Nuclear strength functions from microscopic models:

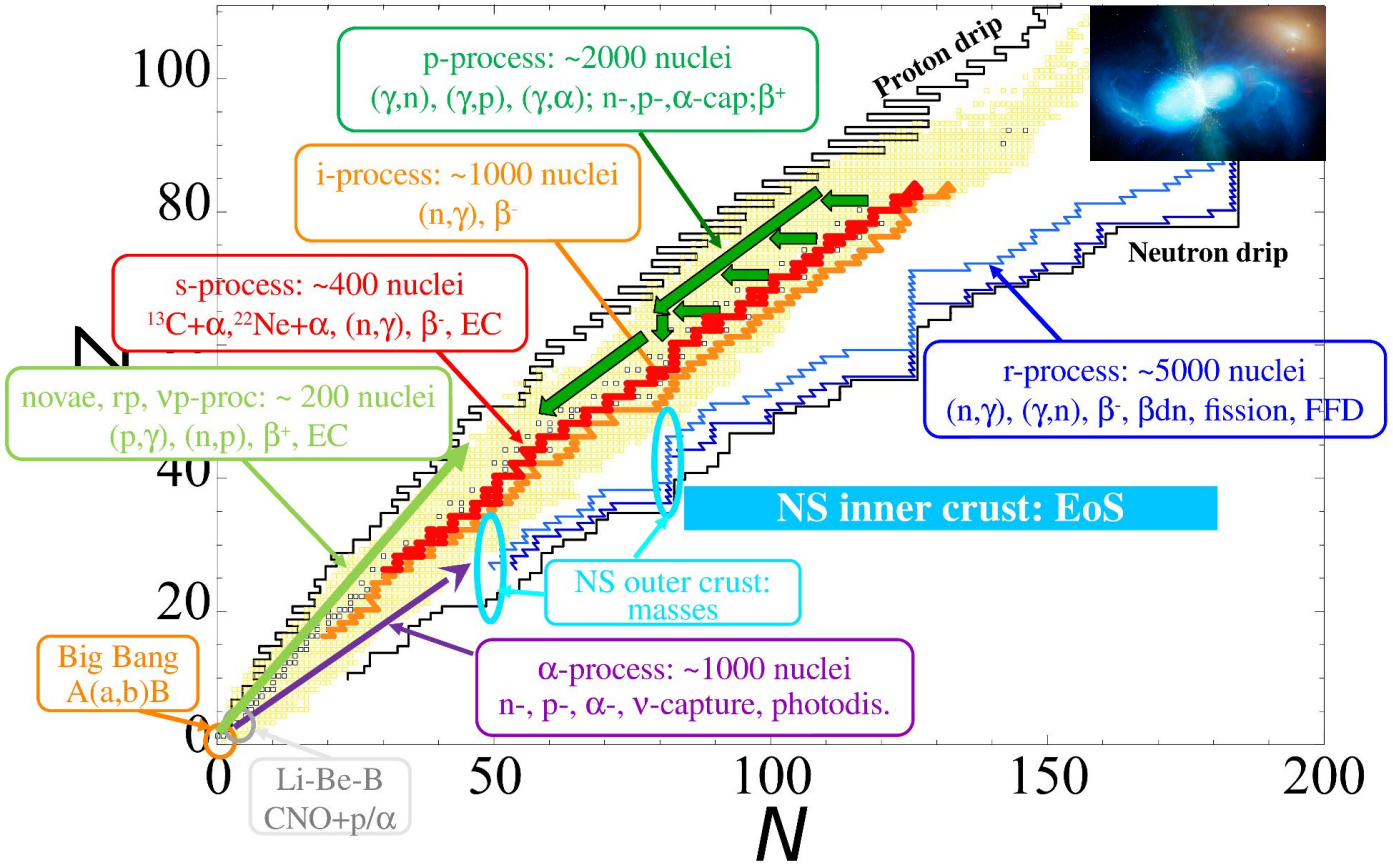
The impact of triaxiality

Pepijn Demol, *S. Goriely, W. Ryssens, E. Vancayseele*

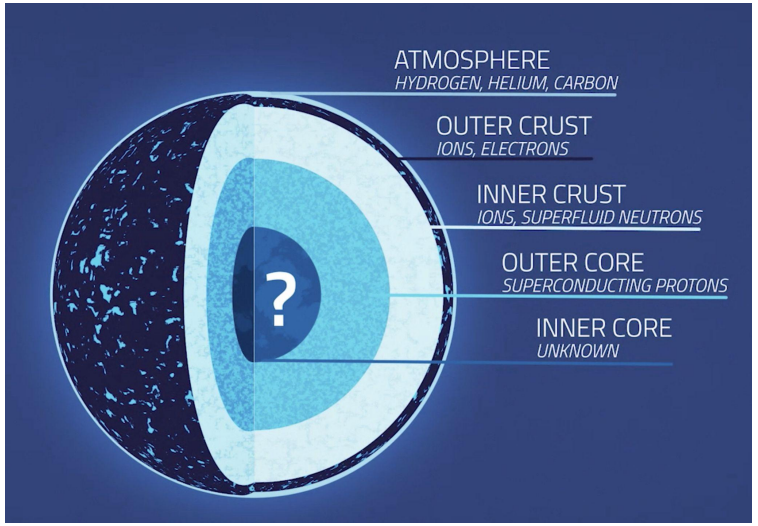
