Threshold-Aware Evolution for Heavy-Quarkonia via Heavy Flavor Non-Relativistic evolution HF-NRevo

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arXiv:2510.10593 [hep-ph]



NRFF1.0: A New Generation of FFs

NRFF1.0 is a public set of collinear Fragmentation Functions (FFs) for quarkonium production at moderate to large p_T.
`https://github.com/FGCeliberto/Collinear_FFs`

Key Features

- **Framework:** Built on the single-parton fragmentation approximation in a Variable Flavor Number Scheme (VFNS).
- **Consistency:** First set to include **all partonic channels** with consistent flavor threshold treatments.
- **Pseudoscalar Sector:** Construction for η_c and η_b uses fully NLO-accurate NRQCD inputs.

- **It replaces** $ZCW19^+$: **F. G. Celiberto and M. Fucilla, Eur. Phys. J. C 82, 929 (2022), 2202.12227**
  It systematically accounts for heavy quark thresholds during DGLAP evolution via the **HF-NRevo** scheme.

Non-Relativistic QCD (NRQCD)

The NRQCD Lagrangian

The total Lagrangian is constructed from all operators permitted by gauge and Lorentz symmetries:

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_g + \mathcal{L}_l + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

where subscripts denote *gluons (g)*, *light quarks (l)*, *Pauli spinors (ψ)*, and *anti-spinors (χ)*.

Expansion at $\mathcal{O}(1/m_Q^2)$

Including the kinetic term $\frac{\mathbf{D}^4}{8m_Q^3}$, the heavy quark sector $\mathcal{L}_\psi$ is:

$$\mathcal{L}_\psi = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m_Q} + c_F \frac{g\boldsymbol{\sigma} \cdot \mathbf{B}}{2m_Q} + c_D \frac{g[\mathbf{D} \cdot \mathbf{E}]}{8m_Q^2} + \dots \right) \psi$$

NLO NRQCD: General Features

Improvements over Leading Order (LO)

- **Scale Stability:** Significantly reduces renormalization and factorization scale dependence.
- **Color-Octet (CO) Mechanisms:** Accounts for $Q\bar{Q}$ pairs produced in CO states that evolve into mesons via soft gluon emission.
- **Universality:** Essential for testing Long-Distance Matrix Elements (LDMEs) across different colliders.

Quarkonium Production: Factorization and Velocity Scaling

NRQCD Power Counting: Theoretical predictions are organized as double expansions in the strong coupling α_s and the heavy-quark relative velocity v.

- **Velocity Scaling Rules:** Estimate the importance of Long-Distance Matrix Elements (LDMEs).
- **Non-Relativistic Limit:** Valid for $v \ll 1$ (where $v^2 \approx 0.3$ for $c\bar{c}$ and $v^2 \approx 0.1$ for $b\bar{b}$).
- **Predictive Power:** Allows for a systematic truncation of the infinite series of LDMEs.

Fock State Expansion and Color-Octet Mechanism

NRQCD Factorization: The $Q\bar{Q}$ pair is produced at short distances in a Fock state $n = {}^{2S+1}L_J^{[a]}$, where $a = 1$ (Singlet) or $a = 8$ (Octet).

- **Transition:** Intermediate CO states evolve into physical quarkonia via *non-perturbative* soft gluon emission.
- **Limit:** As $v \to 0$, NRQCD recovers the Color Singlet Model (CSM) for S-wave states.

Table: Leading velocity scaling of LDMEs

State	$^1S_0^{(1)}$	$^3S_1^{(1)}$	$^1S_0^{(8)}$	$^3S_1^{(8)}$	$^3P_0^{(8)}$	$^3P_1^{(8)}$	$^3P_2^{(8)}$
η_c, η_b	1	–	v_Q^4	v_Q^3	v_Q^4	v_Q^4	v_Q^4
$J/\psi, \Upsilon$	–	1	v_Q^3	v_Q^4	v_Q^4	v_Q^4	v_Q^4

