TMD factorization for diffractive heavy-quark production in $\gamma^* A$ and diffractive D^0 production in UPCs

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Outline | Production of heavy quarks in diffraction

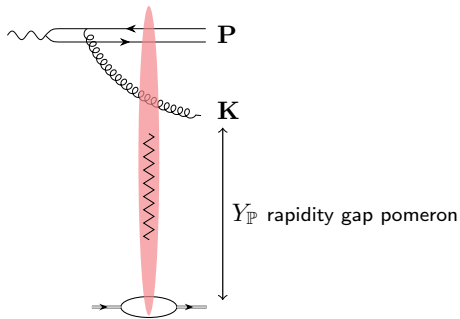
Based on Caucal, G-E, Iancu, Lappi, Salazar (2026)
and Caucal, G-E, Lappi, Salazar (in preparation)

1) Gluon diffractive TMD
    a. Introduction
    b. Color Glass Condensate theory
    c. Light-cone perturbation theory (LCPT)
    d. TMD factorization
2) Quark diffractive TMD
3) Diffractive and inclusive D^0 in UPCs
4) Summary

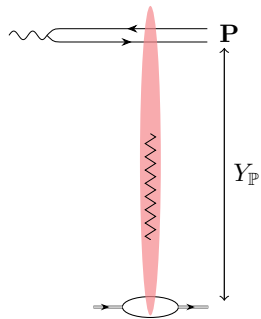
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Diffractive jets: $\boxed{\mathrm{d}\sigma \sim P_{\perp}^{-4}}$



Exclusive jets: $\boxed{\mathrm{d}\sigma \sim P_{\perp}^{-6}}$ → more suppressed

- New experimental data on diffractive $J/\psi, D, \Upsilon$ production

- Correlation limit (back-to-back)

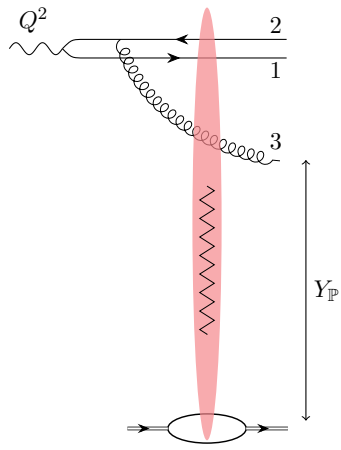
hard jets:	$m,\ k_{1\perp} \simeq k_{2\perp} \sim Q \gg Q_\mathrm{s}$
semi-hard jet:	$k_{3\perp} \sim Q_\mathrm{s} \sim$ GeV

- Goal $\rightarrow$ TMD factorization from dipole picture

"Hard" factor Gluon diffractive TMD

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}^2\mathbf{K}\mathrm{d}^2\mathbf{P}\mathrm{d}Y_\mathbb{P}} = H(z_1, z_2, P_\perp, m)\ \frac{\mathrm{d}xG_\mathbb{P}(x, x_\mathbb{P}, K_\perp)}{\mathrm{d}^2\mathbf{K}}$$

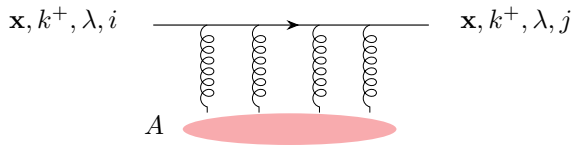
relative momentum $\mathbf{P} \simeq \mathbf{k}_1 \simeq -\mathbf{k}_2$
imbalance $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 = -\mathbf{k}_3$
soft: $z_3 \ll 1$



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- At small x many gluons in the target $\rightarrow$ described as a classical gluon field A
- Eikonal interaction of the probe with the color field in mixed space $(\mathbf{x}, k^+)$:



λ: helicity, i, j: color

Wilson line

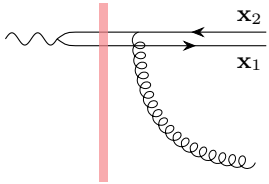
Path ordered exponential $\rightarrow$ resums multiple scatterings (not perturbative)

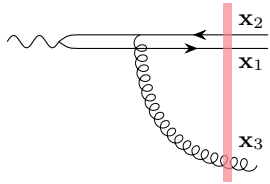
$$V_{F,A}(\mathbf{x}) = \mathcal{P} \exp\left(-ig \int dx^+ \, A_a^-(x^+, \mathbf{x}) \, t^a \right)$$

t^a : $SU(3)$ color matrix

- scattering operator $\hat{S}$ on states $\Rightarrow$ color rotation $(i \to j)$ in mixed space $\Rightarrow$ Wilson line

$$\hat{S}\,|q(k^+,\mathbf{x},i)\rangle = \sum_j \underbrace{V_F(\mathbf{x})_{ji}}_{\text{Wilson line}} |q(k^+,\mathbf{x},j)\rangle$$