Oslo, 10th Workshop on Nuclear Level Density and Gamma Strength

18-22 May 2026

Nucleosynthesis



Neutron stars



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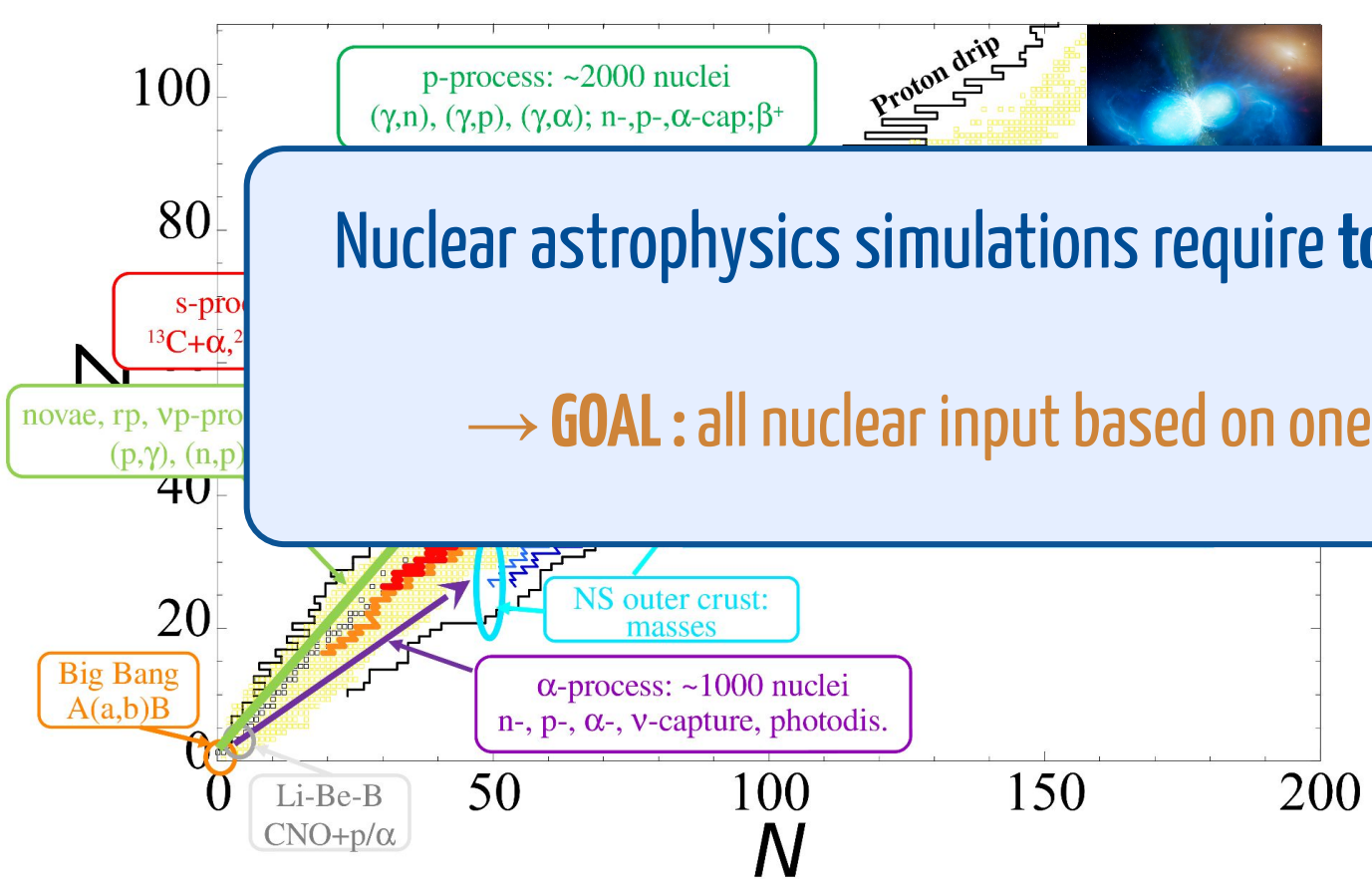

