The Simplest B Decay, Precisely

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JGU Mainz
w/ C. Cornella, M. Ferré, M. Neubert
based on [2601.14361]

Phenomenology Seminar
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European Research Council
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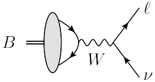
Precision Physics, Fundamental Interactions and Structure of Matter

Why care about $B \to \ell\,\nu$



$$\Rightarrow \quad \mathscr{B}(B \to \ell\,\nu) = \tau_B G_F^2 f_B^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

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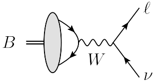

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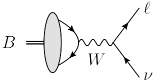

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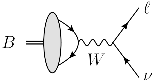

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- **Without** QED, hadronic effects are **well-understood**, with QCD uncertainties $\lesssim 1\%$

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B(p)\rangle = i\,f_B\,p^\mu\,, \qquad f_B = 189.4 \pm 1.4\,\text{MeV}$$

[FNAL/MILC 1712.09262]

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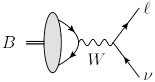

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- In the **exclusive** channel, QED corrections can be **sizeable**, competing with **QCD** uncertainties!

  $\longrightarrow$ **A precise prediction is needed!**

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This reasoning tempts us into a simple **Yukawa-theory** description of the process

$$\mathscr{L}_{\rm yuk} = (D_\mu B^\dagger)(D^\mu B) - m_B^2 B^\dagger B - y_B B \bar{\ell} \nu + \sum_\psi \bar{\psi}(i \not{D} - m_\psi)\psi$$

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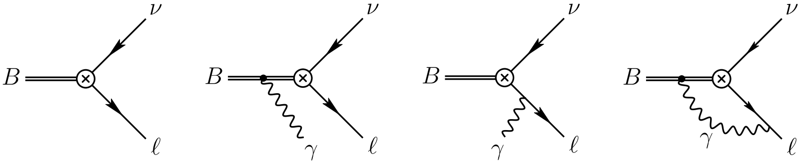

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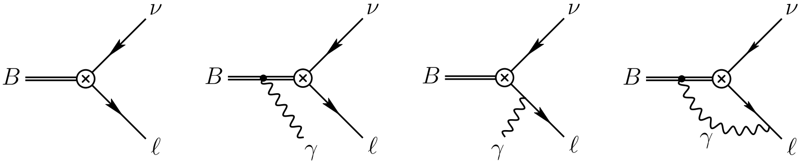

Captures **some** of the important QED corrections, but by far not all.

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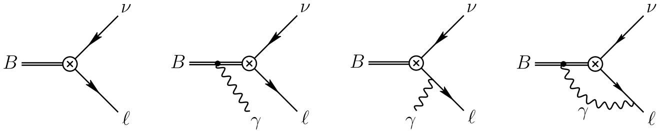

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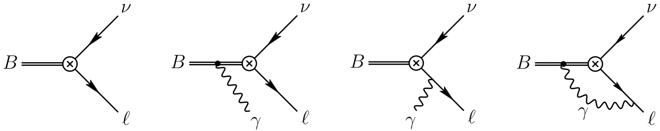

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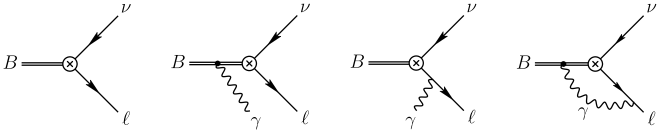

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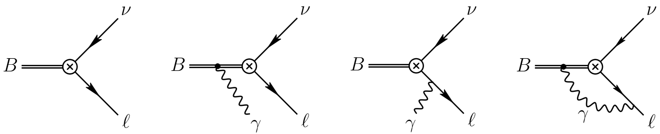

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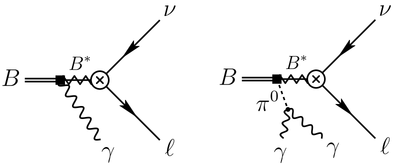
⇒ allows for additional decay topologies:



QED Corrections at High and Low Scales

- Electromagnetic corrections are sensitive to the **lepton mass** and the restriction on **additional radiation**, yielding large (double) logarithmic corrections

$$\alpha_{\mathrm{EM}} \log^{(2)} \frac{m_\ell^2}{m_B^2}\,, \qquad \alpha_{\mathrm{EM}} \log \frac{E_\gamma^2}{m_B^2}\,.$$

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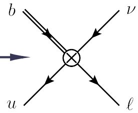

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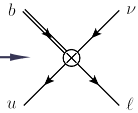
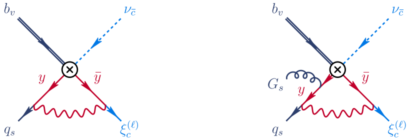


  - Radiation **softer** than Λ_{QCD} sees the meson as a **point-like** object.
$\longrightarrow$ description using meson-effective-theory.

- **Energetic** virtual photons with $E_\gamma \sim m_b$ and $\vec{p}_\gamma \parallel \vec{p}_\ell$ can **recoil** against the light **spectator** quark and transfer momentum between the lepton and the spectator.



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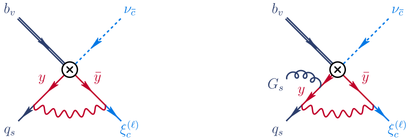

- The light partons are then **displaced** along the **lightcone**, and the hadronic currents become **non-local**. The corresponding hadronic matrix elements

$$\langle 0|\bar{q}(z_-)\ldots h_v(0)|B\rangle\,, \qquad \langle 0|\bar{q}(z_-)\ldots G_{\mu\nu}(y_-)\ldots h_v(0)|B\rangle\,,$$

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QED Corrections at Intermediate Scales

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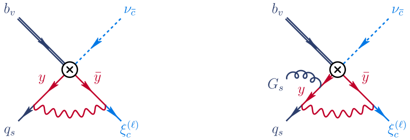

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are **lightcone distributions**.

