}\left[T_a\mathbb{S}(\boldsymbol{x},Q)T_b\mathbb{S}^\dagger(\boldsymbol{y},Q)\right]/N_c$, $\mathbb{D}^{(F)}_{ab}(\boldsymbol{x},\boldsymbol{y},Q) = 2\,\mathrm{tr}\left[t_a\mathbb{V}(\boldsymbol{x},Q)t_b\mathbb{V}^\dagger(\boldsymbol{y},Q)\right]$.

$\frac{\partial}{\partial \ln Q^2}$  $=$  $+$  $+$ 

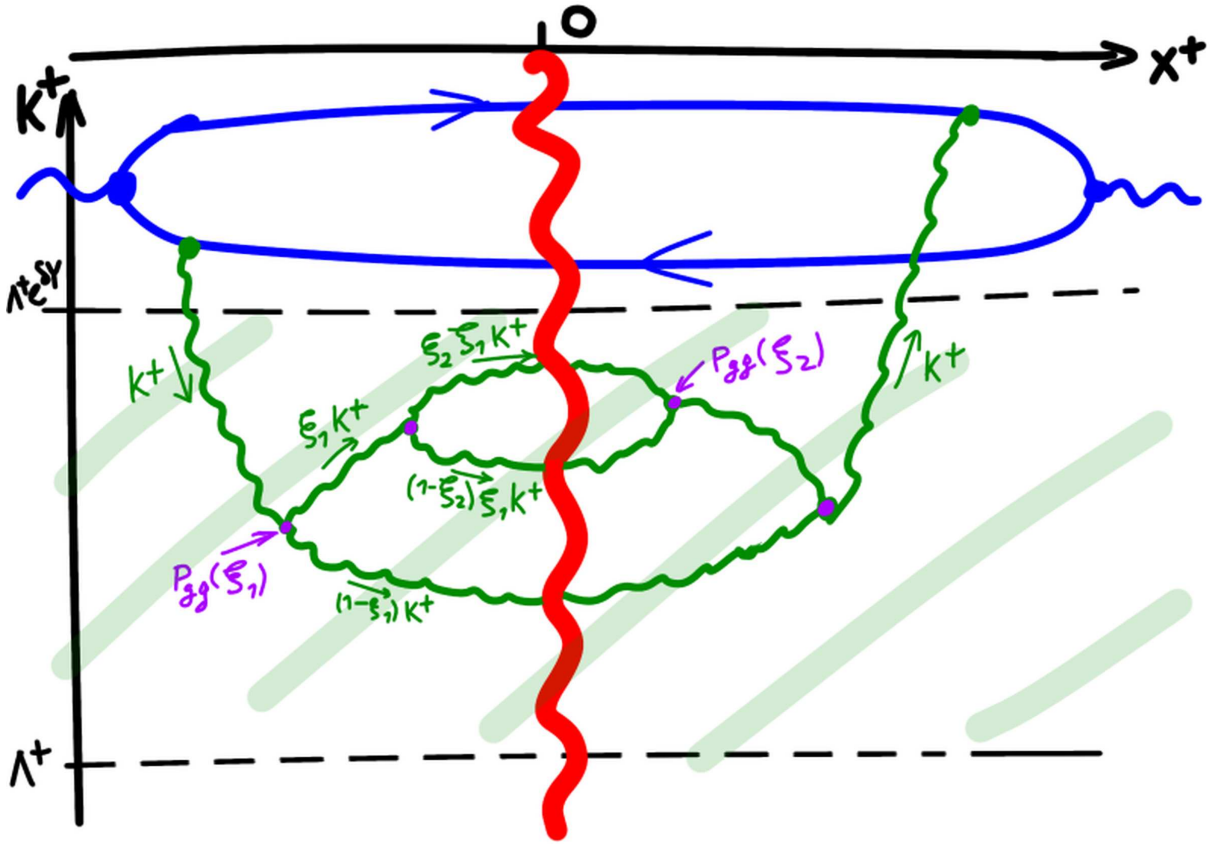
(Local) anti-collinear resummation in the JIMWLK Hamiltonian

