Thoughts on rescattering corrections

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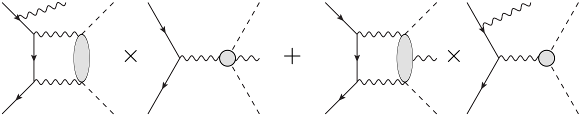
Workshop on Radiative Corrections and Monte Carlo Simulations at Electron–Positron Colliders

Turin, Italy

Motivation



- **Structure-dependent radiative corrections** potentially sizable in $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$
- GVMD estimates for $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$ Talks by Pau and Marco
- In general, matrix elements required for
  1. $\gamma^*\gamma^* \to \pi\pi$: construction of matrix elements known, but how to combine with multiloop techniques and implement in MC generators?
  2. $\gamma^*\gamma^*\gamma \to \pi\pi$: full construction of matrix elements work in progress Talk by Eirini
- One key concern: GVMD does not account for rescattering corrections, but for $\gamma^*\gamma^*\gamma \to \pi\pi$ the pions can be in a *P*-wave

  $\hookrightarrow$ **are we missing effects enhanced by F_π^V?**

Unitarity and Breit–Wigner propagators: scattering

- **"Rescattering"** just refers to **unitarity**, in the simplest case (elastic scattering)

$$\operatorname{Im} t(s) = \sigma(s)|t(s)|^2 \qquad \sigma(s) = \sqrt{1 - \frac{4m^2}{s}}$$

- Single resonance (let's abbreviate $D_i = s - M_i^2 + iM_i\Gamma_i$)

$$t = -\frac{g_1^2}{D_1} \qquad \operatorname{Im} t = \frac{g_1^2 M_1 \Gamma_1}{|D_1|^2} \stackrel{!}{=} \frac{\sigma g_1^4}{|D_1|^2} = \sigma |t|^2 \qquad \Rightarrow \qquad \sigma g_1^2 = M_1 \Gamma_1$$

$\hookrightarrow$ unitarity fulfilled (Γ_1 needs to become momentum dependent)

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$\hookrightarrow$ unitarity fulfilled (Γ_1 needs to become momentum dependent)

- Two resonances, adding Breit–Wigner functions

$$t = -\sum_{i=1}^{2} \frac{g_i^2}{D_i} \qquad \Rightarrow \qquad \mathrm{Im}\, t - \sigma|t(s)|^2 = \sum_{i=1}^{2} \frac{g_i^2(M_i\Gamma_i - \sigma g_i^2)}{|D_i|^2} - 2\mathrm{Re}\left(\frac{\sigma g_1^2 g_2^2}{D_1 D_2^*}\right)$$

- Interference term violates unitarity, negligible only for well-separated resonances

Unitarity and Breit–Wigner propagators: scattering

- One way to save unitarity: add **Breit–Wigner phases** instead Svec 2001

$$t = \frac{e^{2i\delta}-1}{2i\sigma} \qquad \delta = \delta_1 + \delta_2 \qquad t_i = \frac{e^{2i\delta_i}-1}{2i\sigma} = -\frac{g_i^2}{D_i} \qquad \sigma g_i^2 = M_i \Gamma_i$$

$$t = t_1 + t_2 + 2i\sigma t_1 t_2 = c_- t_1 + c_+ t_2 \qquad c_\pm = \frac{M_1^2 - M_2^2 \pm i(M_1\Gamma_1 + M_2\Gamma_2)}{M_1^2 - M_2^2 - i(M_1\Gamma_1 - M_2\Gamma_2)}$$

- Can be expressed as a sum of Breit–Wigner amplitudes, but unitarity fixes the (complex) coefficients!
- **Sums of Breit–Wigner functions are really inefficient parameterizations, since it becomes very tedious to keep track of unitarity constraints**
- There is also the issue of lack of analyticity, not my focus today

