Phenomenological results on pion unpolarised TMDPDF

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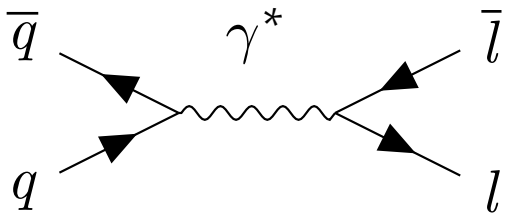
extraction of pion PDFs

- theoretical approaches via model calculations
- determination in LQCD with
  - computation of Mellin moments from local operators
  - highly boosted hadrons (LaMET)
  - short distance factorisation
- phenomenology from experimental data:
  problem : pion cannot be used as target / primary beam
  - pion as **secondary** beam:
    - Drell-Yan (DY)
    - direct photon
    - J/ψ production
  - pion as **virtual** target : $p \to n + \pi^{+,*}$ (Sullivan process)

TMDs in scattering processes:

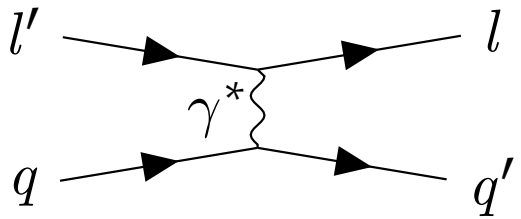
There are **three channels** to access the (quark) TMDs (PDFs & FFs):

Drell-Yan



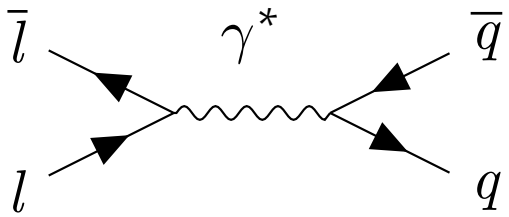
PDF $\otimes$ PDF

SIDIS



PDF $\otimes$ FF

SIA



FF $\otimes$ FF

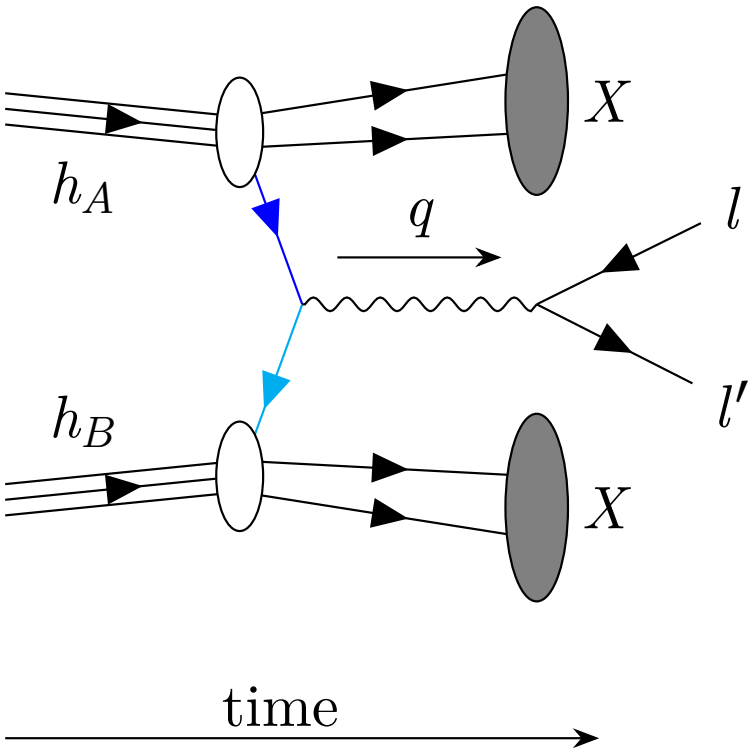
- TMD**PDF** for initial quark states
- TMD**FF** for final quark states

The Drell Yan process

Drell-Yan reaction: **boson** production in hadronic background

$$h_1(p_1) + h_2(p_2) \longrightarrow \boldsymbol{\gamma/Z/W(q)} + X \longrightarrow l(k) + l'(k') + X$$

Two colliding hadrons → two parton distribution functions necessary.



Kinematics:

$$q = k + k' \qquad Q^2 = q^2$$

$$x_{1,2} \approx \frac{Q^2}{2q \cdot p_{1,2}} \qquad \boldsymbol{q_T} = |q \perp \langle p_1, p_2 \rangle|$$

$$x_F = x_1 - x_2 \qquad y = \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right)$$

The Drell Yan process: TMD factorisation

In case of small transverse boson momentum ($q_T^2 \ll Q^2$):

$$d\sigma_{\mathrm{DY}} \sim \mathcal{C}(Q,\mu) \sum_f e_f^2 \int_0^\infty db\, b\, J_0(bq_T) f_{1,f}(x_1, b, \mu, \zeta_1) f_{1,f}(x_2, b, \mu, \zeta_2)$$

where $\zeta_1 \zeta_2 \simeq Q^4$

!!cross-section = convolution of two TMDs!!

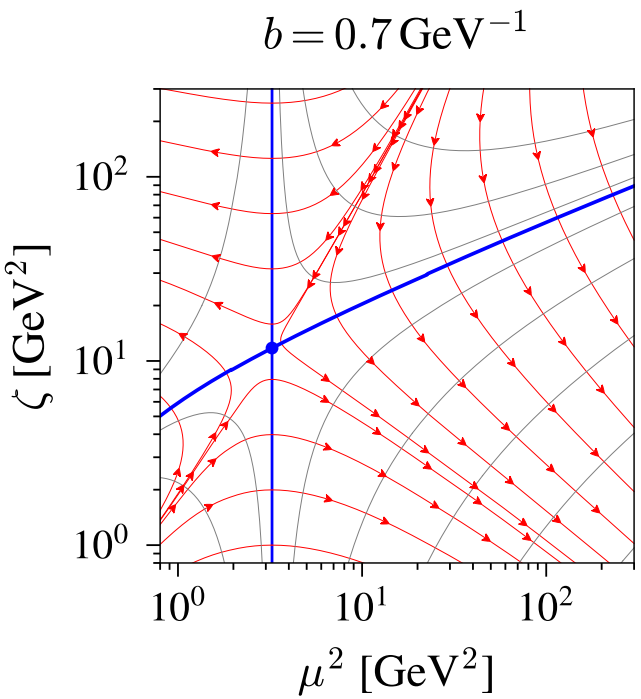
Extract the non-perturbative functions by assuming a functional form and extracting the parameters from experimental data.

