$e^+e^- \to \mu^+\mu^-\gamma$ at NLOPS accuracy

Workshop on Radiative Corrections and Monte Carlo Simulations at Electron-Positron Colliders
Turin 2026

Marco Ghilardi
On behalf of the BabaYaga team

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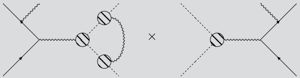
INFN National Institute for Nuclear Physics
UNIVERSITÀ DI PAVIA

NLOPS accuracy

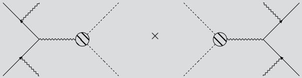
NLO

Exact calculation up to $\mathcal{O}(\alpha)$ w.r.t. the born scattering amplitude.

- Virtual corrections



- Real corrections



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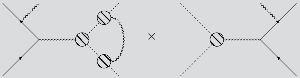

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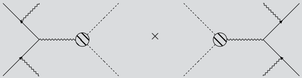
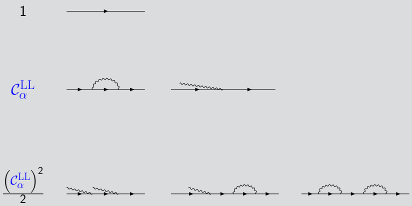


LL resummation

Approximate calculation of soft-virtual corrections to any order.



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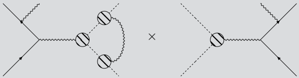

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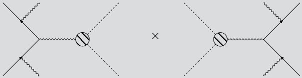

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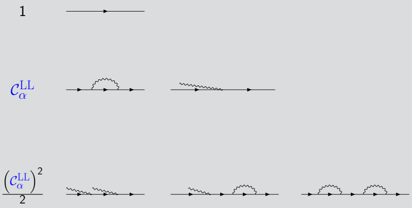

NLO+LL (NLOPS) accuracy

$$d\sigma_{\mathrm{NLOPS}} = \sum_{n=1}^{\infty} \exp\left\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\right\} F_{\mathrm{SV}} \frac{1}{n!} \left|\mathcal{M}_{n}^{X}\right|^2 d\Phi_n(\{p\},\{k\}) \,.$$

LL approximation

$$d\sigma_{\mathrm{LOPS}} = \sum_{\mathrm{n}=1}^{\infty} \Pi(\varepsilon, \mathrm{Q}^2)\, \frac{1}{\mathrm{n}!}\, \left|\mathcal{M}_{\mathrm{n}}^{\mathrm{X}}\right|^2 \, d\Phi_{\mathrm{n}}(\{\mathrm{p}\}, \{\mathrm{k}\})$$

where

- $\Pi(\varepsilon, Q^2)$ is the Sudakov Form factor where ε is the soft-hard separator.
- $\mathcal{M}_n^X$ stands for $\mathcal{M}^{\mathrm{ex}}$ for $n = 1, 2$ and $\mathcal{M}^{\mathrm{LL}}$ for $n \geq 3$.

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$$\left|\mathcal{M}_n^{\mathrm{LL}}\right|^2 = \prod_{i=1}^{n-2} \left[-\frac{\alpha}{2\pi} P(z_i)\, \mathcal{I}_{\mathrm{SF}}(k_i, z_i)\, \frac{4\pi^2\,(1 - z_i)}{z_i \omega_i^2}\right] \underbrace{\left|\mathcal{M}_2^{\mathrm{ex}}(\{\tilde{p}\}, \{\tilde{k}\})\right|^2}_{e^+e^- \to XX\gamma\gamma}$$

LL approximation

If we define

$$\mathcal{I}(k) = \sum_{i,j=1}^{4} \mathcal{I}_{ij}(k) = \sum_{i,j=1}^{4} \eta_i \eta_j \frac{(p_i \cdot p_j)}{(p_i \cdot k)\,(p_j \cdot k)} k_0^2 \,,$$

then

$$\mathcal{I}_{\mathrm{SF}}(k,z) = -\sum_{ij} \mathcal{I}_{ij}(k) \cdot F_{ij}(k,z)$$

where

$$\begin{cases} F_{ij}(k,z) = 1 - \delta_{ij} \dfrac{(1-z)^2}{1+z^2} & i = \text{fermion} \\ F_{ij}(k,z) = 1 & i = \text{scalar}\,. \end{cases}$$