Perturbative Accessibility

Quarkonium $[\mathcal{Q}]$	$g \to \mathcal{Q}$	$Q \to \mathcal{Q}$	$\tilde{q}$ or $\tilde{Q} \to \mathcal{Q}$
$\mathcal{Q} \equiv \boldsymbol{\mathcal{S}_Q^-}\,[^1\boldsymbol{S}_0^{(1)}] \equiv \eta_c, \eta_b$	$\mathcal{O}(\alpha_s^2) \equiv \text{LO}$ $\mathcal{O}(\alpha_s^3) \equiv \text{NLO}$ $\mathcal{O}(\alpha_s^4) \equiv \text{NNLO}$ $\vdots$	$\mathcal{O}(\alpha_s^2) \equiv \text{LO}$ $\mathcal{O}(\alpha_s^3) \equiv \text{NLO}$ $\mathcal{O}(\alpha_s^4) \equiv \text{NNLO}$ $\vdots$	$-$ $\mathcal{O}(\alpha_s^3) \equiv \text{NLO}$ $\mathcal{O}(\alpha_s^4) \equiv \text{NNLO}$ $\vdots$
$\mathcal{Q} \equiv \boldsymbol{\mathcal{V}_Q}\,[^3\boldsymbol{S}_1^{(1)}] \equiv J/\psi, \Upsilon$	$-$ $\mathcal{O}(\alpha_s^3) \equiv \text{NLO}$ $\mathcal{O}(\alpha_s^4) \equiv \text{NNLO}$ $\vdots$	$\mathcal{O}(\alpha_s^2) \equiv \text{LO}$ $\mathcal{O}(\alpha_s^3) \equiv \text{NLO}$ $\mathcal{O}(\alpha_s^4) \equiv \text{NNLO}$ $\vdots$	$-$ $-$ $\mathcal{O}(\alpha_s^4) \equiv \text{NNLO}$ $\vdots$

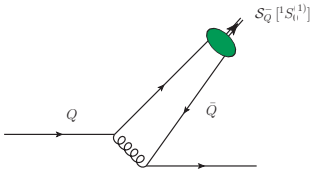
NRFF1.0: Theoretical Foundation at μ_0

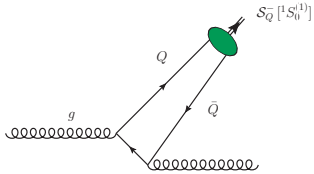
According to NRQCD

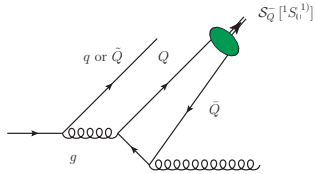
$$D_a^{\mathcal{Q}}(z, \mu_{F,0}) = \sum_{[n]} \mathcal{D}_a^{\mathcal{Q}}(z, [n]) \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle \ .$$

All parton channels receive initial-scale inputs from NRQCD.

NRFF1.0: Theoretical Foundation at μ_0







HF-NRevo Thresholds for η_c and η_b

$\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ quarkonium	$\mu_{F,0}(g \to \mathcal{S}_Q^-)$	$\mu_{F,0}(d,u,s \to \mathcal{S}_Q^-)$	$\mu_{F,0}(Q \to \mathcal{S}_Q^-)$	$\mu_{F,0}(\tilde{Q} \to \mathcal{S}_Q^-)$	$Q_0 \equiv \max(\{\mu_{F,0}\})$
η_c	$2m_c$	$m_{d,u,s} + 2m_c$	$3m_c$	$\boldsymbol{m_b + 2m_c}$	$\boldsymbol{m_b + 2m_c}$
η_b	$2m_b$	$m_{d,u,s} + 2m_b$	$\boldsymbol{3m_b}$	$m_c + 2m_b$	$\boldsymbol{3m_b}$

