Hadronic channels and structure-dependent corrections in BabaYaga@NLO

Workshop on Radiative Corrections and Monte Carlo Simulations at Electron-Positron Colliders Turin 2026

Marco Ghilardi
On behalf of the BabaYaga team

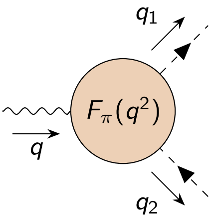
June 4, 2026

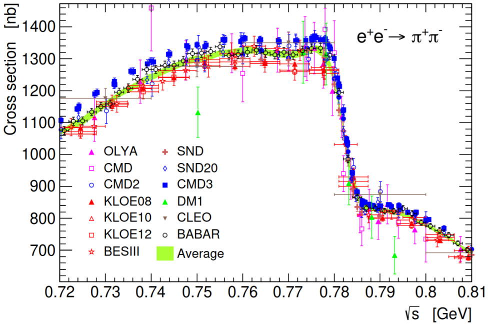
INFN National Institute for Nuclear Physics

UNIVERSITÀ DI PAVIA

The Pion Form Factor

$$\langle \pi^+(q_2)\pi^-(q_1)|J^\mu_\pi(0)|0\rangle = e(q_1 - q_2)^\mu F_\pi\left(q^2\right)$$





https://arxiv.org/pdf/2312.02053

The energy scan

At an e^+e^- collider, the hadronic cross section can be measured directly by varying the center-of-mass energy:

$$e^+e^- \to \pi^+\pi^-, \qquad \sqrt{s} = \text{variable}$$

The energy scan

At an e^+e^- collider, the hadronic cross section can be measured directly by varying the center-of-mass energy:

$$e^+e^- \to \pi^+\pi^-, \qquad \sqrt{s} = \text{variable}$$

For each energy point, the pion production cross section is obtained from:

$$\sigma_{\pi^+\pi^-}(s) = \frac{N_{\pi^+\pi^-}}{\mathcal{L}\,\varepsilon\,(1+\delta_{\mathrm{rad}})},$$

where:

- $\mathcal{L}$ is the integrated luminosity,
- ε is the detection efficiency,
- δ_{rad} accounts for radiative corrections.

The energy scan

At an e^+e^- collider, the hadronic cross section can be measured directly by varying the center-of-mass energy:

$$e^+e^- \to \pi^+\pi^-, \qquad \sqrt{s} = \text{variable}$$

For each energy point, the pion production cross section is obtained from:

$$\sigma_{\pi^+\pi^-}(s) = \frac{N_{\pi^+\pi^-}}{\mathcal{L}\,\varepsilon\,(1+\delta_{\rm rad})},$$

where:

- $\mathcal{L}$ is the integrated luminosity,
- ε is the detection efficiency,
- $\delta_{\rm rad}$ accounts for radiative corrections.

By scanning different energies, the pion form factor can be measured over a wide kinematic range.

The radiative return

At fixed collider energy $\sqrt{s}$, lower-energy pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \quad M_{\pi\pi}^2 = (p_{\pi^+} + p_{\pi^-})^2 < s$$

The radiative return

At fixed collider energy $\sqrt{s}$, lower-energy pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \quad M_{\pi\pi}^2 = (p_{\pi^+} + p_{\pi^-})^2 < s$$

The measured differential cross section relates to the pion production cross section:

$$\frac{\mathrm{d}\sigma_{\pi^+\pi^-\gamma}}{dM_{\pi\pi}} = H(s, M_{\pi\pi}^2)\,\sigma_{\pi^+\pi^-}(M_{\pi\pi}^2),$$

where $H(s, M_{\pi\pi}^2)$ is the radiator function.

The radiative return

At fixed collider energy $\sqrt{s}$, lower-energy pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \quad M_{\pi\pi}^2 = (p_{\pi^+} + p_{\pi^-})^2 < s$$

The measured differential cross section relates to the pion production cross section:

$$\frac{\mathrm{d}\sigma_{\pi^+\pi^-\gamma}}{dM_{\pi\pi}} = H(s, M_{\pi\pi}^2)\,\sigma_{\pi^+\pi^-}(M_{\pi\pi}^2),$$

where $H(s, M_{\pi\pi}^2)$ is the radiator function.

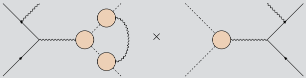
A precise theoretical treatment of radiative corrections is required to extract the pion form factor at the sub-percent level, both in energy scan and radiative return.