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REMARK: In the limit $k \cdot p \to 0$ we correctly reproduce the massive unpolarized splitting function of *S. Dittmaier, hep-ph/9904440*

Matching at NLO

$$\mathrm{d}\sigma^{\alpha}_{\mathrm{LL}} = [1 + \mathrm{C}_{\alpha,\mathrm{LL}}]\, |\mathcal{M}_1^{\mathrm{ex}}|^2\, \mathrm{d}\Phi_1 + |\mathcal{M}_2^{\mathrm{ex}}|^2\, \mathrm{d}\Phi_2$$

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- The hard contribution $|\mathcal{M}_2|^2$ is already exact.

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- The hard contribution $|\mathcal{M}_2|^2$ is already exact.
- To match the exact $\rm NLO\, soft - virtual\, correction$ we introduce

$$F_{\rm SV} = 1 + (\mathcal{C}_\alpha - \mathcal{C}_{\alpha,\rm LL})$$

The master formula for NLOPS accuracy reads

$$d\sigma_{\rm NLOPS} = \sum_{n=1}^{\infty} \Pi(\varepsilon, Q^2)\, F_{\rm SV}\, \frac{1}{n!}\, |\mathcal{M}_n^{\rm X}|^2\, d\Phi_n(\{p\},\{k\})\,.$$

NNLO comparison (analytical)

Considering the inclusive case over photon angles, *V.S. Fadin and R.N. Lee (arxiv:2308.09479)* provides the full expression for the NNLO SS+SV+VV correction for the process $e^+e^- \to \gamma\gamma^*$(ISC).

$$\mathrm{d}\sigma_{\mathrm{ss+sv+vv}} = \mathrm{c}_0 + \mathrm{c}_1\mathrm{L} + \mathrm{c}_2\mathrm{L}^2 + \mathrm{c}_3\mathrm{L}^3 + \mathrm{c}_4\mathrm{L}_\omega + \mathrm{c}_5\mathrm{L}\mathrm{L}_\omega + \mathrm{c}_6\mathrm{L}^2\mathrm{L}_\omega + \mathrm{c}_7\mathrm{L}_\omega^2 + \mathrm{c}_8\mathrm{L}\mathrm{L}_\omega^2 + \mathrm{c}_9\mathrm{L}^2\mathrm{L}_\omega^2$$

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- A comparison with the $\mathcal{O}(\alpha^2)$ expansion (w.r.t. the radiative process) of the master formula for the 1γ sample shows the following.

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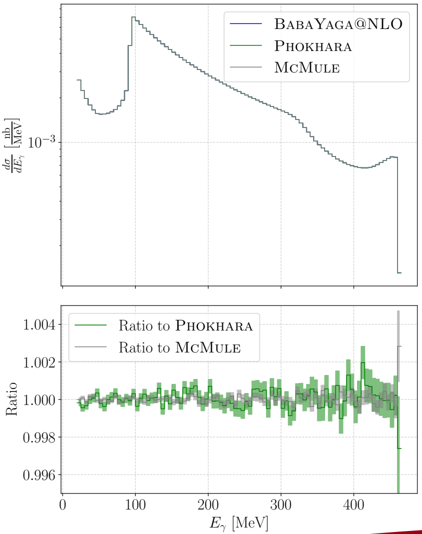
$$d\sigma_{ss+sv+vv} = c_0 + c_1 L + c_2 L^2 + c_3 L^3 + c_4 L_\omega + c_5 L L_\omega + c_6 L^2 L_\omega + c_7 L_\omega^2 + c_8 L L_\omega^2 + c_9 L^2 L_\omega^2$$

- A comparison with the $\mathcal{O}(\alpha^2)$ expansion (w.r.t. the radiative process) of the master formula for the 1γ sample shows an agreement at the level of $\mathcal{O}\left(\alpha^2 \, L\right)$
- Virtual corrections on the 2γ sample are correctly reproduced in the soft limit, due to YFS theorem.