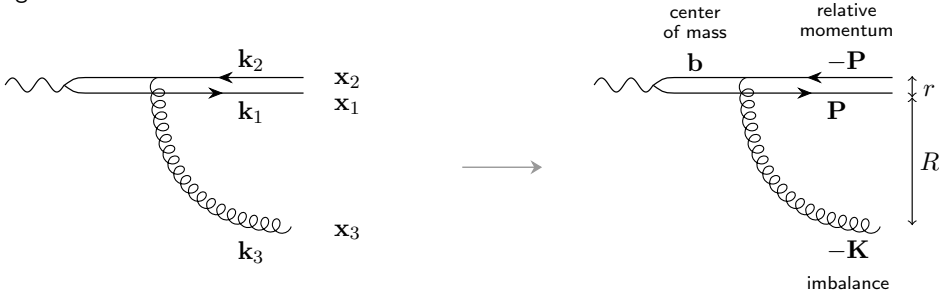




$$S_{q\bar{q}}(\mathbf{x}_1,\mathbf{x}_2) = \tfrac{1}{N_c}\big\langle \operatorname{Tr}\big[\underbrace{V_F(\mathbf{x}_1)}_{q}\underbrace{V_F^\dagger(\mathbf{x}_2)}_{\bar{q}}\big]\big\rangle$$

$$S_{q\bar{q}g}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) = \tfrac{1}{N_c C_F}\big\langle \underbrace{V_A^{ba}(\mathbf{x}_3)}_{g} \operatorname{Tr}\big[t^b \underbrace{V_F(\mathbf{x}_1)}_{q} t^a \underbrace{V_F^\dagger(\mathbf{x}_2)}_{\bar{q}}\big]\big\rangle$$

- Change of variables

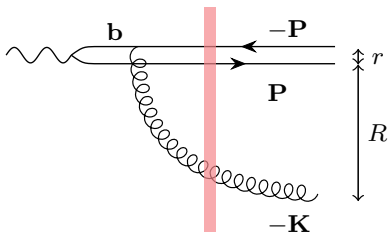


- Correlation limit: $\mathbf{P}, m \sim Q \gg K_\perp = k_{3\perp} \sim Q_\mathrm{s}$ $\implies$ $r \ll R$

  - Exponent of the Fourier transform has $\mathbf{P} \cdot \mathbf{r}$
  - $\Psi^{\gamma^* \to q\bar{q}g}$ from LCPT has Bessel function $K_n(r\sqrt{Q^2 z_1 z_2 + m^2})$

- Hard quarks and semi-hard gluon $\rightarrow$ $\boxed{r \ll R}$

$\mathcal{S}_{q\bar{q}g}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \simeq \mathcal{S}_{q\bar{q}g}(\mathbf{b}, \mathbf{b}, \mathbf{R} + \mathbf{b}) \sim \mathcal{S}_g(\mathbf{R})$ Effective gluon-gluon dipole



color transparency of small dipoles

$S_{q\bar{q}g}(\mathbf{b}, \mathbf{b}, \mathbf{R} + \mathbf{b}) - S_{q\bar{q}}(\mathbf{b}, \mathbf{b}) \simeq S_g(\mathbf{R}) - 1 = -\mathcal{N}_{gg}(\mathbf{R})$ Independent of $\mathbf{r}$

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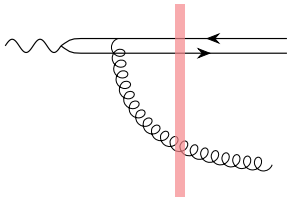
   d. TMD factorization

2) Quark diffractive TMD

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4) Summary

- Fock-state expansion of the projectile (dipole frame)



$$\left|\gamma^*\right\rangle^{\text{in}} \sim \left|\gamma^*\right\rangle_0 + \underbrace{\Psi^{\gamma^*\to q\bar{q}}\left|q\bar{q}\right\rangle_0}_{\text{light-cone wavefunction}} + \Psi^{\gamma^*\to q\bar{q}g}\left|q\bar{q}g\right\rangle_0 + \dots$$

$$\left|q\bar{q}g\right\rangle_{\text{s}}^{\text{out}} \sim \left|q\bar{q}g\right\rangle_0 + \Psi^{q\bar{q}g\to q\bar{q}}\left|q\bar{q}\right\rangle_0 + \dots$$

color singlet
($\mathbb{P}$ exchange)

- Invariant amplitude from scattering matrix element:

$$\mathcal{M}^{\gamma^*\to q\bar{q}g} \sim {}_{\text{s}}^{\text{out}}\left\langle q\bar{q}g\right| 1 - \underbrace{\hat{S}}_{\text{scattering operator}} \left|\gamma^*\right\rangle^{\text{in}}$$

Regular diagrams

Instantaneous diagrams



$$\Psi^{\gamma^* \to q\bar{q}g}\underbrace{(z_1, z_2, z_3, \mathbf{P}, \mathbf{R})}_{\text{Interaction in mixed space}} \simeq \underbrace{H^{in}(z_1, z_2, \mathbf{P}, m, Q)}_{\text{Hard factor same as inclusive DIS}} \frac{1}{2\pi}\left[\frac{\delta^{nm}}{2} - \frac{R^n R^m}{R^2}\right]\Omega^2 K_2(\Omega R)$$

$$\Omega^2 = \frac{z_3}{z_1 z_2}(P_\perp^2 + \tilde{Q}^2)$$
$$\tilde{Q}^2 = z_1 z_2 Q^2 + m^2$$

$$\mathcal{M}^{\gamma^* \to q\bar{q}g} \sim \varepsilon^i_\lambda \varepsilon^{*m}_\sigma H^{in}(z_1, z_2, \mathbf{P}, m, Q) G^{nm}(\mathbf{K}, \Omega, Y_{\mathbb{P}})$$

where

$$G^{nm}(\mathbf{K}, \Omega, Y_{\mathbb{P}}) = \left(\frac{K^n K^m}{K_\perp^2} - \frac{\delta^{nm}}{2} \right) \mathcal{G}(K_\perp, \Omega, Y_{\mathbb{P}})$$

and

$$\mathcal{G}(K_\perp, \Omega, Y_{\mathbb{P}}) = 2(2\pi)\Omega^2 \int_0^\infty \mathrm{d}R R J_2(K_\perp R) K_2(\Omega R) \mathcal{N}_{gg}(R, Y_{\mathbb{P}})$$