Unitarity and Breit–Wigner propagators: form factors

- For a form factor unitarity is less restrictive

$$\text{Im}\, F(s) = \sigma(s) F(s) \big[t(s)\big]^*$$

$\hookrightarrow$ **Watson's final-state theorem** $F = |F| e^{i\delta}$

- Single resonance

$$F = \frac{c_1}{D_1} \qquad \text{Im}\, F = -\frac{c_1 M_1 \Gamma_1}{|D_1|^2} = -\frac{c_1 \sigma g_1^2}{|D_1|^2} = \sigma F t^*$$

$\hookrightarrow$ unitarity fulfilled (goes through for any real function c_1)

- Two resonances, adding Breit–Wigner functions

$$F = \sum_{i=1}^{2} \frac{c_i}{D_i} \qquad \Rightarrow \qquad \text{Im}\, F - \sigma F t^* = \sum_{i=1}^{2} c_i \frac{\sigma g_i^2 - M_i \Gamma_i}{|D_i|^2} + \sigma \left(\frac{c_1 g_2^2}{D_1 D_2^*} + \frac{c_2 g_1^2}{D_2 D_1^*} \right)$$

$\hookrightarrow$ interference term again violates unitarity

Unitarity and Breit–Wigner propagators: form factors

- To save unitarity, can again add Breit–Wigner phases instead
- Use Omnès solution $\Omega[\delta]=\exp\left\{\frac{s}{\pi}\int_{4m^2}^{\infty}ds'\,\frac{\delta(s')}{s'(s'-s)}\right\}$ (using also analyticity) to simplify the argument (P, $\tilde{P}$ polynomials)

$$F=P\Omega[\delta_1+\delta_2]=P\Omega[\delta_1]\Omega[\delta_2]\overset{\text{narrow-width limit}}{\rightarrow}\frac{\tilde{P}}{D_1D_2}=\frac{\tilde{P}}{D_2-D_1}\left(\frac{1}{D_1}-\frac{1}{D_2}\right)$$

- Unitarity again fixes the (complex) coefficients
- **Adding Breit–Wigner functions for overlapping resonances is a bad idea for form factors as well**

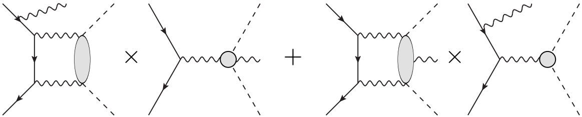
Unitarity and Breit–Wigner propagators: form factors

- Frequently seen $F_\pi^V = \sum_{i=\rho,\omega,\rho',\rho''} \frac{c_i}{D_i}$
  $\hookrightarrow$ can we agree not do to that anymore for ρ–ω mixing?
- A factorized form might still lack analyticity, but at least avoids the unitarity issue

$$F_\pi^V = \frac{c_\rho}{D_\rho}\left(1 + \frac{c_\omega}{D_\omega}\right) + \dots$$

  - Gounaris–Sakurai from Aliberti et al. 2024 (RMCL2) has this form
  - Matches the physical picture that a $\pi^+\pi^-$ pair from ω decay is still subject to final-state interactions controlled by the $\rho(770)$
  - Can always be broken down to sums of propagators by partial fraction decomposition
- Realistic case is more complicated, see Holz et al. 2022 for the full-fledged coupled-channel system of $2\pi/3\pi$

Back to unitarity for $\gamma^*\gamma^*(\gamma) \to \pi\pi$

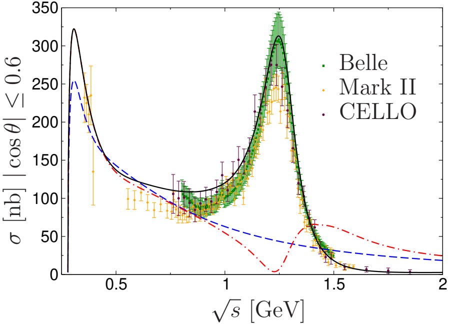


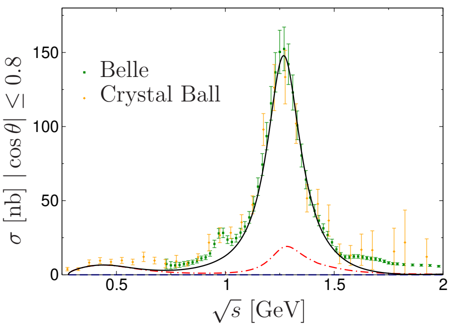
- In GVMD, the gray blobs are replaced by pion propagators, with couplings dressed with pion form factors

