A melonic quantum mechanical model without disorder

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I will describe a quantum mechanical model of interacting fermions *without disorder* that has a melonic approximation when the number of fermions is large.

It has the same low-energy physics as the $\mathcal{N} = 2$ supersymmetric SYK model.[1]

In this sense, it is similar in spirit to previous tensor models of SYK such as the Gurau-Witten model and $O(N)^3$ model.[2]

This model involves $N = 2j + 1$ complex fermions transforming in a spin j representation of $SU(2)$. The supercharge is built from the Wiger 3j symbol:

$$Q \sim (3j \text{ symbol})\psi^3, \qquad Q^\dagger \sim (3j \text{ symbol})(\psi^\dagger)^3 \ , \qquad H \equiv \{Q, Q^\dagger\}$$

Models based on $SU(2)$ 3j symbols were previously discussed by Amit and Roginsky[3], and more recently by Bendetti and Delporte[4], which inspired this work.

[1] Fu et al. 2017.

[2] Witten 2019; Klebanov and Tarnopolsky 2017.

[3] Amit and Roginsky 1979.

[4] Benedetti and Delporte 2021.

Motivations

- Novel mechanism for melonic expansion
- Compared to previous tensor models, this one is more amenable to exact numerical diagonalization
- An interesting toy model for studying supersymmetric black hole microstates and BPS chaos[5]
- Bears resemblance to field theories reduced on spheres (ex. ABJM in the presence of a magnetic field[6])
- In a sector of large $SU(2)$ charge, the model simplifies and becomes equivalent to a 2d CFT

[5] Chang and Y.-H. Lin 2024; S. Choi et al. 2023; J. Choi et al. 2024; Chang, Chen, et al. 2025; Chen, H. W. Lin, and Shenker 2025; Chen 2025.
[6] Benini, Hristov, and Zaffaroni 2016.

Outline

1. Definition and basic properties
2. Melonic approximation
   $\rightarrow$ Nearly conformal IR described by $\mathcal{N} = 2$ Schwarzian action
3. Regime of large $SU(2)$ charge and CFT_2

Definition and Basic Properties

$N = 2j + 1$ complex fermions in a single spin j representation of $SU(2)$

$$\{\psi_m, (\psi_{m'})^\dagger\} = \delta_{m,m'}$$

$$Q \propto \sqrt{JN} \sum_{-j \le m_1, m_2, m_3 \le j} \begin{pmatrix} j & j & j \\ m_1 & m_2 & m_3 \end{pmatrix} \psi_{m_1}\psi_{m_2}\psi_{m_3} \ , \qquad H \ = \{Q, Q^\dagger\}$$

$$\begin{pmatrix} j & j & j \\ m_1 & m_2 & m_3 \end{pmatrix} \equiv \frac{(-1)^{-m_3}}{\sqrt{2j+1}} \langle j m_1 \ j m_2 | j \ (-m_3) \rangle$$

We take j to be an odd integer. For odd j, the 3j symbol is antisymmetric in the indices.

Definition and Basic Properties

The normal-ordered Hamiltonian takes the form

$$H = \frac{J}{3}\Big[(2j+1) - 3(N_\psi + j + \tfrac{1}{2}) + 3\sum_m O^\dagger_{j,m} O_{j,m}\Big]$$

where

$$O_{j,m} = \sqrt{2(2j+1)} \sum_{m_1<m_2} \begin{pmatrix} j & j & j \\ m_1 & m_2 & -m \end{pmatrix} \psi_{m_1}\psi_{m_2}$$

We introduce complex bosonic fields, $b_n \equiv \{Q^\dagger, \psi_n\}$ and $b_n^\dagger \equiv \{Q, \psi_n^\dagger\}$ to linearize the action of the supersymmetry transformations.

In terms of these bosons, the Lagrangian is

$$L = \sum_{-j\le m_1\le j}\left[i\psi^\dagger_{m_1}\partial_t\psi_{m_1} - b^\dagger_{m_1} b_{m_1} + \sqrt{\frac{JN}{2}} \sum_{-j\le m_2,m_3\le j} \begin{pmatrix} j & j & j \\ m_1 & m_2 & m_3 \end{pmatrix} b_{m_1}\psi_{m_2}\psi_{m_3} + c.c.\right]$$

