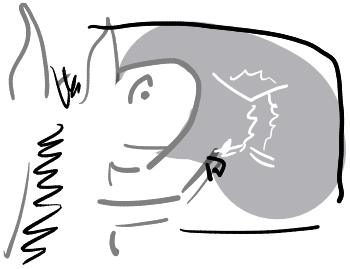
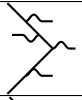
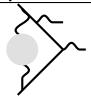
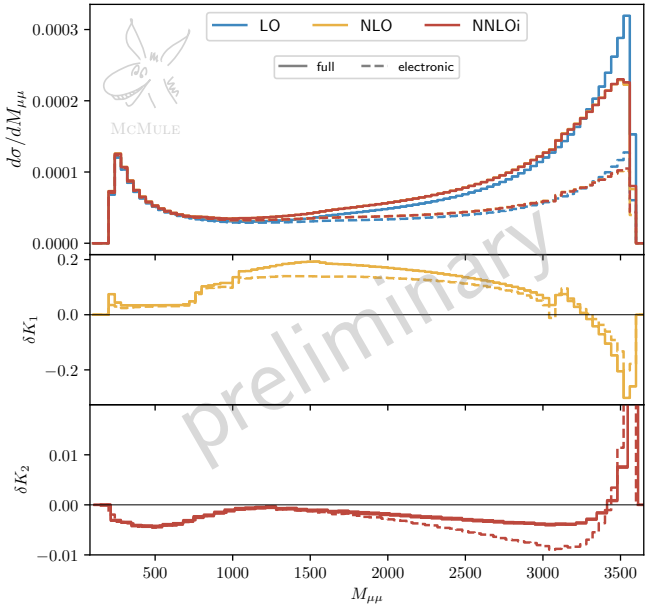
McMule update on $ee \to \mu\mu\gamma$



by Sara Gündogdu

McMule goal: MUonE calculation @ N^{3}LO

	$ee \to \gamma^*\gamma$	methods	challenges	status
VV		massification	KLOE SA ✗	fully massive calc. & jettification
RR		OL	B/BES solved ✓	LBK
RV		OL	KLOE SA/LA ✓ B/BES ✓	LBK
VP		Disperon QED	✓	


LO
NLO
NNLOi
full
electronic
McMule
preliminary
$d\sigma/dM_{\mu\mu}$
0.0003
0.0002
0.0001
0.0000
δK_1
0.2
0.0
−0.2
δK_2
0.01
0.00
−0.01
500
1000
1500
2000
2500
3000
3500
$M_{\mu\mu}$

RV matrix elements need **next-to-soft stabilisation** $\Longrightarrow$ automatized in McMule

Single soft photon (RV):

$$= \frac{1}{E_\gamma^2} \underbrace{\mathcal{E}}_{\text{eikonal}} + \frac{1}{E_\gamma}\left\{ \underbrace{D\left[\quad\right]}_{\text{LBK}} + \underbrace{S}_{\text{soft function}} \right\} + \mathcal{O}(E_\gamma^0)$$

Multi-photon extension @ all orders [Tim Engel, 2024]: $k_1 \sim k_2 \sim \cdots \sim k_n$

$$= \frac{1}{E_\gamma^{2n}} \prod_{a=1}^{n} \underbrace{\mathcal{E}_{k_a}}_{\text{eikonal}} + \frac{1}{E_\gamma^{2n-1}} \left\{ \sum_{a=1}^{n} \prod_{c\neq a} \mathcal{E}_{k_c} \left(\underbrace{D_{k_a}\left[\quad\right]}_{\text{LBK}} + \underbrace{S_{k_a}}_{\text{soft function}} \right) \right.$$

$$\left. + \sum_{a,b=1}^{n} \prod_{c\neq a,b} \mathcal{E}_{k_c} \; G_{k_a}^{k_b} \right\} + \mathcal{O}(E_\gamma^{-2n+2})$$

LBK approximation for $X = 2$ soft photons (RR):

$$\mathcal{M}^{(0)}_{n+2}(\{p_i\}, k_1, k_2) \simeq \left(\mathcal{E}_1\mathcal{E}_2 + 2 \cdot G^{(0)}_{12}\right)\mathcal{M}^{(0)}_{n}(\{p_i''\})$$

[Tim Engel, 2024] [R. Balsach et al., 2023]

Shifted momenta $p_i \mapsto p_i' \mapsto p_i''$:

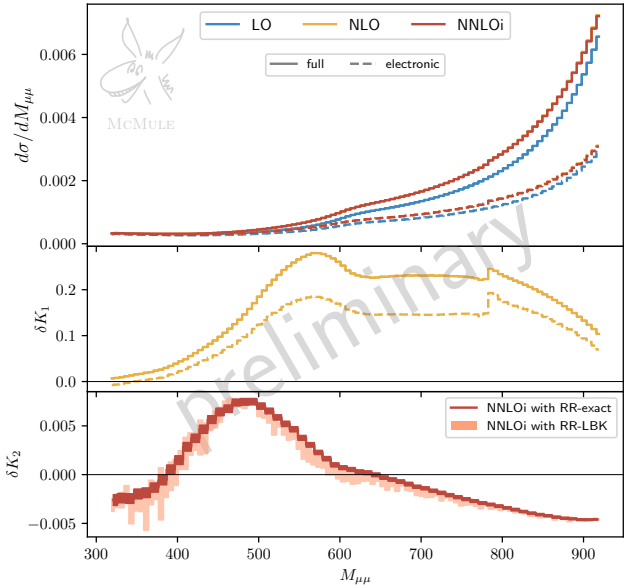
$$p_i' = p_i + \delta p_i(\{p_i\},\, k_1)$$
$$p_i'' = p_i' + \delta p_i(\{p_i'\},\, k_2)$$

where $\delta p_i \sim \mathcal{O}(k)$ constructed such that:

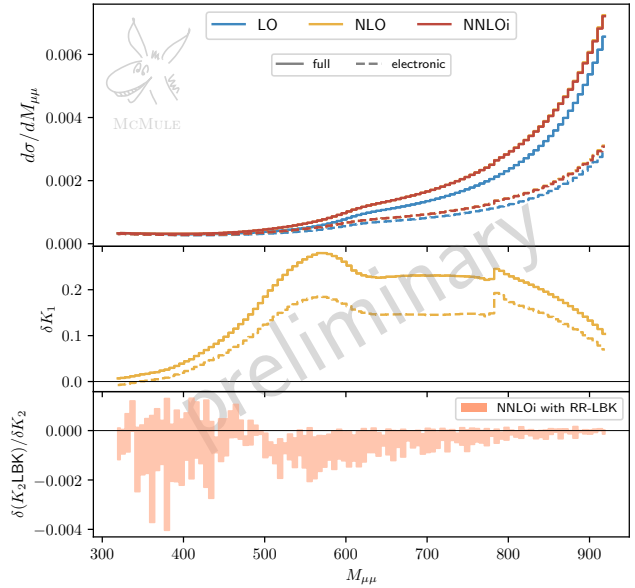
- on-shell: $p_i''^{\,2} = m_i^2$
- momentum conservation: $\sum_i \delta p_i = -k_a$
- LBK validity is not spoiled

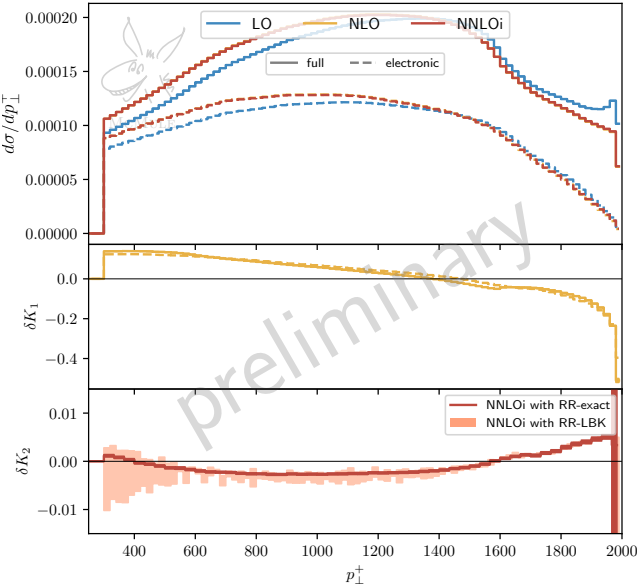
Key feature: The shifted momenta routine can be called iteratively, LP and NLP behaviour is correctly captured

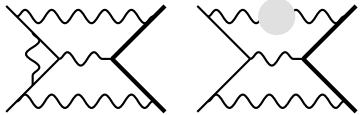
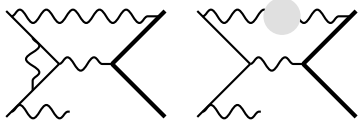
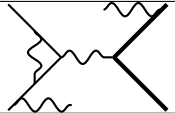
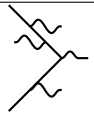
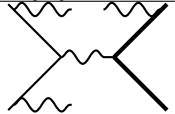
LBK approximation for KLOE-LA



LBK approximation for KLOE-LA




LO
NLO
NNLOi
full
electronic
$d\sigma/dp_\perp^+$
0.00020
0.00015
0.00010
0.00005
0.00000
δK_1
0.0
−0.2
−0.4
δK_2
0.01
0.00
−0.01
NNLOi with RR-exact
NNLOi with RR-LBK
preliminary
400
600
800
1000
1200
1400
1600
1800
2000
$p_\perp^+$

	$ee \to \gamma^*$	$ee \to \mu\mu$ ($e\mu \to e\mu$ MUonE)	methods	challenges
VVV			massification	2-loop Dis-peron QED, HLbL (?)
VVR			from $pp \to 2j + \gamma$ [Badger et al 23] massification, Dis-peron QED, NTS	jettification @ 2-loop, KLOE-SA
VRR			OL, NTS	OL for $ee \to \gamma^*$
RRR			NTS	