Quantized Giant Gravitons
as the
'Periodic Table' of Supersymmetric States

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The giant graviton expansion

- Superconformal indices of various holographic gauge theories admit finite-N expansions:

$$\boxed{\mathcal{I}_N(q) = \mathcal{I}_\infty(q) \sum_m q^{mN} \mathcal{I}_m^{\mathrm{GG}}(q)}$$

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- Giant graviton fluctuation spectrum:
  - Via superconformal algebra [Arai, Fujiwara, Imamura, Mori '20]
  - From probe brane for $\frac{1}{2}$-BPS modes [Lee '23][Murthy '23] and for Schur index

This work: General fluctuations directly from probe brane actions

3d $\mathcal{N} = 8$ ABJM:	M5 in $\mathrm{AdS}_4 \times S^7$
4d $\mathcal{N} = 4$ SYM:	D3 in $\mathrm{AdS}_5 \times S^5$
6d $\mathcal{N} = (2,0)$:	M2 in $\mathrm{AdS}_7 \times S^4$

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$$S = -T_q \int d^{q+1}\xi \sqrt{-g} + T_q \int A_{q+1}$$

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- Quantize Hamiltonian
- Fermions and worldvolume vector (D3) and 2-form (M5) are dimension-dependent

D3 brane modes

	$H\tilde{L}$	J_1	J_2	R_1	$[R_2, R_3]$	$\mathcal{H}$
$\zeta_{1+,(\ell,m_1,m_2)}$	$\ell+1$	1	0	0	$[\ell, 0]$	$\ell-(m_1+m_2)$
$\zeta_{2+,(\ell,m_1,m_2)}$	$\ell+1$	0	1	0	$[\ell, 0]$	$\ell-(m_1+m_2)$
$\zeta_{3+,(\ell,m_1,m_2)}$	$\ell+3$	0	0	1	$[\ell, 0]$	$\ell+2-(m_1+m_2)$
$\zeta_{1-,(\ell,m_1,m_2)}$	$\ell+1$	-1	0	0	$[\ell, 0]$	$\ell+2-(m_1+m_2)$
$\zeta_{2-,(\ell,m_1,m_2)}$	$\ell+1$	0	-1	0	$[\ell, 0]$	$\ell+2-(m_1+m_2)$
$\zeta_{3-,(\ell,m_1,m_2)}$	$\ell-1$	0	0	-1	$[\ell, 0]$	$\ell-(m_1+m_2)$
$A^{(1)}_{\ell,m_1,m_2}$	$\ell+1$	0	0	0	$[\ell, 1]$	$\ell+1-(m_1+m_2)$
$A^{(2)}_{\ell,m_1,m_2}$	$\ell+1$	0	0	0	$[\ell, -1]$	$\ell+1-(m_1+m_2)$
$\bar{\psi}_{++-,(\ell,m_1,m_2)}$	$\ell+1/2$	$1/2$	$1/2$	$-1/2$	$[\ell+1/2, -1/2]$	$\ell-(m_1+m_2)$
$\bar{\psi}_{---,(\ell,m_1,m_2)}$	$\ell+1/2$	$-1/2$	$-1/2$	$-1/2$	$[\ell+1/2, -1/2]$	$\ell+2-(m_1+m_2)$
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Compute single-letter index:

$$\sum_{\text{modes}} (-1)^F e^{-\beta\mathcal{H}} p^{J_1} q^{J_2} y_1^{R_1} y_2^{R_2} y_3^{R_3}$$

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$$\mathcal{I}_N(p,q;y_i) = \mathcal{I}_\infty(p,q;y_i) \sum_{m_1,m_2,m_3\geq 0} y_1^{m_1 N} y_2^{m_2 N} y_3^{m_3 N}\, \mathcal{I}^{\mathrm{GG}}_{(m_1,m_2,m_3)}(p,q;y_i)$$

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- GGE for non-protected BPS partition functions
  - Include interactions in DBI action?