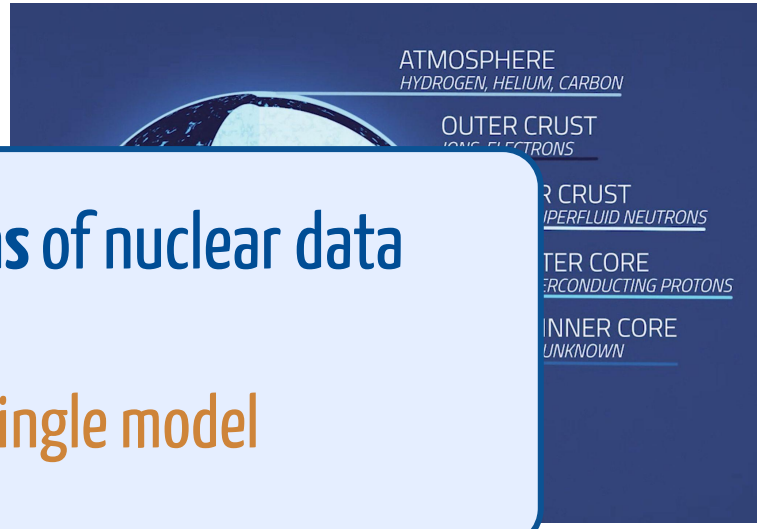

Nuclear astrophysics simulations require **tons** of nuclear data

→ **GOAL :** all nuclear input based on one single model

Skyrme energy density functional (EDF) theory

Coupling constants fitted to data

$$E_{\mathrm{Skyrme}}[\rho] \sim \int \mathrm{d}^3\mathbf{r}\,\left[C^{\rho}\rho(\mathbf{r})\rho(\mathbf{r}) + C^{\tau}\tau(\mathbf{r})\rho(\mathbf{r}) + C^{\gamma}\rho(\mathbf{r})^{\gamma}\rho(\mathbf{r})\rho(\mathbf{r}) + \cdots\right]$$

Local densities and currents of the wavefunction

$$\min_{|\Psi\rangle} \; E_{\mathrm{tot}} = E_{\mathrm{Skyrme}} + E_{\mathrm{kin}} + E_{\mathrm{Coul}} + E_{\mathrm{pair}} + E_{\mathrm{corr.}}$$

Set of trial wavefunctions:

Mean-field states = Slater determinant + **symmetry breaking**

⟶ solving the **Hartree-Fock(-Bogoliubov)** problem

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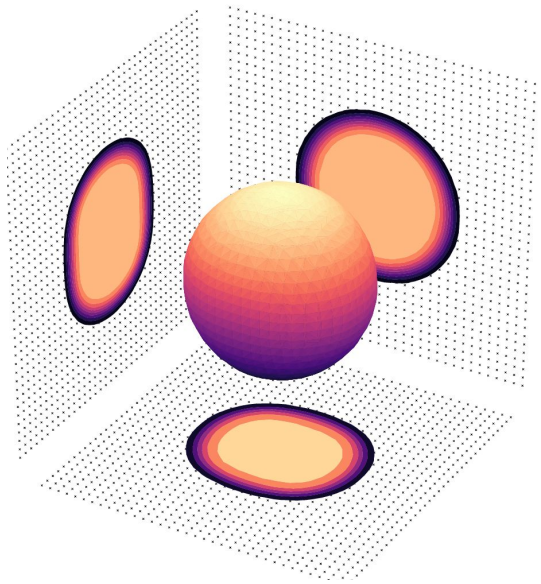
Set of trial wavefunctions:
Mean-field states = Slater determinant + **symmetry breaking**
→ solving the **Hartree-Fock(-Bogoliubov)** problem

- \+ Wavefunction with individual nucleons
- \+ Feasible for 1000s of nuclei (incl. odds)
- \+ Many observables accessible

- – Unclear how to improve functional form
- – Lack of systematic uncertainty quantification

Brussels-Skyrme-on-a-Grid: BSkG

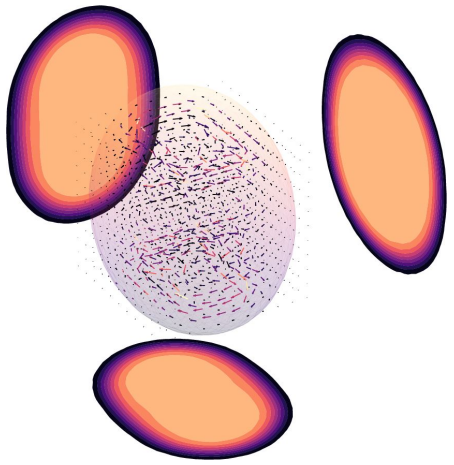


- 3D representation agnostic wrt shape
- shape DOF characterized by multipole moments
- Symmetry breaking enlarges variational space
  - \+ captures collective correlations at modest CPU cost
  - \- BUT loss of quantum numbers

