Dispersive description of the $\gamma^* T \to \pi\pi$ subprocess and its role in $(g-2)_\mu$

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with

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Overview

1. Hadronic spin-2 resonances: in our construction, phenomenology, current estimates.
2. $\gamma^* T \to \pi\pi$, as an ingredient of tensor TFFs.
3. Glimpse at the construction $(g-2)_\mu$: $f_2(1270)$ pole pieces.

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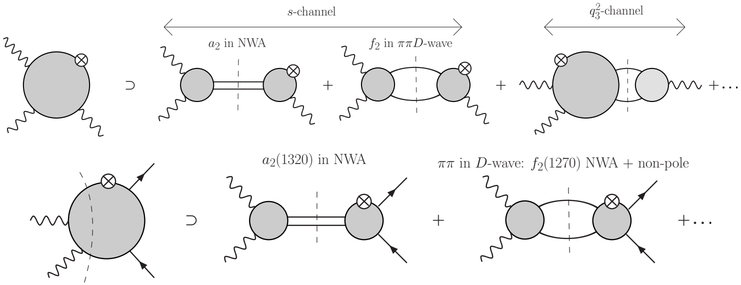
1. Hadronic spin-2 resonances: in our construction, phenomenology, current estimates.
   - Our roadmap to spin-2 contributions in HLbL
   - Pheno of $f_2(1270)$
   - hQCD vs QM predictions and a "sign problem"

2. $\gamma^* T \to \pi\pi$, as an ingredient of tensor TFFs.

3. Glimpse at the construction $(g-2)_\mu$: $f_2(1270)$ pole pieces.

The road to spin-2: $\gamma^* T \to \pi^+\pi^-$ and $T \to \gamma^*\gamma^*$ appear!



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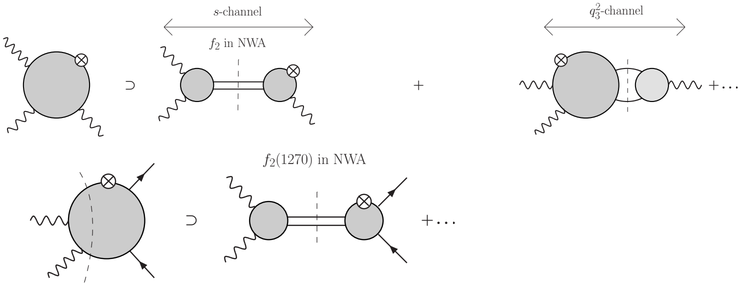

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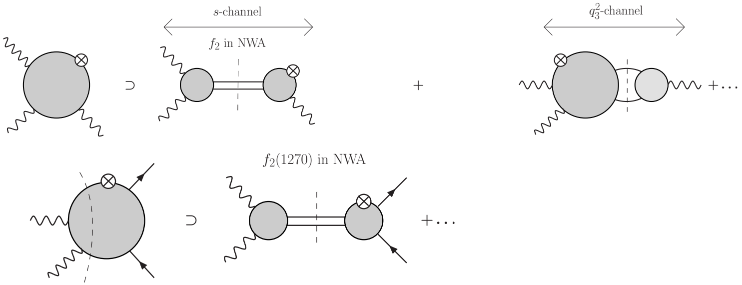

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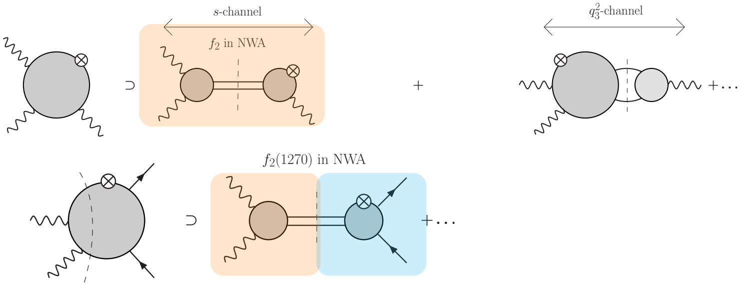

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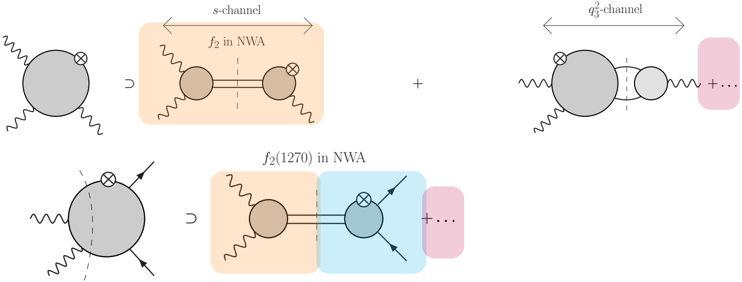

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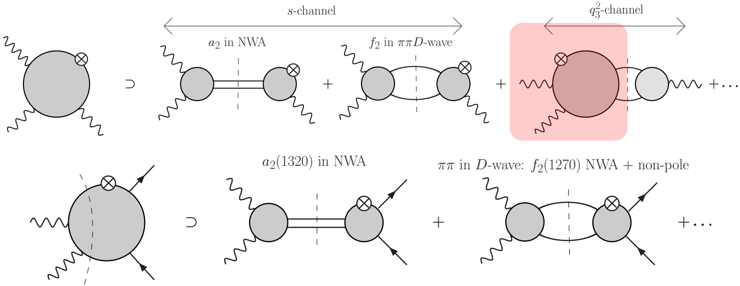