- In the case of QED, due to the **external states** being **charged**, these distributions are **no longer process-universal** because QED is sensitive to the directions of the charged final state. [Beneke et al]

A Multiscale Problem

Beyond leading order in QED, the process $B \to \ell \nu$ depends on a variety of both **static** and **dynamical** scales (here discussing only $\ell = \mu$)

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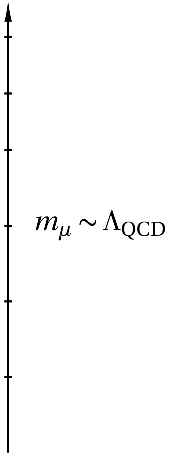

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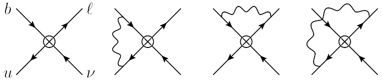

$m_\mu \sim \Lambda_{QCD}$

description of meson by Fock states $|b\bar{q}\rangle + |b\bar{q}g\rangle + \ldots$ [Beneke et al (*2019*), JHEP10 232]

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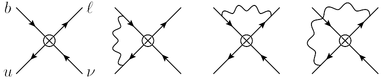

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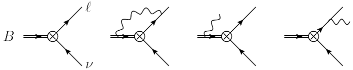

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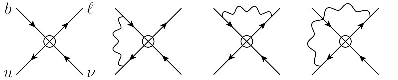

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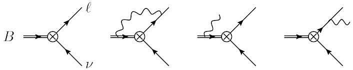

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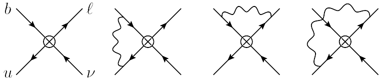

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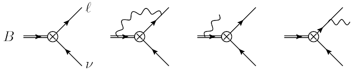

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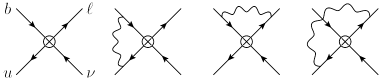

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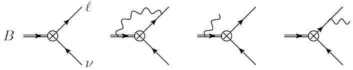
$E_{\mathrm{cut}} \sim \frac{\Lambda_{\mathrm{QCD}}\, m_\mu}{m_B}$



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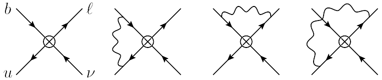

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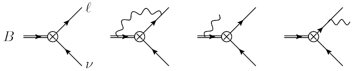

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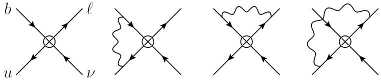

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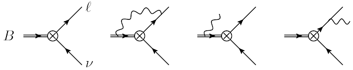

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[Dai et al (2022), Phys.Rev.D 105 3]

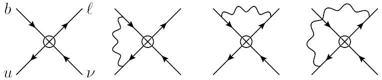
A Multiscale Problem

Beyond leading order in QED, the process $B \to \ell\,\nu$ depends on a variety of both **static** and **dynamical** scales (here discussing only $\ell = \mu$)

m_W

m_B

$\mu_{\text{hc}} = \sqrt{m_B \Lambda_{\text{QCD}}}$

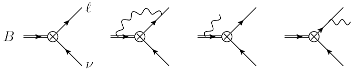


$m_\mu \sim \Lambda_{\text{QCD}}$

description of meson by Fock states $|b\bar{q}\rangle + |b\bar{q}g\rangle + \ldots$ [Beneke et al (*2019*), JHEP10 232]

description of meson by point-like scalar

$E_{\text{cut}} \sim \dfrac{\Lambda_{\text{QCD}}\, m_\mu}{m_B}$



$\mu_{\text{usc}} = \dfrac{E_{\text{cut}}\, m_\mu}{m_B}$

[Dai et al (*2022*), Phys.Rev.D 105 3]

→ **Factorization** of scales requires a number of different **EFT** constructions

(LEFT, HQET, SCET$_{\text{I,II}}$, bHLET, ...)

Weak Effective Theory

Relevant **LEFT** operators at the **weak scale**:

$$
\begin{aligned}
\mathscr{L}_{\mathrm{LEFT}} \ni\ & L_\ell^{V,LL}\left(\bar{\ell}\,\gamma^\mu P_L\,\nu_\ell\right)\left(\bar{u}\,\gamma_\mu P_L\,b\right) + L_\ell^{V,LR}\left(\bar{\ell}\,\gamma^\mu P_L\,\nu_\ell\right)\left(\bar{u}\,\gamma_\mu P_R\,b\right) \\
& + L_\ell^{S,RL}\left(\bar{\ell}\,P_L\,\nu_\ell\right)\left(\bar{u}\,P_R\,b\right) + L_\ell^{S,RR}\left(\bar{\ell}\,P_L\,\nu_\ell\right)\left(\bar{u}\,P_L\,b\right) + L_\ell^{T,RR}\left(\bar{\ell}\,\sigma^{\mu\nu} P_L\,\nu_\ell\right)\left(\bar{u}\,\sigma_{\mu\nu} P_L\,b\right) \\
& + \left[L_{\nu e}^{V,LL}\right]_{2112}\left(\bar{\nu}_\mu \gamma_\mu P_L\,\nu_e\right)\left(\bar{e}\,\gamma^\mu P_L\,\mu\right),
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Matching coefficients in the SM are:

$$L_\ell^{V,LL} = -\frac{4\,G_F^{(\mu)}}{\sqrt{2}}\,V_{ub}\,K_{\text{EW}}(\mu)$$
$$K_{\text{EW}}(\mu) = 1 + \frac{Q_\ell\,\alpha}{4\pi}\left[3\,Q_u\left(\ln\frac{\mu^2}{m_Z^2} + \frac{11}{2}\right) + (Q_b + Q_u)\left(1 + \frac{\kappa}{4}\right)\right] + \mathcal{O}(\alpha^2)$$

[Dekens, Stoffer 2019]

Virtual Corrections at the weak scale

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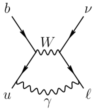
[Dekens, Stoffer 2019]

Here κ is a **scheme-dependent** parameter!

Dirac Reduction Schemes

$$K_{\mathrm{EW}}(\mu) = 1 + \frac{Q_\ell\,\alpha}{4\pi}\left[3\,Q_u\left(\ln\frac{\mu^2}{m_Z^2} + \frac{11}{2}\right) + (Q_b + Q_u)\left(1 + \frac{\kappa}{4}\right)\right] + \mathcal{O}(\alpha^2)$$

It arises from box diagrams in the matching producing additional Dirac structures:



$$\sim \left(\bar{u}_\ell\,\gamma_\mu\gamma_\nu\gamma_\rho\,P_L\,v_\nu\right)\left(\bar{v}_q\,\gamma^\mu\gamma^\nu\gamma^\rho\,P_L\,u_b\right) + \ldots$$

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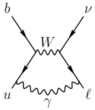

$$\sim \left(\bar{u}_\ell \gamma_\mu \gamma_\nu \gamma_\rho P_L v_\nu\right)\left(\bar{v}_q \gamma^\mu \gamma^\nu \gamma^\rho P_L u_b\right) + \ldots$$

These structures can be reduced **only** in $d = 4$ spacetime dimensions:

$$\gamma_\mu \gamma_\nu \gamma_\rho P_L \otimes \gamma^\mu \gamma^\nu \gamma^\rho P_L \overset{d=4}{=} 16 \gamma_\mu P_L \otimes \gamma^\mu P_L$$

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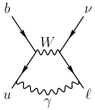

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In **dim-reg**, one generalizes this relation to the **multiplicative** identity

$$\gamma_\mu \gamma_\nu \gamma_\rho P_L \otimes \gamma^\mu \gamma^\nu \gamma^\rho P_L = (16 + \kappa \epsilon) \gamma_\mu P_L \otimes \gamma^\mu P_L$$

with the standard choice in the literature $\kappa = -4(1+\epsilon)$.

