BabaYaga@NLO

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Workshop on Radiative Corrections and Monte Carlo Simulations at e^+e^- Colliders

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Main historical references

BabaYaga is on the way to its thirties 😎

- ⇝ C.M.C.C. et al., Nucl. Phys. B **584** (2000) 459 — BabaYaga 3.5
- ⇝ C.M.C.C., Phys. Lett. B **520** (2001) 16 — improved Parton Shower in BabaYaga
- ⇝ C.M.C.C. et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 — BabaYaga for $\mu^+\mu^-$, $\gamma\gamma$, $\pi^+\pi^-$
- ⇝ G. Balossini et al., Nucl. Phys. **B758** (2006) 227 — BabaYaga@NLO for Bhabha
- ⇝ G. Balossini et al., Phys. Lett. **663** (2008) 209 — BabaYaga@NLO for $e^+e^- \to \gamma\gamma$
- ⇝ L. Barzè et al., Eur. Phys. J. C **71** (2011) 1680 — BabaYaga with dark photon

More recent references

- ⇝ C.M.C.C. et al., Phys. Lett. B **798** (2019) 134976 EW corrections at high energies for $e^+e^- \to \gamma\gamma$
- ⇝ M. Chiesa et al., Phys. Rev. D 112 (2025) BabaYaga@NLO for FCC-ee luminosity (with NP contamination)
- ⇝ E. Budassi et al., JHEP 05 (2025) 196 BabaYaga@NLO for $e^+e^- \to \pi^+\pi^-$
- ⇝ E. Budassi et al., JHEP 05 (2026) 221
  BabaYaga@NLO for $e^+e^- \to \pi^+\pi^-\gamma$ and $\to \mu^+\mu^-\gamma$
- ⇝ C.M.C.C. et al., `arXiv:2603.28621 [hep-ph]` (submitted to PLB)
  BabaYaga@NLO for $e^+e^- \to \pi^+\pi^-\gamma$ with GVMD

] Promised for at the 5th Workstop/Thinkstart in Zurich, 2023 🙂

Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\text{ref}}}{\sigma_{\text{theory}}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\text{ref}}}{N_{\text{ref}}} \oplus \frac{\delta \sigma_{\text{theory}}}{\sigma_{\text{theory}}}$$

- Reference (*normalization*) processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ **Large-angle** QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to achieve a typical precision at the level of $1 \div 0.1\%$

$\hookrightarrow$ QED radiative corrections are mandatory

⋆ At LEP and future high-energy e^+e^- machines **small-angle** Bhabha scattering is the golden process

$\rightsquigarrow$ `BabaYaga` was primarily developed to calculate σ_{theory} with high precision, and providing a true Monte Carlo event generator

Summary of QED (photonic) radiative corrections

Inclusion of QED radiative corrections into MC generators is mandatory.

The corrections to the Leading Order cross section can be arranged **1)** order by order in α or **2)** by resumming (exponentiating) infrared and collinear leading-log corrections or **3)** combining the two (matching).

Schematically ($L \equiv \log \frac{s}{m_e^2}$ = collinear log):

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Blue: Leading-Log PS, Leading-Log YFS, SF

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Red: PS matched to NLO, YFS, SF + NLO

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Schematically ($L \equiv \log \frac{s}{m_e^2}$ = collinear log):

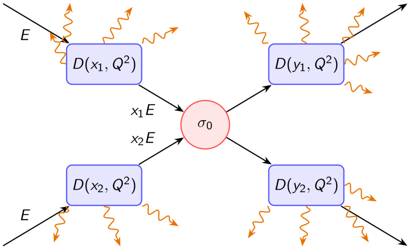
LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

Typically at flavour factories (on integrated Bhabha σ)

possible additional enhancements from infra-red logs induced by events selection

- We include the leading-log *photonic* RC by means of an *in-house* Parton Shower in QED

⇝ PS algorithms rely on QCD-inspired Structure Function approach, exploiting factorization of soft/collinear divergencies (enhancements) in QED, which lead to exponentiation



⇝ Looking only at photon radiation, $D(x, Q^2)$ are the **Leading-Log non-singlet QED SF** which obey the DGLAP equation: it can be **exaclty** and **numerically** solved by a PS algorithm

⇝ In this framework, exclusive events can be generated

- Its theoretical accuracy is limited because it includes only LL RCs.