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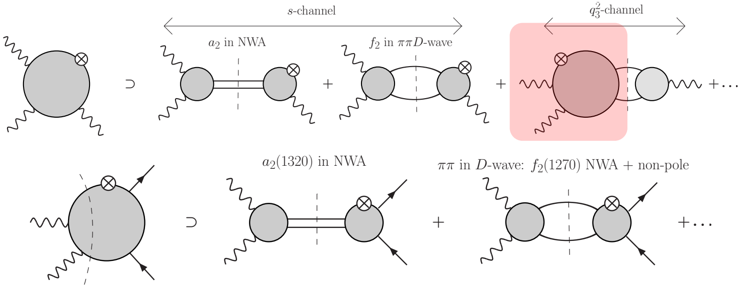

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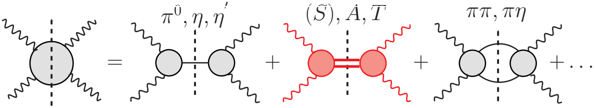
Tensors in HLbL so far: perils of the old approach



- **Old approach:** disperse in s, t, u, then $q_4 \to 0$ at the end:

$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_2 \int_0^\infty dQ_1 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau).$$

- **Tensors (and $J \geq 2$ intermediates)** in 4 pt. kinematics approach - **kinematic singularities** in Q_i^2 (due to redundant basis of HLbL) *M. Hoferichter, P.Stoffer, M. Zillinger, arXiv:2402.14060*

$$\Pi_i \supset \frac{1}{Q_1^2 Q_2^2 (Q_1^2 - Q_2^2)} \mathcal{F}_i^T \mathcal{F}_j^T + \dots$$

- **Sum rules remove all kin. singularities,** but only guarnateed for ∞-tower, not individual intermediates (incl. tensors)! $\Rightarrow$ **Are tensors doomed in the old approach?**

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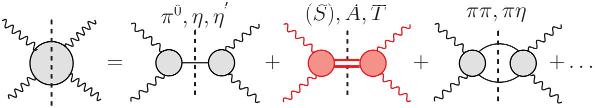

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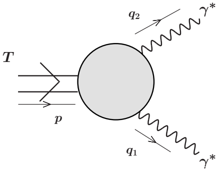
- **Special cases exist, singularities vanish when:**
  1. Only $\mathcal{F}_1^T \neq 0$
  2. When pairs $\mathcal{F}_{1,2}^T \neq 0$ or $\mathcal{F}_{1.3}^T \neq 0$.
- **Model approaches exist** that utilises this.
- **Can not set other TFFs to** 0 **if they are generated (and they are)!**

Triangle kinematics approach resolves this (and more)!

Pheno of $f_2(1270)$: introduction



- General $T \to \gamma^*\gamma^*$ amplitude described by $\times 5$ TFFs:

$$\mathcal{M}^{\mu\nu\alpha\beta} = \sum_{i=1}^{5} T_i^{\mu\nu\alpha\beta} \frac{1}{m_T^{2n}} \mathcal{F}_i^T(q_1^2, q_2^2)$$

- **Known light-cone OPE** (Brodsky-Lepage limit) *Hoferichter, Stoffer, arXiv:2004.06127v2*:

$$\mathcal{F}_1^T(q_1^2, q_2^2) \sim \frac{1}{Q^4} f_1(\omega), \qquad \mathcal{F}_{i\neq 1}^T(q_1^2, q_2^2) \sim \frac{1}{Q^6} f_i(\omega),$$

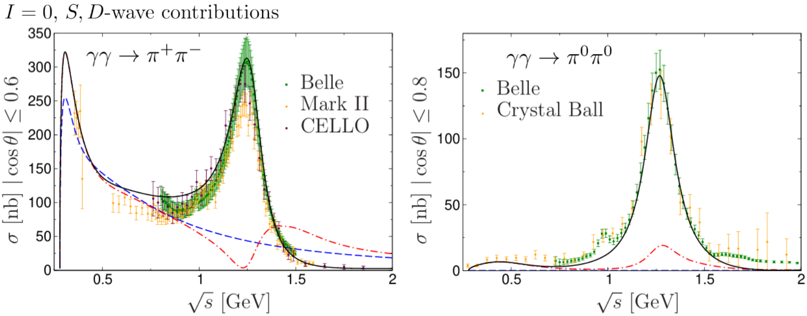
with $Q^2 = 1/2(q_1^2 + q_2^2)$, $\omega = 2/Q^2 \times (q_1^2 - q_2^2)$.