The perturbative accuracy

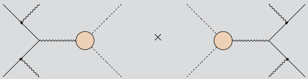
NLO

Exact calculation up to $\mathcal{O}(\alpha)$ w.r.t. the born scattering amplitude.

- Virtual corrections



- Real corrections

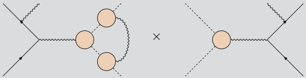


The perturbative accuracy

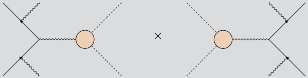
NLO

Exact calculation up to $\mathcal{O}(\alpha)$ w.r.t. the born scattering amplitude.

- Virtual corrections

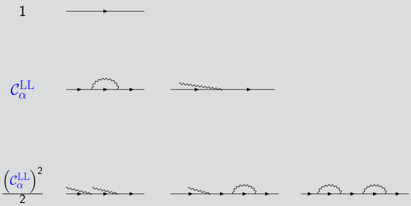


- Real corrections



LL resummation

Approximate calculation of soft-virtual corrections to any order.

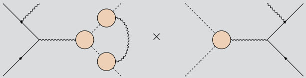


The perturbative accuracy

NLO

Exact calculation up to $\mathcal{O}(\alpha)$ w.r.t. the born scattering amplitude.

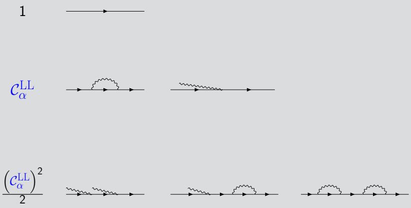
- Virtual corrections



- Real corrections

LL resummation

Approximate calculation of soft-virtual corrections to any order.

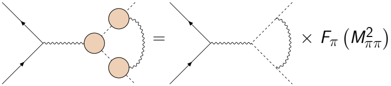


NLO+LL (NLOPS) accuracy

$$d\sigma_{\mathrm{NLOPS}} = \sum_{n=1}^{\infty} \exp\left\{-\mathcal{C}_\alpha^{\mathrm{LL}}\right\} F_{\mathrm{SV}} \frac{1}{n!} \left|\mathcal{M}_n^{X}\right|^2 d\Phi_n(\{p\},\{k\})\,.$$

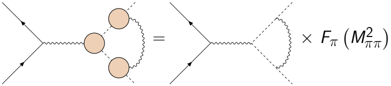
The Pion internal structure

The standard way which respects IR cancellation is F×sQED:



The Pion internal structure

The standard way which respects IR cancellation is F×sQED:

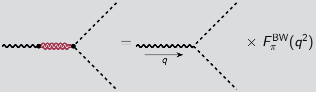


To include the pion form factor into loop integration:

GVMD

$$F_\pi^{\mathrm{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\mathrm{BW}}(q^2)$$

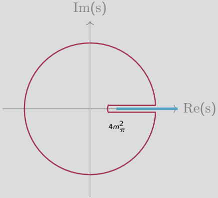
$$F_{\pi,v}^{\mathrm{BW}}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED

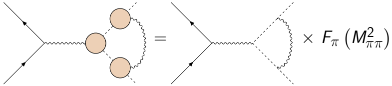
$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\mathrm{Im}F_\pi(s')}{q^2 - s'}$$

$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\mathrm{Im}F_\pi(s') = 1$$



The Pion internal structure

The standard way which respects IR cancellation is F×sQED:

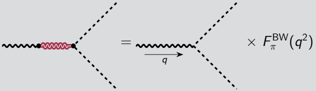


To include the pion form factor into loop integration:

GVMD

$$F_\pi^{\mathrm{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\mathrm{BW}}(q^2)$$

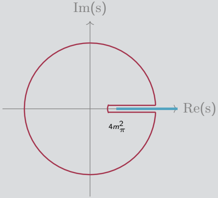
$$F_{\pi,v}^{\mathrm{BW}}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED

$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\mathrm{Im}F_\pi(s')}{q^2 - s'}$$

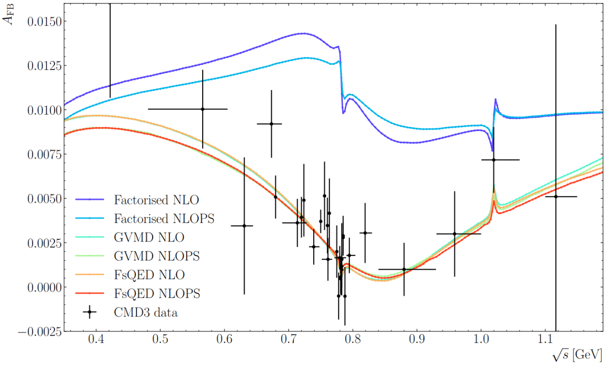
$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\mathrm{Im}F_\pi(s') = 1$$