  $\hookrightarrow$ **the corresponding subamplitudes are not unitary**
- In dispersive language: some variant of pion-pole contributions included, but certainly **no rescattering corrections**
- How relevant are those?
  - $\gamma^*\gamma^* \to \pi\pi$: only *S*- and *D*-waves, so rescattering important for $f_0(500)$ and $f_2(1270)$
  - $f_0(500)$ yields moderate corrections to dominant pion-pole terms
  - $f_2(1270)$ already at relatively high energy
  - $\gamma^*\gamma^*\gamma \to \pi\pi$: selection rules no longer forbid a *P*-wave

    $\hookrightarrow$ **rescattering into $\rho(770)$ possible**, could be sizable

Phenomenology of $\gamma\gamma \to \pi\pi$





- Pion pole dominant effect at low energies $\sqrt{s} \lesssim 1$ GeV

  $\hookrightarrow$ **unitarity corrections** from $\pi\pi$ S-wave rescattering $\Rightarrow$ ***f_0(500) resonance***
- For $\sqrt{s} \gtrsim 1$ GeV, ***f_2(1270) resonance***

  $\hookrightarrow$ D-wave rescattering of $3\pi \simeq \omega$, $2\pi \simeq \rho$ left-hand cuts
- Need to figure out how to implement these (known) rescattering corrections together with multi-loop techniques My talk in Liverpool, see backup

First challenge: form of dispersive amplitudes

- $N_{0,++}(s)$, $N_{0,00}(s)$: **partial-wave projection** of pion-pole terms

$$N_{0,++}(s) = F_\pi^V(q_1^2)F_\pi^V(q_2^2)\bigg[\frac{8}{\sigma_\pi(s)\lambda_{12}^{1/2}(s)}\bigg(\frac{sq_1^2q_2^2}{\lambda_{12}(s)}+M_\pi^2\bigg)Q_0(x_s)+2\frac{(q_1^2-q_2^2)^2-s(q_1^2+q_2^2)}{\lambda_{12}(s)}\bigg]$$

$$N_{0,00}(s) = F_\pi^V(q_1^2)F_\pi^V(q_2^2)\frac{4}{\lambda_{12}(s)}\bigg[\frac{(q_1^2-q_2^2)^2-s^2}{\sigma_\pi(s)\lambda_{12}^{1/2}(s)}Q_0(x_s)+2s\bigg]$$

$$x_s = \frac{s-q_1^2-q_2^2}{\sigma_\pi(s)\lambda_{12}^{1/2}(s)} \qquad \sigma_\pi(s)=\sqrt{1-\frac{4M_\pi^2}{s}} \qquad Q_0(x)=\frac{1}{2}\int_{-1}^{1}\frac{dz}{x-z}$$

- Would like to have a **rational function** in the q_i^2 to incorporate into multiloop machinery, possible strategies:
    1. Revert partial-wave projection, perform z integral numerically
    2. Dispersion relation in q_i^2 possible?
    3. Wick rotation/Gegenbauer decomposition of box diagram?
    4. Dispersive calculation of box diagram?

