

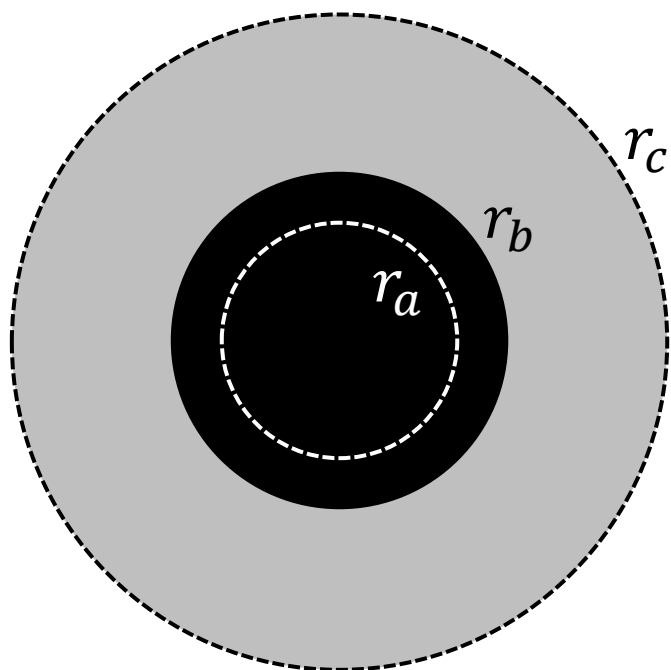
Lukewarm-Nariai Black Hole: a Critical Point of Continuous Phase Transition