- A TMD should be a target distribution. But here (!) $\mathcal{G}$ depends on projectile variables through the gluon energy denominator:

$$\Omega^2 \simeq \frac{z_3}{z_1 z_2}(P_\perp^2 + \tilde{Q}^2)$$

Ω^2: effective virtuality of the gluon

z_3: longitudinal plus momentum fraction of the gluon

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TMD factorization | From plus to minus momenta

LCPT: projectile plus momenta

q^+ photon longitudinal momentum

z fraction of q^+ carried by final partons

↓ change of variables

TMD: target minus momenta

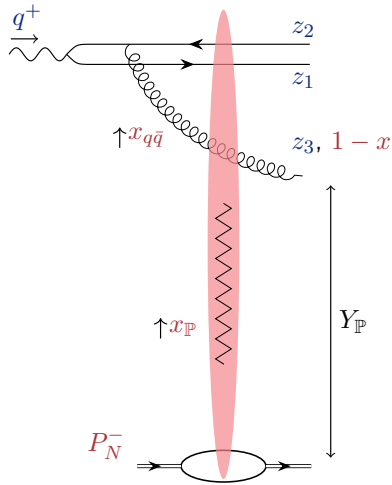
P_N^- target longitudinal momentum

$x_{\mathbb{P}}$ fraction of P_N^- carried by the $\mathbb{P}$

$x_{q\bar{q}}$ fraction of P_N^- transferred to $q\bar{q}$

$x = x_{q\bar{q}}/x_{\mathbb{P}}$ fraction of $x_{\mathbb{P}}P_N^-$ carried by the gluon

$\Rightarrow$ gluon DTMD $\dfrac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp)}{\mathrm{d}^2\mathbf{K}}$ is a target distribution



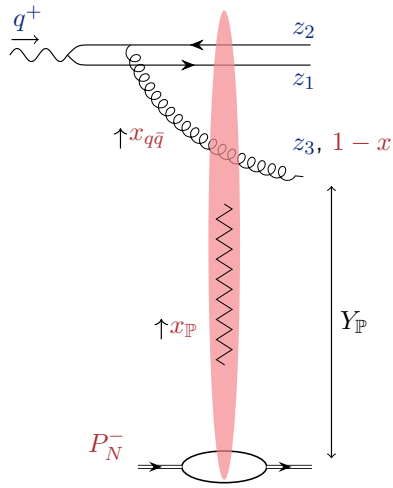
Effective virtuality of the gluon:

$$\Omega^2 \simeq \frac{z_3}{z_1 z_2}(P_\perp^2 + \tilde{Q}^2) \quad \to \quad \Omega^2 = \frac{x}{1-x} K_\perp^2$$

Phase space:

$$\frac{\mathrm{d}z_3}{z_3} = \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}} - x_{q\bar{q}}} = \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}} \frac{1}{1-x}$$

→ Target picture: gluon is seen as emitted from the $\mathbb{P}$.



Hard factor (same as inclusive DIS) Gluon DTMD

$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma_T^* A\to q\bar{q}gA}}{\mathrm{d}z\mathrm{d}^2\mathbf{K}\mathrm{d}^2\mathbf{P}\mathrm{d}Y_{\mathbb{P}}} = \alpha_{\mathrm{e.m.}}\alpha_{\mathrm{s}}e_f^2\left[\left(z^2+(1-z)^2\right)\frac{P_\perp^4+\tilde{Q}^4}{\left[P_\perp^2+\tilde{Q}^2\right]^4}+2m^2\frac{P_\perp^2}{\left[P_\perp^2+\tilde{Q}^2\right]^4}\right]\frac{\mathrm{d}xG_{\mathbb{P}}(x,K_\perp,Y_{\mathbb{P}})}{\mathrm{d}^2\mathbf{K}}$$

$$\tilde{Q}^2 = z(1-z)Q^2+m^2$$

Gluon DTMD

$$\frac{\mathrm{d}xG_{\mathbb{P}}(x,K_\perp,Y_{\mathbb{P}})}{\mathrm{d}^2\mathbf{K}} \equiv \frac{S_\perp(N_{\mathrm{c}}^2-1)}{8\pi^4(1-x)}\left[\Omega^2\int_0^\infty \mathrm{d}R\,R\,J_2(K_\perp R)\,K_2(\Omega R)\,\mathcal{N}_{gg}(R,Y_{\mathbb{P}})\right]^2$$

\# density of gluons with $K_\perp$ and x in the $\mathbb{P}$ wavefunction. $\Omega^2 = K_\perp^2 x/(1-x)$

- Generalizes result of Iancu, Mueller, Triantafyllopoulos, Wei (2022) to heavy quarks

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- Correlation limit (back-to-back)

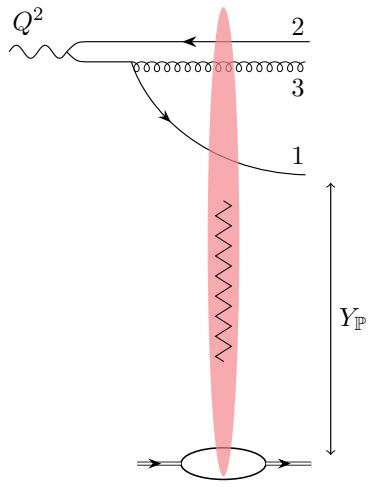
hard jets:	$k_{2\perp} \simeq k_{3\perp} \sim Q \gg Q_s$
semi-hard jet:	$m,\ k_{1\perp} \sim Q_s$