Brussels-Skyrme-on-a-Grid: BSkG

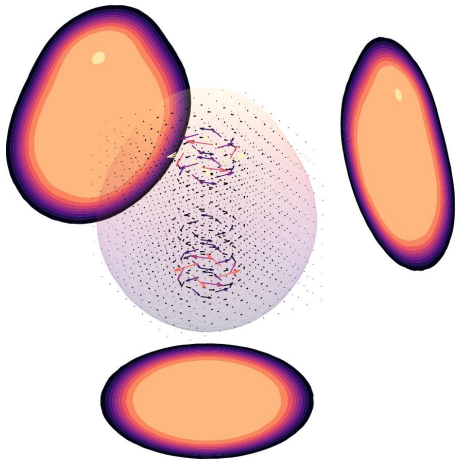
Triaxial

β_{20}, β_{22} or β_2, γ



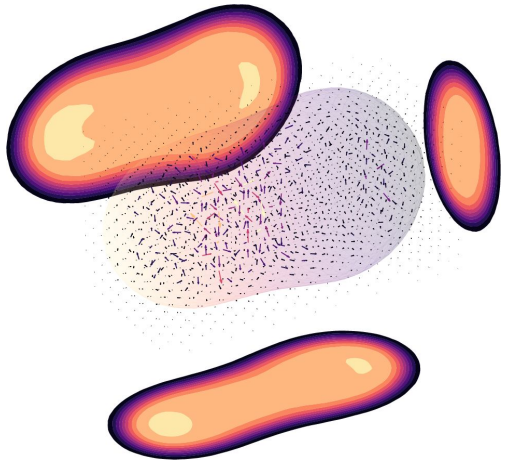
Octupole

β_{20}, β_{30}



Triaxial + octupole

β_{20}, β_{22} **and** β_{30}



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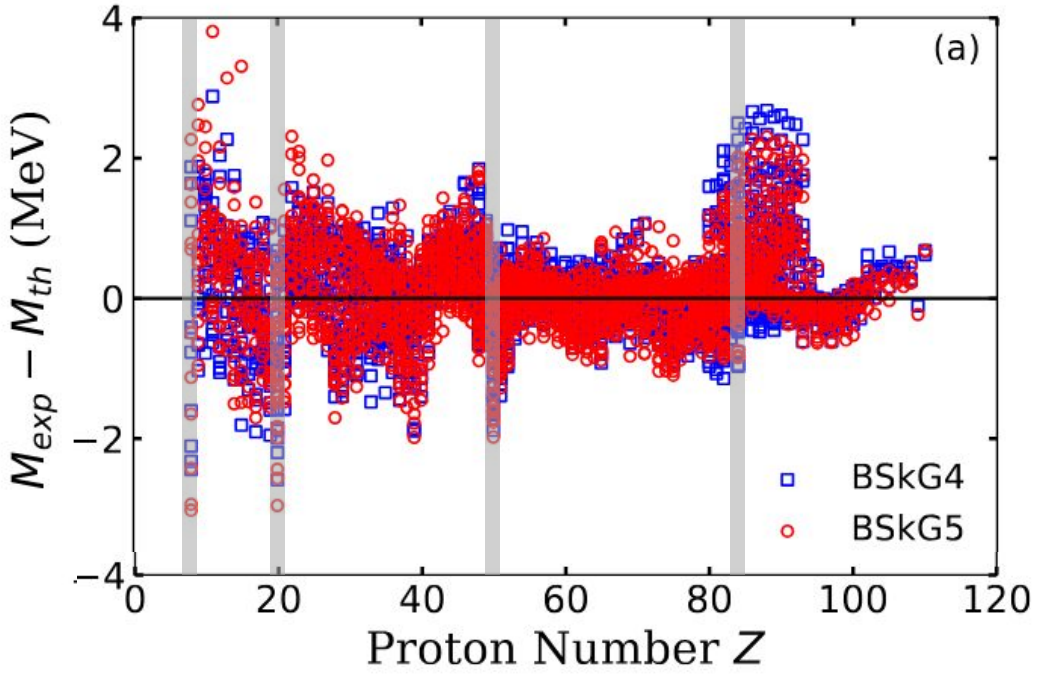
Brussels-Skyrme-on-a-Grid: BSkG

Coupling constants (~**25** parameters) fitted to data:

- **2457** masses
- **884** charge radii
- **45** fission barriers
- Infinite matter properties

$$E_{\mathrm{Skyrme}}[\rho] \sim \int \mathrm{d}^3\mathbf{r}\,[C^{\rho}\rho(\mathbf{r})\rho(\mathbf{r}) + C^{\tau}\tau(\mathbf{r})\rho(\mathbf{r}) + C^{\gamma}\rho(\mathbf{r})^{\gamma}\rho(\mathbf{r})\rho(\mathbf{r}) + \cdots]$$

RMSE (MeV)	**BSkG1**	…	**BSkG5**	HFB-14	FRDM
Masses	0.741	…	0.643	0.729	0.560
Charge radii (fm)	0.027	…	0.027	0.039	0.038
Primary barriers	0.87	…	0.43	0.61	0.79
Secondary barriers	0.86	…	0.49	0.70	1.35
Isomers	0.45	…	0.59	0.93	1.04