$U(1)_R$ and $SU(2)$ currents

We define the fermion number operator N_ψ as

$$N_\psi = \sum_{m=-j}^{j} \psi_m^\dagger \psi_m - \frac{2j+1}{2}$$

which is related to the $\mathrm{U}(1)_R$ charge by

$$R = \frac{N_\psi}{3}$$

The SU(2) currents are

$$J_3 = \sum_m m\psi_m^\dagger \psi_m \ ,$$

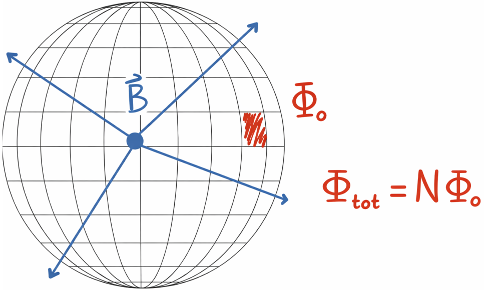
$$J_\pm = \sum_m \sqrt{(j \mp m)(j \pm m + 1)}\psi_{m\pm1}^\dagger \psi_m$$

Another perspective: lowest Landau level on S^2 with magnetic flux N

It can be convenient to view the fermions ψ_m as the modes of a 2+1 dimensional fermion moving on $S^2\times$(time), where the S^2 encloses a magnetic monopole with $N = 2j + 1$ units of magnetic flux.

The lowest energy sector of this system (the lowest Landau level) is N-fold degenerate, and transforms in a spin j representation of $SU(2)$.

This is just another way to view the Hilbert space, but it will provide useful intuition.

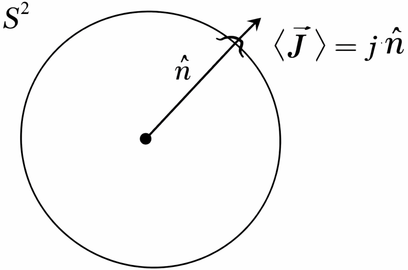


Another perspective: lowest Landau level on S^2 with magnetic flux N

For large j, we can think of the wavefunctions as coherent states localized within one flux quantum on the sphere.

In other words, a spin j irrep of $SU(2)$ can be treated as a classical spin in the $j \to \infty$ limit.

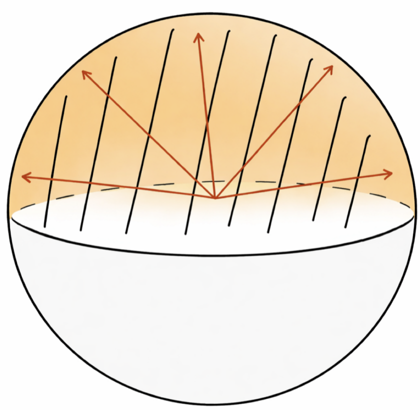
For example, such a wavefunction sitting at some direction $\hat{n}$ on the sphere would have angular momentum $\langle \vec{J} \rangle = j\hat{n}$.



Maximal J_3 states

A set of states which will be important are those with the largest J_3.

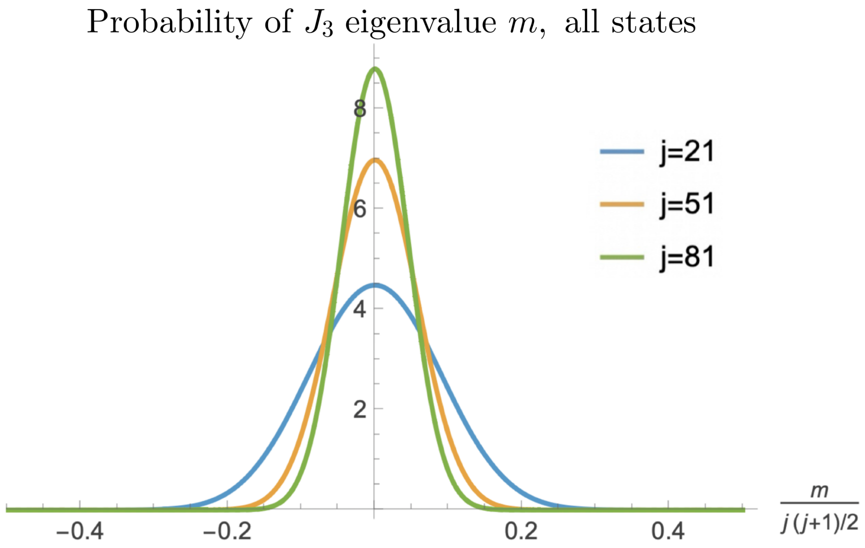
These correspond to adding fermions with angular momenta all pointing up as much as possible, i.e. filling the northern hemisphere. There are 2 such states.