Tuned comparison - E_γ

Scenario KLOE-LA @ 1.02 GeV



arxiv.org/abs/2410.22882

Matrix element for $n \geq 3$

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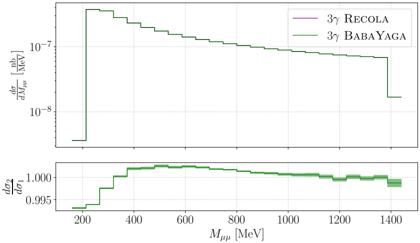
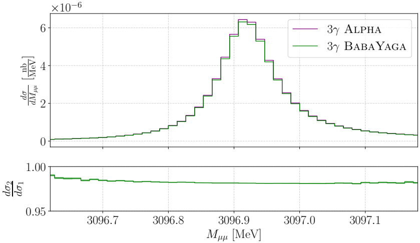
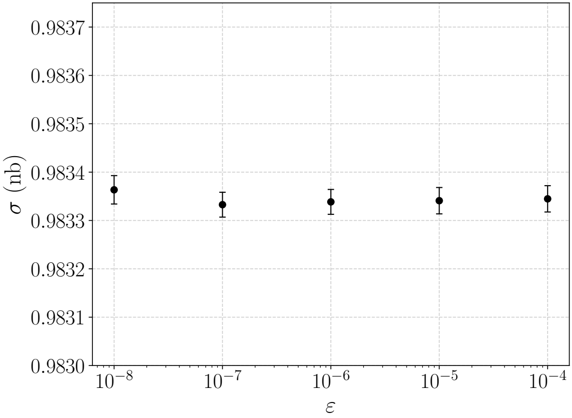
Scenario B with 2 additional photons with $E_\gamma \geq 100$ MeV (Tree-level)





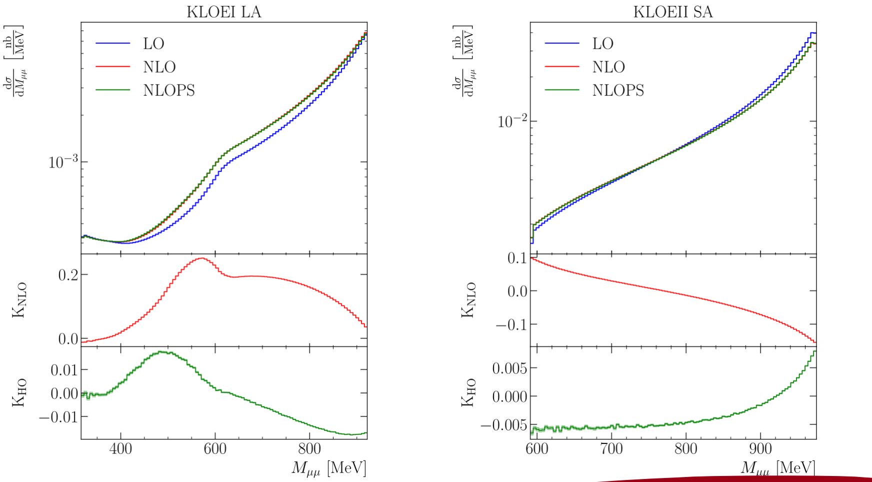
ALPHA arXiv:hep- ph/9507237
RECOLA arxiv.org/abs/1211.6316

