3rd RadioMonteCarLow2 Meeting

Monte Carlo codes: McMule

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for the McMule Team

Università di Torino

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- higher-order predictions and comparison with precision experiments
- focus on $2 \to 2$ and $2 \to 3$ low-energy QED scattering processes
- **input**: matrix elements by us or others (at NNLO + first visits at N3LO)
- **output**: physical cross section for any physical observable at fixed order
- at present an integrator, generator features under testing

McMule

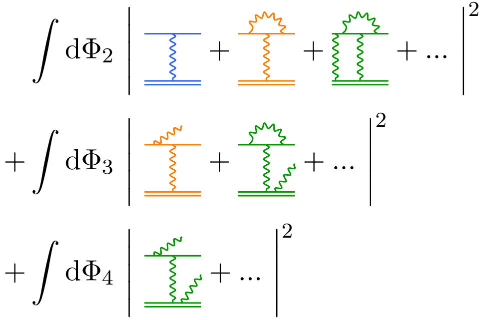
Monte Carlo for MUons and other LEptons

code → `https://mule-tools.gitlab.io/`

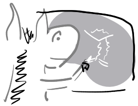
docs → `https://mcmule.readthedocs.io/`

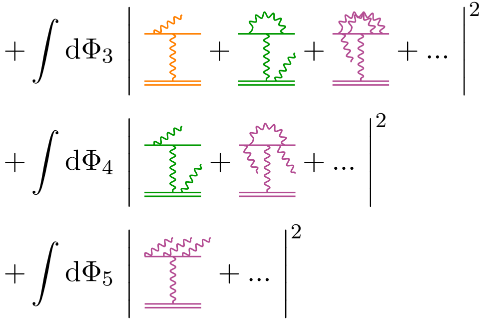
UNIVERSITÀ DI TORINO McMULE

a peek in the ~~stable~~ theory



1. fully-differential PS integration
   $\to$ FKS$^\ell$
2. numerical instabilities due to pseudo-singularities
   $\to$ next-to-soft stabilisation
3. virtual amplitudes with massive particles
   $\to$ one-loop: OpenLoops
   $\to$ two-loop: massification





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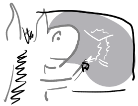
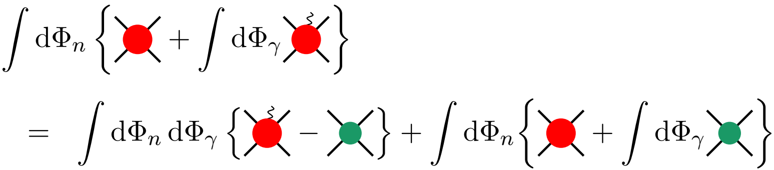
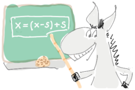


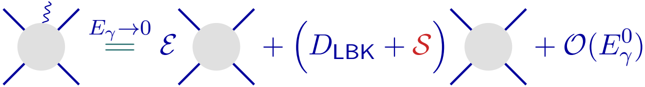
a universal recipe for divergent radiative corrections



- exploits exponentiation of soft singularities [YFS 61]
- works at all orders in QED [Engel, Signer, Ulrich 19]
- singularities are dealt with locally → stable numerical integration
- no resolution parameters → theory error: 0



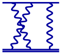
a universal recipe for unstable real-virtual corrections



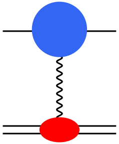
- use OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19] for the bulk of the phase space
- when unstable, use LBK theorem [LBK 58-61, Engel, Signer, Ulrich 21, 2xEngel 23]
  ↪ implemented through momentum shifts [Balsach, Bonocore, Kulesza 23]
- if $E_\gamma < E_{\mathrm{NTS}} \sim 10^{-3}\sqrt{s}/2$ switch to NTS expansion above
- theory error: $\mathcal{O}(10^{-3})\,@\,\mathrm{NNLO} \sim \mathcal{O}(10^{-6})$ for $e\mu \to e\mu$
- different carbon footprint: 7 days vs 3 months

a(n almost) universal recipe for two-loop integrals with masses

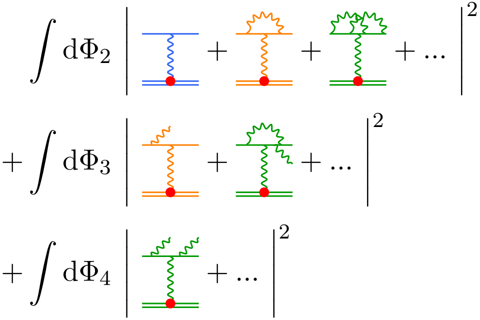
- use $m_\ell^2 \ll s \sim Q^2 \to$ expand in m_ℓ^2/Q^2

 $\sim A \log^2 \dfrac{m_\ell^2}{Q^2} + B \log \dfrac{m_\ell^2}{Q^2} + C + \mathcal{O}\Big(\dfrac{m_\ell^2}{Q^2}\Big)$