Second challenge: anomalous thresholds

- In principle, we know how to deal with anomalous contributions

$$h_0(s)\Big|_{\text{anom}} = \theta\big(q_1^2+q_2^2-4M_\pi^2\big)\,\frac{\Omega_0(s)}{2\pi i}\int\limits_0^1 dx\,\frac{\partial s_x}{\partial x}\,\frac{\text{disc}_{\text{an}}\,h_0(s_x)}{s_x-s}$$

$$\text{disc}_{\text{an}}\,h_0(s) = -\frac{8\pi}{\sqrt{\lambda_{12}(s)}}\,\frac{t_0(s)}{\Omega_0(s)} \qquad s_x = 4M_\pi^2\,x+(1-x)s_+$$

but this makes it even harder to implement the result

- Ultimately, the anomalous contribution is related to the **singularities of the logarithm in the partial-wave projection**

$$Q_0(x_s) \propto \log\frac{s-q_1^2-q_2^2+\sigma_\pi(s)\lambda_{12}^{1/2}(s)}{s-q_1^2-q_2^2-\sigma_\pi(s)\lambda_{12}^{1/2}(s)}$$

- What happens with the anomalous contributions in Idea 1 above?
- Is there a way to account for the anomalous effects in a numerical partial-wave projection?

Towards a cheap estimate of rescattering corrections?

- Current status:
  - $\gamma^*\gamma^* \to \pi\pi$: dispersive representation known, implementation work in progress
  - $\gamma^*\gamma^*\gamma \to \pi\pi$: dispersive representation known in some limits, full description work in progress
- In the meantime: Is there a way to estimate how large *P*-wave rescattering could become?

Towards a cheap estimate of rescattering corrections?

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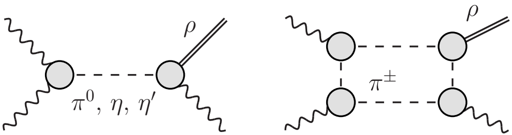
- In the meantime: Is there a way to estimate how large *P*-wave rescattering could become?
- I might have a suggestion:
  - The rescattering ultimately has to come from the *P*-wave phase shift δ_1^1
  - Somehow the Omnès factor $\Omega[\delta_1^1]$ will play a role
  - Let's go to the narrow-resonance limit and identify $\Omega[\delta_1^1]$ with an on-shell $\rho(770)$

$\hookrightarrow$ need the (much) simpler matrix element $\gamma^*\gamma^*\gamma \to \rho$

- This is a special case of hadronic light-by-light scattering, with one virtuality fixed to the mass of the $\rho(770)$

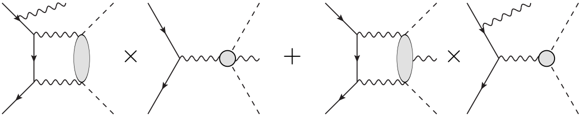
Anatomy of $\gamma^*\gamma^*\gamma \to \rho$



- Possible intermediate states the same as for HLbL
  - **Pseudoscalar poles**: π^0, η, η'
  - Two-meson intermediate states: **pion box**, rescattering corrections, higher left-hand cuts
  - Axial-vector, tensor contributions
  - Perturbative QCD, OPE
- For the leading ones we have "workable" representations

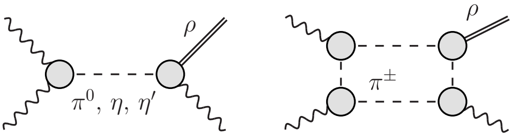
$$F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)=\frac{1}{\pi^2}\int dx\int dy\,\frac{\rho(x,y)}{(x-q_1^2)(y-q_2^2)}\qquad F_\pi^V(q^2)=\frac{1}{\pi}\int dx\,\frac{\operatorname{Im}F_\pi^V(x)}{x-q^2}$$

Conclusions (Liverpool)



- **Pion-pole contributions** only the leading terms, corrections include
  - Rescattering effects
  - Higher left-hand cuts
- **Challenges in the implementation**
  - Functional form of dispersive amplitudes
  - Anomalous thresholds
- Example of ***S*-wave rescattering corrections**
  - Correspond to $f_0(500)$ resonance in $I = 0$
  - Might not be the most relevant ones phenomenologically, but easiest conceptually
  - Would suggest this as test case to learn on how to implement dispersive amplitudes beyond F×sQED