The HF-NRevo Scheme: Bridging the Thresholds for η_c

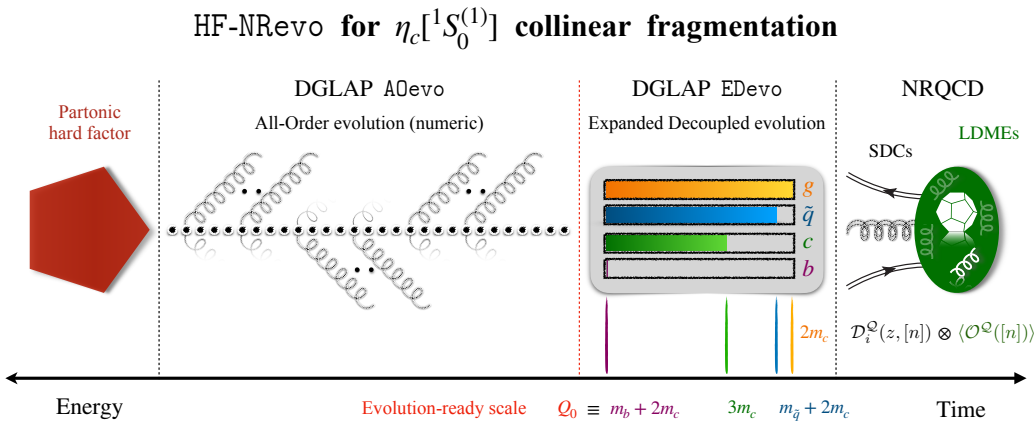


Figure: Evolution workflow within the HF-NRevo scheme

The HF-NRevo Scheme: Bridging the Thresholds for η_b

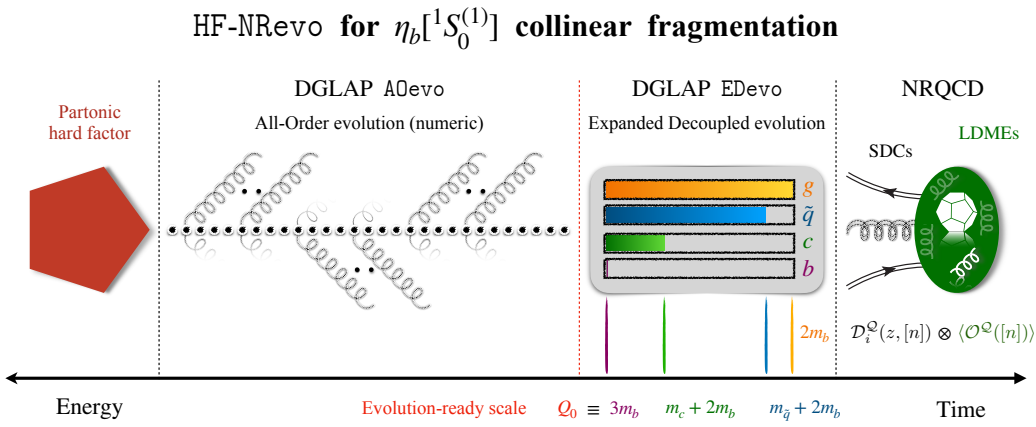
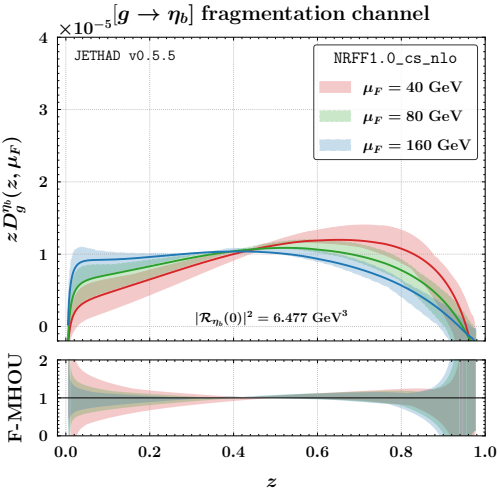
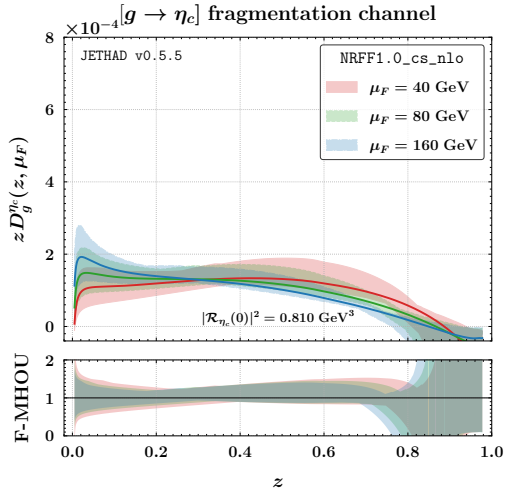