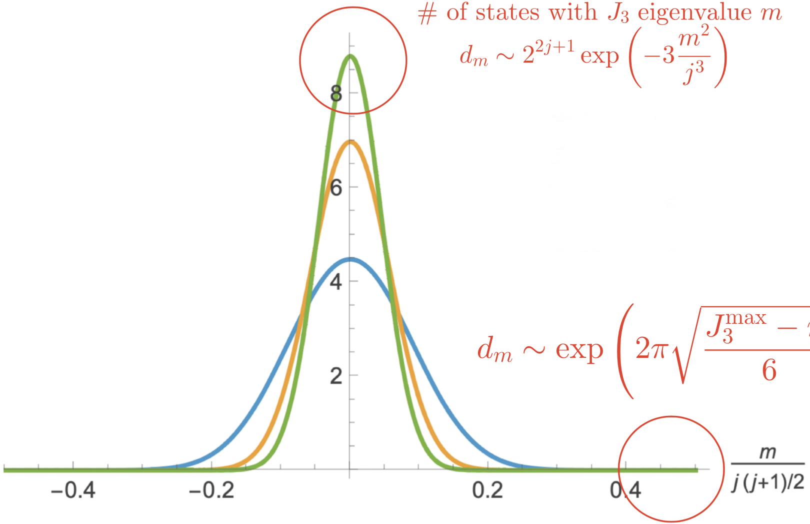
$$2 \text{ states}: \prod_{m=0}^{j} \psi_m^\dagger |0\rangle, \quad \prod_{m=1}^{j} \psi_m^\dagger |0\rangle$$

$$J_3^{\max} = \sum_{m=1}^{j} m = \frac{j(j+1)}{2}$$

Hilbert space decomposition by spin (J_3 eigenvalue)



Hilbert space decomposition by J_3 eigenvalue



# of states with J_3 eigenvalue m

$$d_m \sim 2^{2j+1} \exp\left(-3\frac{m^2}{j^3}\right)$$

$$d_m \sim \exp\left(2\pi\sqrt{\frac{J_3^{\max} - m}{6}}\right)$$

Counts of zero-energy ground states (BPS states) by J_3 eigenvalue

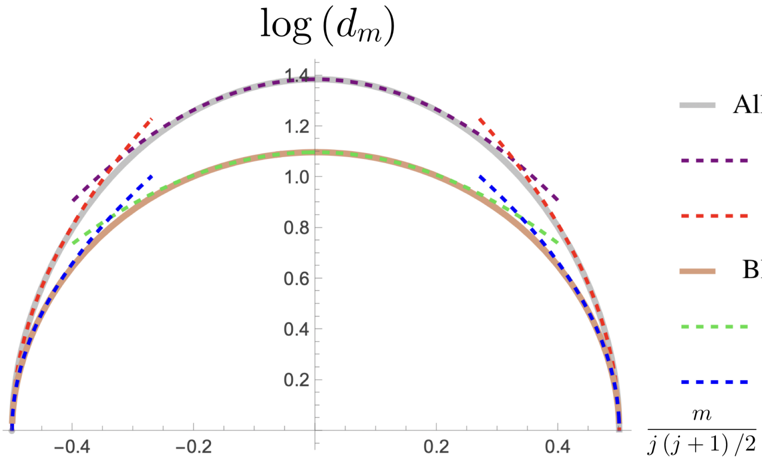
Like $\mathcal{N} = 2$ SYK, this model has a large degeneracy of supersymmetric ground states, or BPS states. We can use Witten index arguments to count the number of BPS states in each J_3 sector.

In total, we find 2×3^j BPS states.

For large j, the degeneracy d_m^{BPS} of BPS states with J_3 eigenvalue m is

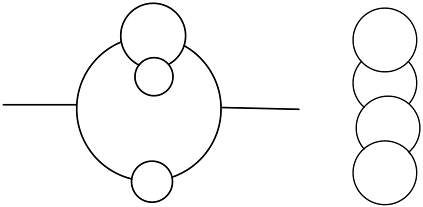
$$d_m^{BPS} \sim \begin{cases} 2 \times 3^j \exp\left(-\frac{9}{4}\frac{m^2}{j^3}\right) & |m| \ll J_3^{max} \\ \exp\left(2\pi\sqrt{\frac{J_3^{max} - |m|}{9}}\right) & 1 \ll J_3^{max} - |m| \ll J_3^{max} \end{cases}$$

State counts by spin



Melonic Expansion – Background

In some field theories and quantum mechanical models, a sub-class of Feynman diagrams, called "melon" diagrams, have the dominant scaling in N.



This is desirable because it means that one can sum all the Feynman diagrams in the large N limit and thereby compute correlation functions at strong coupling.

SYK is one example of a model with a melonic expansion, and this involves doing a "disorder average" over the couplings.