Conclusions (Turin)



- Can we change the benchmarks for F_π^V to **avoid adding ρ and ω Breit–Wigner** amplitudes with arbitrary coefficients?
- While more detailed work is ongoing, can we get useful estimates for ***P*-wave rescattering in $\gamma^*\gamma^*\gamma \to \pi\pi$ from $\gamma^*\gamma^*\gamma \to \rho$**?
  - Amplitudes in principle known from HLbL scattering
  - Pseudoscalar poles (π^0, η, η'), pion loop, ...
  - Could be simple enough to allow for (straightforward?) implementation
    $\hookrightarrow$ known dispersive and double-dispersive representations

Decomposition into scalar functions

- Consider process

$$\gamma^*(q_1,\lambda_1)\gamma^*(q_2,\lambda_2) \to \pi^a(p_1)\pi^b(p_2)$$

$$s=(q_1+q_2)^2=(p_1+p_2)^2 \qquad t=(q_1-p_1)^2=(q_2-p_2)^2 \qquad u=(q_1-p_2)^2=(q_2-p_1)^2$$

- Decompose amplitude into **scalar functions** Bardeen, Tung, Tarrach, see next talk for derivation

$$W_{\mu\nu}=\sum_{i=1}^{5} T^i_{\mu\nu} A_i$$

$$T_1^{\mu\nu}=q_1\cdot q_2 g^{\mu\nu}-q_2^\mu q_1^\nu \qquad T_2^{\mu\nu}=q_1^2q_2^2g^{\mu\nu}+q_1\cdot q_2 q_1^\mu q_2^\nu-q_1^2q_2^\mu q_2^\nu-q_2^2q_1^\mu q_1^\nu$$

$$T_3^{\mu\nu}=(t-u)(\tilde T_3^{\mu\nu}-\tilde T_4^{\mu\nu}) \qquad q_3=p_2-p_1$$

$$\tilde T_3^{\mu\nu}=q_1\cdot q_2 q_1^\mu q_3^\nu-q_1^2q_2^\mu q_3^\nu-\frac{1}{2}(t-u)q_1^2g^{\mu\nu}+\frac{1}{2}(t-u)q_1^\mu q_1^\nu$$

$$\tilde T_4^{\mu\nu}=q_1\cdot q_2 q_3^\mu q_2^\nu-q_2^2q_3^\mu q_1^\nu+\frac{1}{2}(t-u)q_2^2g^{\mu\nu}-\frac{1}{2}(t-u)q_2^\mu q_2^\nu$$

$$T_4^{\mu\nu}=q_1\cdot q_2 q_3^\mu q_3^\nu-\frac{1}{4}(t-u)^2g^{\mu\nu}+\frac{1}{2}(t-u)\left(q_3^\mu q_1^\nu-q_2^\mu q_3^\nu\right)$$

$$T_5^{\mu\nu}=q_1^2q_2^2q_3^\mu q_3^\nu+\frac{1}{2}(t-u)\left(q_1^2q_3^\mu q_2^\nu-q_2^2q_1^\mu q_3^\nu\right)-\frac{1}{4}(t-u)^2q_1^\mu q_2^\nu$$

- Matches onto **5 helicity amplitudes** $H_{\lambda_1\lambda_2}$, $\lambda_1\lambda_2\in\{++,+-,0+,+0,00\}$