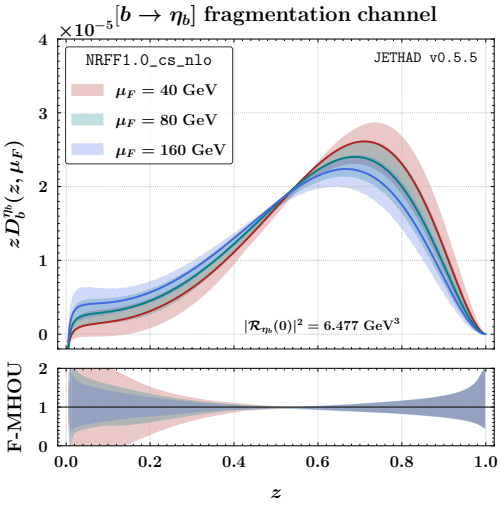
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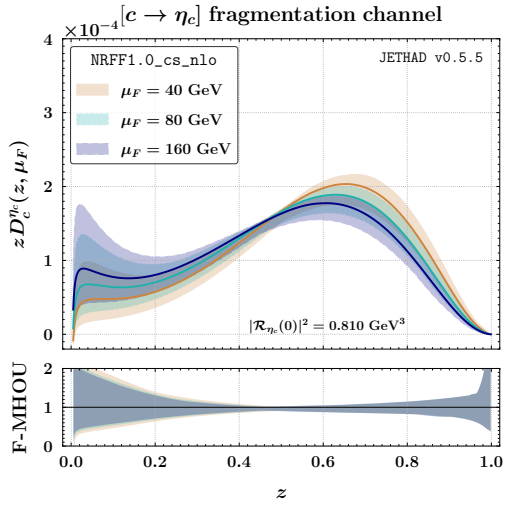
Quarkonium FFs from HF-NRevo evolution - Gluon MHOU uncertainties



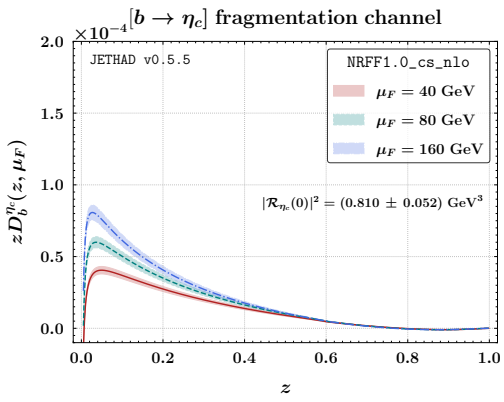


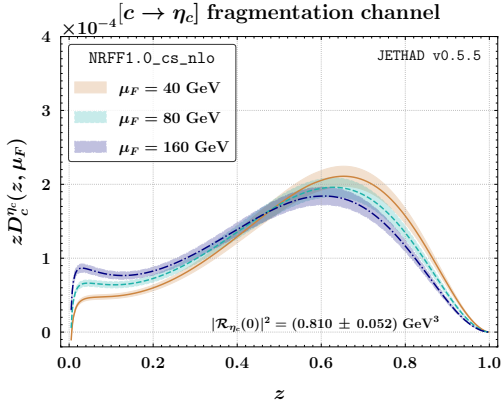
Quarkonium FFs from HF-NRevo evolution - Quark MHOU uncertainties



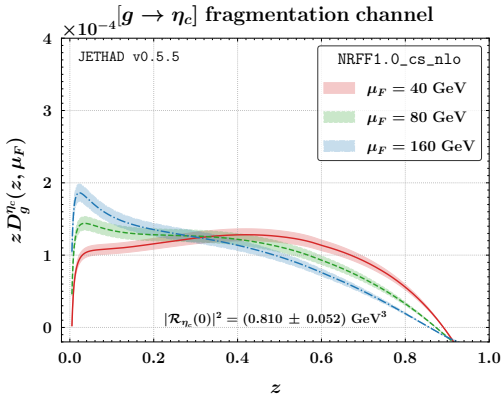


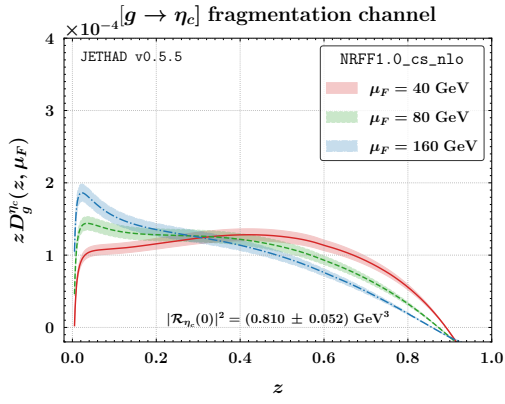
Quarkonium FFs from HF-NRevo evolution - Quark LDME uncertainties