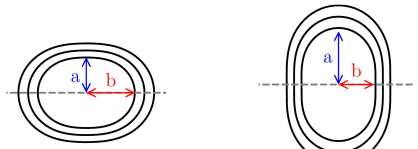


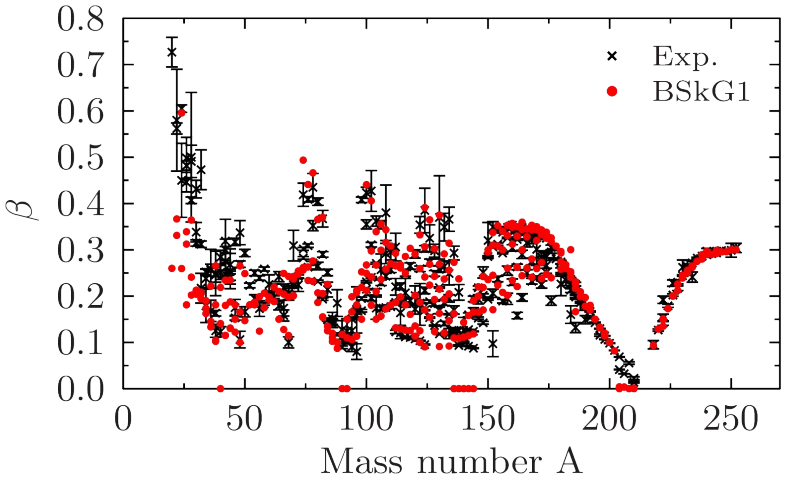
BSkG1: G. Scamps et al., EPJA **57**, 333 (2021).
BSkG5: G. Grams et al. arXiv:2601:05968 (2026).
HFB-14: S. Goriely et al., PRC ***75***, 064312 (2007).
FRDM: P. Möller et al., At. Data Nucl. Data Tables, **109-110** (2016).

BSkG1: G. Scamps et al., EPJA **57**, 333 (2021).
BSkG2: G. Grams et al., EPJA **59**, 270 (2023).

More on shape and charge distributions

"ordinary" quadrupole deformation





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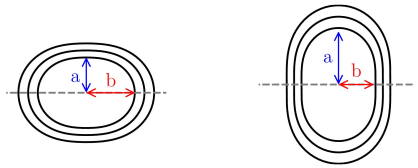
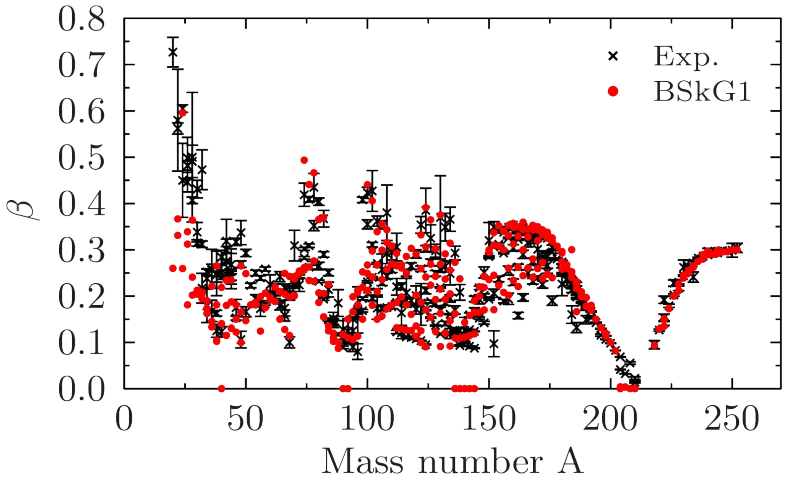
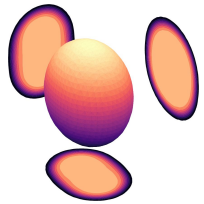
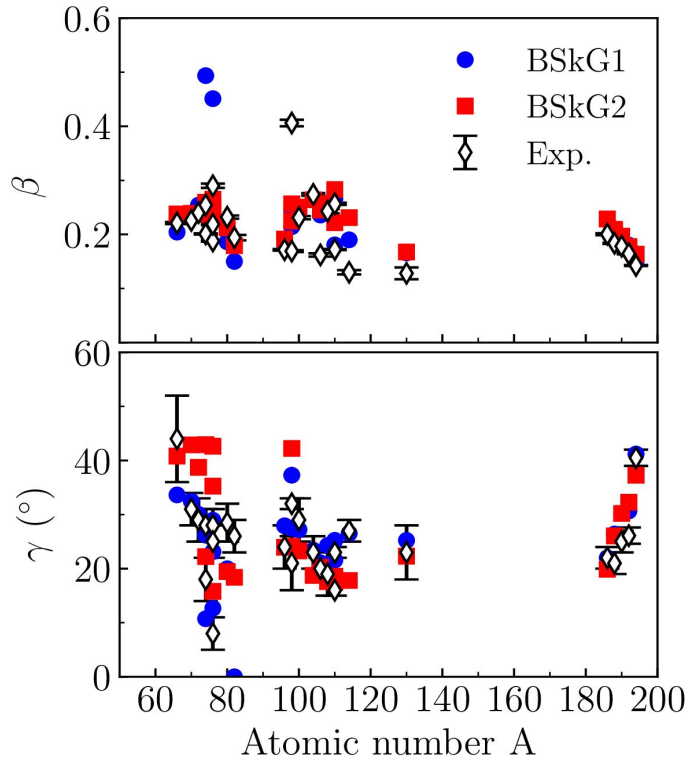