Pion-pole contributions

- **Pion-pole contributions** defined by Cutkosky rules, this gives

$$A_1^\pi = -F_\pi^V(q_1^2)F_\pi^V(q_2^2)\bigg(\frac{1}{t-M_\pi^2}+\frac{1}{u-M_\pi^2}\bigg)$$

$$A_4^\pi = -F_\pi^V(q_1^2)F_\pi^V(q_2^2)\frac{2}{s-q_1^2-q_2^2}\bigg(\frac{1}{t-M_\pi^2}+\frac{1}{u-M_\pi^2}\bigg) = F_\pi^V(q_1^2)F_\pi^V(q_2^2)\frac{2}{(t-M_\pi^2)(u-M_\pi^2)}$$

$$A_2^\pi = A_3^\pi = A_5^\pi = 0$$

- **Coincides with sQED times form factors**: F$\times$sQED $=$ FsQED in this case
- This is not guaranteed to happen, counterexample: nucleon pole vs. nucleon Born term in nucleon Compton scattering
- Result is easy to combine with multiloop techniques by writing

$$\frac{F_\pi^V(s)}{s} = \frac{1}{s}+\frac{1}{\pi}\int_{4M_\pi^2}^\infty ds'\frac{\text{Im}\,F_\pi^V(s')}{s'(s'-s)} \qquad \text{or} \qquad \frac{F_\pi^V(s)}{s} = \frac{1}{s}\times\frac{1}{\pi}\int_{4M_\pi^2}^\infty ds'\frac{\text{Im}\,F_\pi^V(s')}{s'-s}$$

- For $\gamma^*\gamma^*\to\pi\pi$ at low energies this is the dominant effect, but certainly for $\gamma^*\gamma^*\gamma\to\pi\pi$ we need to learn how to go beyond

S-wave rescattering

- Pion-pole amplitude not unitary

  $\hookrightarrow$ **rescattering corrections** via $\pi\pi$ phase shifts $\delta_J(s)$

- Unitarity relation formulated at the level of **helicity amplitudes**

$$\text{Im}\, h_J(s) = h_J(s) e^{-i\delta_J(s)} \sin\delta_J(s)$$

  $\hookrightarrow$ **inhomogenous Muskhelishvili–Omnès problem**

- S-wave solution reads

$$h_{0,++}(s) = N_{0,++}(s) + \frac{\Omega_0(s)}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\sin\delta_0(s')}{|\Omega_0(s')|}\bigg[\bigg(\frac{1}{s'-s} - \frac{s'-q_1^2-q_2^2}{\lambda_{12}(s')}\bigg)N_{0,++}(s') + \frac{2q_1^2q_2^2}{\lambda_{12}(s')}N_{0,00}(s')\bigg]$$

$$h_{0,00}(s) = N_{0,00}(s) + \frac{\Omega_0(s)}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\sin\delta_0(s')}{|\Omega_0(s')|}\bigg[\bigg(\frac{1}{s'-s} - \frac{s'-q_1^2-q_2^2}{\lambda_{12}(s')}\bigg)N_{0,00}(s') + \frac{2}{\lambda_{12}(s')}N_{0,++}(s')\bigg]$$

$$\Omega_0(s) = \exp\bigg\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\delta_0(s')}{s'(s'-s)}\bigg\} = 1 + \frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\text{Im}\,\Omega_0(s')}{s'(s'-s)} = \frac{1}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\text{Im}\,\Omega_0(s')}{s'-s}$$

- $N_{0,++}(s)$, $N_{0,00}(s)$: partial-wave projection of pion-pole terms

  $\hookrightarrow$ complicated dependence on q_i^2

First challenge: form of dispersive amplitudes

- Idea 1 produces

$$A_i(s) = F_\pi^V(q_1^2)F_\pi^V(q_2^2)\frac{\Omega_0(s)}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\sin\delta_0(s')}{(s'-s)|\Omega_0(s')|}\int_{-1}^1 dz\, \frac{\bar{A}_i(s',z)}{D(s',z)}$$

$$\bar{A}_1(s,z) = -2(s-q_1^2-q_2^2)s\lambda_{12}(s) + 2(s-4M_\pi^2)\Big[s(\lambda_{12}(s)+4q_1^2q_2^2) - ((q_1^2+q_2^2)\lambda_{12}(s)+12sq_1^2q_2^2)z^2\Big]$$

$$\bar{A}_2(s,z) = -4(s-4M_\pi^2)\Big[s(s-q_1^2-q_2^2) + (\lambda_{12}(s)-3s(s-q_1^2-q_2^2))z^2\Big]$$

$$D(s,z) = \lambda_{12}(s)\Big[s(\lambda_{12}(s)+4q_1^2q_2^2) - (s-4M_\pi^2)\lambda_{12}(s)z^2\Big] \qquad \lambda_{12}(s) = s^2+(q_1^2-q_2^2)^2-2s(q_1^2+q_2^2)$$