Scan

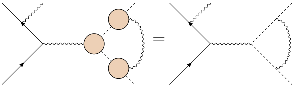
2603.28621

The charge asymmetry



The Pion internal structure

The standard way which respects IR cancellation is F×sQED:



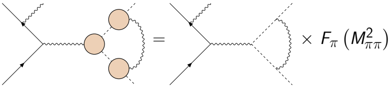
$$= \quad \times \; F_\pi\left(M^2_{\pi\pi}\right)$$

The Pion internal structure

F×sQED

2601.19530

The standard way which respects IR cancellation is F×sQED:



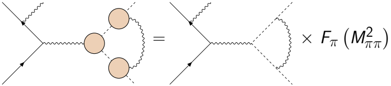
The Pion internal structure

INFN UNIVERSITÀ DI PAVIA

F×sQED

2601.19530

The standard way which respects IR cancellation is F×sQED:

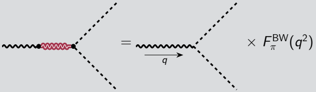


To include the pion form factor into loop integration:

GVMD

$$F_\pi^{\rm BW}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\rm BW}(q^2)$$

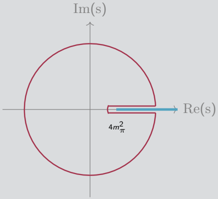
$$F_{\pi,v}^{\rm BW}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED

$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{{\rm d}s'}{s'}\frac{{\rm Im}F_\pi(s')}{q^2 - s'}$$

$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{{\rm d}s'}{s'}{\rm Im}F_\pi(s') = 1$$



The Pion internal structure

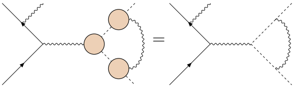
F×sQED

2601.19530

GVMD

2603.28621

The standard way which respects IR cancellation is F×sQED:



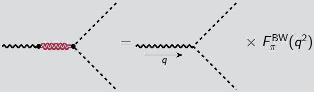
$\times\, F_\pi\left(M^2_{\pi\pi}\right)$

To include the pion form factor into loop integration:

GVMD

$$F_\pi^{\mathrm{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\mathrm{BW}}(q^2)$$

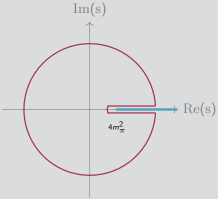
$$F_{\pi,v}^{\mathrm{BW}}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED

$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\mathrm{Im}F_\pi(s')}{q^2 - s'}$$

$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\mathrm{Im}F_\pi(s') = 1$$

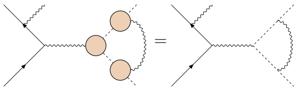


The Pion internal structure

F×sQED

2601.19530

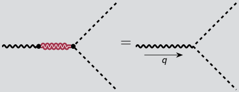
The standard way which respects IR cancellation is F×sQED:



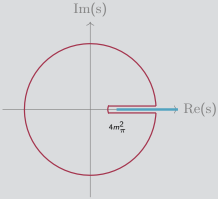
$\times\ F_\pi\left(M^2_{\pi\pi}\right)$

To include the pion form factor into loop integration:

GVMD	FsQED
$F_\pi^{\rm BW}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\rm BW}(q^2)$	$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{{\rm d}s'}{s'}\frac{{\rm Im}F_\pi(s')}{q^2-s'}$
$F_{\pi,v}^{\rm BW}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2-q^2}$	$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{{\rm d}s'}{s'}{\rm Im}F_\pi(s') = 1$



$\times\ F_\pi^{\rm BW}(q^2)$



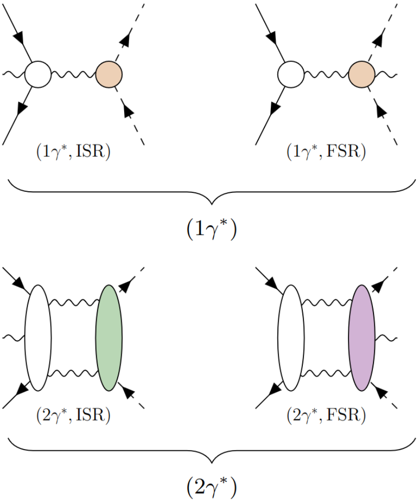
GVMD

2603.28621

FsQED

W.I.P.