The dependence cancels bewteen the **matching coefficient** K_{EW} and the **matrix element**, allowing to **convert** between schemes:

$$\mathscr{A}_{\mathrm{LEFT}} = -\frac{4G_F^{(\mu)}}{\sqrt{2}} V_{ub} K_{\mathrm{EW}}^{[\kappa]} \left[1 - Q_\ell (Q_b + Q_u) \frac{\alpha}{16\pi} \kappa \right] \gamma_\mu \otimes \gamma^\mu + \ldots$$

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When working with EFTs that decompose **Lorentz structures** (like HQET, SCET, ...) working with $\kappa = 0$ is convenient, so we will translate the literature result to $\kappa = 0$.

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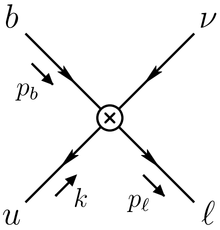
For example in **Sudakov coordinates**,

$$\gamma_\mu = \gamma_\mu^+ + \gamma_\mu^- + \gamma_\mu^\perp$$

reduction identities are **no longer** multiplicative and finding reductions **consistent** with the LEFT scheme is **difficult**.

Soft-Collinear Effective Theory

Next, integrate out dynamics of the "hard scale" $\mu_h \sim m_B$. Consider momenta:



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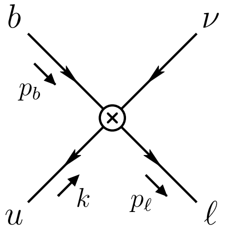

$$p_b^\mu = m_b v^\mu + q^\mu \qquad v^\mu = (1,0,0,0)$$

$$p_\ell^\mu = \frac{m_B}{2} n^\mu + \frac{m_\ell^2}{2m_B} \bar{n}^\mu \qquad n^\mu = (1,0,0,1)$$

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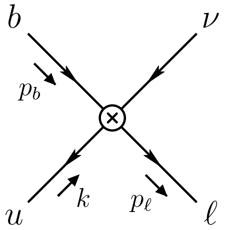

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This motivates an **EFT** construction in which the **fields are split** into **large** and **small** components:

$$\psi_b = e^{-i m_b v\cdot x} \left(\frac{1+\not{v}}{2} + \frac{1-\not{v}}{2}\right) \psi_b \equiv b_v + H_v$$

$$\psi_\ell = \left(\frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4}\right) \psi_\ell \equiv \xi_c + \eta_c$$

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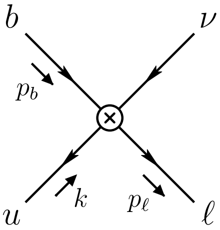

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For the heavy field, this is called **Heavy-Quark Effective Theory**, for the energetic light field it is called **Soft-Collinear Effective Theory**.

Modes and Scales in SCET

The effective theory below m_B has - a priori - the following **degrees of freedom** (and associated **virtualities**):

Light quarks, gluons	q_s, A_s	$k^2 \sim \Lambda_{\rm QCD}^2$	"soft"

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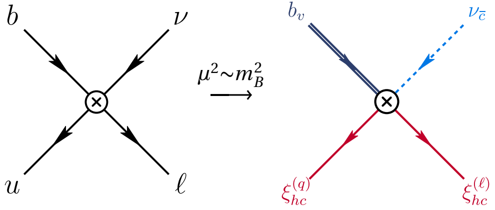

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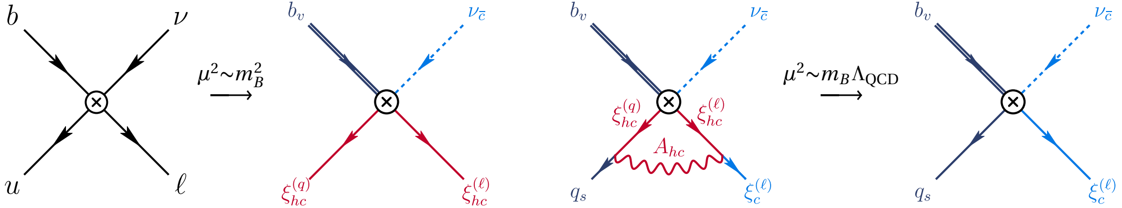

Operators in SCET-1 (excerpt)

Type-A Operators:

$$O_1^A = \frac{m_\ell}{\bar{n}\cdot\mathcal{P}_{hc}}\left(\bar{u}_s\,\bar{\not{n}}\,P_L\,b_v\right)\left(\bar{\mathcal{X}}_{hc}^{(\ell)}\,P_L\,\nu_{\bar{c}}\right)$$

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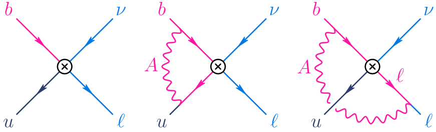

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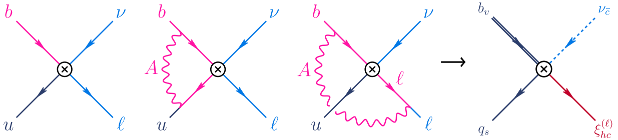

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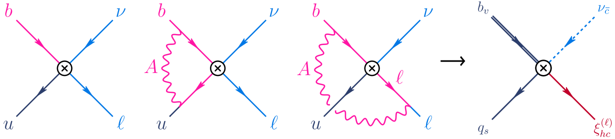

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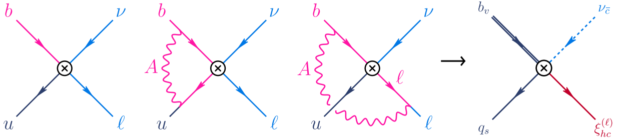

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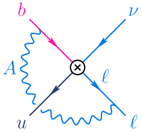

Operators in SCET-1 (excerpt)

Type-A Operators:

$$O_1^A = \frac{m_\ell}{\bar{n}\cdot\mathcal{P}_{hc}}\left(\bar{u}_s\,\bar{\not{n}}\,P_L\,b_v\right)\left(\bar{\mathcal{X}}_{hc}^{(\ell)}\,P_L\,\nu_{\bar{c}}\right)$$