Double scale dependence: evolution equations

$f(x, b; \mu, \zeta)$

$$\mu^2 \frac{d}{d\mu^2} f(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} f(x, b; \mu, \zeta) \quad (1)$$

$$\zeta \frac{d}{d\zeta} f(x, b; \mu, \zeta) = -\mathcal{D}(b, \mu) f(x, b; \mu, \zeta) \quad (2)$$



- define TMDPDF at reference scales: saddle point where $(1) = (2) = 0$
- evolve $(\mu, \zeta)_{\text{saddle}} \longrightarrow (Q, Q^2)_{\text{cross-section}}$

Form of TMDPDFs:

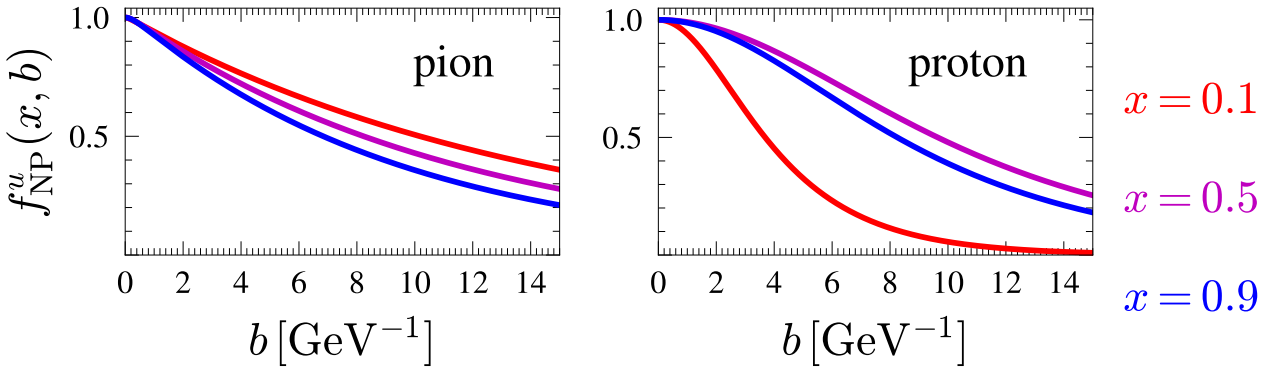
$$f_{1,f}(x,b) = \left(\sum_{f'} C_{f\to f'} \otimes q_{f'} \right)(x,b) \quad \cdot \quad f^{f}_{\mathrm{NP}}(x,b)$$

- TMDPDF ~ *perturbative* if transverse distance $b \ll \Lambda^{-1}_{\mathrm{QCD}}$ is small
- *perturbative* part is expansion and convolution with PDFs
- Fit simpler AND relation TMDs $\leftrightarrow$ PDFs satisfied by construction
- total TMDPDF is product of *perturbative* expression and NP parametrisation

Form of TMDPDFs:

$$f_{1,f}(x,b) = \left(\sum_{f'} C_{f\to f'} \otimes q_{f'}\right)(x,b) \cdot f^{f}_{\mathrm{NP}}(x,b)$$

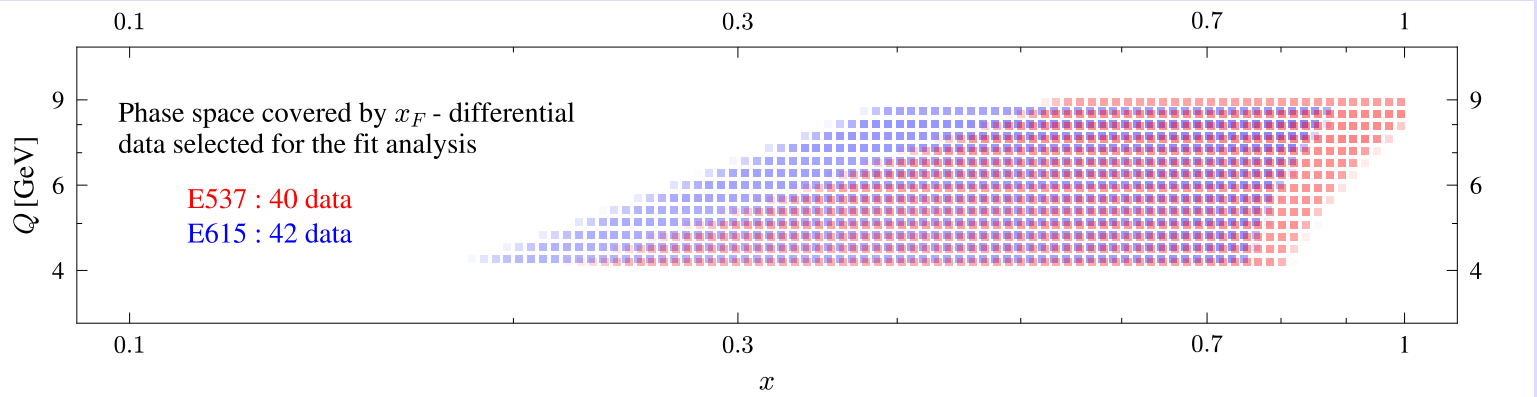
$$f^{u,\pi}_{\mathrm{NP}}(x,b) = \exp\left\{-\left(\lambda_1(1-x)+\lambda_2 x\right)\left(\sqrt{1+\frac{b^2}{x\lambda_3}}-1\right)\right\}$$



features an exponential decay at large-b!

$\pi^- + W \to \mathrm{DY}$ – data review

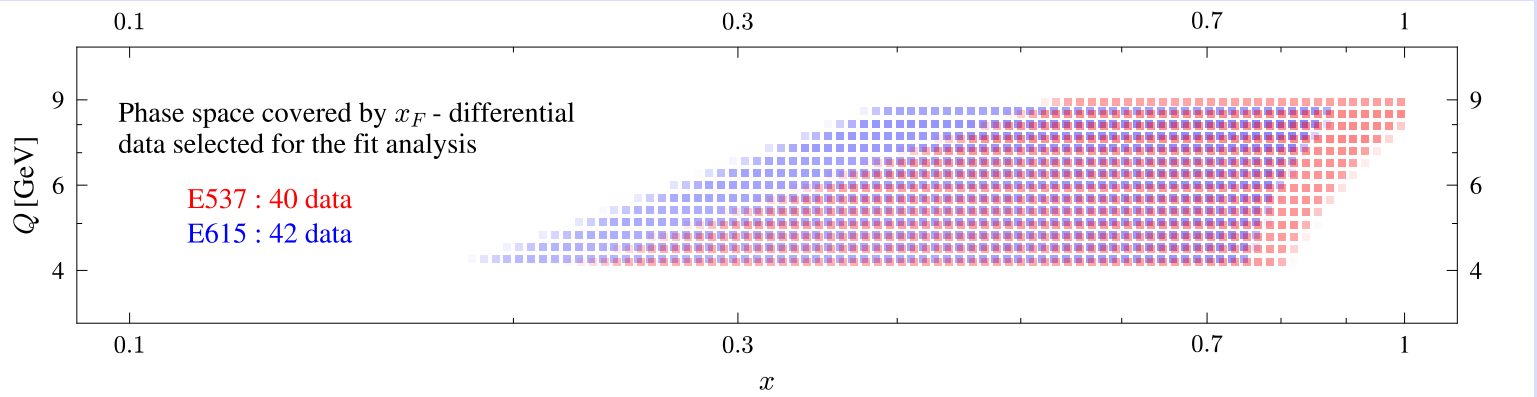
Extraction is challenging – because data are rare.