Subsets of GVMD corrections

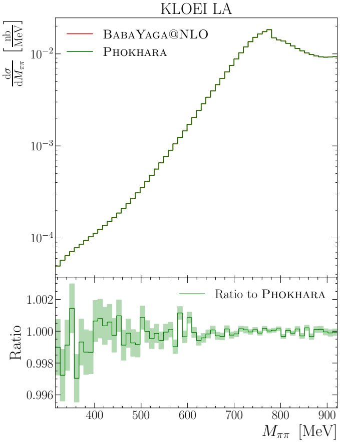


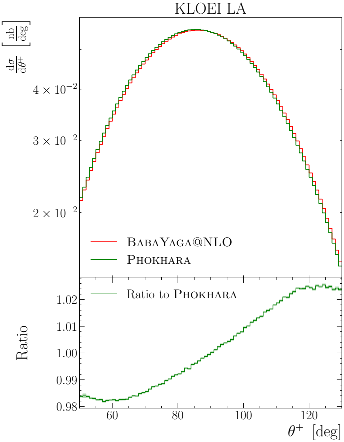
The GVMD Form Factor

	ρ	ω	ϕ	ρ'	ρ''	ρ'''
m_v (MeV)	755.18	781.81	1019.47	1319.9	1732.2	2218.4
Γ_v (MeV)	139.62	7.1726	4.8770	698.94	320.02	123.50
$\|c_v\|$	-	0.0075	0.00074	0.2395	0.0876	0.0064
φ_v (rad)	-	1.67601	5.5833	3.0948	0.5447	-0.2988

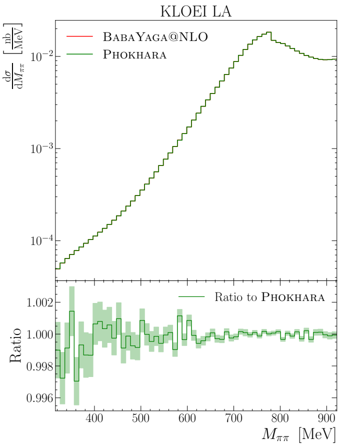
Table 1: Input parameters for our model of the pion form factor $F_\pi^{\mathrm{BW}}(q^2)$.

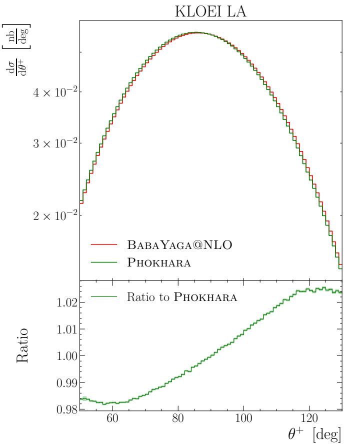
Tuned comparison - F×sQED





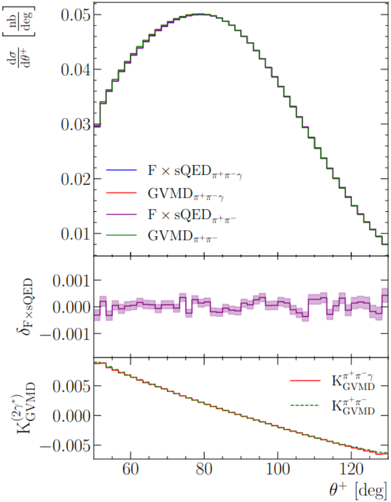
Tuned comparison - F×sQED





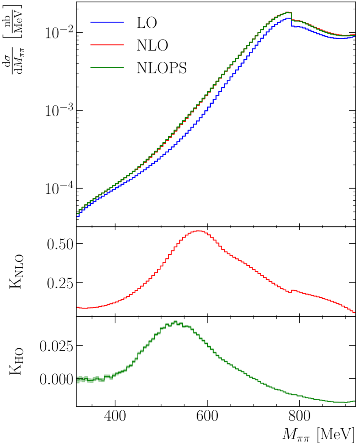
Puzzling...