In this model, which has fixed couplings, the same melon diagrams dominate in the large N limit, but for a different reason than in SYK.

Melonic Expansion – previous work

In the past[7], bosonic field theories with cubic interactions given by the SU(2) 3j symbols were studied.

There, the authors argued that melonic diagrams dominate by using facts about the large j scaling of $SU(2)$ invariants (6j, 9j, 12j symbols etc.)

The general asymptotics of such invariants is an open problem. So, they did not prove melonic dominance, but checked it up to a large number of loops.

Here we will give an alternative, hopefully more intuitive argument for melonic dominance.

[7] Amit and Roginsky 1979; Benedetti and Delporte 2021.

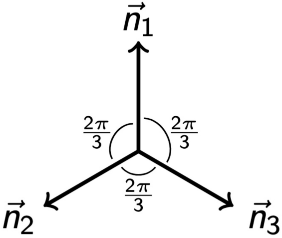
A simple explanation for melonic dominance

At large j, the 3j symbol enforces classical angular momentum conservation.

In other words, at large j, the angular momenta of the fermions, or the vectors $\vec{n}_i$, should sum up to zero:

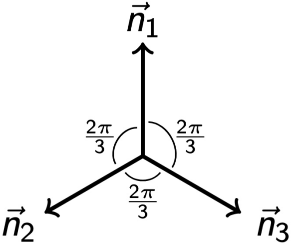
$$\vec{n}_1 + \vec{n}_2 + \vec{n}_3 \sim 0$$

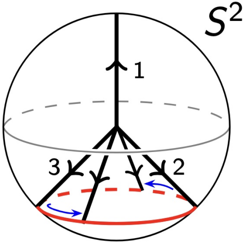
This equation should hold at any vertex.

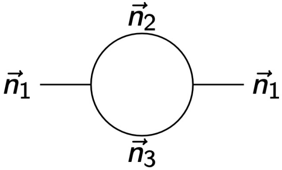


Because each vector is on the unit sphere, $\vec{n}_i^2 = 1$, this means they have a relative angle of $2\pi/3$. They can still be rotated inside the sphere.

A simple explanation for melonic dominance







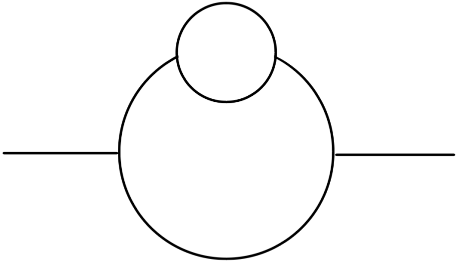
Fixing $\vec{n}_1$, we have a degeneracy of configurations for $\vec{n}_2$ and $\vec{n}_3$.

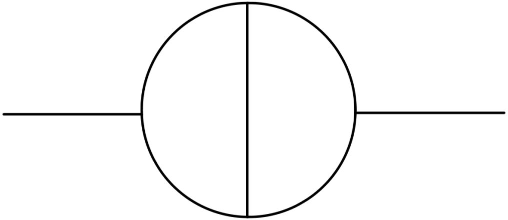
So there is an enhancement for this bubble proportional to the total number of states swept out by this rotation.

The sphere area scales as N, so the circle contributes a factor of $\sqrt{N}$.

An example

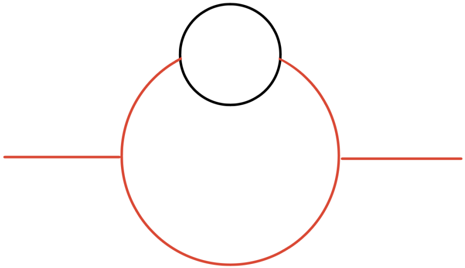
Let's consider the following two corrections to the propagator.

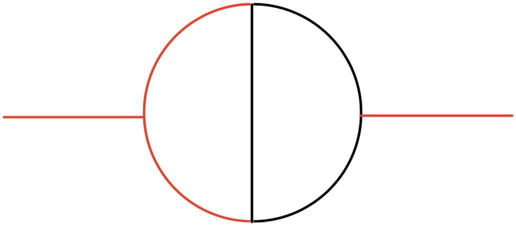




An example

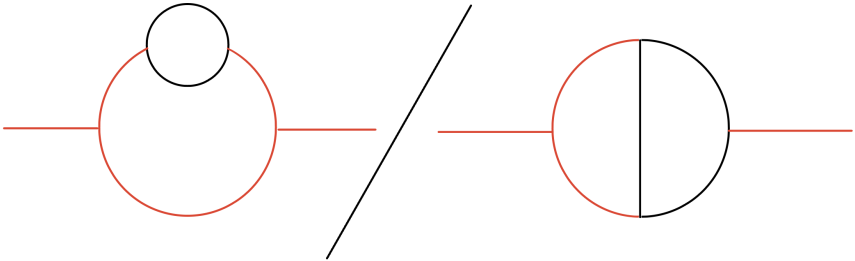
We can use the overall rotational invariance of the S^2 to fix the first three arrows.
(red $=$ fixed)





An example

The non-melon is suppressed relative to the melon by



$$\sim \sqrt{N}$$

General vacuum diagrams

Let us now consider general vacuum diagrams.

