Scalar fields in the presence of an evaporating black hole

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Motivation: Black holes may behave as particles

Black holes behave similar to elementary particles

Study massless scalar fields near horizon of evaporating black hole. How does entanglement structure change due to evaporation?

Use framework of **algebraic QFT and Tomita-Takesaki theory**[1] relating conjugants/causal complements $\mathcal{A}_1, \mathcal{A}_2$:

$$\text{flow: } \Delta^{it}\mathcal{A}_1\Delta^{-it} = \mathcal{A}_1, \quad \text{conjugation: } J\mathcal{A}_1 J = \mathcal{A}_2. \quad (0.1)$$

[1] M. Takesaki, *Theory of operator algebras. II*, Vol. 125, Encyclopaedia of Mathematical Sciences, Operator Algebras and Non-commutative Geometry, 6 (Springer-Verlag, Berlin, 2003).

Basic case: Rindler space

Consider **complex scalar fields in Rindler space** with the conjugant spaces left (L) and right (R) wedges.

$$\phi(x) = \sum_{\lambda} \int_0^{\infty} d\omega \int d^2k \left[c_k^{\lambda} u_k^{\lambda}(x) + d_k^{\lambda\dagger} u_k^{\lambda\dagger}(x) \right], \qquad (0.2)$$

where $\lambda = R, L$ and u_k^{λ} are the solutions of the Klein-Gordon equation. Using modular theory, we obtain the Bogoliubov transformations to the Minkowski particle (antiparticle) operators $G_k^{(1)}$ ($G_k^{(2)}$):

$$\begin{cases} G_k^{(1)} & \leftrightarrow e^{\pi\omega} c_k^R - d_{\tilde{k}}^{L\dagger} \\ G_k^{(2)} & \leftrightarrow e^{-\pi\omega} d_k^{R\dagger} - c_{\tilde{k}}^{L} \end{cases}. \qquad (0.3)$$

Near-horizon linear Vaidya is conformal Rindler

Vaidya metric[2] with quasi-stationary, linearly-evaporating black hole:

$$ds^2 = -\left(1 - \frac{r_0 - \alpha v}{r}\right) dv^2 + 2drdv + r^2 d^2\Omega. \tag{0.4}$$

Coordinate redefinitions, transformations modified from[3] with new coordinates (T, Z, X^2, X^3) to get

$$ds^2 = \Omega(-Z^2(dT)^2 + dZ^2 + (dX^2)^2 + (dX^3)^2), \tag{0.5}$$

$$\Omega = \exp\left[-2\alpha(T + \ln|Z| + \mathcal{O}(\alpha^2)\right]. \tag{0.6}$$

Conformal to Rindler → same causal structure → use AQFT

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[2] P. C. Vaidya, "Newtonian Time in General Relativity", Nature **171**, 260–261 (1953).

[3] P. K. Dahal and S. Maharana, "The Hawking temperature of dynamical black holes via conformal transformations", Eur. Phys. J. C **84**, 783 (2024).

Evaporation induces a phase

Bogoliubov coefficients now

$$\begin{cases} G_k^{(1)} & \leftrightarrow e^{\pi\omega}\, e^{-2i\pi\alpha}\, c_k^R - d_{\tilde{k}}^{L\dagger} \\ G_k^{(2)} & \leftrightarrow e^{-\pi\omega}\, e^{-2i\pi\alpha}\, d_k^{R\dagger} - c_{\tilde{k}}^{L} \end{cases} . \tag{0.7}$$

Also apparent in thermofield double form of Minkowski vacuum:

$$|0\rangle_M \propto \exp\left(\int d\omega \sum_{l,m} e^{-\pi\omega} (e^{2i\alpha\pi}\, c_{l,\omega}^{L\dagger} d_{l,\omega}^{R\dagger} + e^{-2i\alpha\pi}\, d_{l,\omega}^{L\dagger} c_{l,\omega}^{R\dagger}) \right) \times |0\rangle_{L,+} |0\rangle_{R,+} |0\rangle_{L,-} |0\rangle_{R,-} . \tag{0.8}$$

Interpretation of the phase factor

Shapiro time advance due to modular time evolution in the bulk

Appearance of phase factors in the thermofield double state can be considered **wormhole** effect: state picking up a phase through traversing the wormhole