Schwinger-Dyson & topological recursion in 3d gravity

wip w/ Daniel Jafferis, LR, Diandian Wang, Cynthia Yan.

- In [2407.02649] we showed that there is a correspondence at fixed central charge c b/w

partition functions of a matrix / tensor model defined using (constraint)^2 potential V_ε $\longleftrightarrow$ partition functions of 3d manifolds (up to factors of $\hbar$)

GOE; begin at $\frac{c-1}{12}$,

id included separately

deviation from constraint (constraint)^2

$$Z = \prod_{s\in\mathbb{Z}} \int d\Delta_s \, DC_{ijk} \, e^{-V_0(\Delta_s) - \frac{1}{\hbar} V_\varepsilon(\Delta_s, C_{ijk})}$$

$\hbar \to 0$ bootstrap implemented

OPE coeff. of irrational CFTs (no degeneracies apart from id)

single trace potential

Cardy dos for fixed spin

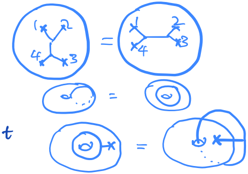
purely tensor

$$V_\varepsilon = \overbrace{V_4} + V_s + V_s^{1\text{-pt}}$$

V_4

V_s

$V_s^{1\text{-pt}}$



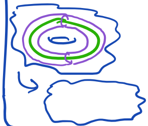
- Focus on V_4. We construct diagrams for the tensor potential & follow the rule that
  - closed loops get hyperbolic surgery done on them

(fill w/ disk = leading)

(from matrix model part)

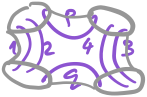
  - open Wilson lines — "trenches" get cut out b/c external insertions fix weights

Ω loop :



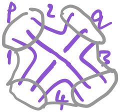
Cardy $\rho_0(P) = \sinh(2\pi b P)\,\sinh(2\pi b^{-1} P)$

- Basic diagrams:

Pillow 

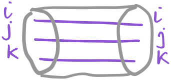
$$\rightarrow \frac{1}{\hbar}\,\frac{\delta_\varepsilon^{(2)}(P_p - P_q)}{|\rho_0(p)\,C_0(12p)\,C_0(34p)|^2}$$

Liouville 3-pt fn

6J 

$$\rightarrow -\frac{1}{\hbar}\,\frac{\left|\begin{Bmatrix} q & 4 & 1 \\ p & 2 & 3 \end{Bmatrix}\right|^2}{|C_0(12p)\,C_0(34p)\,C_0(23q)\,C_0(14q)|^2}$$

Virasoro 6j symbol

C_0 

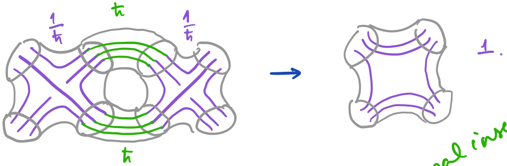
$$\rightarrow \hbar\,|C_0(ijk)|^2$$

Build up links from these. All links generated Likorish $\Rightarrow$ all closed mflds generated.

• The same mfld may be produced in multiple ways

e.g.

keeping track of the ħ factors



vs $\frac{1}{\hbar}$ in the pillow vertex.

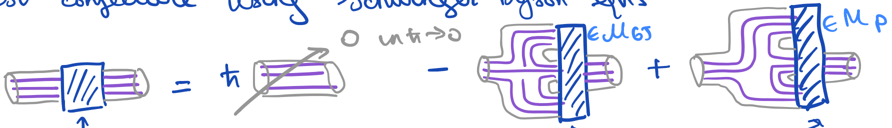
More generally:

external insertions

$$\langle C \dots C \rangle = \sum_{M} Z_{3dG}(M, c, q)\, \underbrace{f(M,\hbar)}_{\text{purely combo}}$$

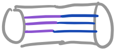
Conjecture: $f(M,\hbar) \xrightarrow[\hbar\to 0]{} 1 \quad \forall M.$

Test conjecture using Schwinger Dyson eqns

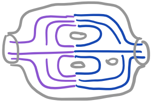


fix mfld on LHS, search for candidates on RHS & check if there is only 1 net contribution

e.g. fix LHS



$\Rightarrow$ $M_{6J} \supset$



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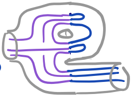
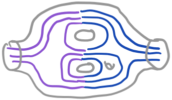


; $M_P \supset$



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