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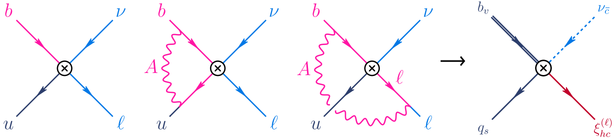

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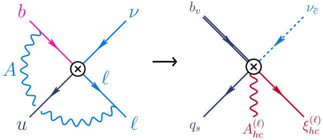

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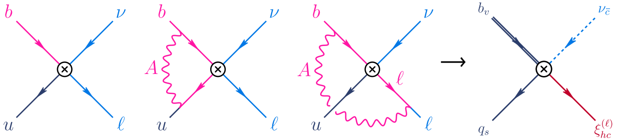

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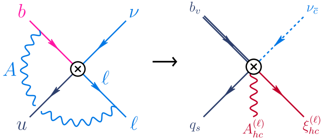

Type-C Operators

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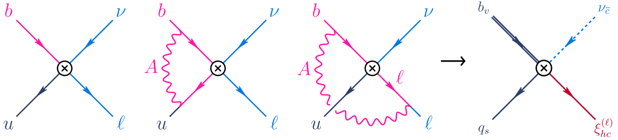

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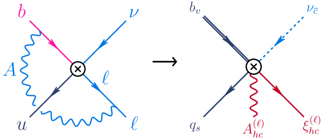

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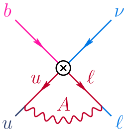

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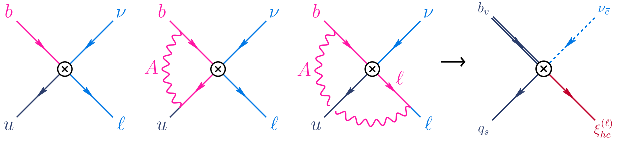

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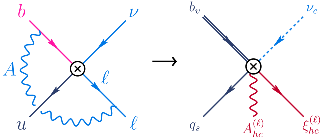

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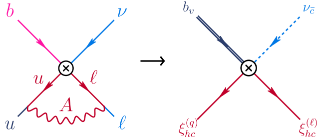

Factorization of Virtual Corrections - Endpoint Divergences

In SCET, operators with **multiple collinear fields** are distributions in the **momentum fraction** of the shared direction, matrix elements are **convolution integrals**:

$$i\mathscr{A}^{\text{virt}}_{B\to\ell\nu} \supset -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\bar{u}(p_\ell)P_L v(p_\nu)$$

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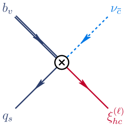

$$S_A = \langle 0 | \bar{u}(0) \not{\bar{n}} P_L b_v(0) | B \rangle$$

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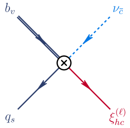

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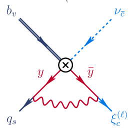
$$S_C(\omega) = \int ds\; e^{i\omega s}\,\langle 0|\bar{u}(sn)\not{\bar{n}}\,P_L\,b_v(0)|B\rangle$$

energetic-photon exchange $\longrightarrow$ LCDAs



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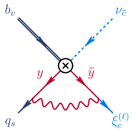

- At $y = 0$ and $\bar{y} = 0$, one **collinear** field has anomalously **low energy** and the mode **separation** between **soft** and **collinear** modes **fails**.

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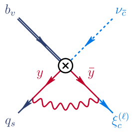

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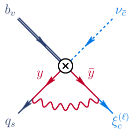

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- This rearrangement begins with **identifying** the small-y region of the **second term** with a contribution to the **first** one, this is called **refactorization**.

[Liu, Neubert (2003.03393); Liu et al (2009.04456, 2009.06779, 2112.00018); Beneke et al (2008.04943, 2205.04479)]
[Bell et al (2205.06021); Feldmann et al (2211.04209); Hurth, Szafron (2312.10450)]

Refactorization-Based Subtraction

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endpoint-divergent

$$S_A = \langle 0 | O_A | B \rangle$$

$$O_A = \bar{q}_s \not{n} P_L h_{v_B} Y_n^{(\ell)\dagger}$$

$$J_C = -Q_\ell Q_u \frac{\alpha}{2\pi} \frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{1-\epsilon} \left(\frac{\mu^2}{m_b \omega y \bar{y}} \right)^{\epsilon} \left(\frac{1}{y} + 1 - 2\epsilon \right)$$

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$$- \theta(\Lambda - m_b y) [\![H_C(m_b, y)]\!] \cdot [\![J_C(m_b\omega, y)]\!] \Big] \cdot S_C(\omega) \Big]$$

subtract small-y

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$$\cdot\Big[H_A(m_b) S_A^{(\Lambda)} + \int d\omega \int_0^1 dy \Big[H_C(m_b,y)\cdot J_C(m_b\omega,y)\cdot S_C(\omega) - \theta(\Lambda - m_b y)[\![H_C(m_b,y)]\!]\cdot[\![J_C(m_b\omega,y)]\!]\Big]\cdot S_C(\omega)\Big]$$

subtract small-y

add it back

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$$S_A^{(\Lambda)} = \langle 0|O_A^{(\Lambda)}|B\rangle$$

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$$\longrightarrow \langle 0|O_A^{(\Lambda)}|B\rangle = -\frac{i}{2}\sqrt{m_B}F_-(\Lambda,\mu)\langle 0|Y_v^{(B)}Y_n^{(\ell)\dagger}|0\rangle$$

Matching at the Hard-Collinear Scale

Integrating out the **hard-collinear** modes yields (showing only a subset) operators that follow directly from SCET-1:

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...and some with **delocalized** quarks:

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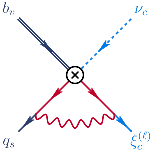

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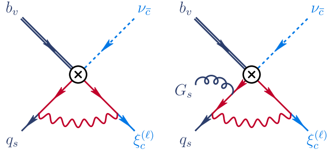

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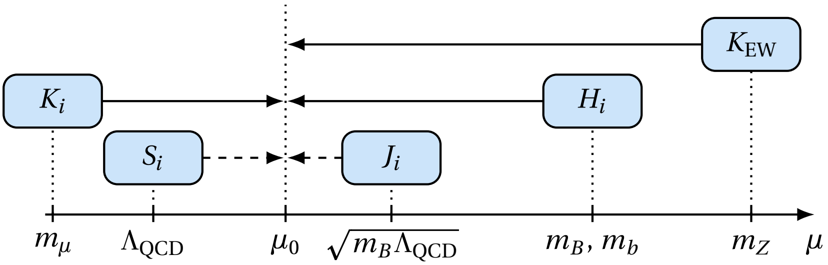

