Universal $\frac{3}{2} \log T$ correction from near-horizon tensor zero modes

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LITP Workshop on Quantum Black Holes

March 3, 2026 arXiv:2602.16767

5-minute overview

Motivation & main result

- Near-extremal black holes develop a long near-horizon AdS_2 region.
- In the $T \to 0$ limit, *tensor would-be zero modes* become important and spoil naive semiclassics.

Euclidean path integral

$$\mathcal{Z} = \mathcal{Z}_0 \int [\mathcal{D}h][\mathcal{D}\phi] e^{-\frac{1}{2}h^* \Delta_L h - \frac{1}{2}\phi^* \Delta_M \phi},$$

$$\mathcal{Z} \sim \mathcal{Z}_0\, T^{3/2} \qquad \Longrightarrow \qquad \delta S = \frac{3}{2} \log T \; + \; \text{const.}$$

Main result: universality ($D \geq 4$, $\phi + A_a$, axially symmetric)

near-extremal configuration $\to$ zero modes $\to$ lifted eigenvalues $\to$ $\frac{3}{2}$ log T

Near-horizon geometry & tensor zero modes

Near-horizon extremal metric in D dimensions

$$ds^2 = g_1(\theta_n)\left[(y^2-1)\,d\tau^2 + \frac{dy^2}{y^2-1}\right] + g_{mm}(\theta_n)\,d\theta_m^2 + g_{ij}(\theta_n)(d\varphi_i + i\,k_i\,(y-1)\,d\tau)\,(d\varphi_j + i\,k_j\,(y-1)\,d\tau)\,, \tag{1}$$

Tensor zero modes at $T = 0$

$$h^{(n)}_{\mu\nu}dx^\mu dx^\nu = C_n\,g_1(\theta_m)e^{in\tau}\left(\frac{y-1}{y+1}\right)^{\frac{|n|}{2}}\left(-d\tau^2 + \frac{dy^2}{(y^2-1)^2} + 2i\,\frac{n}{|n|}\,\frac{d\tau dy}{y^2-1}\right), \tag{2}$$

The only dependence on the explicit background is through $g_1(\theta_n)$.

Key point: these are large diffeomorphisms

$$h^{(n)}_{\mu\nu} = \mathcal{L}_{\xi^{(n)}}\,\bar{g}_{\mu\nu}\,,$$

Higher-dimensional avatars of Schwarzian reparameterization modes.

Small T lifts the spectrum $\Rightarrow$ determinant $\Rightarrow \frac{3}{2}\log T$

Small-T deformation (Empirical/structural Ansatz)

Turn on a mild near-extremal deformation of the background

$$g = \bar{g} + \delta g, \qquad \Phi = \bar{\Phi} + \delta\Phi,$$

Lifted eigenvalues (universal nT behavior)

$$\Lambda^{(n)} = \widetilde{\Lambda}\, n\, T \quad (n \geq 3), \qquad \Lambda^{(2)} = \widetilde{\Lambda}_2\, T.$$

The lifted eigenvalues are independent of the matter sector (Lemma of cancellation in the Lichnerowicz operator).

One-loop determinant from lifted modes

$$\delta Z = \left(\det \Delta_L\right)^{-1/2} = (\widetilde{\Lambda}_2\, T)^{-1} \prod_{n\geq 3} (\widetilde{\Lambda}\, n\, T)^{-1}.$$

$$\delta \log Z = \frac{3}{2}\log T + \text{const} \quad \Rightarrow \quad \delta S = \frac{3}{2}\log T + \text{const}.$$

Explicit calculation in $D = 4, 5, 6$. The prescription works for $D \geq 7$.

Appendix

The Empirical Ansatz:

$$\delta g_{\mu\nu} = T(\delta g_1 + \delta g_2)_{\mu\nu}, \tag{3}$$

$$\begin{aligned}
\delta g_{1\mu\nu} dx^\mu dx^\nu \;&= \Big[y\big(1 - y^2\big)\, \delta g_{\tau\tau}(\theta_n) \Big]\, d\tau^2 \\
&\quad + 2\Big[y(1+y)\, \delta g^{(1)}_{\tau i}(\theta_n) - i\, k_j\, y\, \delta g_{ij}(\theta_n) \Big]\, d\tau\, d\varphi_i \\
&\quad + \Big[\frac{y\, \delta g^{(1)}_{yy}(\theta)}{1 - y^2} \Big]\, dy^2 \;+\; y\, \delta g_{mm}(\theta)\, d\theta_m^2 \;+\; y\, \delta g_{ij}(\theta)\, d\varphi_i d\varphi_j \;,
\end{aligned} \tag{4}$$