Each line corresponds to a point on S^2, so contributes 2 free parameters.

Each vertex imposes 3 constraint equations.

(But, angular momentum conservation will be automatically obeyed at the last vertex it if is obeyed in all the previous ones.)

The number of lines L and vertices V are related by $2L = 3V$.

$$N_{\text{free}} = 2L - 3(V - 1) = 3$$

These 3 degrees of freedom are accounted for by the possibility of performing an overall rotation of all the arrows.

Melonic diagrams are an exception, because some of the vertex constraints are redundant. In particular, we get one free parameter per pair of vertices.

Schwarzian regime ($\beta J \gg 1, \quad R/N \ll 1, \quad J/N \ll 1$)

The Feynman diagrams of this model, and therefore the Schwinger-Dyson equations, are the same as those in $\mathcal{N} = 2$ SYK.[8]

This means that in the IR, this model develops an approximate conformal symmetry. The low-energy effective action is the (super-) Schwarzian,

$$S \propto -\frac{N}{J} \int dt \{f(t), t\} + \text{susy-partners}$$

which gives the leading quantum corrections away from the conformal limit.

Like SYK, this model has a holographic interpretation as describing the near-horizon excitations of certain near-extremal black holes in supergravity.

[8] This is when we have no chemical potential for the $SU(2)$ or $U(1)_R$ symmetries.

Schwarzian regime – $SU(2)$ action

A new feature of this model compared to $\mathcal{N} = 2$ SYK is the global $SU(2)$ symmetry. We also expect some low-action degrees of freedom from this symmetry. Ordinarily we would expect an action like

$$\frac{N}{\mathrm{J}} \int dt Tr[(g^{-1}\dot{g})^2] \qquad \text{(wrong)}$$

where g is an $SU(2)$ group element.

However, this does not appear to be the correct answer in our case because it would raise the energy of modes with non-zero angular momentum.

One possibility is that the action instead contains a first order term of the form

$$\int dt Tr[g^{-1}\dot{g}\rho] \qquad (?)$$

where ρ is a new field in a triplet of $SU(2)$. On shell, ρ becomes constant and equal to the conserved charge $\vec{J}$.

Large $SU(2)$ charge and emergent CFT_2

Now we will discuss a different regime, considering states with nearly maximal $SU(2)$ charge. In this limit, the model simplifies and becomes closely related to a 2d CFT.

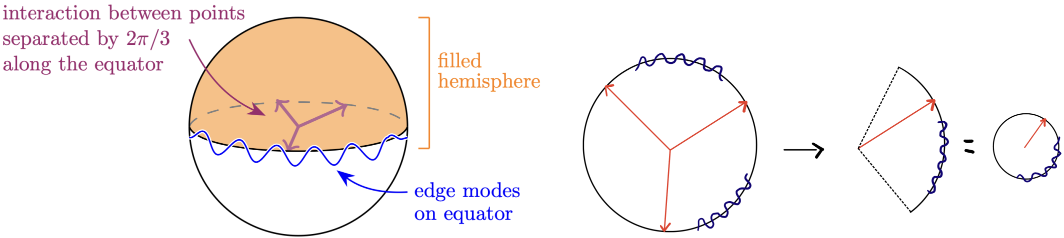
Large $SU(2)$ charge and emergent CFT_2

Consider excitations around the states with maximal J_3.

A quantum hall system like this one has gapless chiral edge modes.

So, for small excitations around these states, the Hilbert space looks like that of a $1+1$d chiral CFT.

When we introduce a local quartic interaction between these modes, we still expect some kind of chiral CFT. We can view our interaction as local by re-scaling the size of the circle.



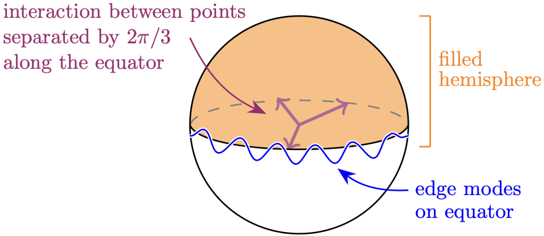
In this regime, the melonic approximation is not valid.