- **Quark Model:**

$$\mathcal{F}_1^T(q_1^2, q_2^2) = \frac{\mathcal{F}_1^T(0,0)\Lambda_T^4}{\left(\Lambda_T^2 - q_1^2 - q_2^2\right)^2}, \qquad \mathcal{F}_{i\neq 1}^T(q_1^2, q_2^2) = 0.$$

$\Rightarrow$ **works in the old approach!**

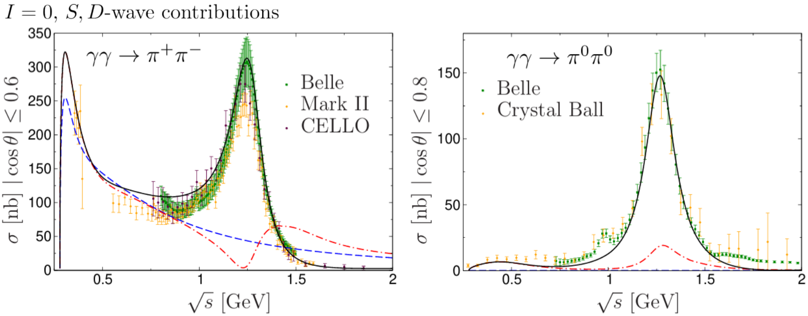
Pheno of $f_2(1270)$: D-wave resonance in $\gamma\gamma \to \pi\pi$



M. Hoferichter, P. Stoffer arXiv:1905.13198

- $\gamma\gamma \to \pi\pi$: arises in $e^+e^- \to e^+e^-\pi\pi$.
- $\gamma^*\gamma^* \to \pi\pi$ ($S-$ and $D-$wave) calculation - good match with data.
- For $\sqrt{s} > 1$GeV - **$f_2(1270)$ resonance dominates**.

Pheno of $f_2(1270)$: D-wave resonance in $\gamma\gamma \to \pi\pi$



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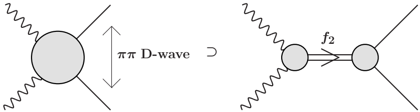

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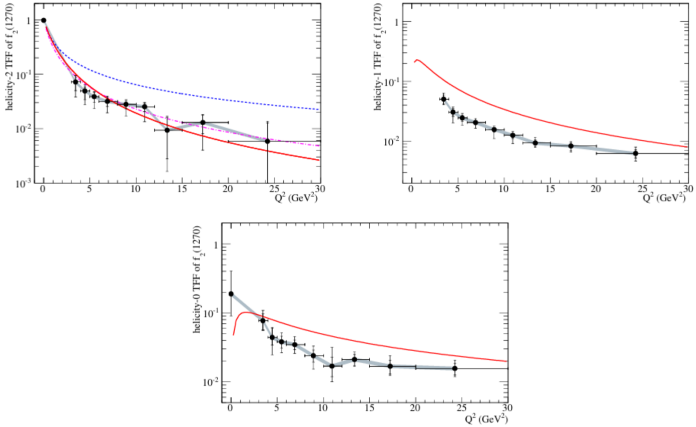
- $\gamma^*\gamma^* \to \pi\pi$ ($S-$ and $D-$wave) calculation - good match with data.
- For $\sqrt{s} > 1$GeV - $\boldsymbol{f_2(1270)}$ **resonance dominates**.
- **But what about strictly $T \to \gamma^*\gamma^*$ data?**

- **Source:** ρ, ω, ϕ (vector) resonances in left-hand cut (LHC) of $\gamma\gamma \to \pi\pi$ D-wave. Different method of TFF extraction $\Rightarrow$ M. Hoferichter, M. Zillinger, in preparation

Pheno of $f_2(1270)$: TFF data is sparse...

M. Masuda *et al.* [Belle Collaboration], Phys. Rev. **D93**, 032003 (2016)



- Only **singly-virtual helicity amplitudes** - $e^+e^- \to e^+e^-\pi^0\pi^0$ data.
- Only consistent with QM for $\lambda = 2$, **can only probe $\mathcal{F}^T_{1,2,5}$**!
- Also know $\Gamma_{T\to\gamma\gamma} = 2.6(5)$ keV, $r_h = 1/24|\mathcal{F}^T_2(0,0)|^2/|\mathcal{F}^T_1(0,0)|^2 = 0.095(20)$.

hQCD prediction: tensions with the QM!

J.Mager, L. Cappielo, J. Leutgeb, A. Rebhan arXiv:2501.19293

$$S = -2k_T \int d^5x \sqrt{g} \left(\mathcal{R} + 2\Lambda\right) + \frac{1}{2g_5^2}\mathrm{tr} \int d^5x \sqrt{g} F_{MN} F^{MN}$$

$$\mathcal{F}_1^T(-Q_1^2, -Q_2^2) = -m_T \frac{1}{g_5^2}\mathrm{tr}\mathcal{Q}^2 \int \frac{dz}{z} h_n(z) \mathcal{J}(z, Q_1)\mathcal{J}(z, Q_2),$$

$$\mathcal{F}_3^T(-Q_1^2, -Q_2^2) = -m_T^3 \frac{1}{g_5^2}\mathrm{tr}\mathcal{Q}^2 \int \frac{dz}{z} h_n(z) \frac{\partial_z \mathcal{J}(z, Q_1)}{Q_1^2} \frac{\partial_z \mathcal{J}(z, Q_2)}{Q_2^2}.$$

- Class of large-N_c models, relies on **4D gauge - 5D gravity duality.**
- Hard wall (finite 5th dimension) model: predicts **two non-zero TFFs.** $\Rightarrow$ **one of the special combinations**
- 3 free parameters (2 fixed from axial vector model) - **fit from SDCs** (possible because of ∞-resonance tower predicted).