Internal checks - Soft limit



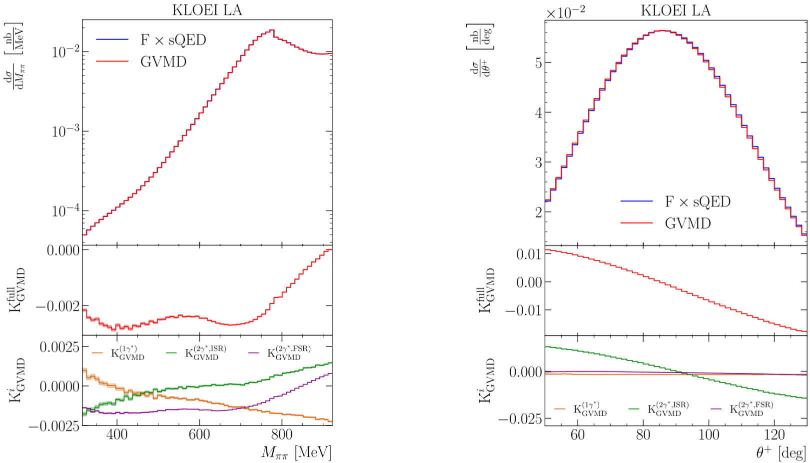
KLOE-LA $M_{\pi\pi}$ - F×sQED

$$K_{NLO} = \frac{d\sigma_{NLO} - d\sigma_{LO}}{d\sigma_{LO}} \qquad K_{HO} = \frac{d\sigma_{NLOPS} - d\sigma_{NLO}}{d\sigma_{LO}}$$

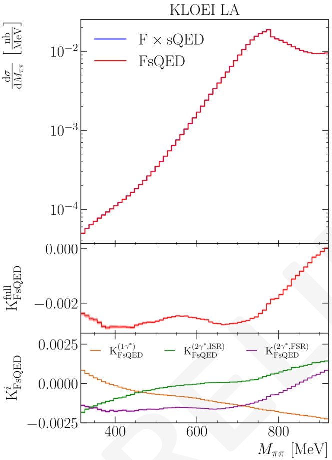


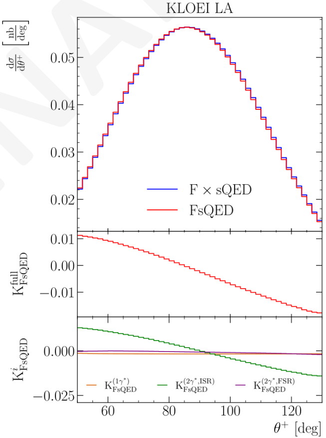
KLOE-LA $M_{\pi\pi}, \theta_{\pi^+}$ - GVMD

$$\mathrm{K_{GVMD}} = \frac{d\sigma_{\mathrm{GVMD}} - d\sigma_{\mathrm{F\times sQED}}}{d\sigma_{\mathrm{F\times sQED}}} \qquad \mathrm{FF} = \mathrm{GVMD}$$

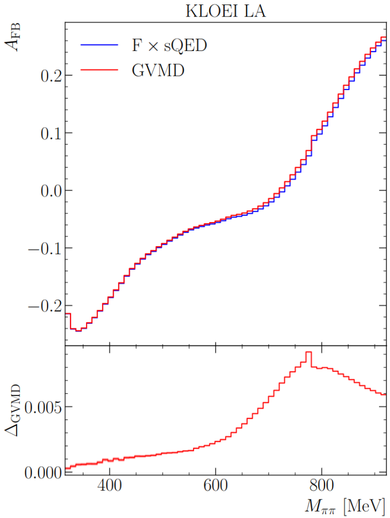


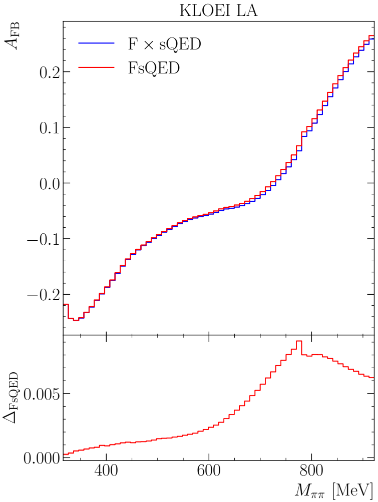
KLOE-LA $M_{\pi\pi}, \theta_{\pi^+}$ - FsQED