Quarkonium FFs from HF-NRevo evolution - Gluon LDME uncertainties





Factorization scale dependence

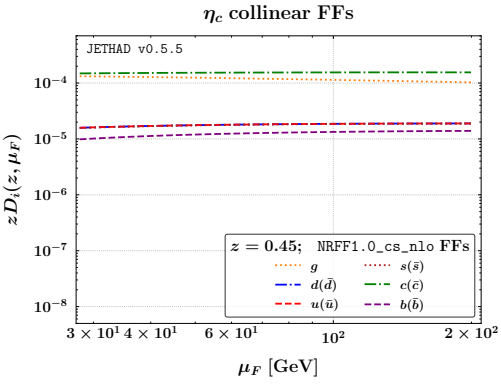


Figure: Factorization scale dependence of NRFF1.0 NLO FFs depicting $\eta_c[^1S_0^1]$

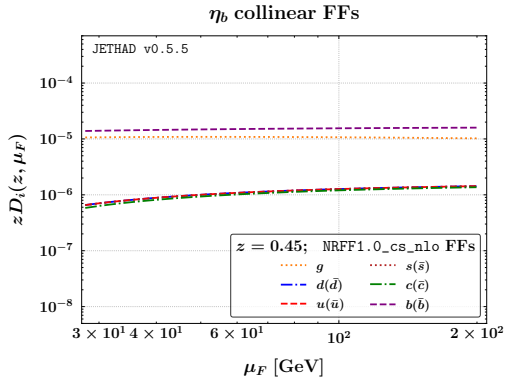


Figure: Factorization scale dependence of NRFF1.0 NLO FFs depicting $\eta_b[^1S_0^1]$

Natural Stability $\rightarrow$ **F. G. Celiberto, Acta Phys. Polon. Supp. 16, 41 (2023), 2211.11780**

Phenomenological Application: Hybrid Factorization (HyF) Framework

Concept: Combines High-Energy (BFKL) and Collinear factorization to describe processes at NLL/NLO+ accuracy.

- **High-Energy Resummation:** Incorporates BFKL Green's functions with singly off-shell emission functions (forward production impact factors).

The differential cross section is expressed as a Fourier expansion:

$$\frac{(2\pi)^2 d\sigma}{dy_1 dy_2 d|\vec{q}_{T1}| d|\vec{q}_{T2}| d\phi_1 d\phi_2} = \mathcal{C}_0 + 2\sum_{n=1}^{\infty} \cos(n\phi)\mathcal{C}_n$$

where $\phi \equiv \phi_1 - \phi_2 - \pi$.

Hybrid factorization for the semi-inclusive hadroproduction of a η_c and η_b quarkonium accompanied by a hadron or a light jet via JETHAD.

F.G. Celiberto, Eur. Phys. J. C 81 (2021) 8, 691 [2008.07378 [hep-ph]]
F.G. Celiberto, Phys. Rev. D 105 (2022) 11, 114008 [2204.06497 [hep-ph]]