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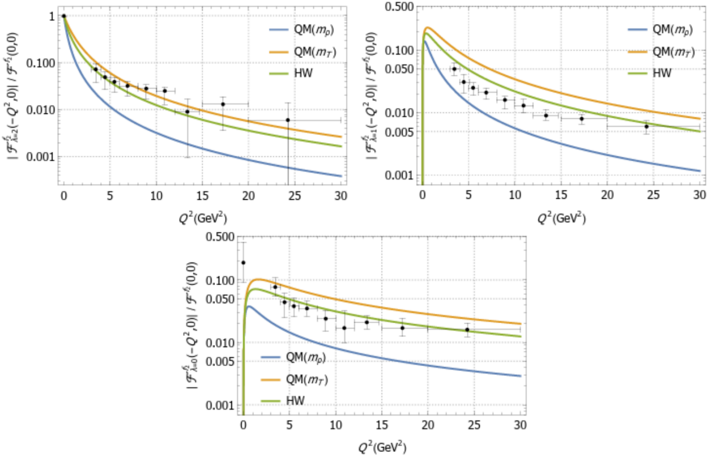

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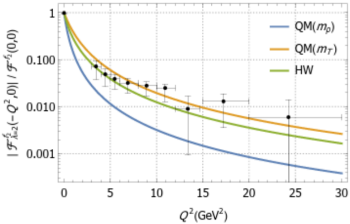
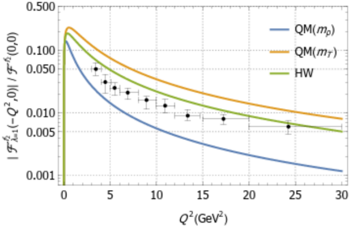
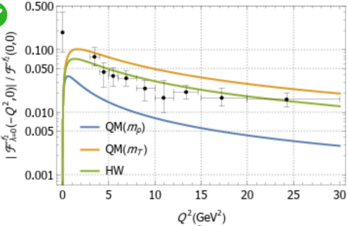




$\Rightarrow$ Predictions: ✓

$m_T = 1.235$ GeV

$\Gamma = 2.3$ keV



$\Rightarrow$ Drawback: improving uncertainties? ✗

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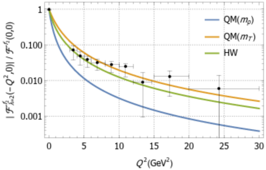

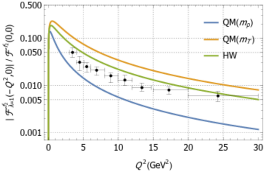

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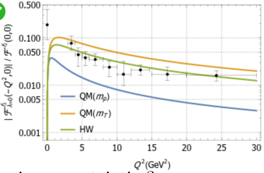


⇒ T-pole contributions:

$HW : a_\mu(f_2 + a_2 + f_2') = 2.9(4) \times 10^{-11}$

$QM : a_\mu(f_2 + a_2 + f_2') = -2.5(8) \times 10^{-11}$

M. Hoferichter, P. Stoffer, M. Zillinger arXiv:2412.00190

⇒ Drawback: improving uncertainties?

⇒ "sign problem" - disp. evaluation needed to reduce uncertainties!

Overview

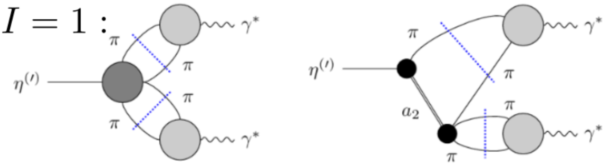
1. Hadronic spin-2 resonances: in our construction, phenomenology, current estimates.

2. $\gamma^* T \to \pi\pi$, as an ingredient of tensor TFFs.
   - Gold standard: $\eta^{(\prime)} \to \gamma^*\gamma^*$
   - $(I = 1)$ TFF construction recipe
   - The $\gamma^* T \to \pi\pi$ subprocess: Inhomogeneous Omnès problem, $q_1 \to 0$ limit

3. Glimpse at the construction $(g-2)_\mu$: $f_2(1270)$ pole pieces.

S. Holz, M. Hoferichter, B.-L. Hoid, B. Kubis arXiv:2412.16281v2

$I = 1:$



$$F_{\eta^{(\prime)}\gamma^*\gamma^*}(q_1^2, q_2^2) = F^{I=1}(q_1^2, q_2^2) + F^{I=0}(q_1^2, q_2^2) + F^{\text{eff}}(q_1^2, q_2^2) + F^{\text{asym}}(q_1^2, q_2^2)$$

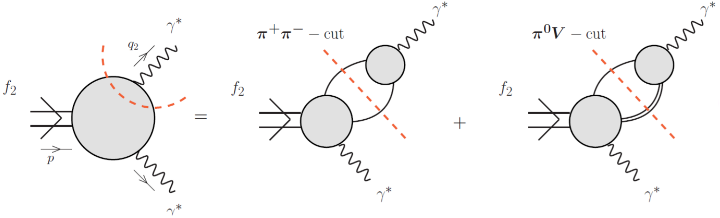
Tensors: This talk! TBD!

$$\mathcal{F}_i^T(q_1^2, q_2^2) = \mathcal{F}_i^{I=1}(q_1^2, q_2^2) + \mathcal{F}_i^{I=0}(q_1^2, q_2^2) + \mathcal{F}_i^{\text{eff}}(q_1^2, q_2^2) + \mathcal{F}_i^{\text{asym}}(q_1^2, q_2^2)$$

- Both $\eta^{(\prime)}$, $f_2(1270)$ - $I = 0$, same isospin decomposition.
- $I = 0$ construction: different channels - $\eta^{(\prime)} \to 2(\pi^+\pi^-)$, but $f_2 \to \pi\pi$.
- **Challenges in T case:**

  1. **$I = 0$ case:** small for $\eta^{(\prime)} \to \gamma^*\gamma^*$, VMD. Hard to estimate for $T \to \gamma^*\gamma^*$ (couplings unavailable).
  2. **No intermediate checks:** $\eta^{(\prime)} \to \pi\pi\gamma$ data available (KLOE, BESIII), but no $T \to \pi\pi\gamma$.