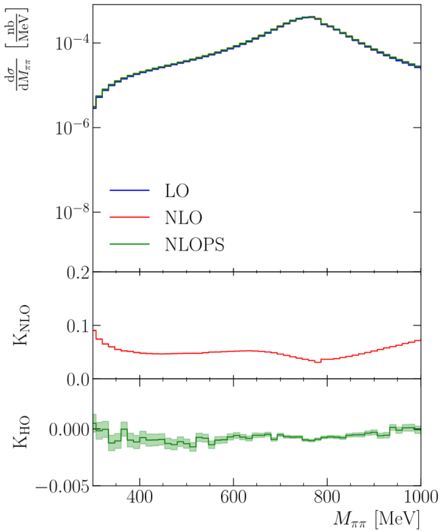


KLOE-LA asymmetry

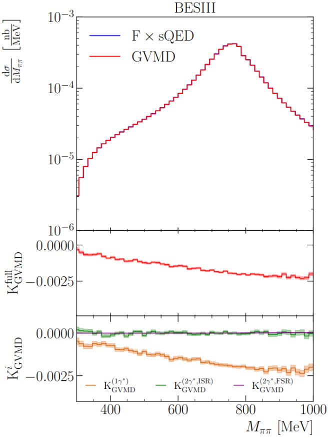


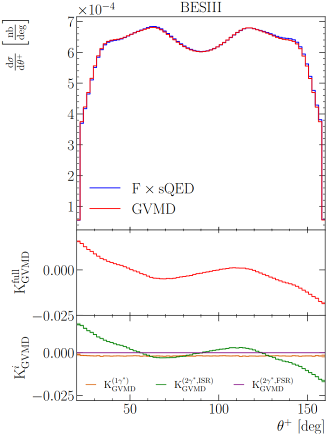


BESIII $M_{\pi\pi}$ - F×sQED

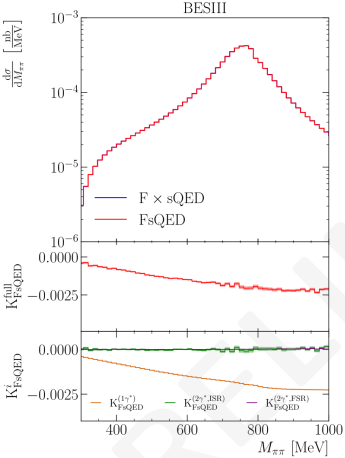


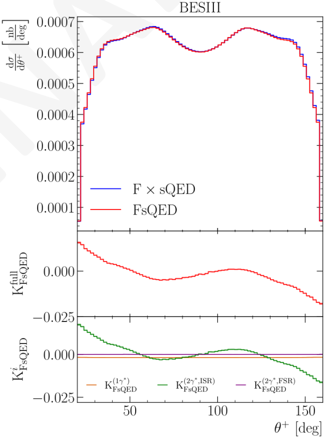
BESIII $M_{\pi\pi}, \theta_{\pi^+}$ - GVMD





BESIII $M_{\pi\pi}, \theta_{\pi^+}$ - FsQED





Summary and Outlook

- Multiple photon contributions are in general important for radiative return experiment.

Summary and Outlook

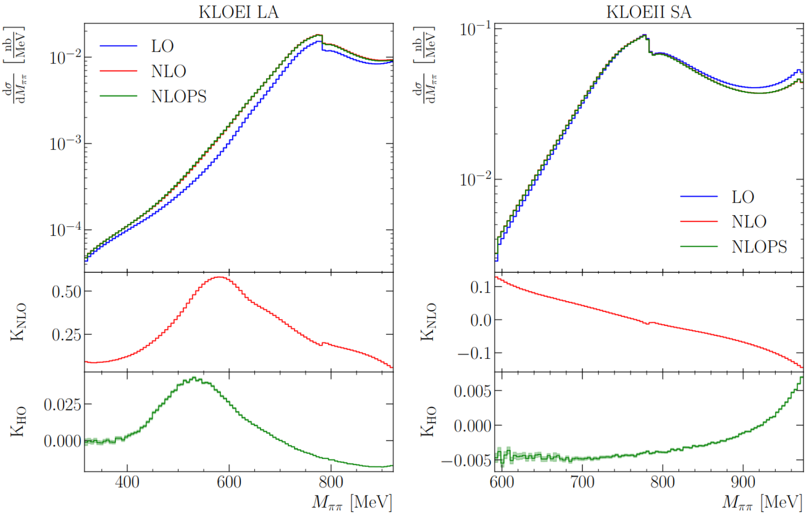
- Multiple photon contributions are in general important for radiative return experiment.
- GVMD structure dependent corrections can be of the same order of HO radiative corrections.
    - An independent study of a subset of GVMD corrections has appeared recently in arxiv:2603.13171.