HyF Anatomy

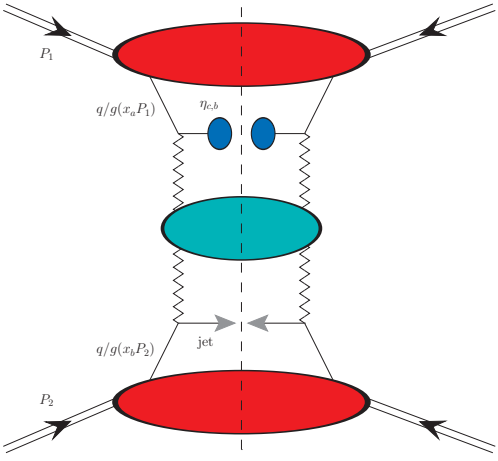


Figure: Quarkonium accompanied by light jet

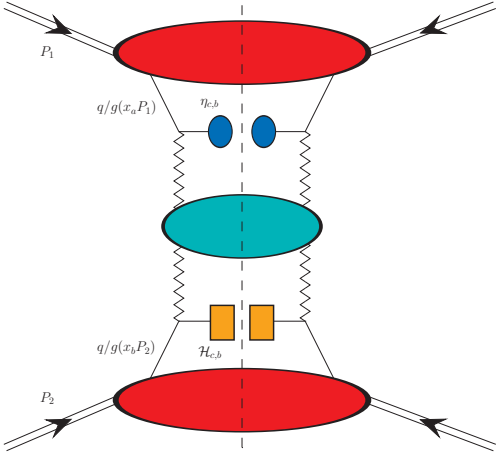


Figure: Quarkonium accompanied by a $H_{c,b}$ hadron

HyF Results

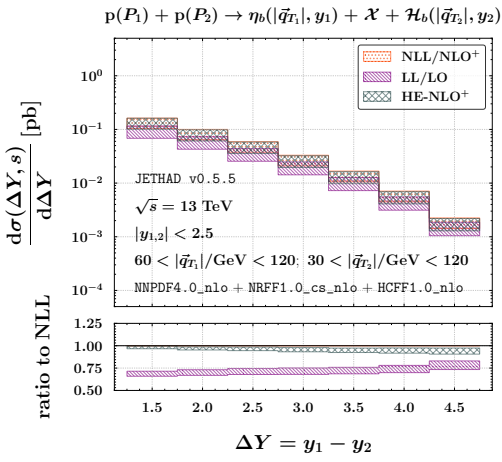


Figure: Rapidity distributions for the semi-inclusive hadroproduction of η_b with a hadron

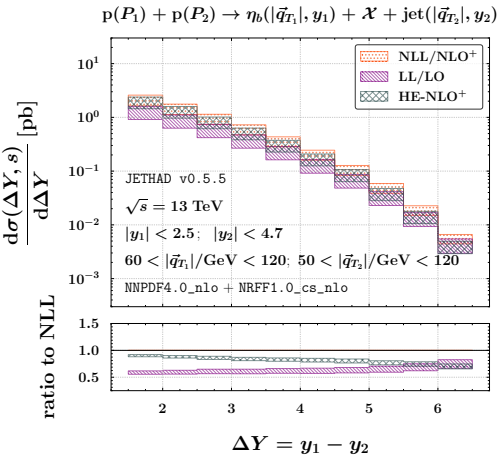


Figure: Rapidity distributions for the semi-inclusive hadroproduction of η_b with a jet

HyF Results

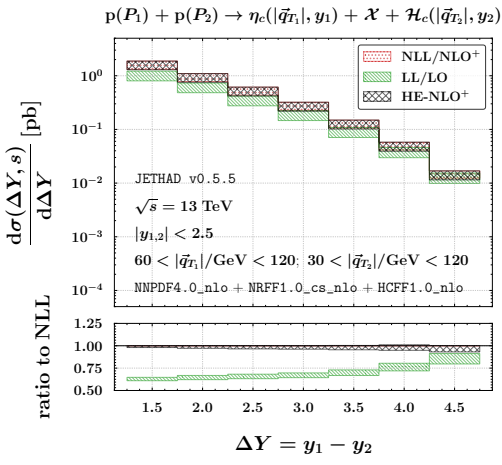


Figure: Rapidity distributions for the semi-inclusive hadroproduction of η_c with a hadron

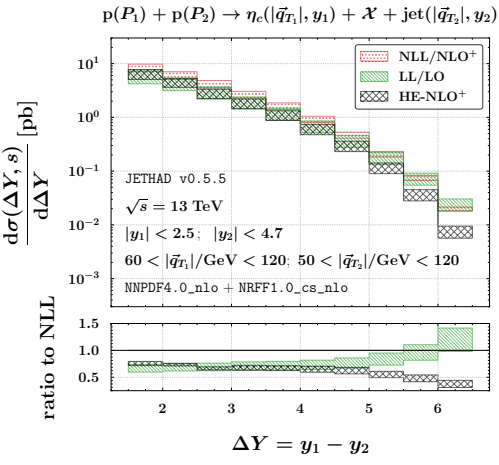


Figure: Rapidity distributions for the semi-inclusive hadroproduction of η_c with a jet

Outlook: The Next Generation

- **Vector States:** The HF-NRevo scheme is currently being extended to J/ψ and Υ, where the color-octet contributions are more complex.
  `https://indico.global/event/16406/contributions/166873/` **by J. P. Lansberg**
- **TMD Evolution & Matching:** Future work involves moving beyond collinear factorization to Transverse Momentum Dependent (TMD) fragmentation.
  `https://indico.global/event/16406/contributions/167179/` **by P. Taels**
- **Exotic States:** This framework provides the consistent basis for studying the **compact/diquark** fragmentation of tetraquarks and other exotics.
- Investigating how **intrinsic charm** manifests itself in high-energy hadronic processes.
  `https://indico.global/event/16406/contributions/162120/` **by F. G. Celiberto**
- Production of pseudoscalar quarkonia in **jets and Quark Gluon Plasma**.
- Towards **NRFF1.1**: vector quarkonia, color octet and uncertainties

Thank you!