ε independence



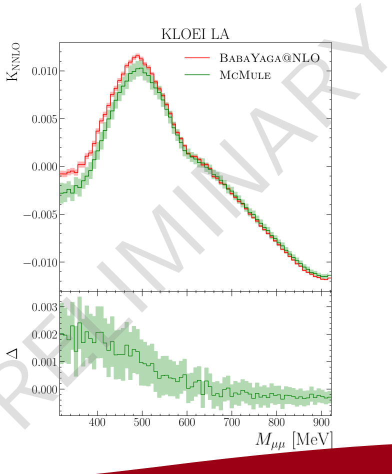
KLOE scenario - $M_{\mu\mu}$

$$K_{NLO} = \frac{d\sigma_{NLO} - d\sigma_{LO}}{d\sigma_{LO}} \qquad K_{HO} = \frac{d\sigma_{NLOPS} - d\sigma_{NLO}}{d\sigma_{LO}}$$

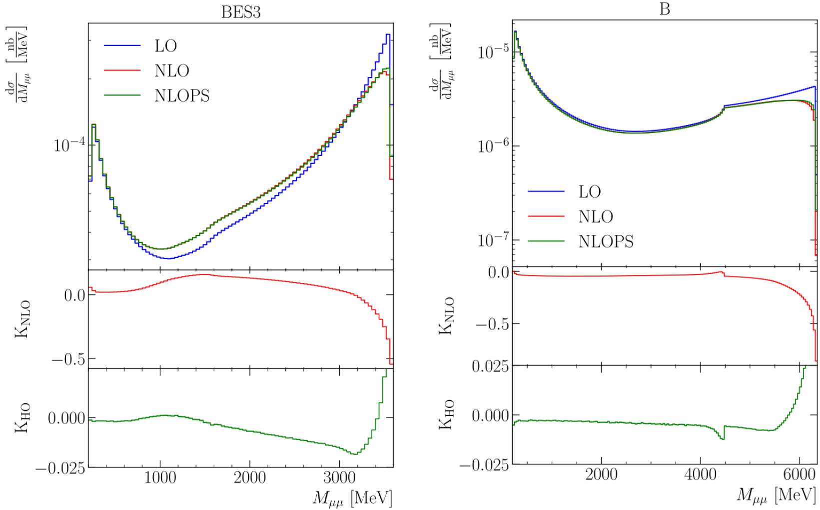


NNLO-ISR comparison KLOEI LA

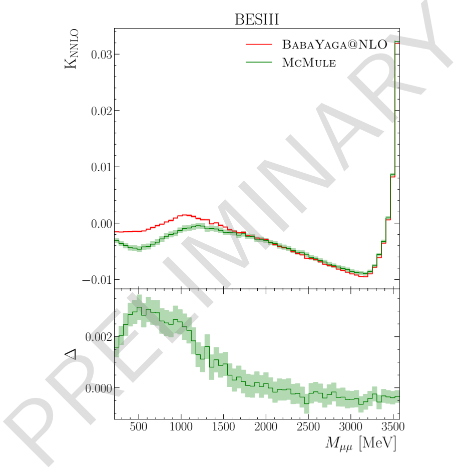
$$\mathrm{K}_{\mathrm{NNLO}} = \frac{d\sigma_{\mathrm{NNLO}} - d\sigma_{\mathrm{NLO}}}{d\sigma_{\mathrm{LO}}} \qquad \Delta = \mathrm{K}^{\mathrm{BabaYaga}}_{\mathrm{NNLO}} - \mathrm{K}^{\mathrm{McMule}}_{\mathrm{NNLO}}$$



BESIII-B scenarios - $M_{\mu\mu}$

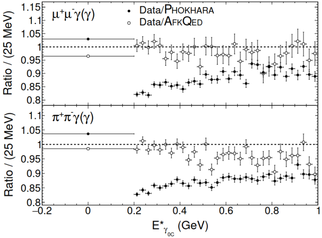


NNLO-ISR comparison BES3



BaBar test (arXiv:2308.05233)