- Goal $\rightarrow$ TMD factorization from dipole picture

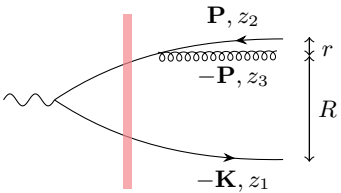
"Hard" factor Quark diffractive TMD

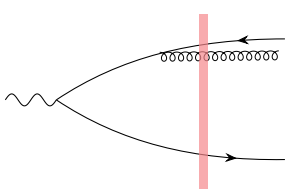
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_2\mathrm{d}z_3\mathrm{d}^2\mathbf{K}\mathrm{d}^2\mathbf{P}\mathrm{d}Y_{\mathbb{P}}} = H(z_2, z_3, P_\perp, Q)\,\frac{\mathrm{d}xq_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp, m)}{\mathrm{d}^2\mathbf{K}}$$

relative momentum $\mathbf{P} \simeq \mathbf{k}_2 \simeq -\mathbf{k}_3$
imbalance $\mathbf{K} = \mathbf{k}_2 + \mathbf{k}_3 = -\mathbf{k}_1$
soft: $z_1 \ll 1$

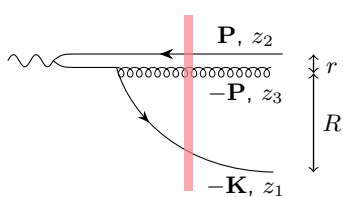


$(\bar{q} \to \bar{q}g)$





$(q \to qg)$



Gluon emission after shockwave is zero by color transparency

- Calculation in LCPT + change of variables:

  longitudinal momentum fraction z_1 of the projectile $\to$ target longitudinal momentum x.

$$\frac{\mathrm{d}\sigma^{\gamma^*\to q\bar{q}g}}{\mathrm{d}z_2\mathrm{d}z_3\mathrm{d}^2\mathbf{K}\mathrm{d}^2\mathbf{P}\mathrm{d}Y_{\mathbb{P}}} = H(z_2,z_3,P_\perp)\frac{\mathrm{d}xq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp,m)}{\mathrm{d}^2\mathbf{K}}$$

- Quark diffractive TMD depends on target variables and the quark mass:

$$\frac{\mathrm{d}xq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp,m)}{\mathrm{d}^2\mathbf{K}} = \frac{S_\perp N_c}{4\pi^3}\frac{1}{2\pi}\frac{x}{1-x}\frac{K_\perp^2+m^2}{(xK_\perp^2+m^2)}\left(\mathcal{Q}_1^2(x,x_{\mathbb{P}},K_\perp)+\frac{(1-x)m^2}{xK_\perp^2+m^2}\mathcal{Q}_0^2(x,x_{\mathbb{P}},K_\perp)\right)$$

\# density of quarks with $K_\perp$, x and m in the $\mathbb{P}$ wavefunction.

$$\mathcal{Q}_1(x,x_{\mathbb{P}},K_\perp) \equiv \omega^2\int_0^\infty \mathrm{d}R\,R\,J_1(KR)\,K_1(\omega R)\,\mathcal{N}_{q\bar{q}}(R)$$

$$\mathcal{Q}_0(x,x_{\mathbb{P}},K_\perp) = \omega^2\int_0^\infty \mathrm{d}R\,R\,J_0(KR)\,K_0(\omega R)\,\mathcal{N}_{q\bar{q}}(R)$$

$$\omega^2 = (m^2+xK_\perp^2)/(1-x)$$

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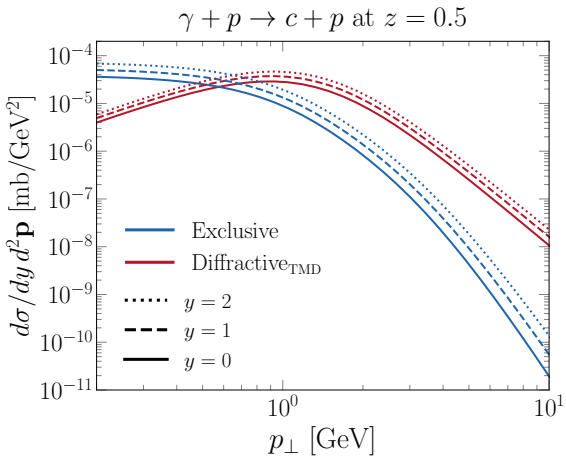
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Charm photoproduction ($Q^2 \approx 0$) | Exclusive vs. diffractive$_{\text{TMD}}$

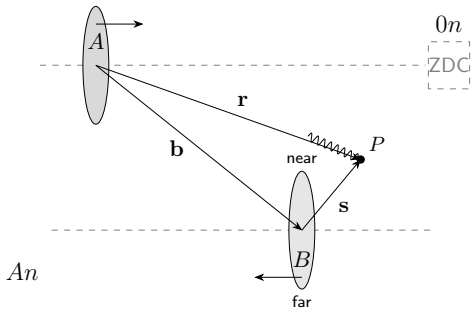
- Coherent diffraction: two contributions
  - exclusive $c\bar{c}$