Large $SU(2)$ charge and emergent CFT_2

In this regime, using the asymptotic expression of the 3j symbol in the limit $j \gg 1, \quad |m_i| \ll j$, one can show that the supercharge has the form

$$Q \propto \int d\varphi\ \psi(\varphi)\psi\left(\varphi + \frac{2\pi}{3}\right)\psi\left(\varphi - \frac{2\pi}{3}\right)$$

Here ψ is a single fermion in $1+1$d which lives on the equator. (The ψ_n correspond to its momentum modes.) φ is the coordinate on the equator.



Large $SU(2)$ charge and emergent CFT_2

We could describe the CFT_2 in terms of ψ, but it turns out to be simpler if we bosonize the fermion, so $\psi =: e^{i\phi} :$, and then

$$Q \propto \int d\varphi \ \exp\left\{ i \left[\phi(\varphi) + \phi\left(\varphi + \frac{2\pi}{3}\right) + \phi\left(\varphi - \frac{2\pi}{3}\right)\right]\right\}$$

This is simpler because Q only depends on the momentum modes of ϕ which are integer multiples of 3.

$$\phi(\varphi) + \phi\left(\varphi + \frac{2\pi}{3}\right) + \phi\left(\varphi - \frac{2\pi}{3}\right) \propto \sum_k \alpha_{3k} e^{ik(3\varphi)} \equiv \phi_z(\tilde{\varphi})$$

We can think of this as a single boson, which we call ϕ_z, defined on a smaller circle $\tilde{\varphi} = 3\varphi$.

Large $SU(2)$ charge and emergent CFT_2

If we now compute $H = \{Q, Q^\dagger\}$ in terms of ϕ_z, we find

$$H \propto \int d\tilde{\varphi}\ (\partial\phi_z)^2$$

This is expected. The supercurrent $G^\pm(z) \sim : e^{\pm i\sqrt{3}\phi} :$ is known[9] to furnish a representation of the $\mathcal{N} = 2$ superconformal algebra, with stress tensor $T(z) \propto (\partial\phi)^2$ and $U(1)$ current $J(z) \propto \partial\phi$.

So, in this regime, the Hamiltonian of our model is just that of the free boson ϕ_z.

[9] Ginsparg 1988.

Large $SU(2)$ charge and emergent CFT_2

We also have the other momentum modes of the original boson ϕ, those which were ± 1 mod 3.

Let us write

$$\phi = \phi_+ + \phi_- + \phi_z \quad \text{where} \quad \phi_\pm(\varphi) = \sum_k \alpha_{3k\pm 1} e^{i(3k\pm 1)\varphi}$$

When we move to the smaller circle $\tilde{\varphi} = 3\varphi$, ϕ_z will be periodic, $\phi_z(\tilde{\varphi} + 2\pi) = \phi_z(\tilde{\varphi})$, while $\phi_\pm$ have twisted boundary conditions:

$$\phi_\pm(\tilde{\varphi} + 2\pi) = e^{\pm i \frac{2\pi}{3}} \phi_\pm(\tilde{\varphi})$$

$\phi_\pm$ do not contribute to the physical energy of our model, which only depends on ϕ_z. However, they do contribute to J_3.

Large $SU(2)$ charge and emergent CFT_2

Writing J_3 in terms of the bosonic fields,

$$J_3^{max} - J_3 \propto \int d\tilde{\varphi}\left((\partial\phi_z)^2 + (\partial\phi_+)^2 + (\partial\phi_-)^2\right)$$

(To see this, recall that the expression for J_3 was $J_3 = \sum_m m\psi_m^\dagger\psi_m = \int d\varphi\ \psi^\dagger\partial\psi$).

So excitations of the bosons $\phi_\pm$ change J_3 but not the energy.

The Hilbert space of the theory is

$$\mathcal{H} = \mathcal{H}^z \otimes \mathcal{H}^\pm$$

where $\mathcal{H}_z$ describes ϕ_z and $\mathcal{H}_\pm$ describes $\phi_\pm$. We can view it as two decoupled chiral CFTs.

Large $SU(2)$ charge and emergent CFT_2

This gives a very simple description of the entire spectrum, and in particular the BPS states, in this limit.

The two states with $J_3 = J_3^{max}$ are always BPS, and correspond to the CFT vacuum.