The error starts at $\mathcal{O}(\alpha)$ ⇝ a consistent **matching to NLO** must be implemented for precise predictions

QED PS in BabaYaga

- We unfold the iterative $D(x, Q^2)$ MC solution, so that the LL corrected cross section can be cast in the form

$$d\sigma_{LL}^{\infty} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \, |\mathcal{M}_{n,LL}|^2 \, d\Phi_n,$$

$$|\mathcal{M}_{1,LL}|^2 = \frac{\alpha}{2\pi} \frac{1+z^2}{1-z} I(k) \, |\mathcal{M}_0|^2 \, \frac{8\pi^2}{E^2 z(1-z)}$$

⇝ The multi-differential phase-space is kept **exact** for the emission of an arbitrary number of photons

⇝ Any approximation is shifted on matrix elements

⇝ $\Pi(Q^2, \varepsilon)$ exponentiates LL virtual and soft emissions **up to all orders in** α

PS: matching in BabaYaga@NLO

- A LL PS-corrected differential cross section can be expanded at $\mathcal{O}(\alpha)$

$$
\begin{aligned}
d\sigma^{\alpha}_{LL} &= \left[1 - \frac{\alpha}{2\pi} I_+ \log\frac{Q^2}{m^2}\right] |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \\
&\equiv [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1,
\end{aligned}
$$

while the NLO cross section can be always cast as

$$
d\sigma^{\alpha} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 .
$$

By defining the factors

$$
F_{SV} = 1 + (C_{\alpha} - C_{\alpha,LL}), \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}
$$

the NLO cross section can be re-written (up to terms of $\mathcal{O}(\alpha^2)$) as

$$
d\sigma^{\alpha} = F_{SV}(1 + C_{\alpha,LL})|\mathcal{M}_0|^2 d\Phi_0 + F_H |\mathcal{M}_{1,LL}|^2 d\Phi_1
$$

which brings to the master formula...

PS: matching in BabaYaga@NLO

$$d\sigma^{\infty}_{matched} = F_{SV}\,\Pi(Q^2,\varepsilon)\,\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{H,i}\right)\,|\mathcal{M}_{n,LL}|^2\,d\Phi_n,$$

- ⇝ it's based on LO and NLO building blocks
- ⇝ F_{SV} and F_H are collinear and infrared safe, no double counting of LL terms
- ⇝ the cross-section is still fully differential (matching applied on an event-by-event basis)
- ⇝ its $\mathcal{O}(\alpha)$ expansion coincides by construction with $d\sigma^{\alpha} = d\sigma^{\text{NLO}}$
- ⇝ resummation of LL higher-orders, beyond NLO, is preserved
- ⇝ it can be expanded at $\mathcal{O}(\alpha^2)$ and compared to exact NNLO calculations

✓ Successfully applied to match QED NLO to PS in BabaYaga@NLO and a bunch of other MCs

↪ The theoretical error is shifted to NNLO $\mathcal{O}(\alpha^2 L)$ terms (typically of $\mathcal{O}(0.1\%)$)

This approach achieves the so-called NLOPS accuracy

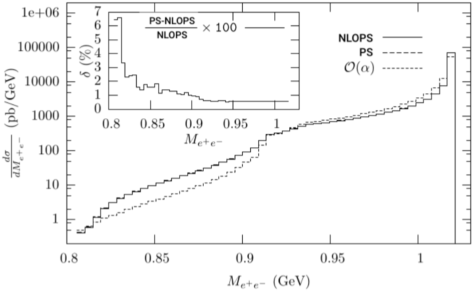
Theoretical precision

The size of radiative corrections has been studied in typical flavour factories setups

ϕ-factories
- **A** $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $20° < \theta_\pm < 160°$, $\xi_{max} = 10°$
- **B** $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $55° < \theta_\pm < 125°$, $\xi_{max} = 10°$

B-factories
- **C** $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $20° < \theta_\pm < 160°$, $\xi_{max} = 10°$
- **D** $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $55° < \theta_\pm < 125°$, $\xi_{max} = 10°$