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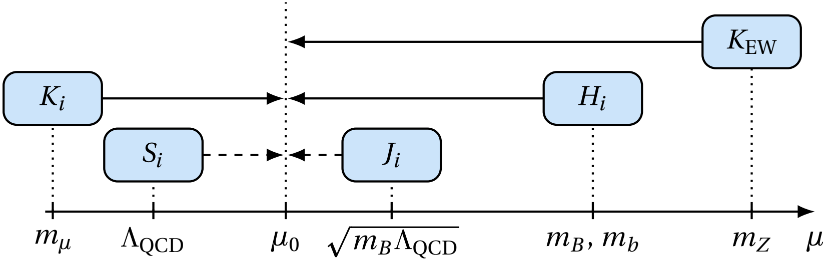

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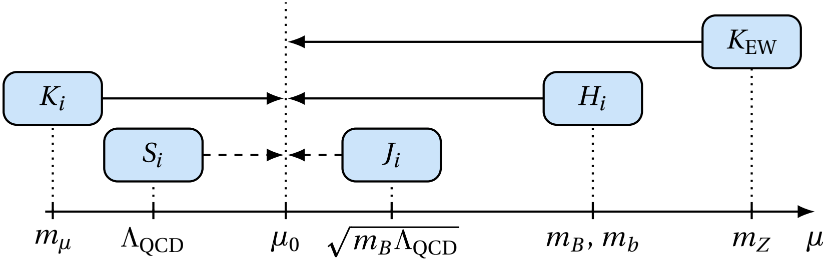

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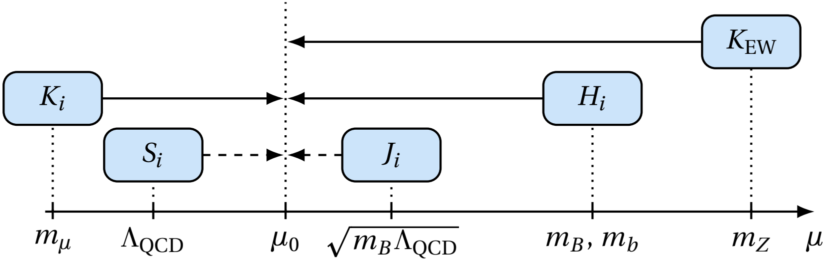

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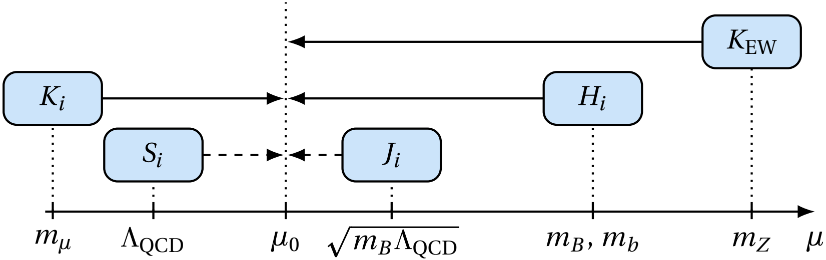

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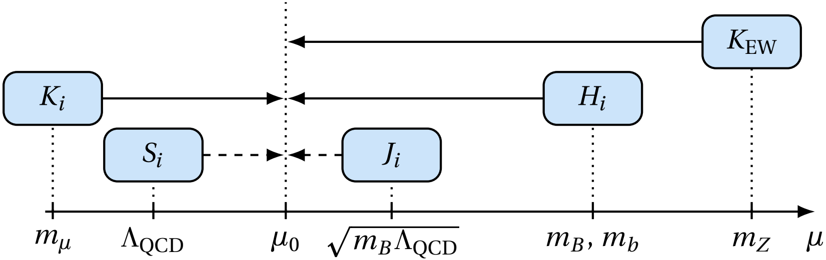

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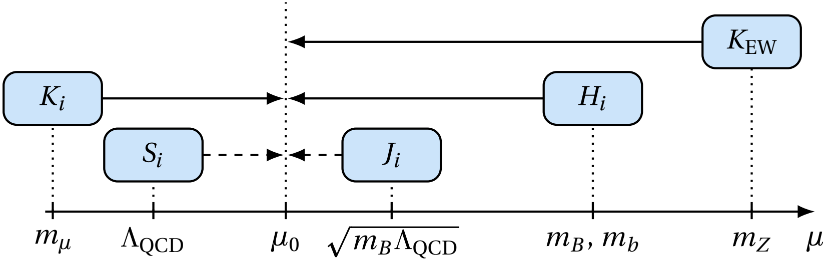

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SCET-2 Matrix Elements

Matrix elements in SCET-2 factorize into **soft, hadronic** and **collinear, leptonic**:

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$$K_1(\mu) = 1 + Q_\ell^2 \frac{\alpha}{2\pi} \left(\log^2 \frac{\mu}{m_\ell} - \frac{5}{2} \log \frac{\mu}{m_\ell} + \frac{\pi^2}{24} - 1 \right)$$

$$K_2(x,\mu) = -Q_\ell \frac{\alpha}{\pi}\, x \left(\log \frac{\mu}{m_\ell} - \log x - \frac{1}{2} \right)$$

The **hadronic soft functions** of the subtracted operators, defined as

$$\left\langle 0 \left| \bar{u}_s \left[1 - \theta_T \left(\frac{-i\,\bar{n} \cdot \overleftarrow{D}_s}{\bar{n} \cdot v} - \Lambda \right) \right] \not{n}\, P_L\, b_v\, Y_n^{(\ell)\dagger} \right| B \right\rangle = -\frac{i\sqrt{m_B}}{2} F_{-}(\Lambda, \mu) \,,$$

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$\Rightarrow$ to be explored in more detail **in future**!