$(I = 1)$ TFF construction: death by 1000 cuts



- Requires $\gamma^* T \to \pi\pi$ (in P-wave) and $\gamma^* T \to \pi^0 V$ $\Rightarrow$ **generate $\times 5$ TFFs!**
- Need to solve in $q_i^2 < 0$, $q_1^2 = m_T^2$, $q_2 \to 0$ limits: different analytic behaviour.

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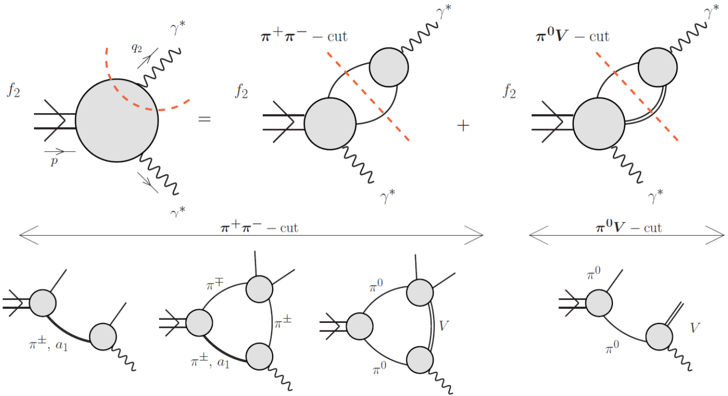

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- Need to solve in $q_i^2 < 0$, $q_1^2 = m_T^2$, $q_2 \to 0$ limits: different analytic behaviour.
- **Input:** $P-$wave phase $\delta_1(s)$, $f_2 \to \pi\pi$, $V \to 3\pi$, $V \to \pi^0\gamma$.
  Fit from $\Gamma_{T\to\gamma\gamma}$ (if needed): $f_2 \to \pi a_1$.

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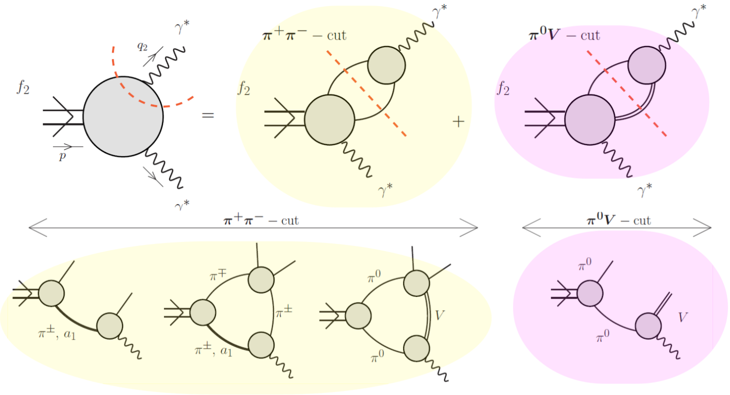

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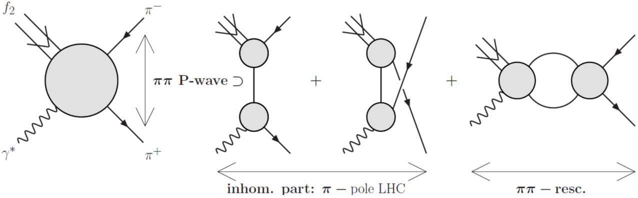
- **Input:** $P-$wave phase $\delta_1(s)$, $f_2 \to \pi\pi$, $V \to 3\pi$, $V \to \pi^0\gamma$.
  Fit from $\Gamma_{T\to\gamma\gamma}$ (if needed): $f_2 \to \pi a_1$.
- **Status:** $\gamma^* T \to \pi\pi$ for π, a_1 solved $\Rightarrow$ Discuss now!, incorporating V intermediates.

The $\gamma^* T \to \pi\pi$ subprocess: Inhom. Omnès problem

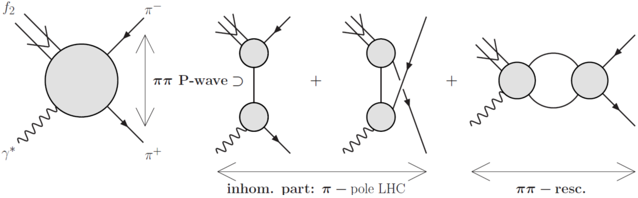


- $\boldsymbol{\gamma^* T \to \pi\pi}$ **definition:** $\times 8$ scalar funcs - $\mathcal{M}^{\mu\nu\alpha\beta} = \sum_i T^{\mu\nu\alpha\beta} B_i$.
- Start with $\pi-$pole LHC contribution (similar for other intermediates.) and **enforce Watson's theorem (rescattering).**
- Generally - hyperbolic disp. rels:

$$B_i(s,t,u) = B_i^{\text{Born}}(s,t,u) + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im} B_i(s',z')}{s'-s}$$
$$+ \frac{1}{\pi}\int_{9M_\pi^2}^{\infty} dt' \, \mathrm{Im} B_i(t',u') \left(\frac{1}{t'-t} + \frac{1}{t'-u} - \frac{1}{t'-a} \right).$$

- **Solution setup:** $B_i(s,t,u) \Rightarrow$ partial waves $h_i^J(s) \Rightarrow$ Roy-Steiner equations (coupling of different J-waves) - complicated!