Summary and Outlook

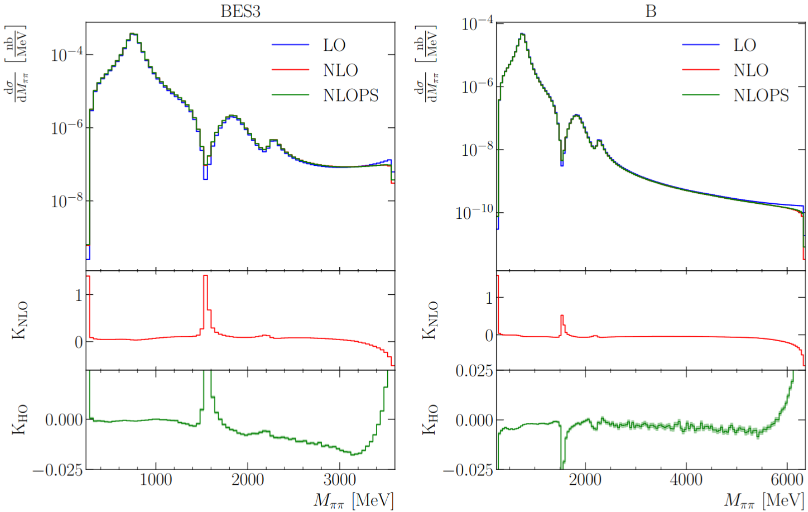
- Multiple photon contributions are in general important for radiative return experiment.
- GVMD structure dependent corrections can be of the same order of HO radiative corrections.
    - An independent study of a subset of GVMD corrections has appeared recently in arxiv:2603.13171.
- Future developments
    - $\pi\pi\gamma$ with FsQED at NLOPS accuracy.
    - Extension to other hadronic channels $(K^+K^-, K_S K_L, \dots)$

Thank you !

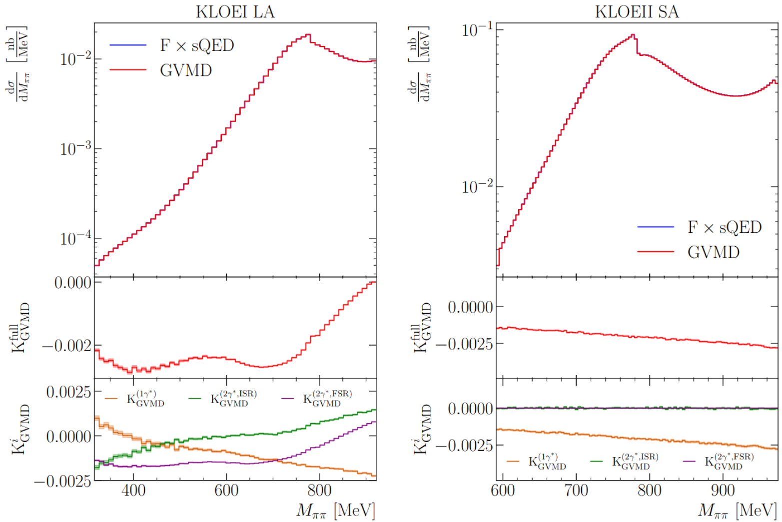
$M_{\pi\pi}$


KLOEI LA
KLOEII SA
LO
NLO
NLOPS
$\frac{d\sigma}{dM_{\pi\pi}}$ $\left[\frac{nb}{MeV}\right]$
10^{-2}
10^{-3}
10^{-4}
10^{-1}
K_{NLO}
0.50
0.25
0.1
0.0
−0.1
K_{HO}
0.025
0.000
0.005
−0.005
400
600
800
700
900
$M_{\pi\pi}$ [MeV]

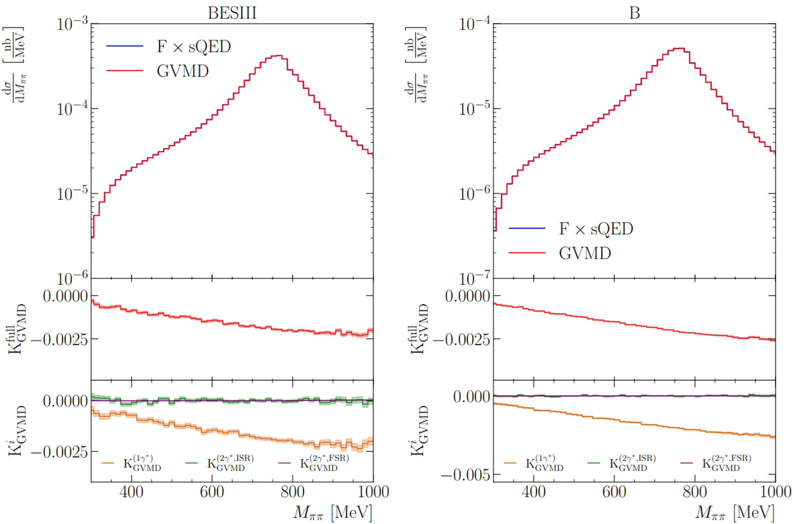
$M_{\pi\pi}$


BES3
B
LO
NLO
NLOPS
$\frac{d\sigma}{dM_{\pi\pi}}$ $\left[\frac{nb}{MeV}\right]$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
K_{NLO}
1
0
K_{HO}
0.025
0.000
−0.025
1000
2000
3000
4000
6000
$M_{\pi\pi}$ [MeV]