- $E^*_\gamma > 4\,\mathrm{GeV}$ and $0.35 < \theta_\gamma < 2.4$ rad
- $p_\perp > 0.1\,\mathrm{GeV}$ and $0.4 < \theta_\pm < 2.45$ rad
- $M_{\mu\mu} < 1.4\,\mathrm{GeV}$ and $0.6 < M_{\pi\pi} < 0.9$ GeV
- p_{0c} missing momentum with $E^*_{0c} > 200$ MeV



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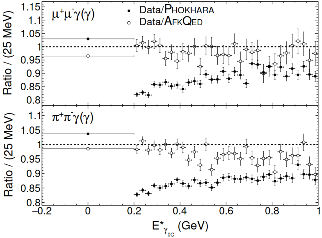
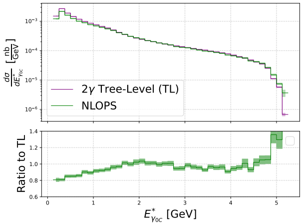


$\pi\pi\gamma$



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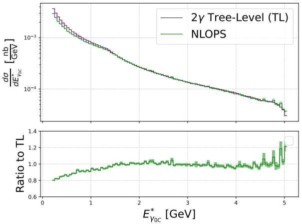

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  $$\mathrm{NLOPS} \quad \mathrm{vs} \quad \mathrm{NNLO} \Rightarrow \text{Assessment of theoretical uncertainty}$$

- The BabaYaga@NLO updated version for $\mu\mu\gamma$ and $\pi\pi\gamma$ is available on github.com/cm-cc/BabaYagaNLO.

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- Multiple photon contributions are in general important for radiative return experiment.
- Relevance of NLOPS MC event generators.

  NLOPS vs NNLO ⇒ Assessment of theoretical uncertainty

- The BabaYaga@NLO updated version for $\mu\mu\gamma$ and $\pi\pi\gamma$ is available on github.com/cm-cc/BabaYagaNLO.
- Future developments
  - *ee*γ at NLOPS accuracy.
  - NNLOPS matching for $2 \to 2$ processes

Thank you !

Scenarios Strong2020

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(a) KLOEI Large-Angle scenario

$$\sqrt{s} = 1.02\,\text{GeV}$$
$$50^\circ \le \vartheta_\pm \le 130^\circ$$
$$|\mathbf{p}^z_\pm| \ge 90\,\text{MeV} \;\vee\; |\mathbf{p}^\perp_\pm| \ge 160\,\text{MeV}$$
$$50^\circ \le \vartheta_\gamma \le 130^\circ \;\wedge\; E_\gamma \ge 20\,\text{MeV}$$
$$0.1\,\text{GeV}^2 \le M^2_{XX} \le 0.85\,\text{GeV}^2$$

(b) KLOEII Small-Angle scenario

$$\sqrt{s} = 1.02\,\text{GeV}$$
$$50^\circ \le \vartheta_\pm \le 130^\circ$$
$$|\mathbf{p}^z_\pm| \ge 90\,\text{MeV} \;\vee\; |\mathbf{p}^\perp_\pm| \ge 160\,\text{MeV}$$
$$\vartheta_{\tilde\gamma} \le 15^\circ \;\vee\; \vartheta_{\tilde\gamma} \ge 165^\circ$$
$$0.35\,\text{GeV}^2 \le M^2_{XX} \le 0.95\,\text{GeV}^2$$