- single observable (DY)
- exclusively access valence region $x \gtrsim 0.2$
- normalisation uncertainties

$\pi^- + W \to \mathrm{DY}$ – data review

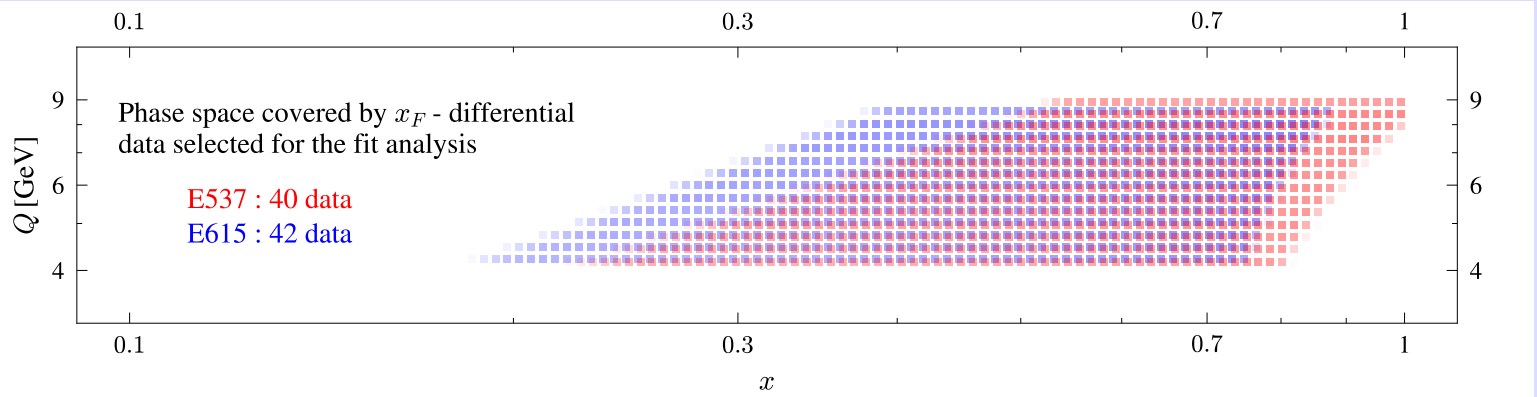
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- normalisation uncertainties
- analysis of kinematic binning
  + normalisation uncertainties (dQ vs dx_F)

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Extraction is challenging – because data are rare.



- single observable (DY)
- exclusively access valence region $x \gtrsim 0.2$
- normalisation uncertainties
- analysis of kinematic binning
  + normalisation uncertainties (dQ vs dx_F)
- extract TMDPDF of valence flavours

definition of the test function – what to minimise

χ^2-test function:

$$\chi^2 = (t-d)^T \, C^{-1} \, (t-d)$$

Covariance matrix

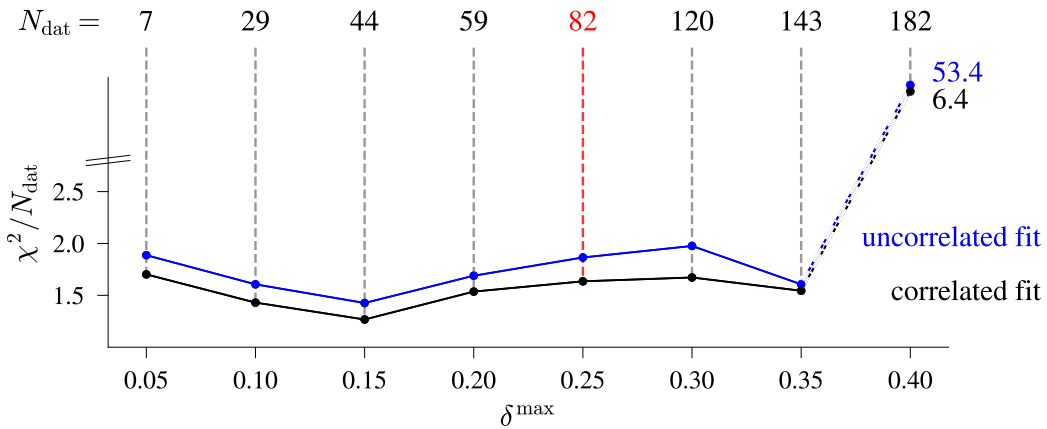
$$C_{ij} = \delta_{ij}\sigma^2_{\text{uncorr},i} + \sigma_{\text{corr},i}\sigma_{\text{corr},j}$$

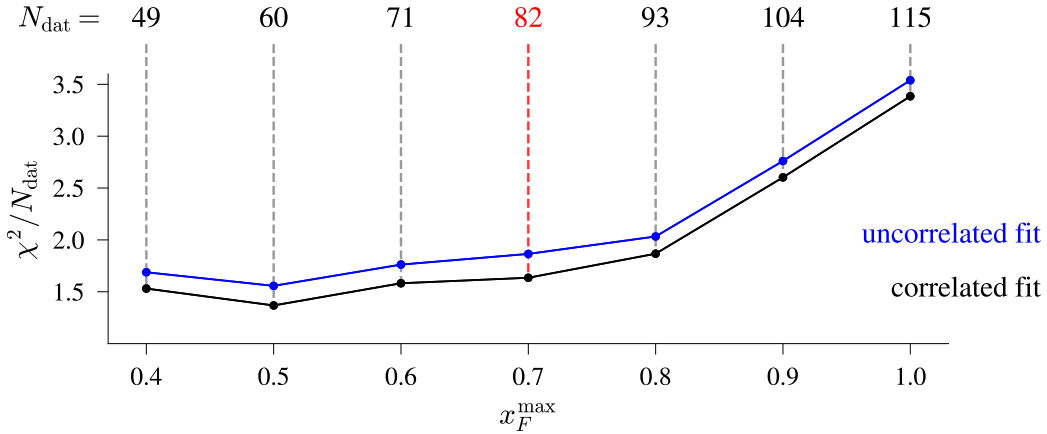
The source of correlation is the normalisation uncertainty:

$$\sigma_{\text{corr},i} = \sigma_{\text{norm}} \cdot d_i$$

Data selection: (kinematic cuts)

E537 + E605 in dx_F binning, and:

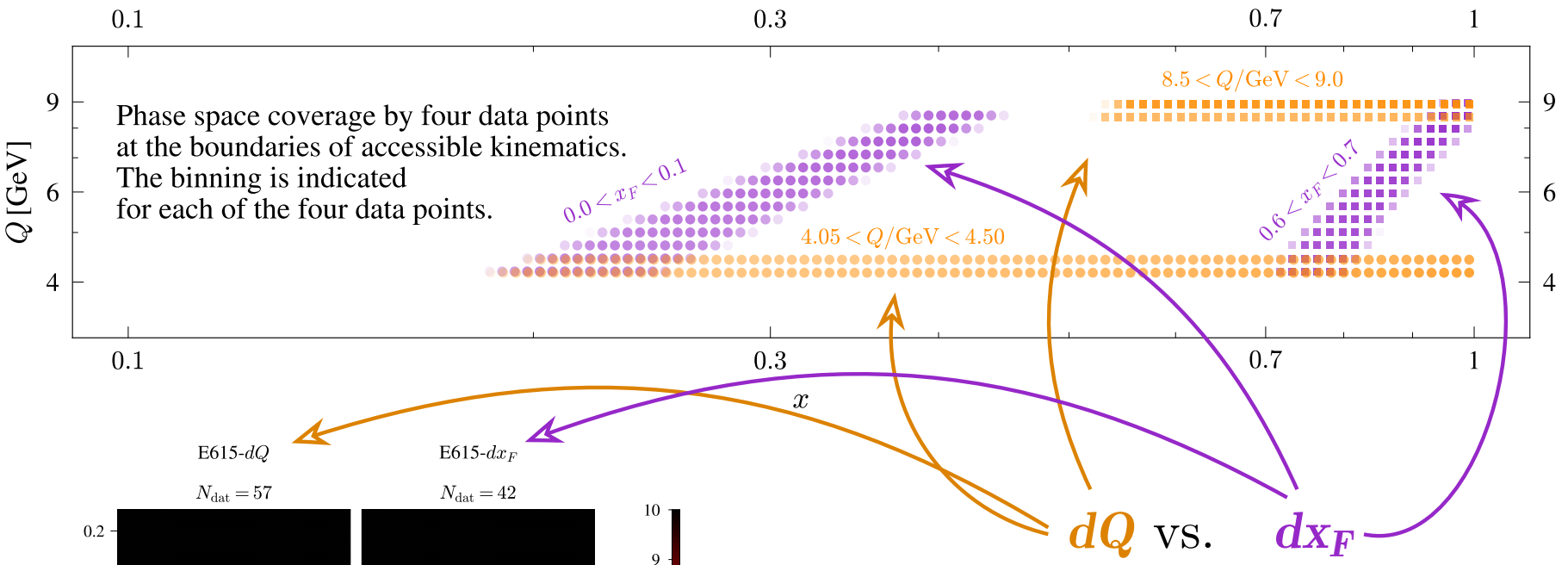




$$\delta = \frac{\langle q_T \rangle}{\langle Q \rangle}$$

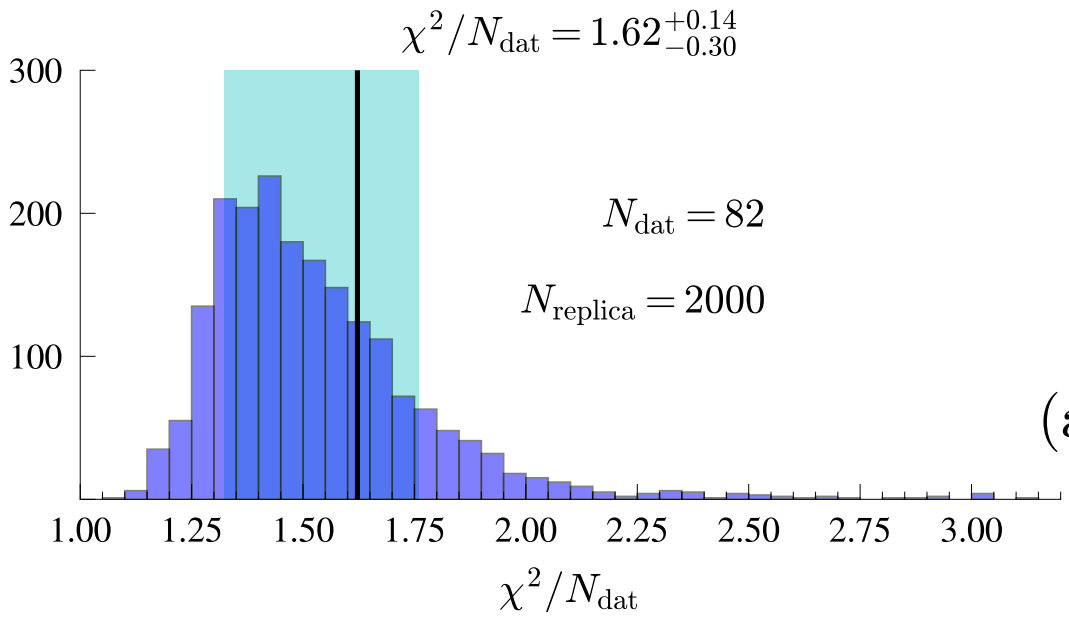
$$x_F = x_\pi - x_N$$

Issues: kinematic binning and normalisation uncertainty



- x_F bins cover smaller x regions $\longrightarrow$ better choice to determine $F(x, b)$
- x_F is a stronger constraint in the fit
- $\longrightarrow$ use dx_F data

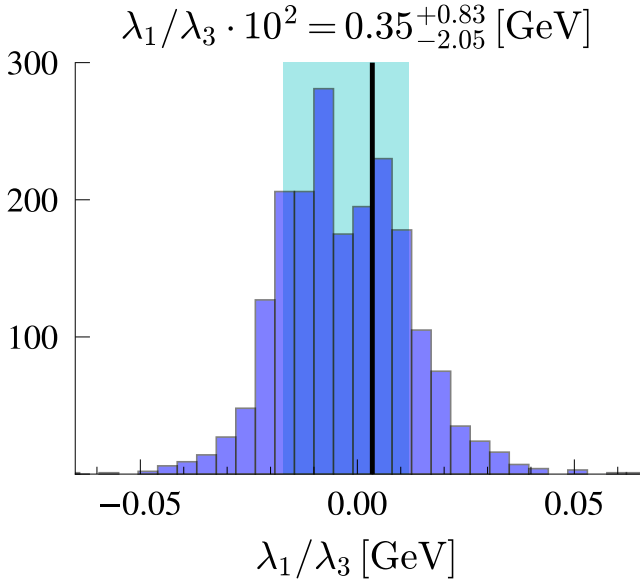
fit characteristics: χ^2 distribution

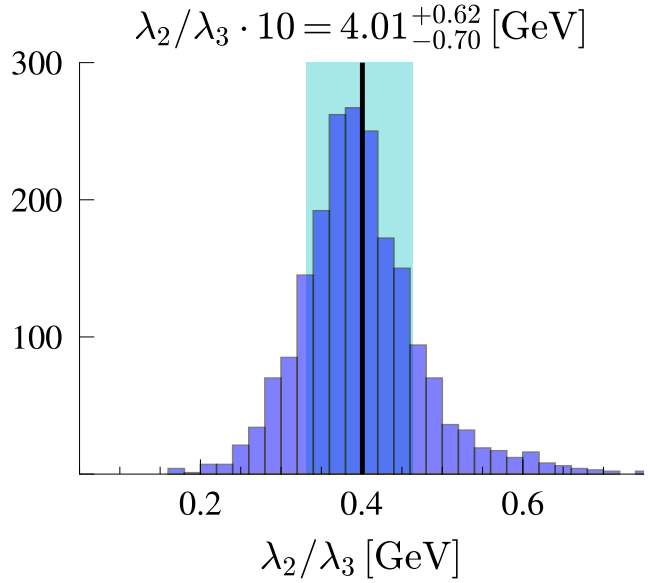


$\chi^2/N_{\rm dat} = 1.40$

(average prediction)