The $\gamma^* T \to \pi\pi$ subprocess: Inhom. Omnès problem



- **Simplification:** only need $\pi\pi$ $P-$wave! Mapping $B_i(s,t,u)$ $\Rightarrow$ partial waves $h_i^1(s)$ $\Rightarrow$ back to $B_i^{\text{P-wave}}(s,t,u)$ - **no t, u-dep.** - single-var. disp. rels!

- Asymptotic ansatz (only $\times 5$ B_i funcs. survive) -unsubtracted disp. rels.:

$$B_{1,2} \asymp \frac{1}{s^2}, \quad B_{3,4,5} \asymp \frac{1}{s},$$

- **Inhomogeneous Omnes problem:**

$$B_i^{\text{P-wave}}(s) = B_{i,\text{Born}}^{\text{P-wave}}(s) + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{B_{i,\text{Born}}^{\text{P-wave}}(s') \sin\delta_1(s')}{(s'-s-i\epsilon)|\Omega_1(s')|}.$$

$$B_{i,\text{Born}}^{\text{P-wave}}(s) = \frac{F_\pi^V(q_1^2)}{((s-s_L)(s-s_R))^{m_i}} \left(P_i + Q_i \text{Disc}_s\, C_0(s, q_1^2, m_T^2)\right),$$

with $s_{R,L} = (m_T \pm \sqrt{q_1^2})^2$ - **soft-singular at $s = m_T^2$ as $q_1^2 \to 0$**. Numerical instabilities - how to deal with them?.

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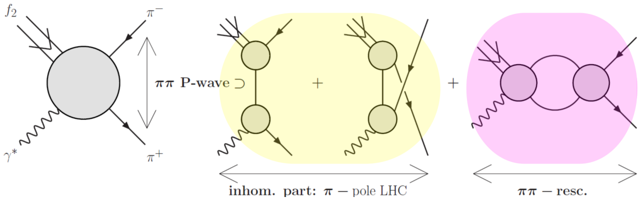

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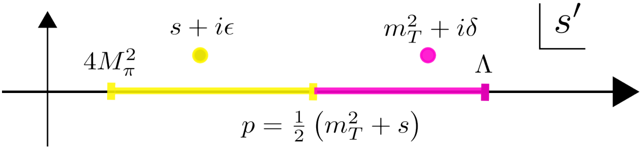
The $\gamma^* T \to \pi\pi$ subprocess: taming numerical instabilities

Inspired by: S. Mutke, M. Hoferichter, B. Kubis arXiv:2406.14608



- **ONS limit:** most complicated case (soft-γ^*) singularity. Work with $m_T^2 + i\delta$ prescription:

$$B_i^{\text{resc.}}(s) = \int_{s_0}^{\infty} ds' \, \frac{T_i(s')}{(s' - m_T^2 - i\delta)^n (s' - s - i\epsilon)},$$

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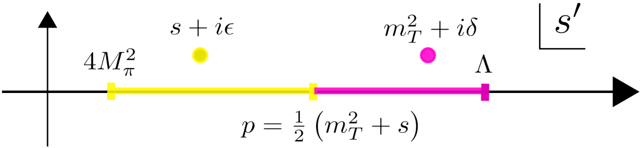

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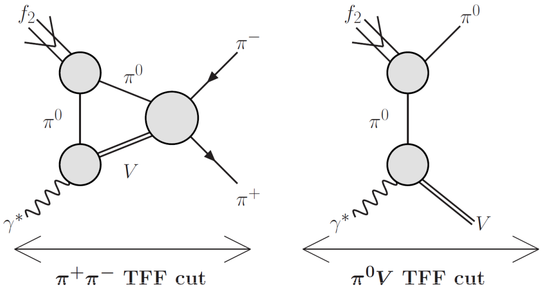
Idea: split integration region - isolate singularities, subtract n derivatives at $s' = s$ and $s' = m_T^2$ s.t. integrands finite even in soft limit: $p = s = m_T^2$.

$$B_i^{\text{resc.}}(s) = \text{PV}_s \left(\int_{s_0}^{p} ds' \, \frac{T(s') - T_n(s', s)}{(s' - m_T^2)^n (s' - s)} \right) + \text{PV}_{m_T^2} \left(\int_{p}^{\Lambda} ds' \, \frac{T(s') - T_n(s', m_T^2)}{(s' - m_T^2)^n (s' - s)} \right) + K_n(s) + R_n(s),$$

- **Remainders** K_n, R_n singular, but evaluated analytically -**singularities cancel, stable numerics!** (Other kinematic cases more simple).