    $\rightarrow$ dominates at $p_\perp \sim$ GeV
  - $c\bar{c}g$ with gluon DTMD (diffractive$_{\text{TMD}}$)

    $\rightarrow$ dominates at large $p_\perp$



Diffractive D^0 in UPCs | Cross section



UPC in the $An0n$ class

$$\frac{\mathrm{d}\sigma^{\text{diffractive}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}_D} = \int_0^1 \frac{\mathrm{d}z_h}{z_h^2}\, D(z_h) \int_{x_{\mathbb{P}\,\min}}^{x_{\mathbb{P}\,\max}} \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}}$$

$$\times \left(\int_{z_\gamma^{\min}}^1 \mathrm{d}z_\gamma\, F(z_\gamma)\, \frac{\mathrm{d}\sigma^{\text{diffractive}_{\text{TMD}}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}\,\mathrm{d}\ln x_{\mathbb{P}}} + \frac{\mathrm{d}\sigma^{\text{exclusive}}_{\text{UPC}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}\,\mathrm{d}\ln x_{\mathbb{P}}} \right)$$

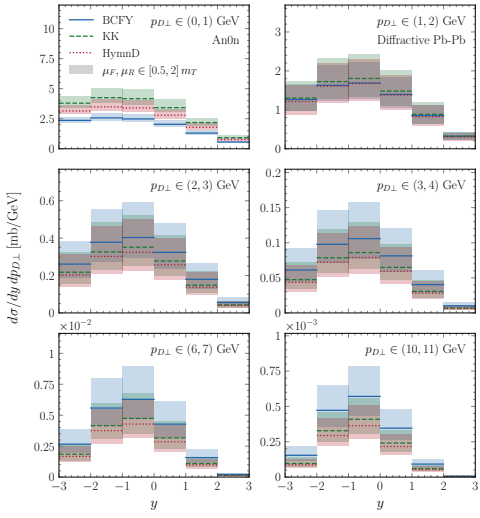
$c \to D^0$ Fragmentation function

Photon flux

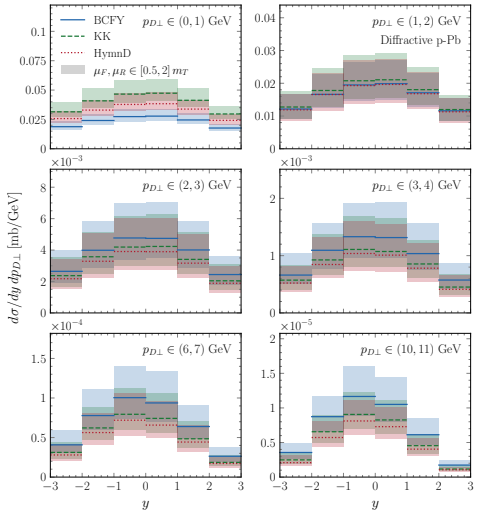
Diffractive D^0 in UPCs | $p_{D\perp}$ bins

See also Guzey, Innocenti, Stasto, Strikman (2026)

Pb–Pb, $\sqrt{s_{NN}} = 5.36$ TeV, $An0n$

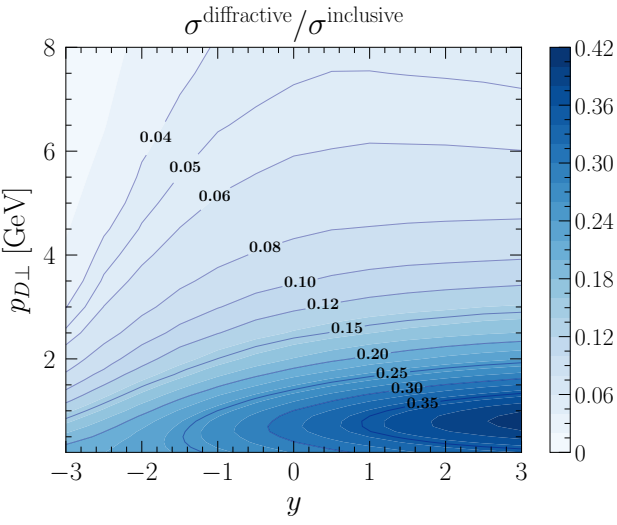


p–Pb, $\sqrt{s_{NN}} = 8.16$ TeV



Diffractive vs. inclusive D^0 | Pb–Pb, $\sqrt{s_{NN}} = 5.36$ TeV, $An0n$, BCFY

- Low $p_{D\perp}$ and forward y: $\sim 40\%$
- High $p_{D\perp}$ and backward y: below 5%

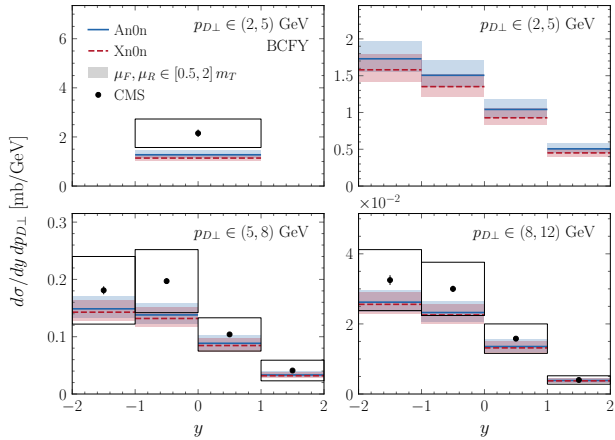


Inclusive D^0: G-E, Lappi, Mäntysaari (2025)