$$\hat{H}^{(\text{res.})} = \frac{\alpha_s(\mu^2)}{2\pi^2} \int_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} K(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \left[J_L^a(\boldsymbol{x}) J_L^a(\boldsymbol{y}) + J_R^a(\boldsymbol{x}) J_R^a(\boldsymbol{y}) - \mathbb{S}_Q^{ab}(\boldsymbol{z}) \left(J_L^a(\boldsymbol{x}) J_R^b(\boldsymbol{y}) + J_L^a(\boldsymbol{y}) J_R^b(\boldsymbol{x}) \right) \right],$$

with the scale-choice [Kovner, Lublinsky, Skokov 2023] :

$$\boxed{Q^2 = \max((\boldsymbol{x}-\boldsymbol{z})^{-2}, (\boldsymbol{y}-\boldsymbol{z})^{-2}, Q_0^2)}$$

Longitudinal phase-space



$$\Sigma^{\parallel} = \int_{2\Lambda^+}^{2\Lambda^+ e^{\delta Y}} \frac{dk^+}{k^+} \int_{\Lambda^+/k^+}^{1-\Lambda^+/k^+} d\xi_1\, p_{gg}(\xi_1) \int_{\Lambda^+/(k^+\xi_1)}^{1-\Lambda^+/(k^+\xi_1)} d\xi_2\, p_{gg}(\xi_2)$$

$$p_{gg}(\xi) = \tfrac{1-\xi}{\xi} + \tfrac{\xi}{1-\xi} + \xi(1-\xi).$$

Longitudinal phase-space

$$\Sigma^{\parallel} = \int_{2\Lambda^+}^{2\Lambda^+ e^{\delta Y}} \frac{dk^+}{k^+} \int_{\Lambda^+/k^+}^{1-\Lambda^+/k^+} d\xi_1 \left(p_{gg}^{(\mathrm{NE})}(\xi_1) + \frac{1}{\xi_1} + \frac{1}{1-\xi_1} \right)$$

$$\times \left[2\theta\left(\xi_1 > \frac{2\Lambda^+}{k^+}\right) \ln \frac{k^+\xi_1}{\Lambda^+} + \int_0^1 d\xi_2\, p_{gg}^{(\mathrm{NE})}(\xi_2) + O(\Lambda^+/(k^+\xi_1)) \right].$$

where $p_{gg}^{(\mathrm{NE})}(\xi) = -2 + \xi(1-\xi)$.

$$\int_{2\Lambda^+/k^+}^{1-\Lambda^+/k^+} \frac{d\xi_1}{1-\xi_1} \ln \frac{k^+\xi_1}{\Lambda^+} = \ln^2 \frac{\Lambda^+}{k^+} - \zeta(2) + O(\Lambda^+/k^+).$$

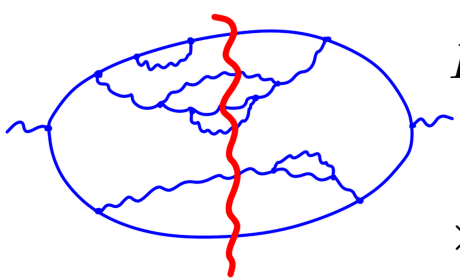
Non-local resummation equation: "Heisenberg form"

$$\mathbb{S}_Q^{ab}(\boldsymbol{x}) = S^{ab}(\boldsymbol{x}) - 2N_c a_s \int_{\Lambda^2}^{Q^2} \frac{d\bar{Q}^2}{\bar{Q}^2} \int_0^1 d\xi\, p_{gg}^{(+)}(\xi) \int_0^{2\pi} \frac{d\phi}{2\pi}$$

$$\times \Bigg[\frac{1}{N_c} \mathrm{tr} \Big[T^a \Big(e^{-\hat{H}_{\bar{Q}}^{(\mathbb{S})} \ln \xi} \mathbb{S}_{\bar{Q}}(\boldsymbol{x} + (1-\xi)\bar{Q}^{-1}\boldsymbol{n}_\phi) \Big)$$

$$\times T^b \Big(e^{-\hat{H}_{\bar{Q}}^{(\mathbb{S})} \ln(1-\xi)} \mathbb{S}_{\bar{Q}}^\dagger(\boldsymbol{x} - \xi\bar{Q}^{-1}\boldsymbol{n}_\phi) \Big) \Big] - \mathbb{S}_{\bar{Q}}^{ab}(\boldsymbol{x}) \Bigg],$$

with $p_{gg}^{(+)}(\xi) = \frac{1-\xi}{\xi_+} + \frac{\xi}{(1-\xi)_+} + \xi(1-\xi)$.