Puxin Lin

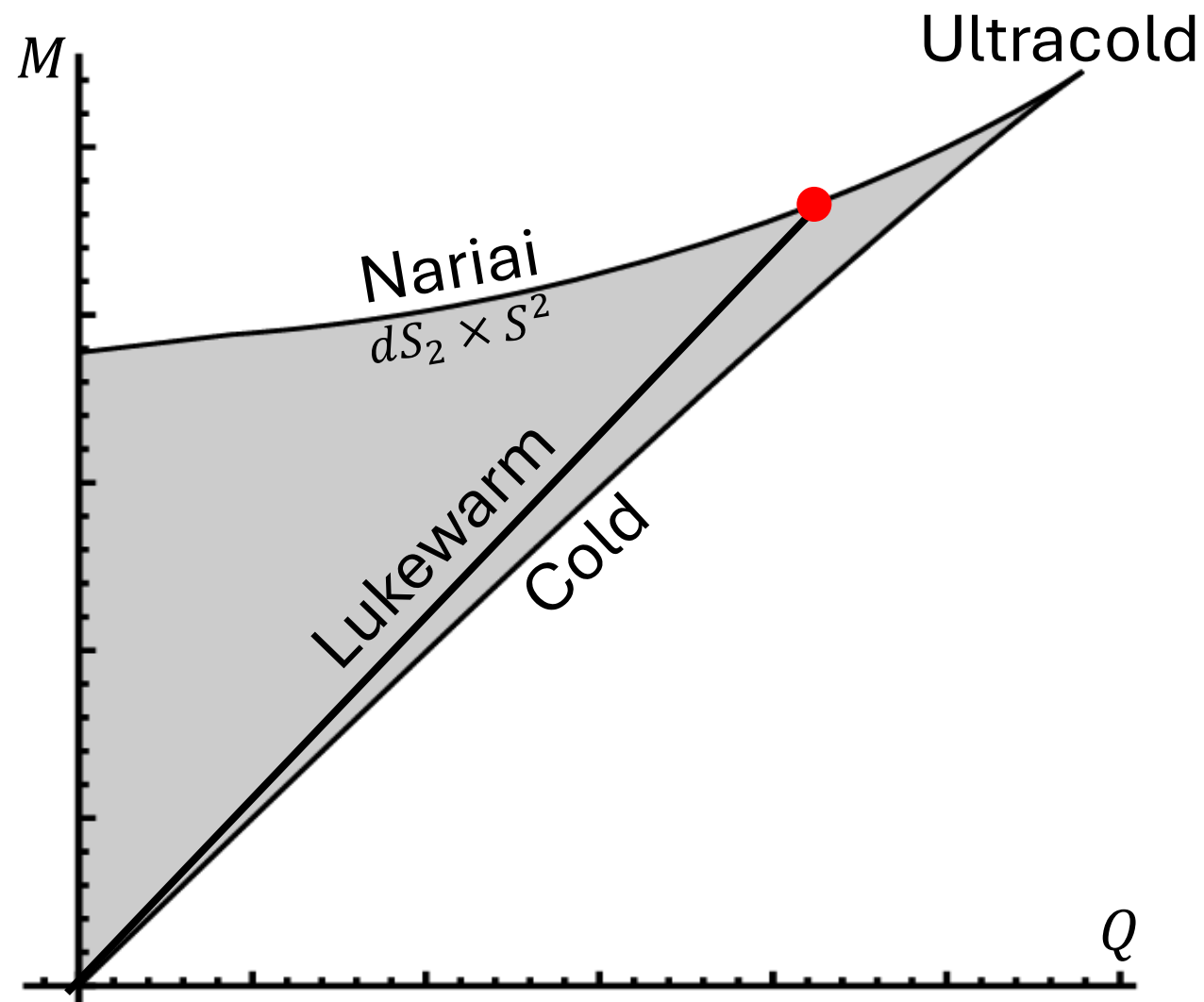
arXiv:2511.03867
& on-going

de Sitter black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$
$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{l^2}$$



Shark-fin



Black hole temperature

Surface gravity $\xi^\mu \nabla_\mu \xi^\nu = \kappa \xi^\nu$

Time-like Killing vector: $\xi^\mu = \partial_t$

$$T = \frac{\kappa}{2\pi} = \frac{f'(r_b)}{4\pi}$$

Observer

$$\xi^\mu = \lambda \partial_t, \quad \xi^2(x_{Obs}^\mu) = 1$$

\Rightarrow Killing vector coincides with (static) observer worldline

\Rightarrow physics in observer's rest frame

Empty dS: $r_{Obs} = 0, \lambda = 1$

Asym. flat: $r_{Obs} = \infty, \lambda = 1$

$$\mathbf{dS BH:} \quad \lambda = \frac{1}{\sqrt{f(r_{Obs})}}, \quad f'(r_{Obs}) = 0$$

(Normalization) (Static Observer)

$$T = \frac{f'(r_b)}{4\pi \sqrt{f(r_{Obs})}}$$

Thermodynamics at the lukewarm-Nariai intersection

Hint for continuous phase transition at ● :
Power law specific heat (macroscopic)

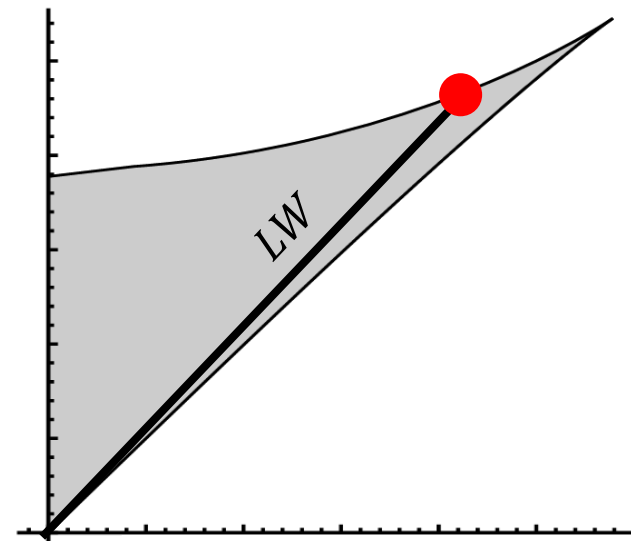
$$C_{LW} = \left(T \frac{\delta S}{\delta T} \right)_{LK} \sim \left| \frac{T - T_c}{T_c} \right|^{-\frac{1}{2}}$$

Critical exponent → **universality class**

To appear: (microscopic)

Correlation length: $\langle \Phi(x)\Phi(y) \rangle \sim e^{-\frac{|x-y|}{\zeta}}$, $\zeta \rightarrow \infty$ → “massless” (zero) modes

Order parameter



Consistent with sign change
of near-Nariai modes in 1-
loop Z for $dS_2 \times S^2$