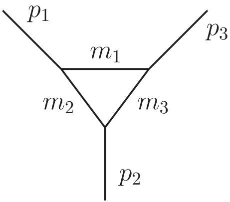
$\hookrightarrow$ rational function of q_i^2, but not quite of standard propagator form

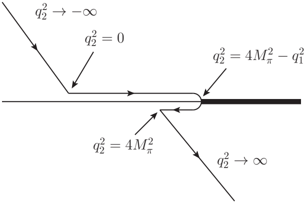
Second challenge: anomalous thresholds

- Consider the scalar loop function $C_0(s)$, $s = p_2^2$
- Fulfills the dispersion relation

$$C_0(s) = \frac{1}{2\pi i}\int_{(m_2+m_3)^2}^{\infty} ds' \frac{\text{disc}\, C_0(s')}{s'-s}$$
$$+\theta\Big[m_3 p_1^2 + m_2 p_3^2 - (m_2+m_3)(m_1^2+m_2 m_3)\Big]$$
$$\times \frac{1}{2\pi i}\int_0^1 dx \frac{\partial s_x}{\partial x}\frac{\text{disc}_{\text{an}}\, C_0(s_x)}{s_x - s}$$

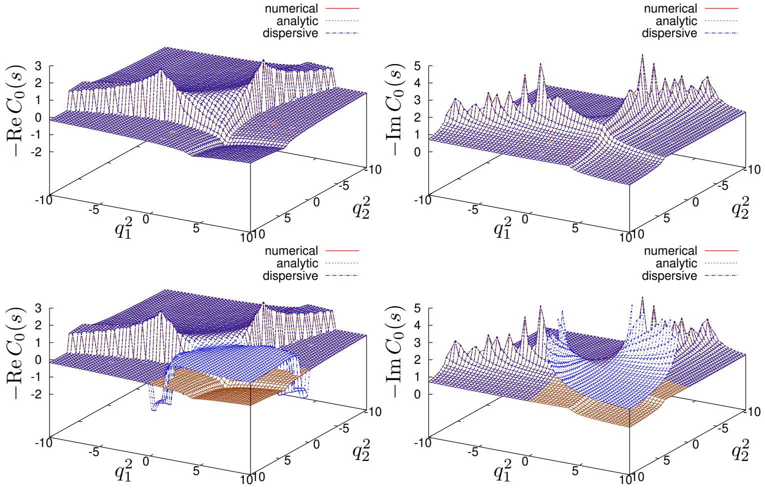
$$s_x = x(m_2+m_3)^2 + (1-x)s_+$$
$$s_+ = p_1^2\frac{m_1^2+m_3^2}{2m_1^2} + p_3^2\frac{m_1^2+m_2^2}{2m_1^2} - \frac{p_1^2 p_3^2}{2m_1^2} - \frac{(m_1^2-m_2^2)(m_1^2-m_3^2)}{2m_1^2}$$
$$+\frac{1}{2m_1^2}\sqrt{\lambda(p_1^2,m_1^2,m_2^2)\lambda(p_3^2,m_1^2,m_3^2)}$$





- Anomalous piece parameterizes the contour deformation from threshold to s_+

Second challenge: anomalous thresholds



- Example for $m_1 = m_2 = m_3 = M_\pi \equiv 1$ (upper: full, lower: without anomalous term)

  $\hookrightarrow$ this is exactly what we need here, **anomalous contribution** for $q_1^2 + q_2^2 \geq 4M_\pi^2$