RAINBOW TRUNCATION

$$\hat{H}_Q^{(\mathbb{S})} = -4a_s \int_0^\infty d\bar{Q} \int_{\Lambda^2}^{\bar{Q}^2} \frac{dq^2}{q^2} \theta(q^2 > Q^2) \int_{\boldsymbol{x}} \int_0^{2\pi} \frac{d\theta}{2\pi}$$

$$\times \Big[C_A \mathbb{S}_q^{ab}(\boldsymbol{x}) - \mathrm{tr}[T^a \mathbb{S}_q(\boldsymbol{x}) T^b \mathbb{S}_q^\dagger(\boldsymbol{x} + q^{-1}\boldsymbol{n}_\theta)] \Big] \frac{\delta}{\delta \mathbb{S}_{\bar{Q}}^{ab}(\boldsymbol{x})}.$$

Non-local resummation: "Schroedinger form"

To get rid of exponents, introduce $\mathbb{S}_Q^{ab}(\boldsymbol{x}, x)$:

$$\mathbb{S}_Q^{ab}(\boldsymbol{x}) = \mathbb{S}_Q^{ab}(\boldsymbol{x}, x = 1).$$

$$\frac{d\mathbb{S}_Q^{ab}(\boldsymbol{x}, x)}{d\ln Q^2} = -2N_c a_s \int_0^1 d\xi \int_0^{2\pi} \frac{d\phi}{2\pi}$$
$$\times \Bigg\{ p_{gg}^{(+)}(\xi) \Bigg[\frac{1}{N_c} \operatorname{tr} \left[T^a \left(\mathbb{S}_{\bar{Q}}(\boldsymbol{x} + (1-\xi)\bar{Q}^{-1}\boldsymbol{n}_\phi, x\xi) \right) \right.$$
$$\left. \times T^b \left(\mathbb{S}_{\bar{Q}}^\dagger(\boldsymbol{x} - \xi\bar{Q}^{-1}\boldsymbol{n}_\phi, x(1-\xi)) \right) \right] - \mathbb{S}_{\bar{Q}}^{ab}(\boldsymbol{x}, 1) \Bigg]$$
$$- \frac{2\theta(\xi - x)}{\xi} \left[\frac{1}{N_c} \operatorname{tr}[T^a \mathbb{S}_q(\boldsymbol{x}, \xi) T^b \mathbb{S}_q^\dagger(\boldsymbol{x} + q^{-1}\boldsymbol{n}_\phi, \xi)] - \mathbb{S}_q^{ab}(\boldsymbol{x}, \xi) \right] \Bigg\},$$

solved with initial condition: $\mathbb{S}_\Lambda^{ab}(\boldsymbol{x}, x) = S^{ab}(\boldsymbol{x})$.

Linearisation

To obtain resummed BFKL equation, we expand the resummed Wilson line in terms of Reggeised gluons $\alpha_c(\boldsymbol{z})$ [Kovner, Lublinsky, MN, Skokov 2026] :

$$\mathbb{S}_{ab}(\boldsymbol{z},x,Q)=\delta_{ab}+igT^{c_1}_{ab}\int\limits_{\boldsymbol{z}_1}R^{(1)}_Q(\boldsymbol{z}-\boldsymbol{z}_1,x)\alpha_{c_1}(\boldsymbol{z}_1)$$

$$+\frac{(ig)^2}{2}(T^{c_1}T^{c_2})_{ab}\int\limits_{\boldsymbol{z}_1\boldsymbol{z}_2}R^{(2,1)}_Q(\boldsymbol{z}-\boldsymbol{z}_1,\boldsymbol{z}-\boldsymbol{z}_2,x)\alpha_{c_1}(\boldsymbol{z}_1)\alpha_{c_2}(\boldsymbol{z}_2)+\ldots,$$