fit characteristics: parameter distributions



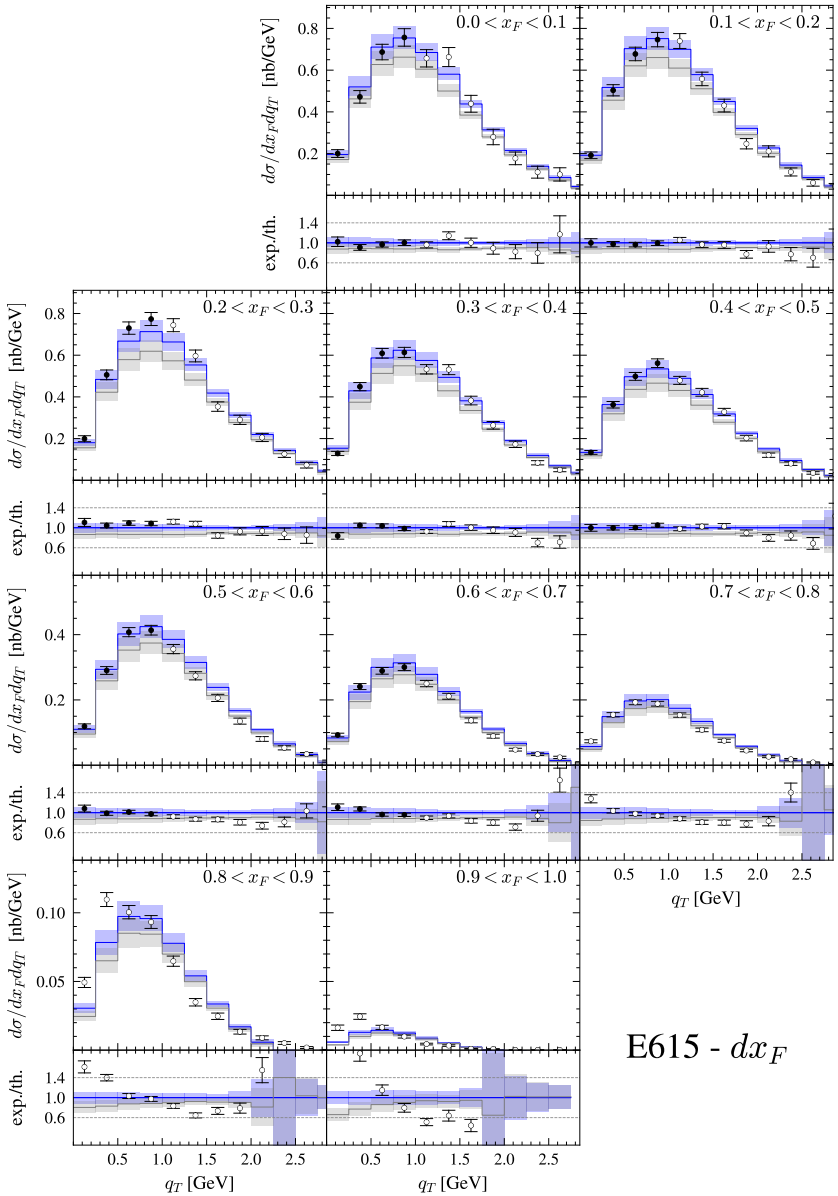


$$f_{\mathrm{NP}}^{f,\pi}(x,b) = \exp\left\{-\left(\lambda_1(1-x)+\lambda_2 x\right)\left(\sqrt{1+\frac{b^2}{(x\lambda_3)^2}}-1\right)\right\}$$

for large b:

$$f_{\mathrm{NP}}^{f,\pi}(x,b) \approx \exp\{-\lambda_2/\lambda_3 b\}$$

pion TMDPDFs: Results (0) data description

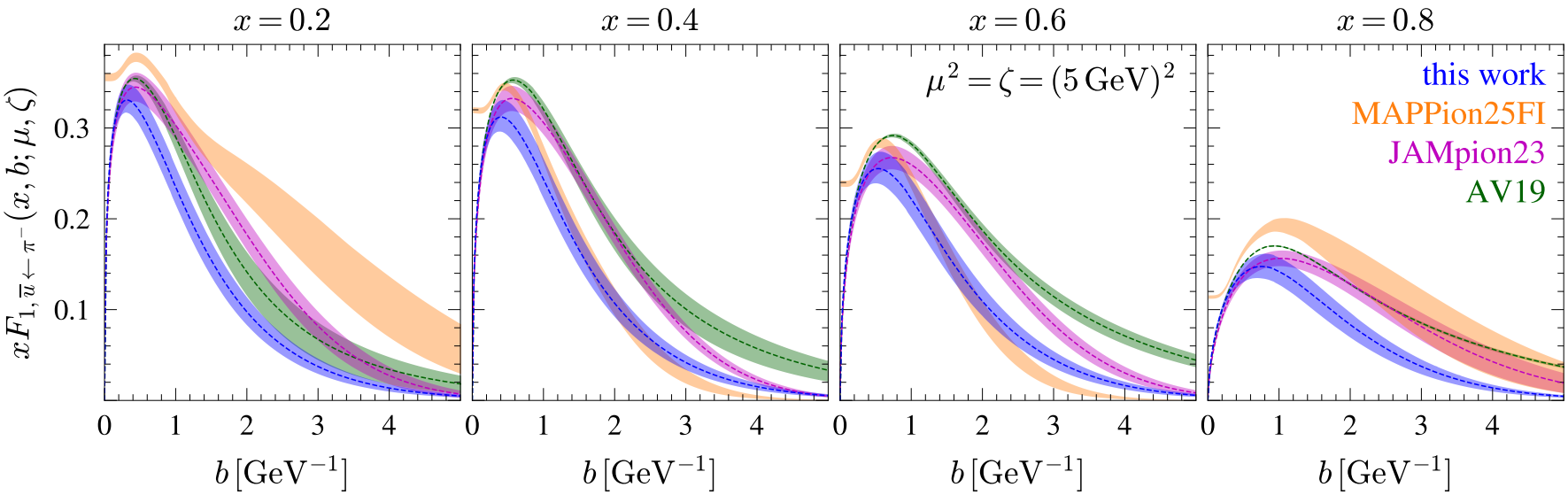


E615 - dx_F

Data set		N_{pt}	χ^2/N_{pt}
E615	dx_F	42	**1.99**
	dQ	52	1.75
E537	dx_F	40	**0.77**
	dQ	57	0.57