$(I=1)$ TFF construction: the $\gamma^* T \to \pi^0 V$ contributions



- Need dispersive π^0-pole description in $\gamma^*(q_1)T(q_2) \to V(q_3)\pi^0(q_4)$, with 23 non-reduntant structures (# of hel. amplitudes).
- Basis decomposition - 300+ rank-7 structures T_i:

$$\{L_i^{\mu\alpha\beta\gamma}\} = \{\epsilon_{abcd} T_i^{\mu\alpha\beta\gamma abcd}\},$$

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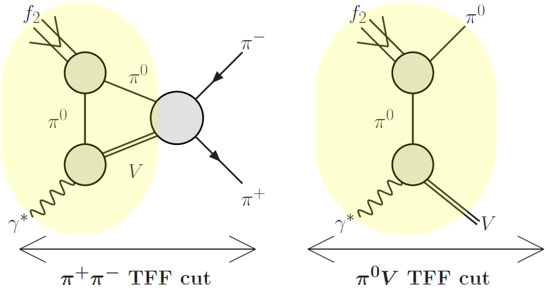

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- **BTT+imposing γ^*, T, V symmetries:** 75 (physical) + 73 (unphysical) strucs. $\Rightarrow$ getting to 23?

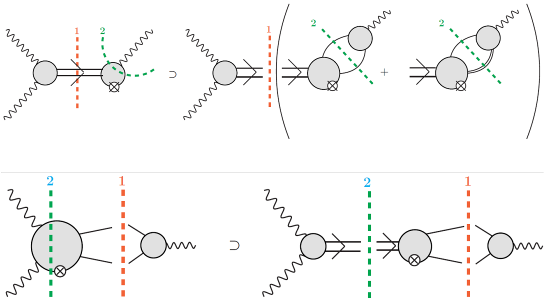
- **Schouten redundancies**- $D=4$: $\epsilon^{[\mu\nu\rho\sigma}g^{\alpha]\beta}=0$. Final result:

$$\mathcal{N}^{\mu\alpha\beta\gamma}_{\pi-\text{pole}}(q_1,q_2,q_3) = -(q_1-q_3)^\alpha (q_1-q_3)^\beta \epsilon^{\mu\gamma\rho\sigma} q_{1\,\rho} q_{3\,\sigma} \frac{\mathcal{F}_0^\Gamma f^*_{V\pi^0}(q_1^2)}{t-M_\pi^2}.$$

Overview

1. Hadronic spin-2 resonances: in our construction, phenomenology, current estimates.
2. $\gamma^* T \to \pi\pi$, as an ingredient of tensor TFFs.
3. Glimpse at the construction $(g-2)_\mu$: $f_2(1270)$ pole pieces.

A real $f_2(1270)$ contribution to $(g-2)_\mu$: pole pieces



- **Need tensor contribution to $(g-2)_\mu \in \mathbb{R}$.**
- Group subprocesses to c.c. pairs?

$$\mathrm{Im}\hat{\Pi}_{gi} = \lim_{s\to q_3^2} \left[\mathrm{Im}_s\check{\Pi}_i + \left(\mathrm{Im}_3\check{\Pi}_i\right)^*\right]$$

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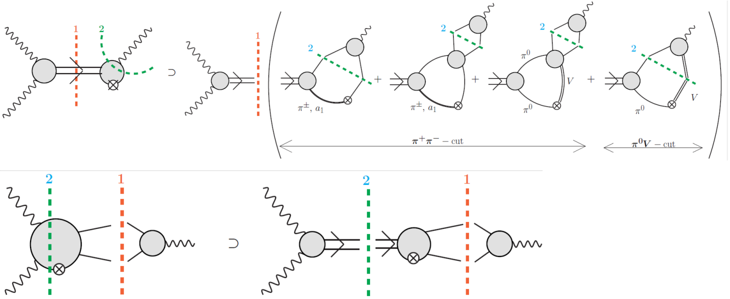

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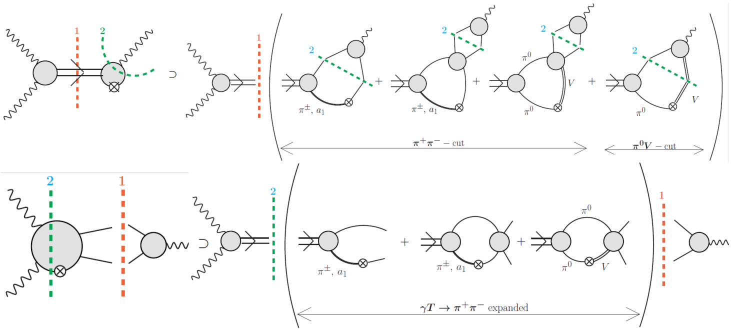

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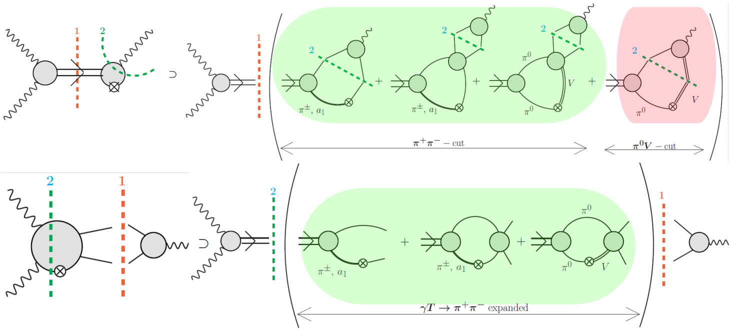