LLA equation for $R^{(2,1)}_Q$ in momentum representation:

$$\frac{d}{d\ln Q^2}R^{(2,1)}_{\mathrm{LLA},Q}(\boldsymbol{p},x)+2N_ca_s\int\limits_0^1 d\xi\left[2p^{(+)}_{gg}(\xi)R^{(2,1)}_{\mathrm{LLA},Q}(\boldsymbol{p},x\xi)-2\frac{\theta(\xi-x)}{\xi}R^{(2,1)}_{\mathrm{LLA},Q}(\boldsymbol{p},\xi)\right]$$

$$+a_s\beta^{(g)}_0R^{(2,1)}_{\mathrm{LLA},Q}(\boldsymbol{p},x)=-2N_ca_s\theta(Q>|\boldsymbol{p}|)\left(\frac{\beta^{(g)}_0}{2N_c}-2\ln x\right)\theta(1-x),$$

solved with initial condition $R^{(2,1)}_{\mathrm{LLA},\Lambda}(\boldsymbol{p},x)=1$ and limit $\Lambda\to\infty$ is taken.

Linearisation: equivalence with ABF

Contribution to the BFKL characteristic function $\chi(\gamma)$ is $R_{\pm}^{(2,1)}(\gamma, x=1)$, where:

$$R_{\pm}^{(2,1)}(\gamma, x) = \int_{Q^2}^{\infty} \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2} \left(\frac{\boldsymbol{p}^2}{Q^2}\right)^{\gamma-1} R_{\pm,Q}^{(2,1)}(\boldsymbol{p}, x).$$

Look for power-law solution $R_{\pm}^{(2,1)}(\gamma, x) \propto x^{N(\gamma)}$:

$$\boxed{2a_s\gamma_{gg}(N_\star(\gamma)) = 1-\gamma,}$$

$$R_{\mathrm{LLA}}^{(2,1)}(\gamma, x) = \frac{N_\star(\gamma)}{4N_c a_s} x^{N_\star(\gamma)}.$$

The framed equation is exactly the ABF equation in gluodynamics!

Conclusions and outlook

- The non-linear generalisation of the (J)ABF resummation for the BFKL kernel **in the anti-collinear case** has been found!
- Including quarks and running coupling is conceptually easy!
- The solution of the full non-linear equation is possible with the (PS-)Monte-Carlo method, at least at large N_c. So we can have fully resummed BK! (Stay tuned)

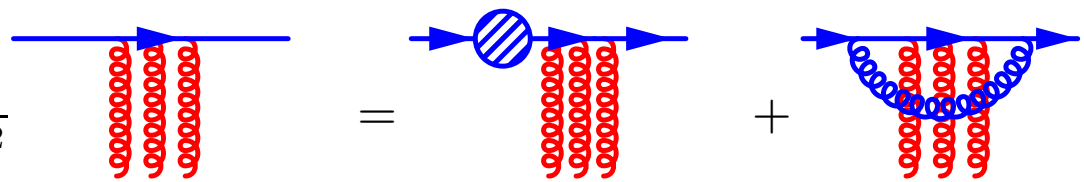
Thank you for your attention!

(anti-)Collinear resummation in JIMWLK (quarks)

In [Kovner,Lublinsky,Skokov 2023] the method to resum large corrections to JIMWLK Hamiltonian, arising when $Q_T \gg Q_P$ was derived. One evolves the "bare" Wilson lines (V^{ij}) at the scale Q_T down to the scale Q_P ($\Rightarrow \mathbb{V}^{ij}_{Q_P}$):

$$\frac{\partial}{\partial \ln Q^2}\mathbb{V}_{ij}(\boldsymbol{x},Q) = -\frac{3C_F}{2}\frac{2a}{\beta_0}\mathbb{V}_{ij}(\boldsymbol{x},Q)$$
$$-\frac{2a}{\beta_0}\int_0^1 d\xi \int_0^{2\pi}\frac{d\phi}{2\pi} p_{gq}(\xi)\mathbb{D}^{(FA)}_{ij}\left(\boldsymbol{x}+(1-\xi)Q^{-1}\boldsymbol{n}_\phi, \boldsymbol{x}-\xi Q^{-1}\boldsymbol{n}_\phi, Q\right),$$

where $a = \alpha_s\beta_0/(4\pi)$, $\mathbb{D}^{(FA)}_{ij}(\boldsymbol{x},\boldsymbol{y},Q) = \left[t_a\mathbb{V}(\boldsymbol{y},Q)t_b\right]_{ij}\mathbb{S}_{ab}(\boldsymbol{x},Q)$.

$\frac{\partial}{\partial \ln Q^2}$  $=$ $+$