- $q_T \ll Q$ ✓
- consistent norm.✓
- large x_F ✗

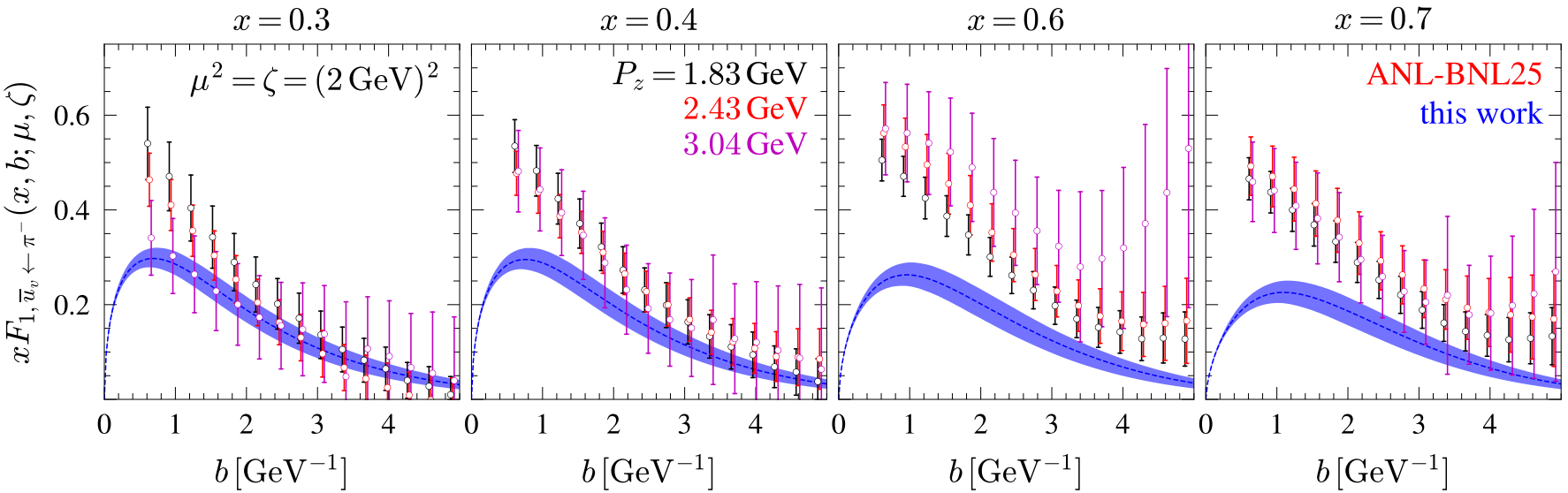
pion TMDPDFs: Results (1)
valence flavour ($\overline{u}$) compared with other extractions



differences in

- proton TMDPDFs, CS-kernel (rapidity scale evolution)
- kinematic binning of data (Q or x_F)
- fit strategy and (resulting) normalisation
- perturbative order and technicalities

pion TMDPDFs: Results (2)
valence combination $(\overline{u} - u)$ compared with LaMET

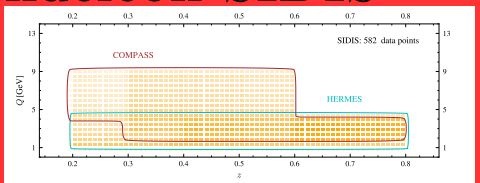


- Large Momentum Effective Theory needs to extrapolate to large hadron momenta P_z
- Promising complementary method to phenomenology (this talk) to determine pion distributions!

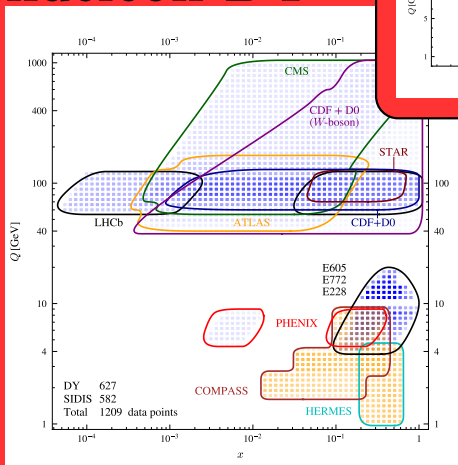
- large b trend ~ overall similar, agreement good for small x
- small b difference can be due to collinear PDF (normalisation)
- x behaviour different (normalisation/other pheno)

Inputs to the determination of the n / π - TMDPDFs

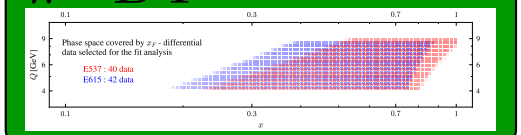
nucleon SIDIS



nucleon DY



π^--DY



nucleon PDF

collinear FF

π^--PDF

pQCD

assumptions

ART25

nucleon TMDPDF, TMDFFs,**CS-kernel**

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π^--TMDPDF

Summary on a technical level: π-TMDPDF

data input

- $\frac{q_T}{Q} = \delta < 0.25$
  (TMD regime: $\delta \ll 1$)
- $x_F = x_\pi - x_{\text{tar.}} < 0.7$
  $\longrightarrow$ 82 data

non-perturbative input:

- `ART25` (TMDs:target
  (PDFs:`MSHTPDF20`)
  and CS-kernel)
- `xfitter` (NLO)
  (PDFs:pion)

perturbative input

Γ_{cusp}	γ_V	$\mathcal{D}_{\text{pert}}$	$C_{f\to f'}$	C_{h}
N^{4}LO	N^{3}LO	N^{3}LO	N^{3}LO	N^{4}LO
α_s^5	α_s^4	α_s^4	α_s^3	α_s^4

human footprint (!)

