Holographic Equidistribution

based on [2602.12265]

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LITP Workshop on Quantum Black Holes Gong Show
March 6th, 2026

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- So a whole sector gets integrated out!

New Direction

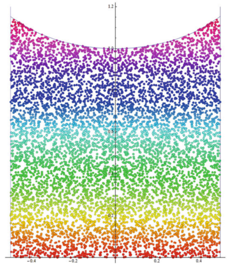

Figure: Hecke points for $N = 7919$ in the (S-transformed) $SL(2, \mathbb{Z})$ fundamental domain. Uniform distribution of horocycle images implies equidistribution of Hecke images (Figure from Terras - Harmonic Analysis on Symmetric Spaces (2016))

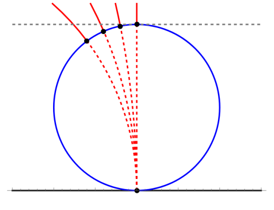

Figure: Instead of cutting off at a uniform radial distance, geodesics in AdS_3 are cut off at a *horocycle* of radius $\epsilon \to 0$, which yields regulator-independent entropies/geodesic lengths. (Figure from [2410.00950])

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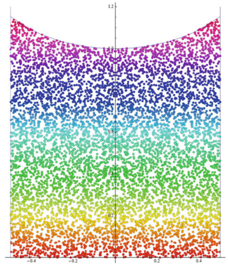

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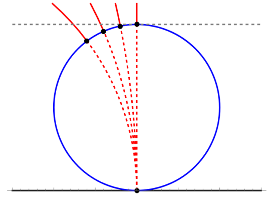

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- Removing the regulator ($\epsilon \to 0$) may correspond to Hecke equidistribution ($N \to \infty$)