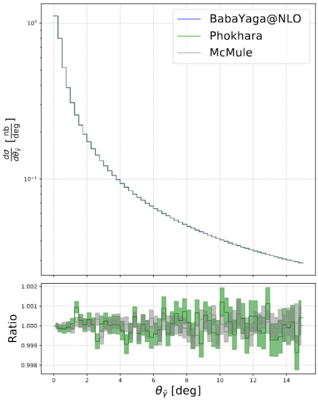
(c) BES3 scenario

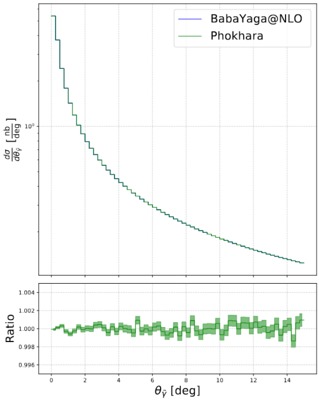
$$\sqrt{s} = 4\,\text{GeV}$$
$$|\cos\vartheta_\pm| \le 0.93 \;\wedge\; |p^\perp_\pm| \ge 300\,\text{MeV}$$
$$[|\cos\vartheta_\gamma| \le 0.8 \;\wedge\; E_\gamma \ge 25\,\text{MeV}] \;\vee$$
$$[0.86 \le |\cos\vartheta_\gamma| \le 0.92 \;\wedge\; E_\gamma \ge 50\,\text{MeV}]$$
$$\exists!\,\gamma \;\text{ s.t. }\; E_\gamma \ge 400\,\text{MeV}$$

(d) B scenario

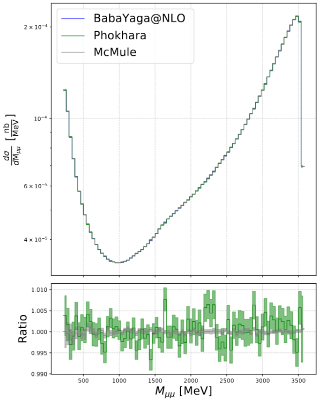
$$\sqrt{s} = 10\,\text{GeV}$$
$$0.65\,\text{rad} \le \vartheta_\pm \le 2.75\,\text{rad} \;\wedge\; |\mathbf{p}_\pm| \ge 1\,\text{GeV}$$
$$0.6\,\text{rad} \le \vartheta_\gamma \le 2.7\,\text{rad} \;\wedge\; \text{E}_\gamma \ge 3\,\text{GeV}$$
$$\vartheta_{\tilde\gamma,\gamma^{(h)}} \le 0.3\,\text{rad}$$
$$M_{XX\gamma} \ge 8\,\text{GeV}$$