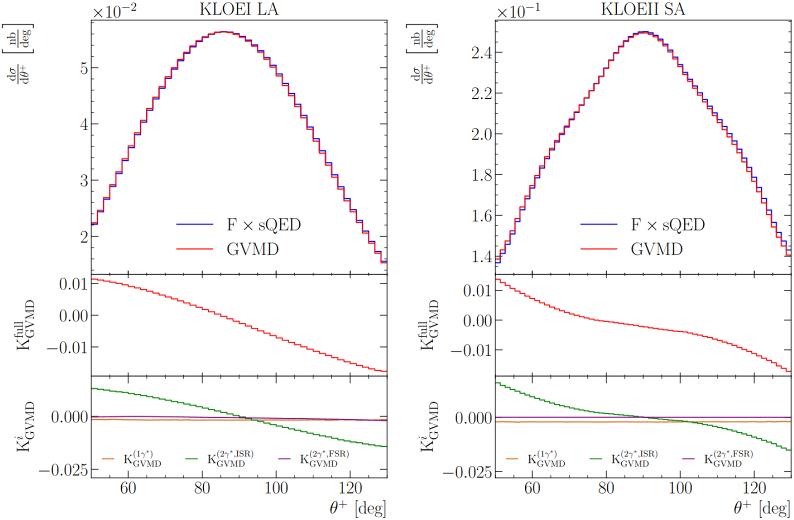
$M_{\pi\pi}$



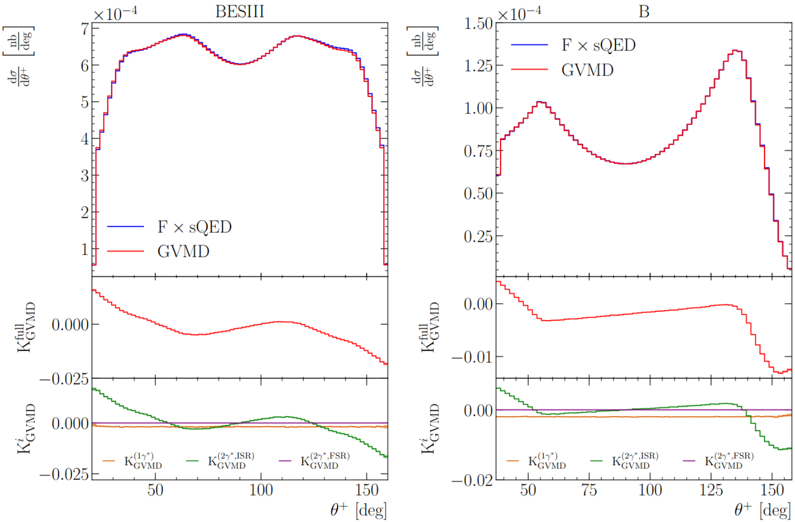
$M_{\pi\pi}$


BESIII
B
F × sQED
GVMD
$\frac{d\sigma}{dM_{\pi\pi}}$ $\left[\frac{nb}{MeV}\right]$
K^{full}_{GVMD}
K^{i}_{GVMD}
$K^{(1\gamma^*)}_{GVMD}$
$K^{(2\gamma^*,ISR)}_{GVMD}$
$K^{(2\gamma^*,FSR)}_{GVMD}$
$M_{\pi\pi}$ [MeV]

θ_{π^+}



θ_{π^+}


BESIII
×10⁻⁴
F × sQED
GVMD
$\frac{d\sigma}{d\theta^+}$ [nb/deg]
K^{full}_{GVMD}
K^{i}_{GVMD}
$K^{(1\gamma^*)}_{GVMD}$
$K^{(2\gamma^*,ISR)}_{GVMD}$
$K^{(2\gamma^*,FSR)}_{GVMD}$
θ^+ [deg]
B
×10⁻⁴
F × sQED
GVMD
$\frac{d\sigma}{d\theta^+}$ [nb/deg]
K^{full}_{GVMD}
K^{i}_{GVMD}
$K^{(1\gamma^*)}_{GVMD}$
$K^{(2\gamma^*,ISR)}_{GVMD}$
$K^{(2\gamma^*,FSR)}_{GVMD}$
θ^+ [deg]