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\begin{aligned}
\mathscr{R}_{\text{virt}} = \exp\Bigg[-Q_\ell\, Q_u\, \frac{\alpha}{\pi} \int_{m_B}^{\mu_0} \frac{d\mu'}{\mu'} \int_{m_B}^{\mu'} \frac{d\bar{\omega}}{\bar{\omega}}\, \widetilde{U}_C(\mu', \bar{\omega})\Bigg] \frac{F_-(m_B, \mu_0)}{F_{\text{QCD}}(\mu_0)} \\
\times \Bigg\{ 1 + \frac{\alpha}{4\pi} \Bigg[\frac{3}{2} Q_\ell^2 \ln \frac{\mu_0^2}{m_\ell^2} - \frac{3}{2} Q_b^2 \ln \frac{\mu_0^2}{m_B^2} - (2+z)\, Q_\ell\, Q_b \ln \frac{m_B^2}{m_\ell^2} + \left(\frac{\pi^2}{12} - \frac{27}{2} \right) Q_\ell^2 \\
+ \left(\frac{21}{2} + \frac{z^2 \ln z}{z-1} + z - 2\,\text{Li}_2(1-z) - \frac{\pi^2}{12} \right) Q_\ell\, Q_b - (2 + 3\ln z)\, Q_b^2 \Bigg] \\
+ Q_\ell\, Q_u\, \frac{\alpha}{\pi} \int_{m_B}^{\mu_0} \frac{d\mu'}{\mu'} \frac{\widetilde{U}_C(\mu', m_B)}{1-\delta(\mu')} \\
- Q_\ell\, Q_u\, \frac{\alpha}{2\pi} \Bigg[\frac{1}{1-\delta(\mu_0)} \ln \frac{\mu_0^2}{m_B\, \omega_-(\mu_0)} - h_1\big(\delta(\mu_0)\big) \Bigg] \widetilde{U}_C(\mu_0, m_B) \\
- Q_\ell\, Q_u\, \frac{\alpha}{\pi} \frac{\widetilde{U}_C(\mu_0, m_B)}{1-\delta(\mu_0)} \int_0^\infty d\omega \int_0^\infty d\omega_g\, \phi_{3g}^B(\omega, \omega_g, \mu_0) \Bigg[\frac{1}{\omega_g} \ln \frac{\omega + \omega_g}{\omega} - \frac{1}{\omega + \omega_g} \Bigg] \Bigg\}
\end{aligned}
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Heavy-Hadron Chiral Perturbation Theory

After **integrating out** pyhsics above Λ_{QCD} and m_ℓ, relevant modes are those feeling the **radiation veto** $\longrightarrow$ modes with $p_0 \sim E_{\text{cut}}$ in the B-frame

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Note: **virtual** corrections are **scaleless** since they do **not** feel the radiation **veto** E_{cut}!

EFT below the hadronic scale: Leptons

Since $m_\mu \sim \Lambda_{\text{QCD}} \gg E_{\text{cut}} \to$ leptons also considered **infinitely heavy** at the **low scale**!

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$$\bar{\ell}\,\gamma^\mu P_L\, \nu_\ell \quad \to \quad e^{i m_\ell\, v_\ell \cdot x}\, C_{v_\ell}^{(\ell)\dagger}(x) \left[\, \bar{h}_{v_\ell}^{(0)} \gamma_\perp^\mu P_L\, \nu_{\bar{c}} + \frac{m_\ell}{\bar{n}\cdot p_\ell}\, \bar{n}^\mu\, \bar{h}_{v_\ell}^{(0)} P_L\, \nu_{\bar{c}} \,\right]$$

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with an **ultrasoft-collinear** Wilson line

$$C_{v_\ell}^{(\ell)}(x) = \exp\left[-i\, Q_\ell\, e \int_0^\infty ds\, n \cdot A_{usc}(x + s\, v_\ell)\, e^{-\epsilon|s|} \right]$$

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$$\bar{\ell}\,\gamma^\mu P_L\, \nu_\ell \quad \longrightarrow \quad e^{i m_\ell\, v_\ell\cdot x}\, C_{v_\ell}^{(\ell)\dagger}(x) \left[\, \bar{h}_{v_\ell}^{(0)} \gamma_\perp^\mu P_L\, \nu_{\bar{c}} + \frac{m_\ell}{\bar{n}\cdot p_\ell}\, \bar{n}^\mu\, \bar{h}_{v_\ell}^{(0)} P_L\, \nu_{\bar{c}} \,\right]$$

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- Superscript "$^{(0)}$" denotes **decoupling** (leading-power photon couplings in $C_{v_\ell}^{(\psi)}$)
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- B and B^* meson nearly mass-degenerate $\longrightarrow$ collect into **superfield**

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with a **common rephasing** by $\bar{m}_B = m_B + \Lambda_c$.

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- $SU(2)_v$**-breaking spurions** give subleading operators:

$$-\frac{\lambda_2}{8m_b} \mathrm{Tr}\left[\bar{H}\left(C_{\mathrm{mag}} \sigma^{\mu\nu}\right) H \sigma_{\mu\nu}\right] + \ldots$$

  generate the mass-splitting: $\delta_{B*} = 2\lambda_2 C_{\mathrm{mag}}/m_b = \dfrac{m_{B*}^2 - m_B^2}{2m_B}.$

Low-Energy Lagrangian

Gathering our results **so far**, the **low-energy Lagrangian** takes the form:

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$$+ \frac{e\, c_{BB*\gamma}}{2\Lambda_c}\left(v^\mu \rho_v^{\dagger\,\nu} \varphi_v\, \tilde{F}_{\mu\nu}^{us} + \text{h.c.}\right) + \dots$$

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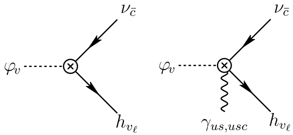
$$\mathscr{L}_{H\ell} = i\sqrt{2}\, G_F^{(\mu)}\, V_{ub}\, \overline{Y}_v^{(B)}\, \overline{C}_{\bar{n}}^{(B)}\, Y_n^{(\ell)\dagger}\, C_{v_\ell}^{(\ell)\dagger} \times \left\{ \frac{y_B f_B^{\text{QCD}}\, m_\ell}{\sqrt{2\, m_B}}\, \varphi_v\, \bar{h}_{v_\ell} P_L\, \nu_{\bar{c}} - \frac{f_{B*}^{\text{QCD}}\, m_{B*}}{\sqrt{2\, m_{B*}}}\left[y_{B*}^\perp\, \bar{h}_{v_\ell}\, \not{\rho}_v^\perp P_L\, \nu_{\bar{c}} + y_{B*}^\parallel \frac{m_\ell}{m_{B*}}\, \bar{n}\cdot\rho_v\, \bar{h}_{v_\ell} P_L\, \nu_{\bar{c}} \right] \right\}$$

... plus many more.