Inclusive D^0 in Pb–Pb UPCs | Diffractive correction to the inclusive $Xn0n$ selection

- CMS data inclusive D^0 in $Xn0n$ class
- Coherent diffraction leaves the target intact
  $\rightarrow$ removed by the Xn condition

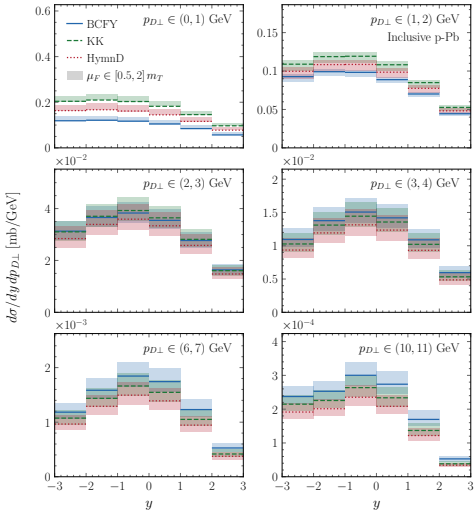
$$\mathrm{d}\sigma^{\mathrm{incl}}_{Xn0n} = \mathrm{d}\sigma^{\mathrm{incl}}_{An0n} - \mathrm{d}\sigma^{\mathrm{diff}}_{0n0n}$$



Inclusive D^0: G-E, Lappi, Mäntysaari (2025)

Inclusive D^0 in p–Pb UPCs | $p_{D\perp}$ bins

p–Pb: no EMD factor



Summary

- TMD factorization for diffractive heavy-quark pairs in the CGC
  - → Hard $q\bar{q}$, semi-hard g: gluon DTMD, independent of the mass 🔗
  - → Hard $\bar{q}g$, semi-hard q: new quark DTMD, depends on the mass 🔗
- Diffractive D^0 in UPCs at the LHC
  - → Predictions for Pb–Pb and p–Pb
  - → Diffraction relevant in inclusive D^0 $Xn0n$ events at low $p_\perp$
- Outlook
  - → Quarkonium in UPCs, diffraction at the EIC

- Change of variables

$$\begin{aligned} \text{relative momentum:} \quad & \mathbf{P} \equiv z_2\mathbf{k}_1 - z_1\mathbf{k}_2 \\ \text{total momentum:} \quad & \mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2 \end{aligned}$$

$$\begin{aligned} \text{center of mass:} \quad & \mathbf{b} \equiv z_1\mathbf{x}_1 + z_2\mathbf{x}_2 \\ q\bar{q} \text{ size:} \quad & \mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2 \\ \text{separation } g - q\bar{q}: \quad & \mathbf{R} = \mathbf{x}_3 - \mathbf{b} \end{aligned}$$

- Hard quarks $\to r \ll R$: the $q\bar{q}g$ interaction reduces to an effective gluon-gluon dipole

$$\begin{aligned} \mathcal{S}_{q\bar{q}g}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \simeq \mathcal{S}_{q\bar{q}g}(\mathbf{b}, \mathbf{b}, \mathbf{R} + \mathbf{b}) &\sim \left\langle V_A^{ba}(\mathbf{R} + \mathbf{b}) \, \mathrm{Tr}\left[t^b V_F(\mathbf{b}) t^a V_F^\dagger(\mathbf{b})\right]\right\rangle \\ &\sim \left\langle \, \mathrm{Tr}\left[V_A(\mathbf{R} + \mathbf{b}) V_A^\dagger(\mathbf{b})\right]\right\rangle \sim \mathcal{S}_g(\mathbf{R})\,. \end{aligned}$$

$$\Psi^{\gamma^*\to q\bar{q}g} = \Psi^{\gamma^*\to q\bar{q}g}_{\text{regular}} + \Psi^{\gamma^*\to q\bar{q}g}_{\text{instantaneous}} = \varepsilon^i_\lambda \varepsilon^{*m}_\sigma H^{in}(z_1, z_2, \mathbf{P}, m, Q) \frac{k_3^n k_3^m + \delta^{nm}\Omega^2/2}{k_{3\perp}^2 + \Omega^2}$$

$$= \varepsilon^i_\lambda \varepsilon^{*m}_\sigma \left(H^{in}(z_1, z_2, \mathbf{P}, m, Q) \frac{k_3^n k_3^m - \delta^{nm} k_{3\perp}^2/2}{k_{3\perp}^2 + \Omega^2} + \frac{H^{in}(z_1, z_2, \mathbf{P}, m, Q)}{2} \right)$$

traceless 2D tensor ($\mathbb{P}$) — diagonal term

- H^{in}: hard factor

$$H^{in}(z_1, z_2, \mathbf{P}, m, Q) = \frac{\varphi^{ij}(z_1, h_1)\delta^{jn}\delta_{h_1 - h_2}}{P_\perp^2 + \tilde{Q}^2} - \frac{\varphi^{ij}(z_1, h_1) 2P^j P^n \delta_{h_1 - h_2} - 2\sqrt{2} m \varepsilon^{i*}_{h_1} P^n \delta_{h_1 h_2}}{(P_\perp^2 + \tilde{Q}^2)^2}$$

$\tilde{Q}^2 = Q^2 z_1 z_2 + m^2$

$\Omega^2 = \frac{z_3}{z_1 z_2}(P_\perp^2 + \tilde{Q}^2)$

- Mixed space wavefunction and scattering
  - Fourier transform Pomeron term (traceless 2D tensor):

    small dipole $\mathbf{r}$ (color transparency)$\rightarrow$ neglect the change due to the scattering of $\mathbf{P}$.