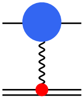
- can be applied up to three-loop level: $\mathcal{A}(m_\ell) = \mathcal{S}' \times Z \times Z \times \mathcal{A}(0) + \mathcal{O}(m_\ell)$
  [Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18, Ulrich 23]
- use e.g. for 2-loop $ee \to \gamma^*\gamma$ with $m_e = 0$ [Badger, Kryś, Moodie, Zoia 23]
- theory error: $\mathcal{O}(10^{-2})\,@\,\mathrm{NNLO} \sim \mathcal{O}(10^{-5})$ for $e\mu \to e\mu$ [Bonciani et al. 21]



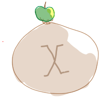
beyond pure QED

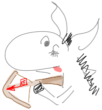


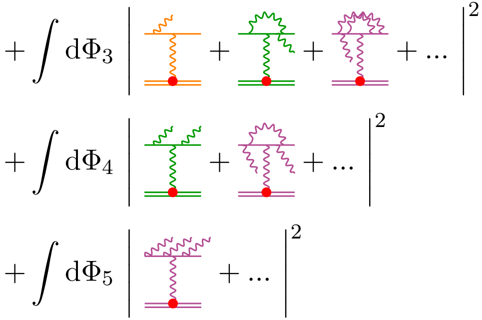
1. FKS$^\ell$
2. next-to-soft stabilisation
3. massification
4. form factors for one-photon exch.
   $X \in \{\pi,\ ^{12}C,\ ^{1}H,\ ^{2}H, \dots\}$



$$= L^{\text{QED}}_{\mu\nu} \left(\sum_h j^\mu_X j^\nu_X \right)$$







1. FKS$^\ell$
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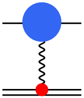
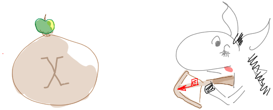
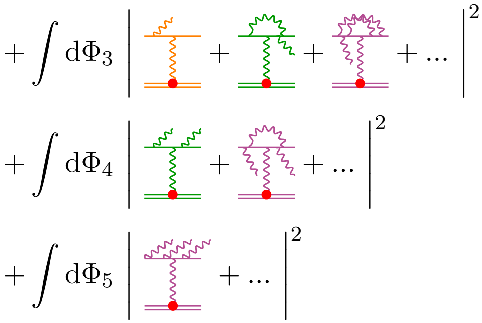
4. form factors for one-photon exch.
   $X \in \{\pi,\ {}^{12}C,\ {}^{1}H,\ {}^{2}H, \ldots\}$

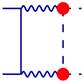


$$= L_{\mu\nu}^{\text{QED}} \left(\sum_h j_X^\mu j_X^\nu \right)$$





1. FKS$^\ell$
2. next-to-soft stabilisation
3. massification
4. form factors for one-photon exch.
5. two-photon exchange (NLO)



[McMule 25]

- FsQED [Colangelo+ 22]
- disperon model in OpenLoops
- threshold subtraction
- customised EFT for large disperons

- $\mu \to \nu\bar{\nu}e\,(\gamma\,/\,ee)$ @ (N)NLO + BSM [1611.03617] [1705.03782] [1712.05633] [2211.01040]
- $*$ $e\nu \to e\nu$ @ NNLO*
- $e^-e^+ \to e^-e^+$ @ NNLO [2106.07469]
- $ee \to ee$ @ NNLO + EW [2107.12311] [2507.17652]
- $e\mu \to e\mu$ @ NNLO [2007.01654] [2212.06481]
- $\ell p \to \ell p$ @ ℓ NNLO + elastic TPE [2007.01654] [2307.16831]
- $*$ $\ell N \to \ell N$ @ ℓ NNLO
- $ee \to \ell\ell(\gamma)$ @ (N)NLO [2210.17172] [2410.22882]
- $ee \to \pi\pi$ @ e NNLO + FsQED @ NLO [2512.10709]
- $ee \to \gamma\gamma$ @ NNLO [2512.22929]
- $*$ $ee \to \mu\mu\gamma$ @ NNLO [see Sara's talk]
- $*$ $ee \to \pi\pi\gamma$ @ e NNLO + FsQED @ NLO [see Sophie and Yizhou's talk]

left to right: M.Rocco (Torino), *Tarzan*, M.Ronchi (Mainz), *Mula*, A.Signer (Zürich & PSI), *Franco*, *Foscolo*, Y.Fang (Zürich & PSI), S.Gündogdu (Zürich & PSI), *Chrigel*, J.Wilson (Liverpool), Y.Ulrich (Liverpool), *Orfeo*, F.Hagelstein (Mainz), S.Kollatzsch (Zürich & PSI); not shown: P.Banerjee (Bhubaneshwar), A.Coutinho (IFIC), V.Sharkovska (Zürich & PSI), D.Radic (Zürich & PSI), G.Billis (PSI), T.Oruç (ETH Zürich)

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`mule-tools.gitlab.io`