“Direct contribution”



- from the $B\ell\nu$ interactions,

$$\frac{y_B f_B^{\mathrm{QCD}}\, m_\ell}{\sqrt{2\, m_B}}\, \overline{Y}_v^{(B)}\, \overline{C}_{\bar n}^{(B)}\, Y_n^{(\ell)\dagger}\, C_{v_\ell}^{(\ell)\dagger}\, \varphi_v\, \bar h_{v_\ell}\, P_L\, \nu_{\bar c}$$

Decay Topologies in the HMET

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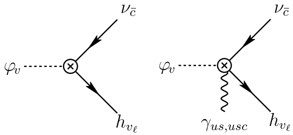

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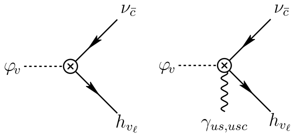

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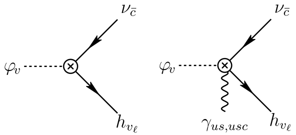

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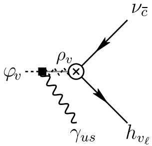

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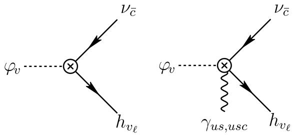

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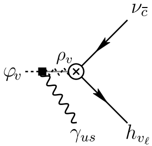

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$$-\frac{y_{B^*}^\perp f_{B^*}^{\mathrm{QCD}}\, m_{B^*}}{\sqrt{2\, m_{B^*}}}\, \overline{Y}_v^{(B)}\, \overline{C}_{\bar n}^{(B)}\, Y_n^{(\ell)\dagger}\, C_{v_\ell}^{(\ell)\dagger}\, \bar h_{v_\ell}\, \not{\phi}_v^\perp\, P_L\, \nu_{\bar c}$$

Chiral Perturbation Theory

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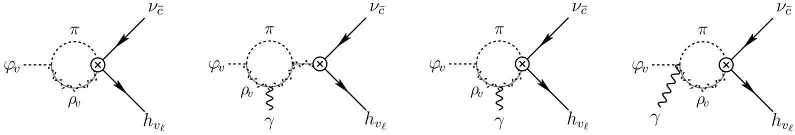
The **relevant terms** can be written as:

$$\mathscr{L}_{\mathrm{HH}\chi\mathrm{PT}} = -\frac{1}{2}\mathrm{Tr}\left[\bar{H}\, i v\cdot D\, H\right] + \frac{1}{4}\mathrm{Tr}\left[\bar{H} H\, v\cdot L\right] - \frac{g}{4}\mathrm{Tr}\left[\bar{H} H \slashed{L}\gamma_5\right] + \ldots$$

with the left-handed meson current $L^\mu = \Sigma\, i D^\mu \Sigma$.

Using our newly acquired Lagrangian, we can compute **virtual corrections**



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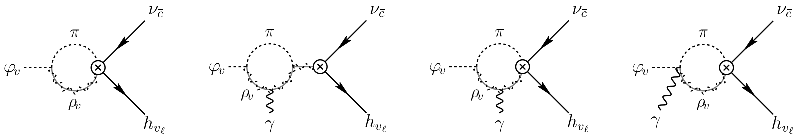

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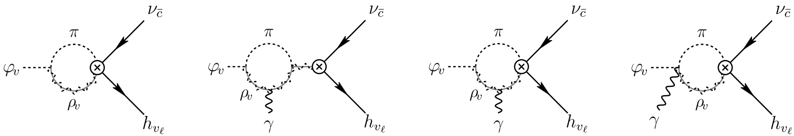

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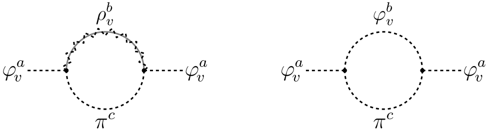

Pion Loops

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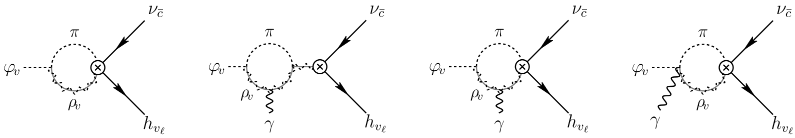

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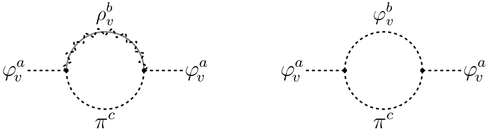

But these are absorbed by the **decay constant**.

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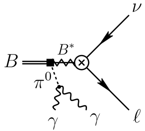

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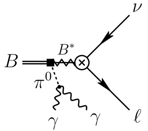

At first glance $O(\alpha^2)$, but the pion is **extremely narrow** & **almost exclusively** decays to $\gamma\gamma$:

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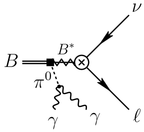

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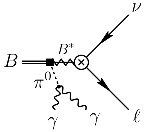

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$\longrightarrow$ Turns out to be **leading contribution** once $E_{\mathrm{cut}} > m_\pi$!

Real corrections to the **direct** contribution are given by matrix elements of the **ultrasoft** and **ultrasoft-collinear** Wilson lines

$$W_{us}(\omega_{us}) = \sum_{n=0}^{\infty} \left[\int \prod_{i=1}^{n} \frac{d^{d-1}\boldsymbol{q}_i}{(2\pi)^{d-1}\, 2E_i} \left| \langle \gamma^n_{us} | \overline{Y}^{(B)}_{v}\, Y^{(\ell)\dagger}_{n} |0\rangle \right|^2 \delta\Big(\omega_{us} - \sum_{j=1}^{n} E_j\Big) \right]$$

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Resummation of Real Radiation

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$$W_{usc}(\omega_{usc}) = \sum_{n=0}^{\infty} \left[\int \prod_{i=1}^{n} \frac{d^{d-1}\boldsymbol{q}_i}{(2\pi)^{d-1}\, 2E_i} \left| \langle \gamma^n_{usc} | \overline{C}^{(B)}_{\bar{n}} \, C^{(\ell)\dagger}_{v_\ell} |0\rangle \right|^2 \delta\Big(\omega_{usc} - \sum_{j=1}^{n} E_j\Big) \right]$$

convoluted in a radiation function

$$R(E_{\text{cut}}, \mu) = \int_0^\infty d\omega_{us} \int_0^\infty d\omega_{usc}\, \theta\big(E_{\text{cut}} - \omega_{us} - \omega_{usc}\big)\, W_{us}(\omega_{us}, \mu)\, W_{usc}(\omega_{usc}, \mu)$$

Resummation of Real Radiation

Real corrections to the **direct** contribution are given by matrix elements of the **ultrasoft** and **ultrasoft-collinear** Wilson lines

$$W_{us}(\omega_{us}) = \sum_{n=0}^{\infty}\left[\int \prod_{i=1}^{n} \frac{d^{d-1}\boldsymbol{q}_i}{(2\pi)^{d-1}\,2E_i}\left|\langle \gamma_{us}^{n}|\,\overline{Y_v^{(B)}}\,Y_n^{(\ell)\dagger}\,|0\rangle\right|^2 \delta\Big(\omega_{us}-\sum_{j=1}^{n}E_j\Big)\right]$$