BACK UP

Phenomenological Analysis

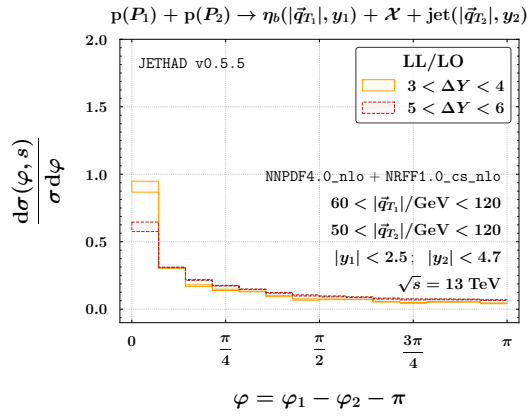


Figure: Angular multiplicities for the semi-inclusive hadroproduction

Phenomenological Analysis

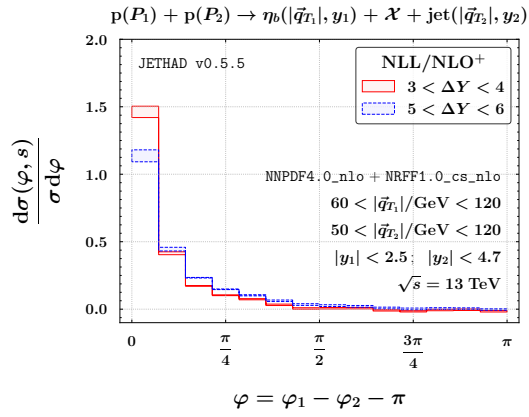


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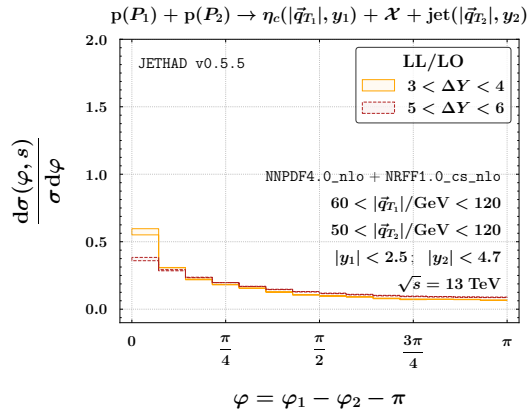


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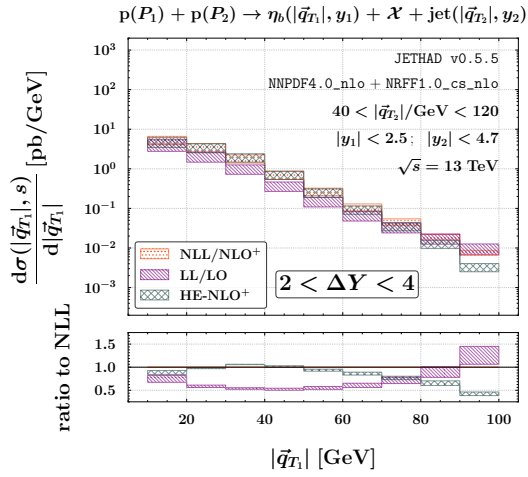


Figure: Transverse momentum distributions for the semi-inclusive hadroproduction

Phenomenological Analysis

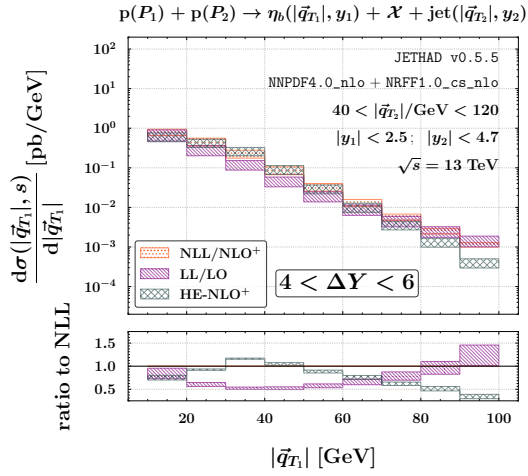


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