Axion wormholes and the AdS/CFT factorization problem

Zhencheng Wang
(University of Illinois Urbana-Champaign)

2601.02507, 2601.09932 with Held, Kaplan, Marolf

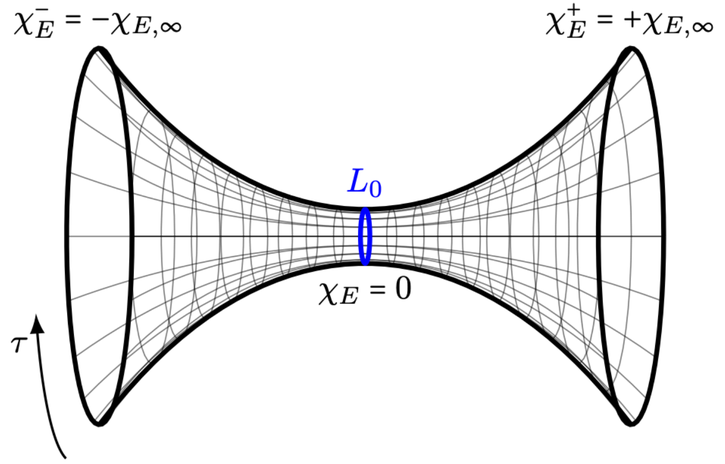
- **Spacetime wormholes** that connect different asymptotically AdS regions are important, but they may lead to **the factorization puzzle** in AdS/CFT:

$$Z(M_1 \sqcup M_2) \neq Z(M_1)Z(M_2)$$

- One simple example is the axion wormholes in AdS: they are wormhole solutions supported by the flux $q \sim \partial_r \chi$ of a massless scalar field $\chi \in i\mathbb{R}$ (the 'axion').

- **Question:** Do the axion wormholes dominate the gravitational path integral computation of the two-boundary partition function as saddle points?

- We analyzed their relevance in the **Lorentzian gravitational path integral**, avoiding the conformal factor problem in Euclidean signature.

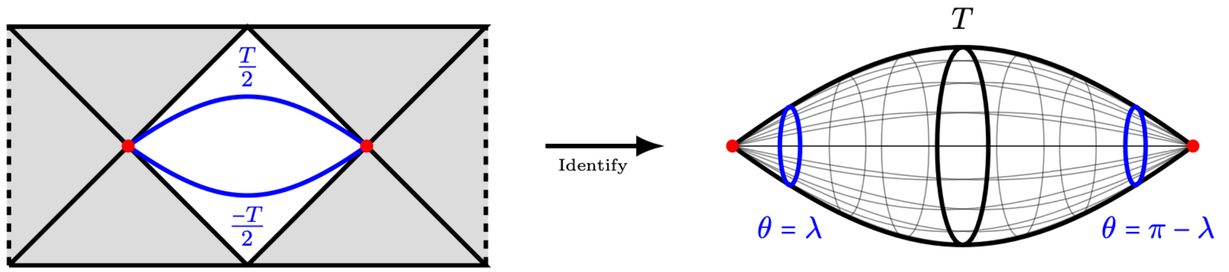


- **Main idea:** Following [Marolf '22], for Hamiltonian $H > E_0$, write the CFT partition function as

$$Z(\beta) = \mathrm{Tr}(e^{-\beta H}) = \int dT\, f_\beta(T)\mathrm{Tr}(e^{-iTH}) \qquad f_\beta(T) = \frac{e^{iTE_0 - \beta E_0}}{2\pi i(T + i\beta)}$$

  For each T, $\mathrm{Tr}(e^{-iHT}) = \int \mathscr{D}g\mathscr{D}\phi e^{iS[g,\phi]}$ in the bulk with $t \sim t + T$ as boundary conditions.

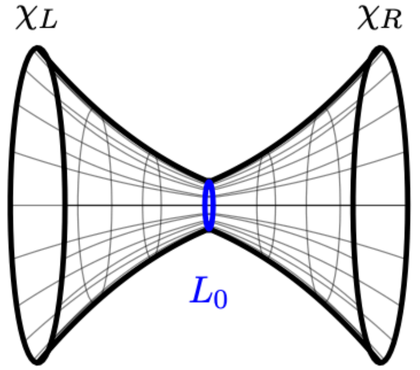
- **On the boundaries:** S^d in Euclidean $\rightarrow$ dS_d static patch in Lorentzian. $H = H_{\text{static patch}}$



- Assuming all other variables are integrated out, except the size of the wormhole throat L_0:

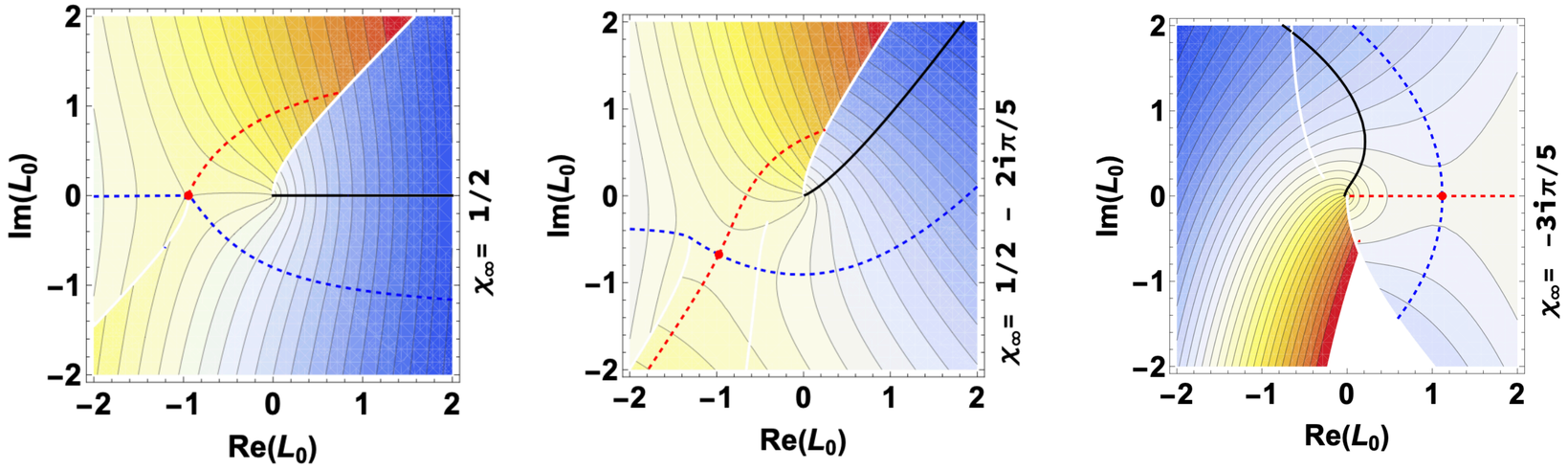
$$Z(\beta) = \int dT \int dL_0\; f_\beta(T) e^{iS_{\text{constrained WH}}[T, L_0]}$$

- For each L_0, evaluate the action of the constrained wormhole solution:
- Integrate over T first, then analyze the L_0 integral.



We define the contour to be $L_0 \in [0, +\infty)$ when χ_∞ is real, and analytically continue χ_∞.

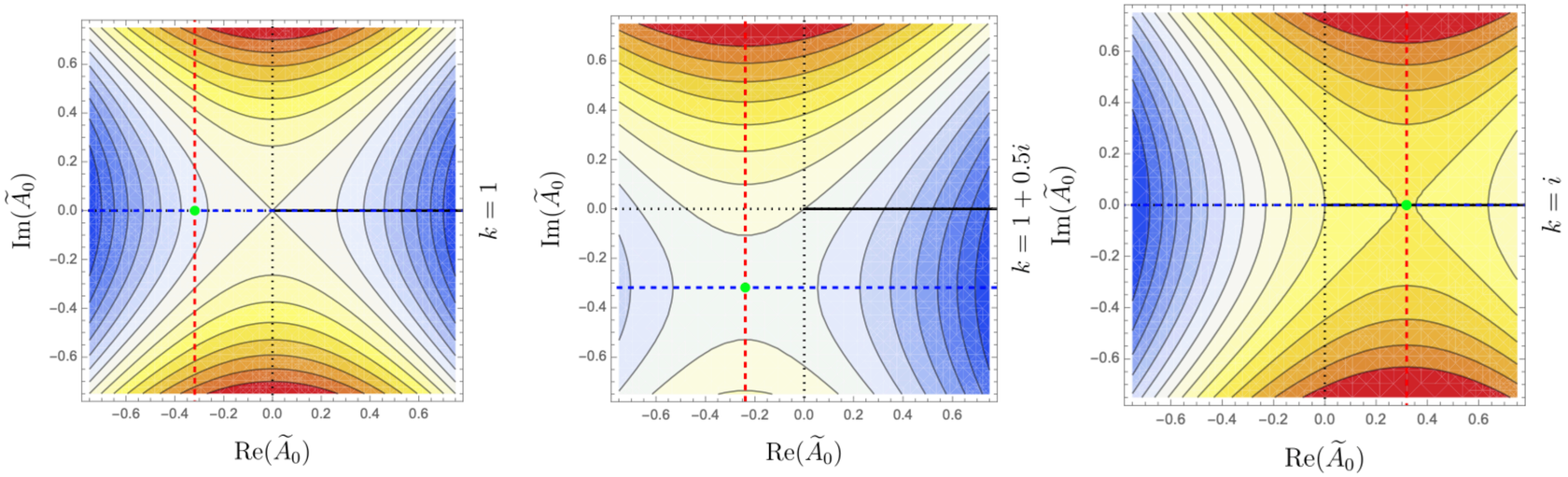
Case 1: (2+1)d Einstein gravity + axion:



The wormhole saddle (red dot) at imaginary χ is not a relevant saddle! The integral is dominated by the endpoint.

Blue: steepest descent contour
Red: steepest ascent contour.
Black: original integration contour.

Case 2: JT gravity + "axion" [Garcia-Garcia, Godet '20]



The wormhole solution (green dot) at imaginary χ dominates the path integral!