$$\begin{aligned}
\delta g_{2\mu\nu} dx^\mu dx^\nu \;&= \Big[2\, i\, k_i\, (1-y)\big(y\, \delta g^{(21)}_{\tau i}(\theta_n) + \delta g^{(22)}_{\tau i}(\theta_n)\big) \;-\; (1-y)\, \delta g^{(2)}_{yy}(\theta_n) \Big]\, d\tau^2 \\
&\quad + 2\Big[y\, \delta g^{(21)}_{\tau i}(\theta_n) + \delta g^{(22)}_{\tau i}(\theta_n) \Big]\, d\tau\, d\varphi_i \\
&\quad + \Big[\frac{\delta g^{(2)}_{yy}(\theta_n)}{(1-y)(1+y)^2} \Big]\, dy^2 \;,
\end{aligned} \tag{5}$$

Appendix

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\begin{aligned}
&\text{Term 1a}: h^*_{\alpha\beta}\ \delta(\tfrac{1}{2} g^{\alpha\mu} g^{\beta\nu} \Box) h_{\mu\nu} && \text{Term 1b}: h^*_{\alpha\beta}\ \delta(-\tfrac{1}{4} g^{\alpha\beta} g^{\mu\nu} \Box) h_{\mu\nu} \\
&\text{Term 2}: h^*_{\alpha\beta}\ \delta(R^{\alpha\mu\beta\nu}) h_{\mu\nu} && \\
&\text{Term 3a}: h^*_{\alpha\beta}\ \delta(R^{\alpha\mu} g^{\beta\nu}) h_{\mu\nu} && \text{Term 3b}: h^*_{\alpha\beta}\ \delta(-R^{\alpha\beta} g^{\mu\nu}) h_{\mu\nu} \\
&\text{Term 4a}: h^*_{\alpha\beta}\ \delta(-\tfrac{1}{2} R g^{\alpha\mu} g^{\beta\nu}) h_{\mu\nu} && \text{Term 4b}: h^*_{\alpha\beta}\ \delta(\tfrac{1}{4} R g^{\alpha\beta} g^{\mu\nu}) h_{\mu\nu} \\
&\text{Term 5a}: h^*_{\alpha\beta}\ \delta(\tfrac{1}{2} V\, g^{\alpha\mu} g^{\beta\nu}) h_{\mu\nu} && \text{Term 5b}: h^*_{\alpha\beta}\ \delta(-\tfrac{1}{4} V\, g^{\alpha\beta} g^{\mu\nu}) h_{\mu\nu} \\
&\text{Term 6a}: h^*_{\alpha\beta}\ \delta(\tfrac{1}{2}\, \mathcal{C}_{IJ} F^I_{\rho\sigma} F^{J\rho\sigma} g^{\alpha\mu} g^{\beta\nu}) h_{\mu\nu} && \text{Term 6b}: h^*_{\alpha\beta}\ \delta(-\tfrac{1}{4}\, \mathcal{C}_{IJ} F^I_{\rho\sigma} F^{J\rho\sigma} g^{\alpha\beta} g^{\mu\nu}) h_{\mu\nu} \\
&\text{Term 7}: h^*_{\alpha\beta}\ \delta(-2\, \mathcal{C}_{IJ} F^{I\alpha\mu} F^{J\beta\nu}) h_{\mu\nu} && \\
&\text{Term 8a}: h^*_{\alpha\beta}\ \delta(-4\, \mathcal{C}_{IJ} F^{I\alpha\gamma} F^{J\mu}{}_{\gamma} g^{\beta\nu}) h_{\mu\nu} && \text{Term 8b}: h^*_{\alpha\beta}\ \delta(2\, \mathcal{C}_{IJ} F^{I\alpha\gamma} F^{J\beta}{}_{\gamma} g^{\mu\nu}) h_{\mu\nu} \\
&\text{Term 9a}: h^*_{\alpha\beta}\ \delta(\tfrac{1}{4}\, f_{AB} \partial_\rho \phi^A \partial^\rho \phi^B\, g^{\alpha\mu} g^{\beta\nu}) h_{\mu\nu} && \text{Term 9b}: h^*_{\alpha\beta}\ \delta(-\tfrac{f_{AB}}{8} \partial_\rho \phi^A \partial^\rho \phi^B\, g^{\alpha\beta} g^{\mu\nu}) h_{\mu\nu} \\
&\text{Term 10a}: h^*_{\alpha\beta}\ \delta(-\, f_{AB} \partial^\alpha \phi^A \partial^\mu \phi^B\, g^{\beta\nu}) h_{\mu\nu} && \text{Term 10b}: h^*_{\alpha\beta}\ \delta(\tfrac{1}{2}\, f_{AB} \partial^\alpha \phi^A \partial^\beta \phi^B\, g^{\mu\nu}) h_{\mu\nu}
\end{aligned}
\tag{6}
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