- parametrisation
- fit strategy

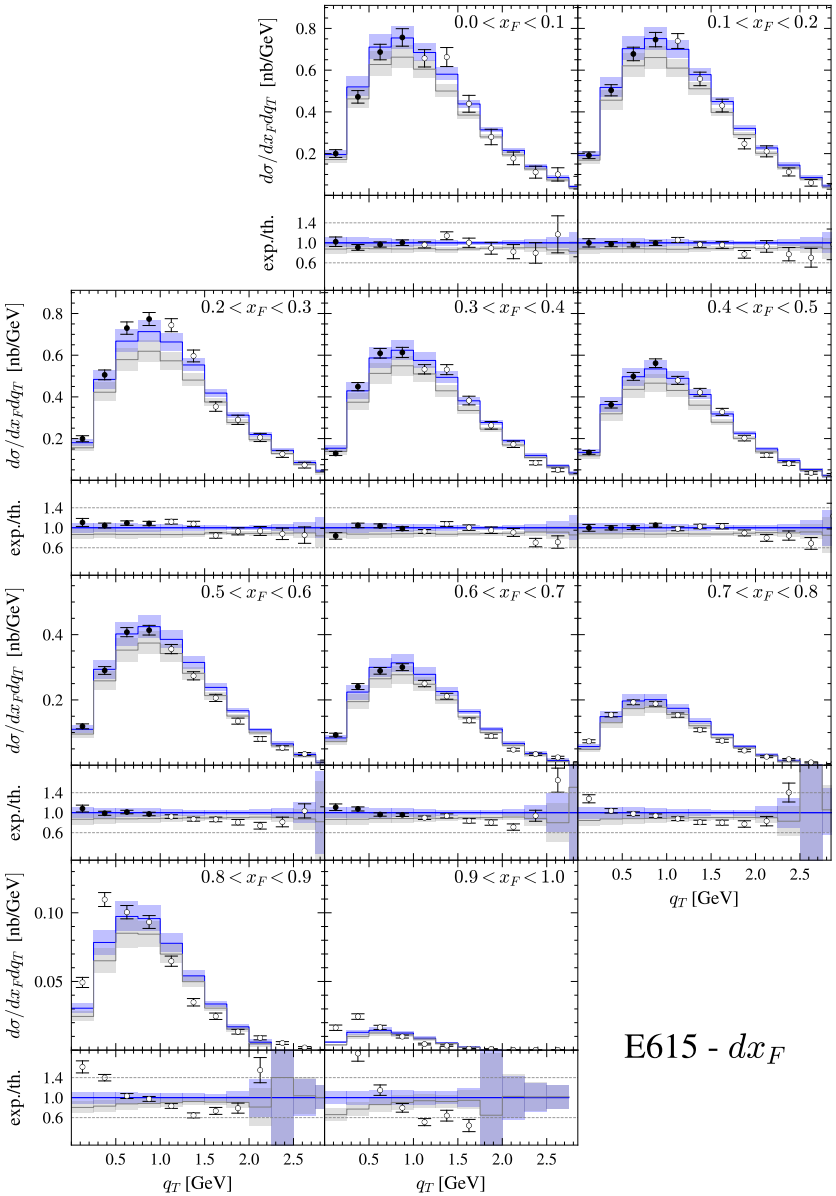
output

unpolarised π^- TMDPDFs

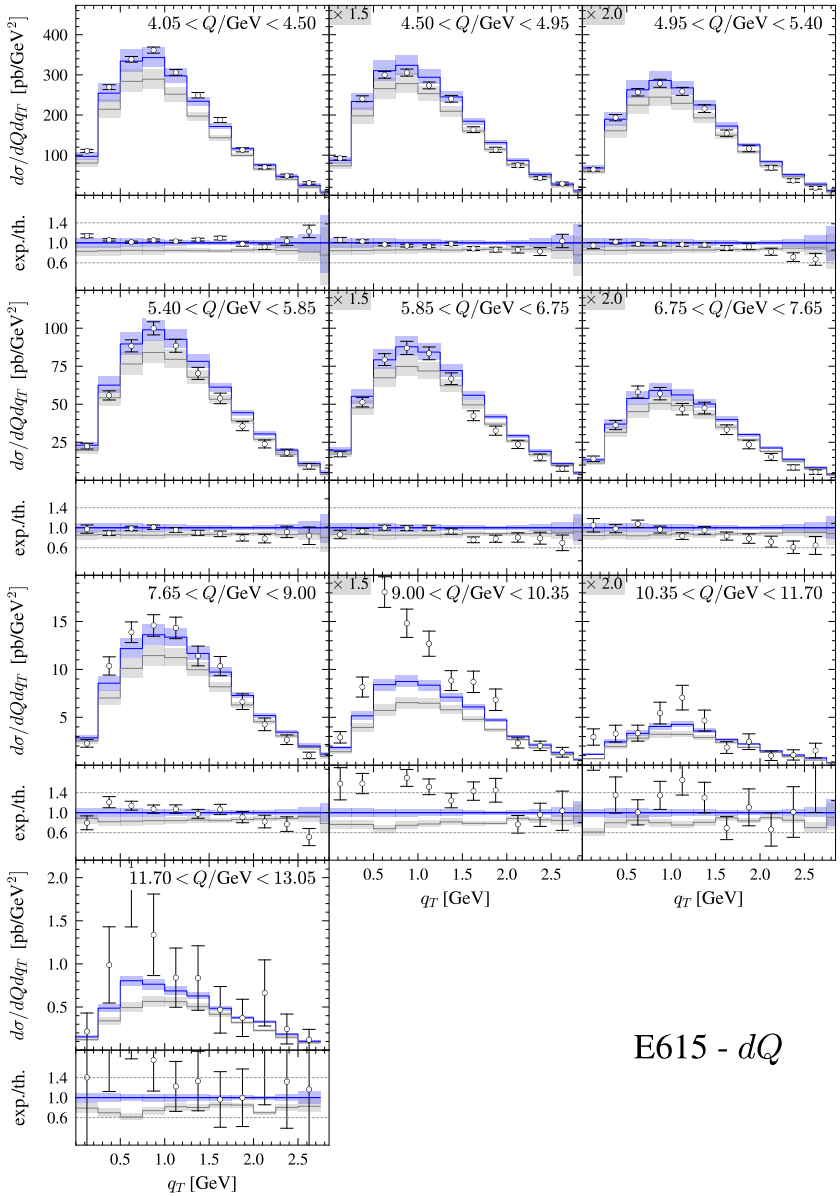
$\chi^2/N_{\text{pt}} = 1.40$

BACKUP SLIDES

E615 data

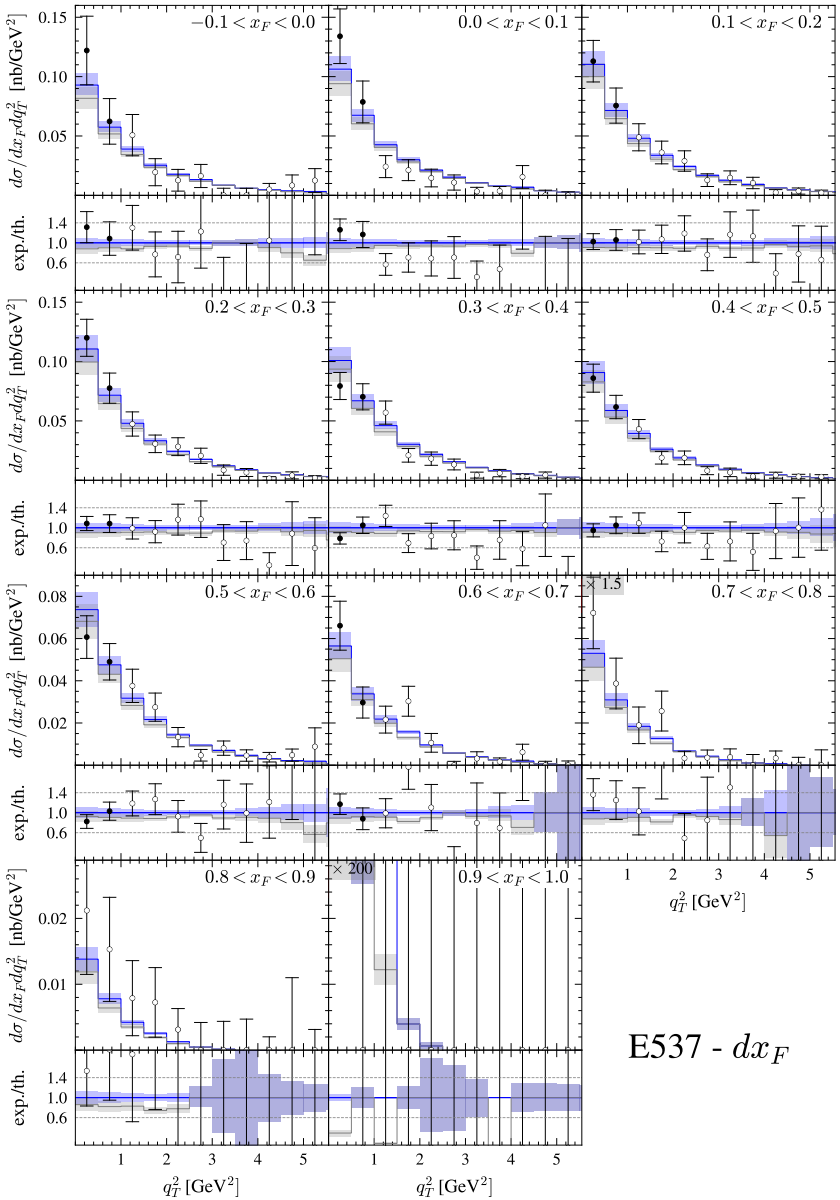