Correction vs Setup	A	B	C	D
$\delta_\alpha = \frac{\sigma_{\mathrm{NLO}} - \sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}}$	-11.61	-14.72	-16.03	-19.57
$\delta_{\mathrm{HO}} = \frac{\sigma_{\mathrm{NLOPS}} - \sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$	0.39	0.82	0.73	1.44
$\delta_{\mathrm{HO}}^{\mathrm{PS}} = \frac{\sigma_{\mathrm{PS}}^{\infty} - \sigma_{\mathrm{PS}}^{\alpha}}{\sigma_{\mathrm{LO}}}$	0.35	0.74	0.68	1.34
$\delta_\alpha^{\mathrm{non-log}} = \frac{\sigma_{\mathrm{NLO}} - \sigma_{\mathrm{PS}}^{\alpha}}{\sigma_{\mathrm{LO}}}$	-0.34	-0.57	-0.34	-0.56
$\delta_\infty^{\mathrm{non-log}} = \frac{\sigma_{\mathrm{NLOPS}} - \sigma_{\mathrm{PS}}}{\sigma_{\mathrm{LO}}}$	-0.30	-0.49	-0.29	-0.46



$\delta_{\mathrm{HO}}^{\mathrm{PS}} \simeq \delta_{\mathrm{HO}}$

The higher orders are added to the NLO

$\delta_\infty^{\mathrm{non-log}} \simeq \delta_\alpha^{\mathrm{non-log}}$

The missing NLO finite corrections are included

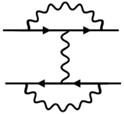
from F. Ucci's talk at MTTD 2025

Theoretical error

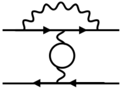
To quantify the theoretical error of BabaYaga, we can compare the exact NNLO versus the PS matched expanded

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{SV} + \sigma^{\alpha^2}_{SV,H} + \sigma^{\alpha^2}_{HH} \quad \text{vs} \quad \text{BabaYaga@NLO } \mathcal{O}(\alpha^2)$$

$\sigma^{\alpha^2}_{SV}$

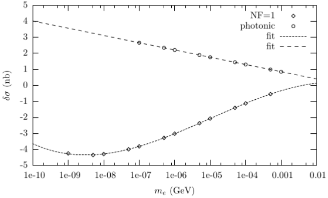


Photonic



Fermionic $N_f = 1$

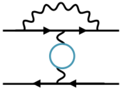
A. Penin, PRL 95 (2005) 010408
Nucl. Phys. B734 (2006) 185



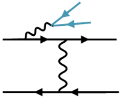
$$\frac{\delta\sigma(\text{Photonic})}{\sigma_{LO}} \propto \alpha^2 L$$

$$\sigma^{\alpha^2}_{SV}(\text{Penin}) - \sigma^{\alpha^2}_{SV}(\text{BY@NLO}) < 0.02\,\%\,\sigma_{LO}$$

$\sigma^{\alpha^2}_{HH}$



Photonic $f \neq e$



Real pairs

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

$$\delta\sigma_{\text{pairs}} \sim 10^{-4}$$

Carloni, Czyż, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann et al., JHEP 1107 (2011) 126

from F. Ucci's talk at MTTD 2025

What's new in BabaYaga@NLO v2.x.y

Besides new processes on the physics side (see Marco's talks later on), new **under-the-hood** features include:

- ✓ Definitely more stable and more general version of

$$\left(\prod_{i=1}^{n} F_{H,i}\right) |\mathcal{M}_{n,LL}|^2$$

- ✓ Largely revised and optimized phase space generation and integration:
  - ⇝ now it samples resonances (also narrow resonances) and Coulomb enhancement at low $\hat{s}$, also in presence of multiple photon emission;
  - ⇝ better phase space coverage when running with "loose" cuts, improving also unweighted events generation.
- ✓ Now we have a github repository (finally!)

github.com/cm-cc/BabaYagaNLO

Possible future developments

- Looking at FCC-ee physics case: luminometry with Bhabha & $e^+e^- \to \gamma\gamma$, including EWK corrections, studying NP sensitivity/contamination within a SMEFT approach

  e.g. Chiesa *et al.*, Phys. Rev. D 112 (2025) 1, 1

- at flavour factories:
  - revise elastic Bhabha and include $e^+e^- \to e^+e^-\gamma$ at NLOPS accuracy
  - study background processes due to the emission of an extra lepton pair, for all the already implemented processes. Formally these are NNLO corrections, and usually strongly depend on event selection criteria.

- Introducing polarized fermions...

- Long-desired matching at NNLO to reach NNLOPS accuracy