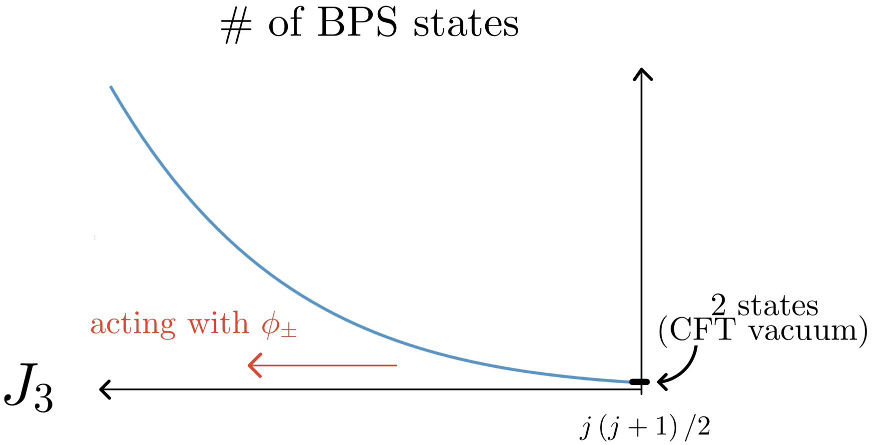
We get all other BPS states by starting from those two states and adding the oscillator modes of the $\phi_\pm$ bosons, while staying in the zero energy state of ϕ_z.

This explains why the counts of states by J_3 eigenvalue we discussed at the beginning, d_m and d_m^{BPS}, have the form of a Cardy density near J_3^{max}.

The partition function which counts the BPS states is

$$Z_{BPS} = 2\prod_{k=0}^{\infty} \frac{1}{(1-q^{1+3k})(1-q^{2+3k})}$$

Large $SU(2)$ charge and emergent CFT_2



Comparing Schwarzian and CFT_2

The CFT_2 regime and Schwarzian regime are valid for very different values of the angular momentum. However, they have some features in common.

1. The CFT_2 predicts a lowest energy for a given R charge:

$$E_{\text{min}}^{\text{CFT}} \propto \frac{\text{J}}{N}\left(R^2 - \frac{1}{6^2}\right)$$

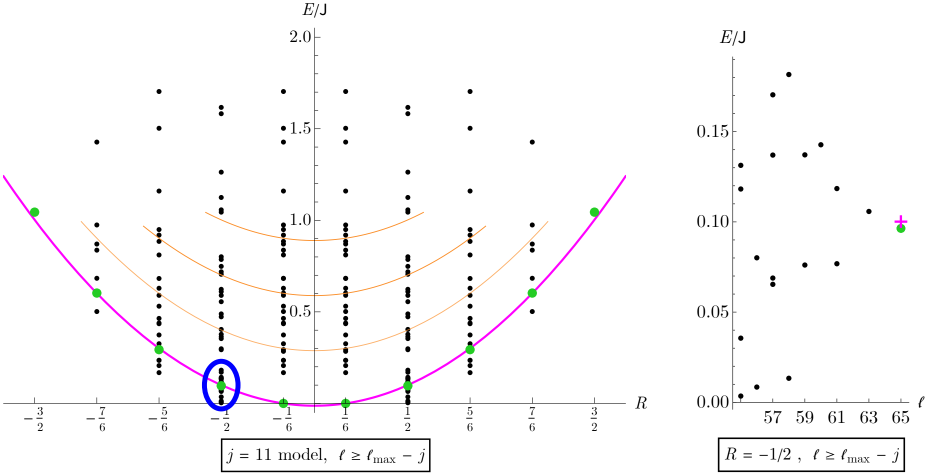
   Separately, the Schwarzian predicts a gap between the BPS and non-BPS states:

$$E_{\text{gap}} \propto \frac{\text{J}}{N}\left(|R| - \frac{1}{2}\right)^2$$

2. Taking the Cardy density of BPS states and extrapolating to the regime where $J_3 \sim 0$, we get $\log(d_m^{\text{BPS}}) \propto j$ which is the correct j dependence. Similarly, the entropy of non-BPS states scales as $S \sim \sqrt{jE}$