E615 - dx_F

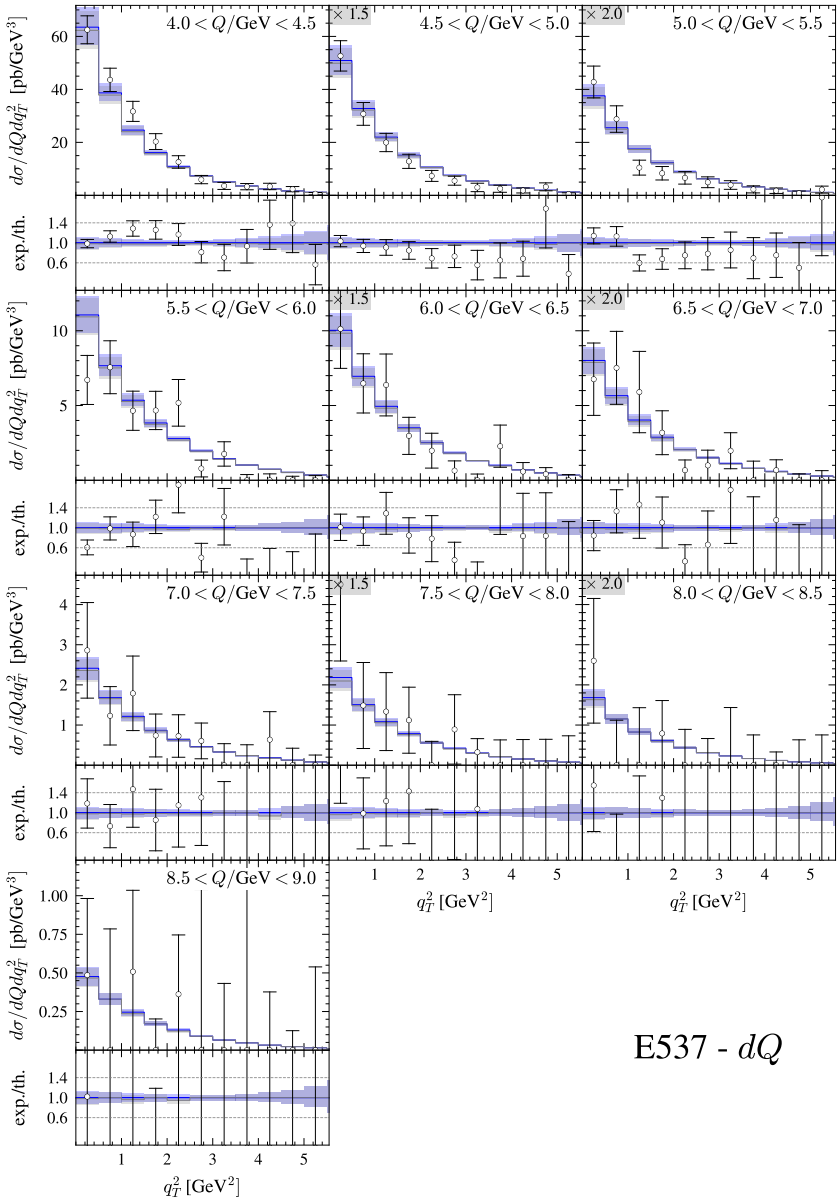


E615 - dQ

E537 data

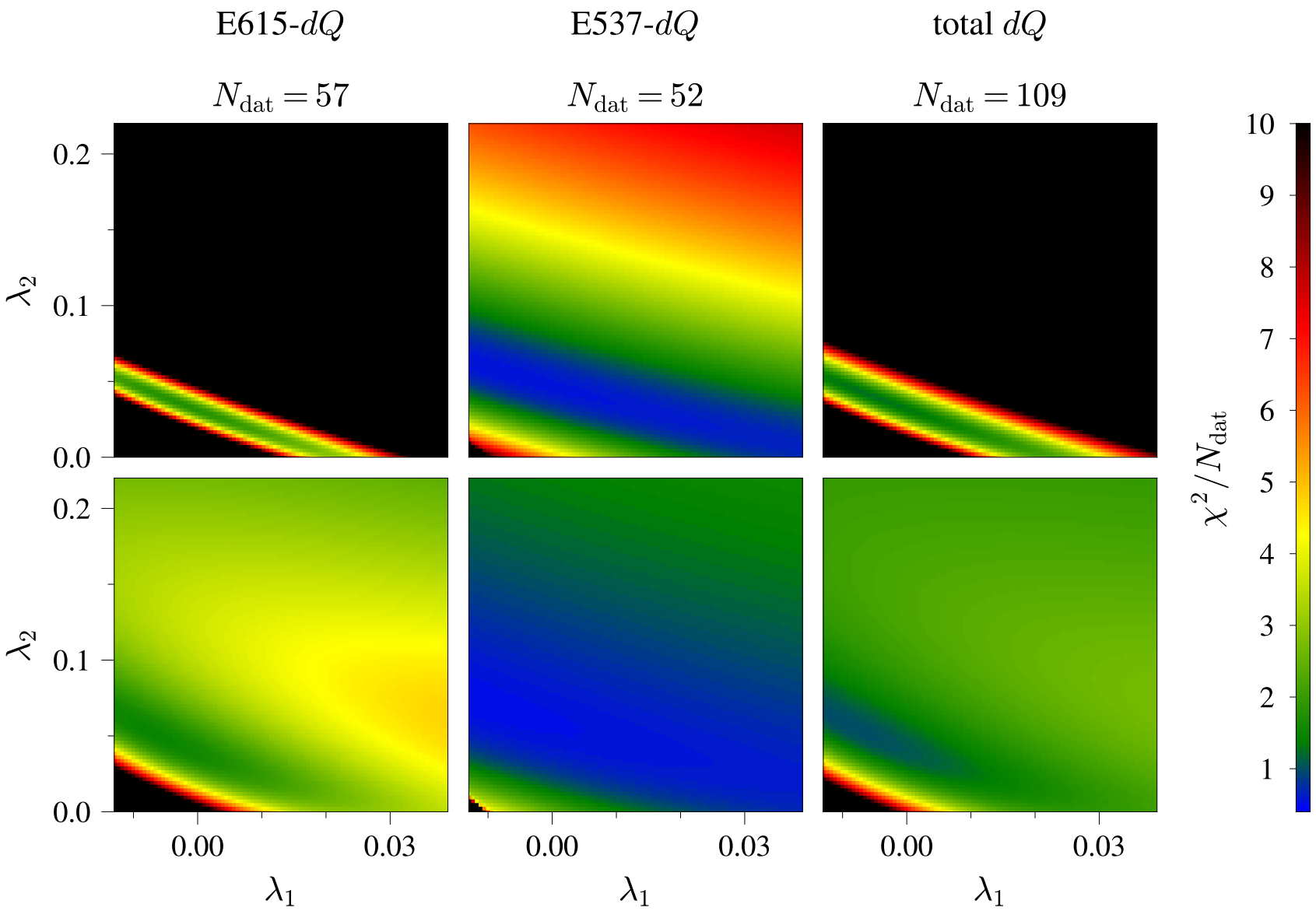


E537 - dx_F



E537 - dQ

Predictive power of data: Q binning



Predictive power of data: x_F binning