$$\begin{aligned}\Psi_{\mathbb{P}}^{im}(z_1,z_2,z_3,\mathbf{P},\mathbf{R}) &= \int \frac{\mathrm{d}^2\mathbf{k}_3}{(2\pi)^2} e^{i\mathbf{k}_3\cdot\mathbf{R}} \Psi_{\mathbb{P}}^{im}(z_1,z_2,z_3,\mathbf{P},\mathbf{k}_3) \\ &= H^{in}(z_1,z_2,\mathbf{P},m) \int \frac{\mathrm{d}^2\mathbf{k}_3}{(2\pi)^2} e^{i\mathbf{k}_3\cdot\mathbf{R}} \frac{k_3^n k_3^m - \delta^{nm}\frac{k_{3\perp}^2}{2}}{k_{3\perp}^2+\Omega^2} \\ &= H^{in}(z_1,z_2,\mathbf{P},m) \frac{1}{2\pi}\left[\frac{\delta^{nm}}{2} - \left(\frac{R^n R^m}{R^2}\right)\right]\Omega^2 K_2(\Omega R)\,.\end{aligned}$$

  - The Fourier transform of $\Psi_{\text{diag}}(\mathbf{P})$ gives a $\delta^2(\boldsymbol{R})$.

$$\delta^2(\boldsymbol{R})\mathcal{N}_{gg}(\mathbf{R}) \rightarrow 0 \text{ due to color transparency}$$

    $\rightarrow$ We keep only the $\Psi_{\mathbb{P}}$ term.

Backup

- Invariant mass

$$M_{q\bar{q}}^2 = (k_1 + k_2)^2 = \left(\frac{k_{1\perp}^2}{z_1} + \frac{k_{2\perp}^2}{z_2} + \frac{m^2}{z_1 z_2} - K_\perp^2 \right)$$

$$M_{q\bar{q}g}^2 = (k_1 + k_2 + k_3)^2 = \left(\frac{k_{1\perp}^2}{z_1} + \frac{k_{2\perp}^2}{z_2} + \frac{k_{3\perp}^2}{z_3} + \frac{m^2}{z_1 z_2} \right) = M_{q\bar{q}}^2 + K_\perp^2 + \frac{k_3^2}{z_3}$$

- β variable

$$\beta \equiv \frac{Q^2}{Q^2 + M_{q\bar{q}g}^2}$$

- Target longitudinal momentum fraction

$$x_{q\bar{q}} = \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}{2P_N \cdot q} \qquad x_{\mathbb{P}} = \frac{Q^2 + M_{q\bar{q}g}^2}{2q \cdot P_N}$$

$$x \equiv \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2 + k_{3\perp}^2/z_3},$$

- Change of variables: $z_3 \to x$ or $x_{\mathbb{P}}$ using

$$\Omega^2 \simeq \frac{z_3}{z_1 z_2}(P_\perp^2 + \tilde{Q}^2) \qquad\qquad x \simeq \frac{P_\perp^2 + \tilde{Q}^2}{P_\perp^2 + \tilde{Q}^2 + z_1 z_2 \frac{K_\perp^2}{z_3}}$$

we get

$$\boxed{\Omega^2 = \frac{x}{1-x} K_\perp^2}$$

Using also $x_{\mathbb{P}}$:

$$x_{\mathbb{P}} = \frac{Q^2 + M_{q\bar{q}g}^2}{2q \cdot P_N} \qquad x_{q\bar{q}} = \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}{2P_N \cdot q} \qquad x \equiv \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} \simeq \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2 + k_{3\perp}^2/z_3}$$

we have the change of variables:

$$\frac{x_{\mathbb{P}} - x_{q\bar{q}}}{x_{q\bar{q}}} = \frac{x_{\mathbb{P}}}{x_{q\bar{q}}} - 1 = \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2 + k_{3\perp}^2/z_3}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2} - 1 = \frac{k_{3\perp}^2/z_3}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}$$

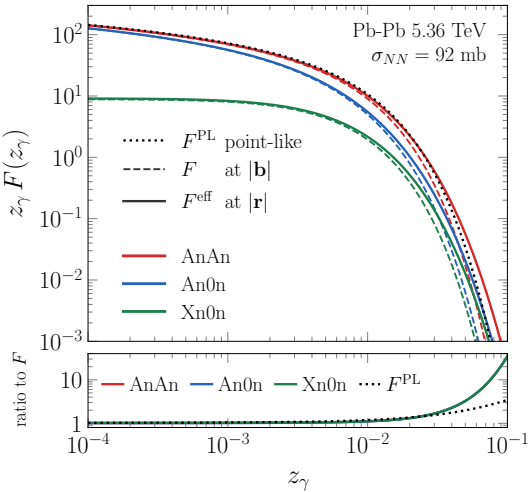
Taking the logarithmic differential:

$$\boxed{\frac{\mathrm{d}z_3}{z_3} = \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}} - x_{q\bar{q}}} = \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}} \frac{1}{1-x}}$$