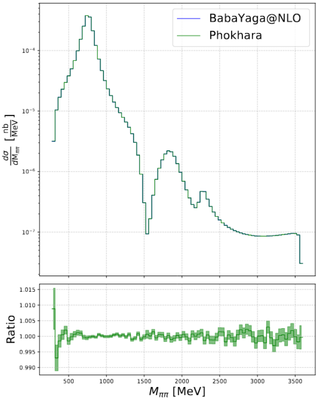
Tuned comparison - KLOE-SA



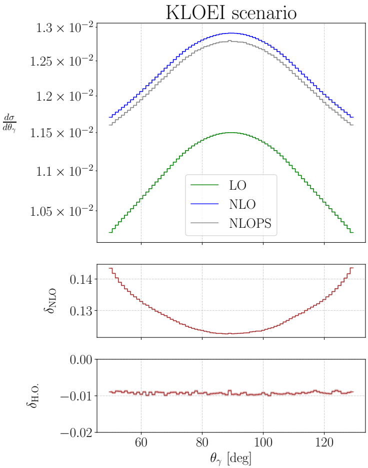


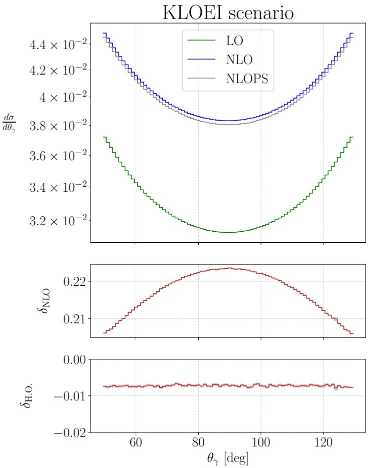
Tuned comparison - BES3



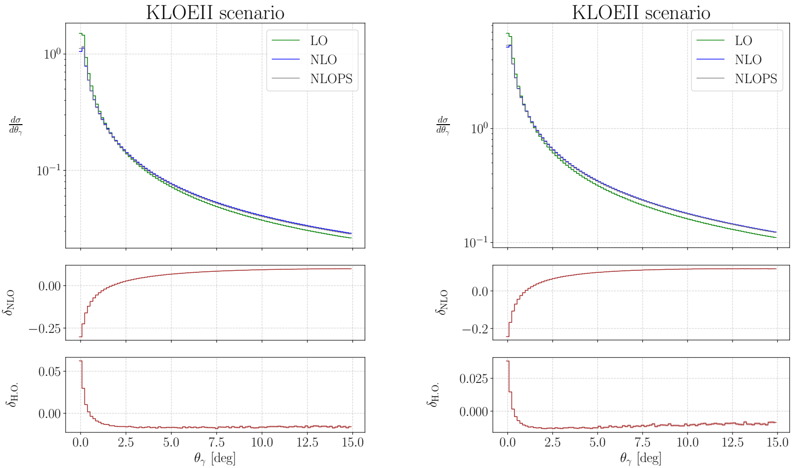


KLOE-LA scenario - θ_γ

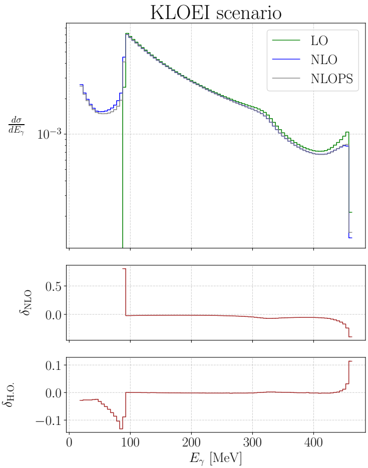


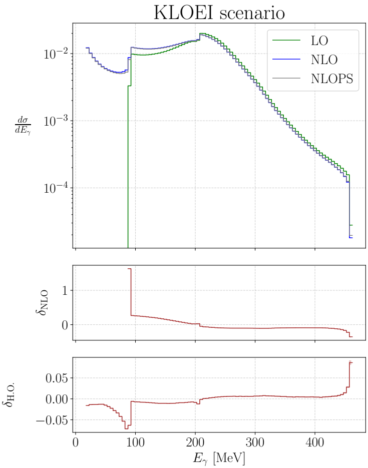


KLOE-SA scenario - $\theta_{\tilde{\gamma}}$

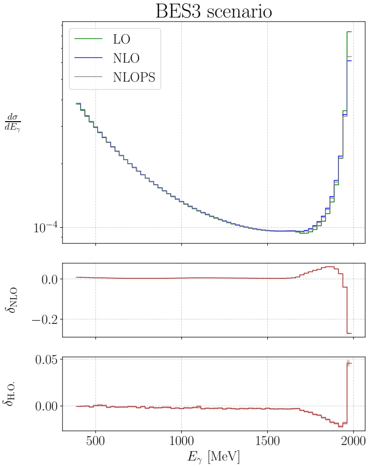


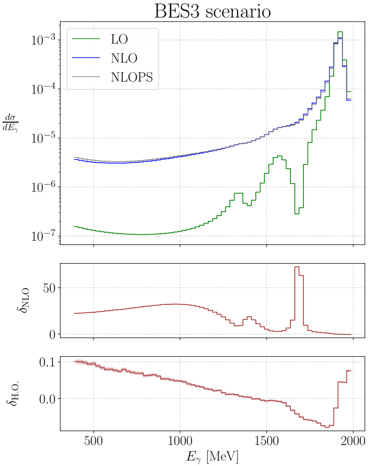
KLOE-LA scenario - E_γ





BES3 scenario - E_γ





B scenario - E_γ