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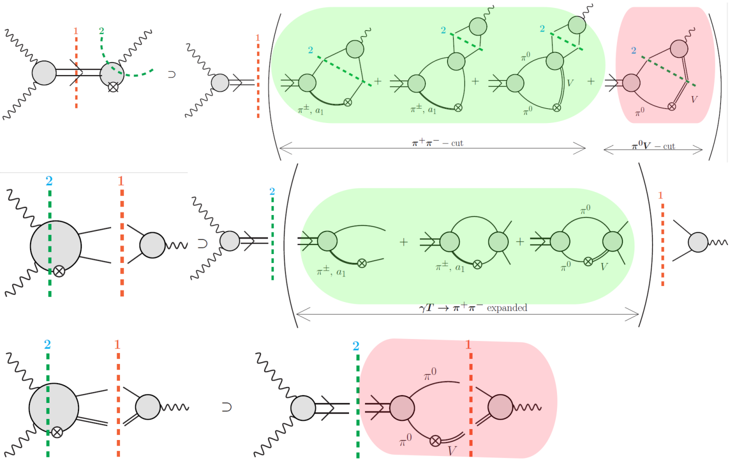

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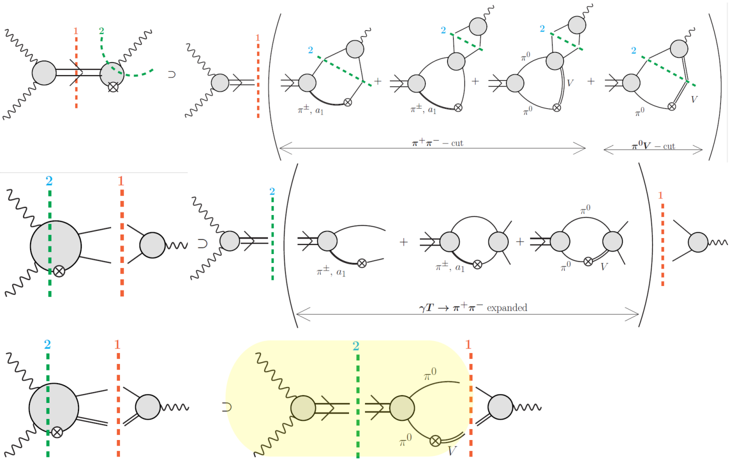

- **Problem:** unpaired subprocess in $\pi^0 V$ secondary cut!
- **Solution:** also consider a $\pi^0 V$ intermediate in q_3^2 cut!
- Now need $\mathbf{3\gamma \to \pi^0 V}$ - doable in soft limit. **Even more complex for full $3\gamma \to \pi\pi$, $\gamma\gamma \to \pi\pi$ in $D-$wave.**

Summary: tensor resonances $\nearrow$, $(g-2)_\mu$ uncertainty $\searrow$!

- **Hadronic subprocesses of spin-2 resonances:** $\gamma^* T \to \pi\pi$, $T \to \gamma^*\gamma^*$ - **crucial ingredients** for $\gamma^*\gamma^*\gamma \to \pi\pi$ and HLbL spin-2 contribution in triangle kinematics.
- **Gold standard:** similarities with $\eta' \to \gamma^*\gamma^*$ exist - $I = 0$ contributions, effective poles, matching - TBD.
- **Replicating $f_2(1270)$ pheno:** $I = 1$ part - need V-intermediates, $\gamma^* T \to \pi\pi$ with $\boldsymbol{\pi}$-pole LHC **solved** - approach adaptable for the rest of contributions.

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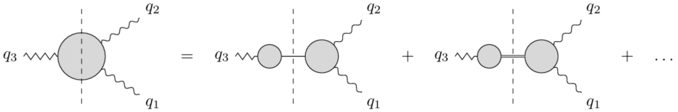
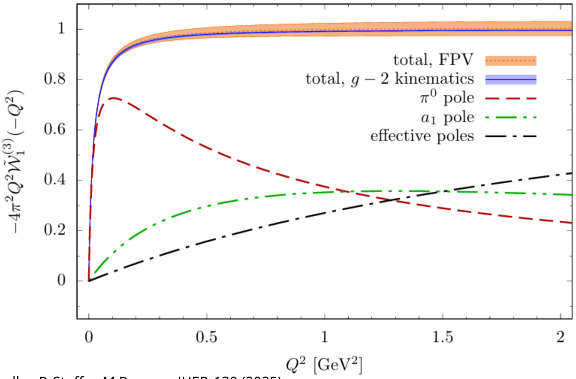
Thank you for your attention!

Appendix: VVA correlator - testing ground for HLbL



Normal kinematics, then $q_2 \to 0$



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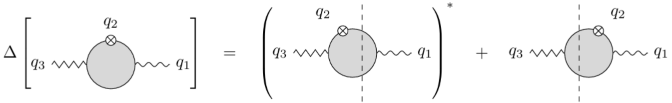
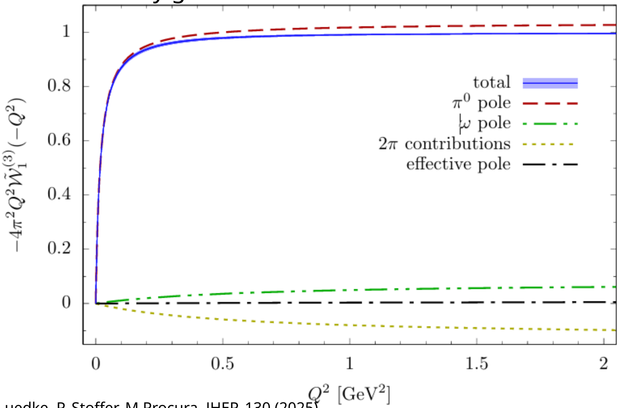


Directly g-2 kinematics



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