- Hard factor for hard $\bar{q}g$, semi-hard q

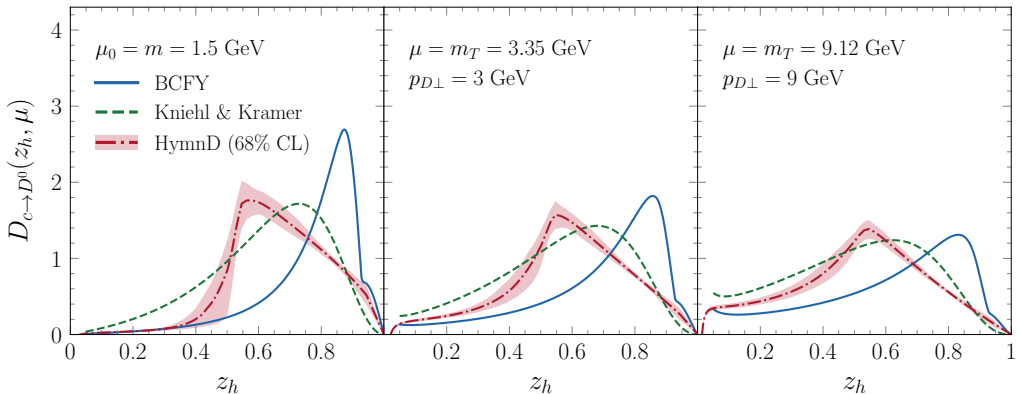
$$H(z_2, z_3, P_\perp, Q) = \delta(1 - z_2 - z_3)\frac{\alpha_s C_F}{2\pi^2}\frac{1}{z_3}\frac{\hat{Q}^2\left[(P_\perp^2 + \hat{Q}^2)^2 + P_\perp^4 z_3^2 + \hat{Q}^4 z_2^2\right]}{P_\perp^2(P_\perp^2 + \hat{Q}^2)^3}$$

$$\hat{Q}^2 = z_2 z_3 Q^2$$



Pb–Pb, $\sqrt{s_{NN}} = 5.36$ TeV. Effective flux at the interaction point P (solid), flux at the target centre (dashed)

Backup | $c \to D^0$ fragmentation functions



LO DGLAP evolution with EKO

- No gluon: the $c\bar{c}$ pair takes all the momentum of the $\mathbb{P} \Rightarrow x_{\mathbb{P}}$ is fixed, $x_{\mathbb{P}} = \dfrac{M^2_{c\bar{c}}}{W^2}$ $(Q^2 = 0)$

- Back-to-back pair (neglecting the momentum transfer), $m_T^2 = p_\perp^2 + m^2$:

$$M^2_{c\bar{c}} = \frac{m_T^2}{z(1-z)}\,, \qquad W^2 = 2q \cdot P_N = z_\gamma\, s_{NN}\,, \qquad z = \frac{p^+}{q^+} = \frac{m_T\, e^y}{z_\gamma \sqrt{s_{NN}}}$$

$$p^+ = m_T e^y/\sqrt{2}, \quad q^+ = \sqrt{2}\, z_\gamma E_A, \quad \sqrt{s_{NN}} = 2E_A$$

- Fixed $x_{\mathbb{P}}$ of the exclusive process:

$$x_{\mathbb{P}} = \frac{m_T^2}{z(1-z)\, z_\gamma\, s_{NN}} = \frac{m_T\, e^{-y}}{(1-z)\sqrt{s_{NN}}}$$

At $z = 1/2$: $x_{\mathbb{P}} = 2m_T\, e^{-y}/\sqrt{s_{NN}}$, the lower limit $x_{\mathbb{P}\,\min}$ of the diffractive$_{\mathsf{TMD}}$ $x_{\mathbb{P}}$ integral at fixed z

- Exclusive $\gamma + A \to c\bar{c} + A$ at fixed photon energy

$$\frac{\mathrm{d}\sigma^{\text{exclusive}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}} = \frac{\alpha_{\text{e.m.}} e_f^2 N_c S_\perp m^2}{2\pi^2}\, z\Big[\big(z^2+(1-z)^2\big)\mathcal{Q}_1^2(p_\perp, x_{\mathbb{P}}) + \mathcal{Q}_0^2(p_\perp, x_{\mathbb{P}})\Big]$$

$$\mathcal{Q}_{1,0}(p_\perp, x_{\mathbb{P}}) = \int \mathrm{d}r\, r\, J_{1,0}(p_\perp r)\, K_{1,0}(mr)\, \mathcal{N}(r, x_{\mathbb{P}})$$

- Single value of $x_{\mathbb{P}}$: $\dfrac{\mathrm{d}\sigma^{\text{exclusive}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}\,\mathrm{d}\ln x_{\mathbb{P}}} = x_{\mathbb{P}}\,\delta\big(x_{\mathbb{P}} - x_{\mathbb{P}}(q^+, y)\big)\,\dfrac{\mathrm{d}\sigma^{\text{exclusive}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}}$

- In UPCs: integrate over the photon flux $F(z_\gamma)$, with $\left|\dfrac{\mathrm{d}z_\gamma}{\mathrm{d}\ln x_{\mathbb{P}}}\right| = z_\gamma\,\dfrac{1-z}{z}$

$$\frac{\mathrm{d}\sigma^{\text{exclusive}}_{\text{UPC}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}\,\mathrm{d}\ln x_{\mathbb{P}}} = z_\gamma^*\,\frac{1-z_*}{z_*}\,F(z_\gamma^*)\,\frac{\mathrm{d}\sigma^{\text{exclusive}}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{p}}$$

z_γ^*: solution of $x_{\mathbb{P}}(q^+, y) = x_{\mathbb{P}}$, $\quad q^+ = \sqrt{2}\, z_\gamma^* E_A$, $\quad z_* = p^+/q^+$