On the four derivative completion of $5d$ $\mathcal{N} = 2$ supergravity coupled to vector multiplets

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WiP: with Yide Cai, James T Liu, Yi Pang and Robert Saskowski

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Bosonic sector is described by the two derivative Lagrangian:

$$\boxed{e^{-1}\mathcal{L}_{\partial^2} = R - \frac{3}{2}a_{IJ}\partial_\mu X^I \partial^\mu X^J - \frac{3}{4}a_{IJ}F^I_{\mu\nu}F^{J\mu\nu} + \frac{1}{4}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda,}$$

where

$$a_{IJ} = -\frac{1}{3}\frac{\partial}{\partial X^I}\frac{\partial}{\partial X^J}\log\mathcal{V}\Big|_{\mathcal{V}=1} = 3X_I X_J - 2C_{IJK}X^K, \qquad X_I = C_{IJK}X^J X^K.$$

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Models of interest:

- STU Model: $n_v = 2$ and $C_{123} = \frac{1}{6}$
- C_{112} Model: $n_v = 1$ and $C_{112} = \frac{1}{3}$

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  - The Weyl squared invariant: $\lambda_M X^M R^2_{\mu\nu\rho\sigma} + D_{IJ} R^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} + \cdots$

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Top down: Dimensional reduction

- Reduction of 10d heterotic on T^5 followed by an $\mathcal{N} = 2$ truncation (or a reduction on $K3$):
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- Reduction from $6d$: Riemann and Gauss bonnet invariants.
  - Symmetric combination: Equivalent to the heterotic reduction
  - Antisymmetric combination: $X^3[((\tilde{F}^3 - \tilde{F}^2)^2)^2 - 4(\tilde{F}^3 - \tilde{F}^2)^4 + \cdots]$

Vector invariants and BPS black holes

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Consider the limit $X^I \to \alpha\lambda^I(1+\kappa\delta\,\varphi^I)$ and $V \to \kappa V$,

$$\mathcal{L}_{4\partial} = \mathcal{L}_{G^4} + \mathcal{L}_{V^4} + \mathcal{L}_{G^2V^2}.$$

The vector invariant in this generic case have a richer structure that the one above

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BPS Black holes For static BPS black holes, surprisingly, 4∂ EOM from these vector couplings vanish and these couplings do no corrections! The Wald Entropy and other thermodynamic properties also remain unchanged with the addition of the vector couplings.

$$S_W = \frac{\pi^2}{2G_N}\sqrt{(Q_1+24\lambda_1)(Q_2+24\lambda_2)(Q_3+24\lambda_3)},$$