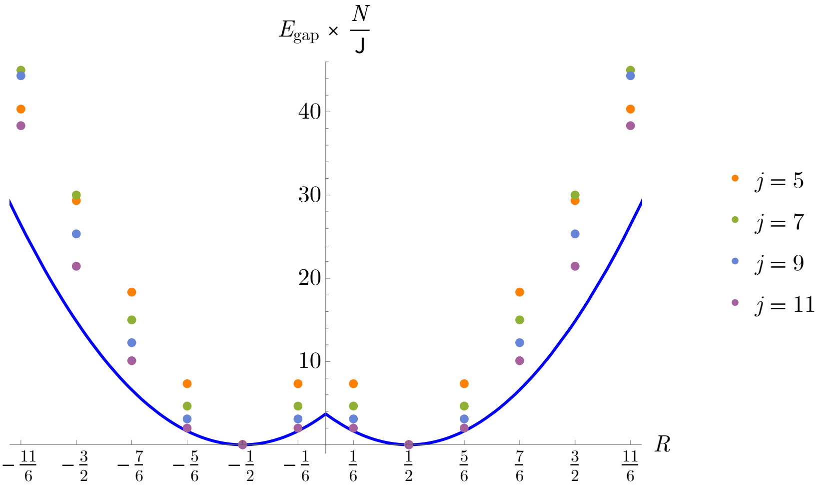
An important difference: The non-BPS states are highly degenerate in the CFT_2 approximation. In the Schwarzian description near $J_3 \sim 0$, they form a continuum.

Comparing Schwarzian and CFT_2



Pink: CFT_2 prediction $E^{\text{CFT}}_{\text{min}}(R)$. Orange: CFT_2 prediction for higher-energy levels. Green: state with largest J_3 for a given R charge. Right: as we decrease the spin, the CFT_2 degeneracy is increasingly broken.

Convergence to Schwarzian prediction for E_{gap}



Blue: Schwarzian prediction for E_{gap}. The numerical points are approaching the Schwarzian prediction as j increases, and the agreement is better for small $|R|$.

Other things we did not have time to discuss

- There are operators in nontrivial $SU(2)$ representations which were not present in SYK, and one can calculate their anomalous dimensions.
- Here we discussed the regime of extreme spin. The model also simplifies at other extreme values of the quantum numbers.
  - Extreme values of the R charge. This is also the high-energy regime. (There are a small number of states in this regime)
  - States with the largest spin for a given R charge.

Possible generalizations

One simple generalization would be a model with different types of fermions, each with a different angular momentum.

$$Q \sim \sum_{m_i} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \psi^1_{m_1} \psi^2_{m_2} \psi^3_{m3}$$

where j_1, j_2, j_3 are all large.

We could also consider a non-supersymmetric model of the form

$$H = \mathrm{J} \sum_m O^\dagger_{j_3 m} O_{j_3 m} \ , \qquad O_{j_3 m} = \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m \end{pmatrix} \psi_{j_1 m_1} \psi_{j_2 m_2}$$

Another simple generalization would be to change the group from $SU(2)$ to $SU(N)$, and consider representations with large quantum numbers or large Dynkin labels.

We also expect these cases to give rise to melon diagrams.

Other future directions

BPS black hole microstates?

We found an explicit description of the BPS states in a solvable corner of the theory. We do not have a description for most of the BPS states, which occur when $J_3 \sim 0$, where the Schwarzian approximation is valid. Given the simple structure of the theory, one might expect that this is possible (but difficult, if they are indeed "chaotic.")

Applications of this melonic expansion?

This model exhibits a new mechanism for melonic dominance. Could these ideas be used to solve any other theories we care about? (Problem: the interaction is non-local on S^2...)

Thank you for your attention.

BPS state partition function: finite j v.s. infinite j

The partition function which counts BPS states in the CFT regime is

$$Z_{BPS}^{\infty} = 2 \prod_{k=0}^{\infty} \frac{1}{(1-q^{1+3k})(1-q^{2+3k})} = 2 \prod_{m=1}^{\infty} (1 + q^m + q^{2m}) \tag{1}$$

From Witten index arguments, we also have a finite j partition function:

$$Z_{BPS}^{j} = 2 \prod_{m=1}^{j} (1 + q^m + q^{2m}) \tag{2}$$

In the CFT approximation, the state counts are j-independent. However, they are j-dependent in the full theory.

Z_{BPS}^{j} starts to deviate from Z_{BPS}^{∞} when we lower J_3 from J_3^{max} by an amount of order j. This is when the CFT realizes that the "Fermi sea" is not infinitely deep.

BPS state partition function: finite j v.s. infinite j

In terms of bosonic oscillator modes, the finite j partition function can be expressed as

$$Z_{BPS}^{j} = \frac{2\prod_{m=j/3+1}^{j}(1-q^{3m})}{\prod_{m=0}^{j/3-1}(1-q^{3m+1})(1-q^{3m+2})} \qquad (j = 0 \bmod 3) \tag{3}$$

Once we lower J_3 by $\sim j$, "ghost" modes in the numerator begin to contribute negatively to the count, decreasing Z_{BPS}^{j} relative to Z_{BPS}^{∞}. This is a little bit like a CFT_2 with an energy cutoff.