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Convolution and renormalization best done in **Laplace space**, we obtain:

$$R(E_{\rm cut},\mu) = \left(\frac{\mu^2\,m_B}{(2E_{\rm cut})^2\,m_\ell}\right)^{-\gamma_{\rm soft}} \frac{e^{-2\gamma_E\gamma_{\rm soft}}}{\Gamma(1+2\gamma_{\rm soft})}\left[1+\frac{\alpha Q_\ell^2}{2\pi}\left(2-\frac{\pi^2}{3}\right)\right] \equiv \left(\frac{\mu^2\,m_B}{(2E_{\rm cut})^2\,m_\ell}\right)^{-\gamma_{\rm soft}}\mathscr{W}$$

Phenomenology

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- The **subtracted decay constant** $F_-(\Lambda,\mu)$ is a **new and unknown** quantity:

$$F_-(\Lambda,\mu) = F_{\rm QCD}(\mu)\left[1+\frac{\alpha}{\pi}f^{(1)}(\Lambda,\mu)+\mathcal{O}\left(\alpha^2\right)\right] \qquad f^{(1)}(\mu_0,\mu_0)=0\pm1$$

Decay Rate - Factorization Formula

$$\frac{\Gamma(E_{\text{cut}})}{\Gamma_{\text{tree}}} = \left(\frac{\alpha(m_Z)}{\alpha(m_B)}\right)^{\frac{9}{20}} \left(\frac{2E_{\text{cut}}}{m_B}\right)^{2\gamma_{\text{soft}}} \cdot \mathscr{W} \cdot \mathscr{R}_{\text{virt}}^2$$

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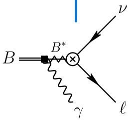
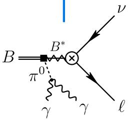
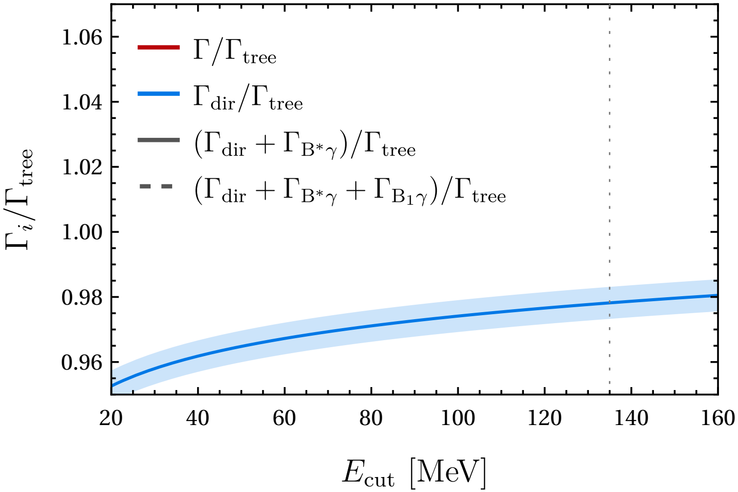


indirect contributions



Decay Rate vs Radiation Veto



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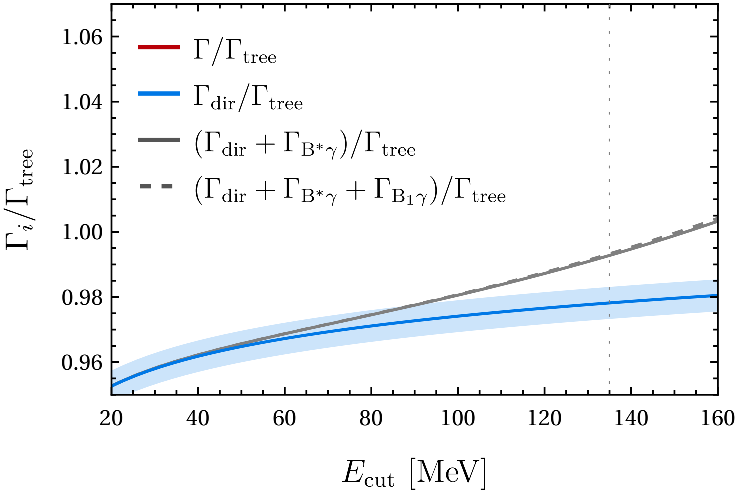

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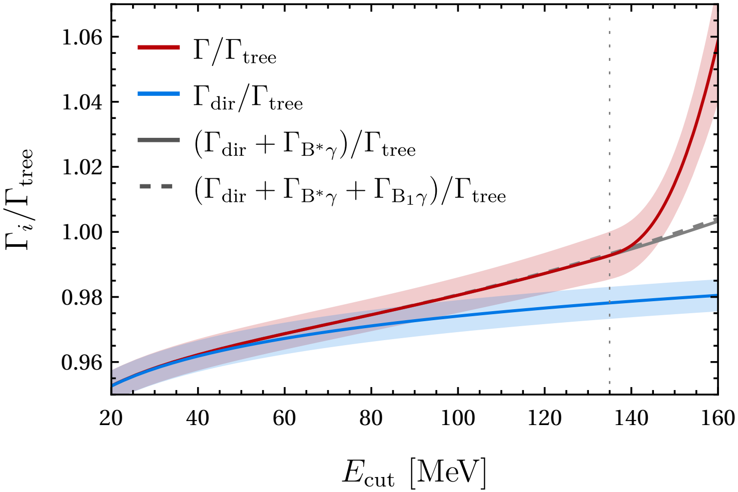

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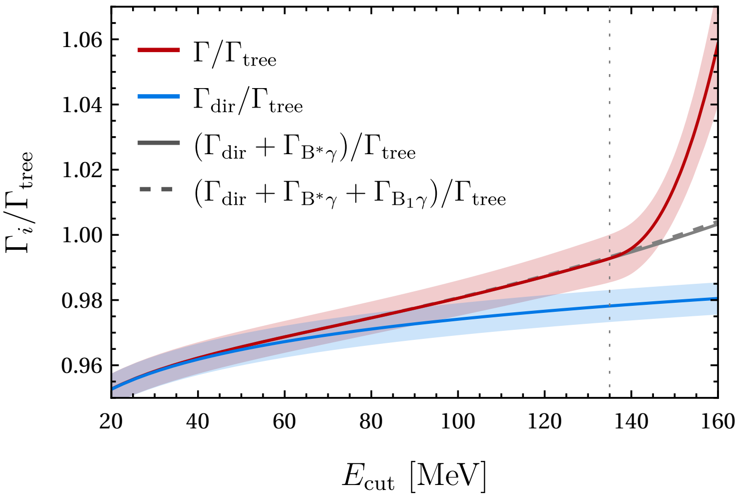

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Implementation of **radiation veto** makes a **crucial difference**!

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