Triaxial deformation





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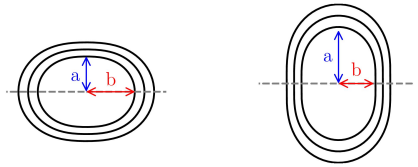

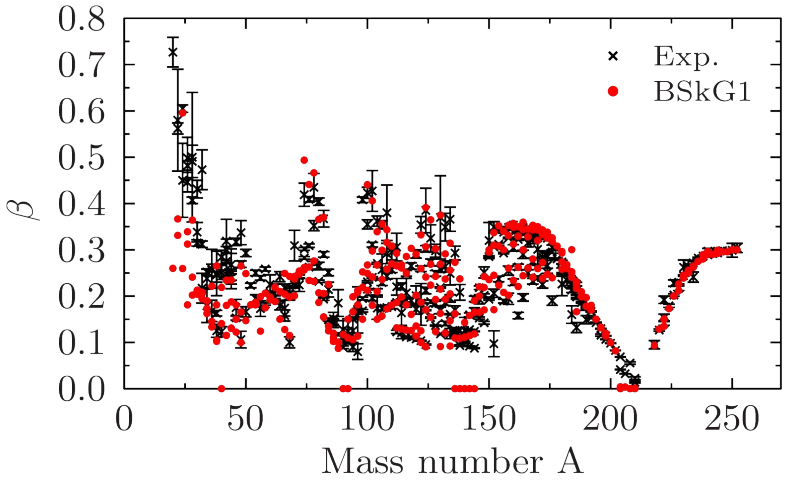

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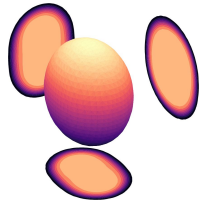
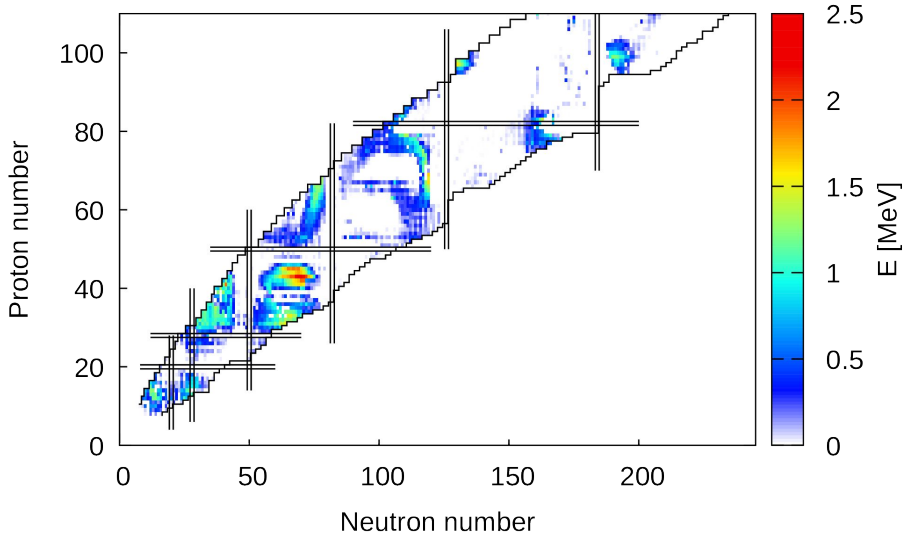




Rotational correction in ^{24}Mg with BSkG1

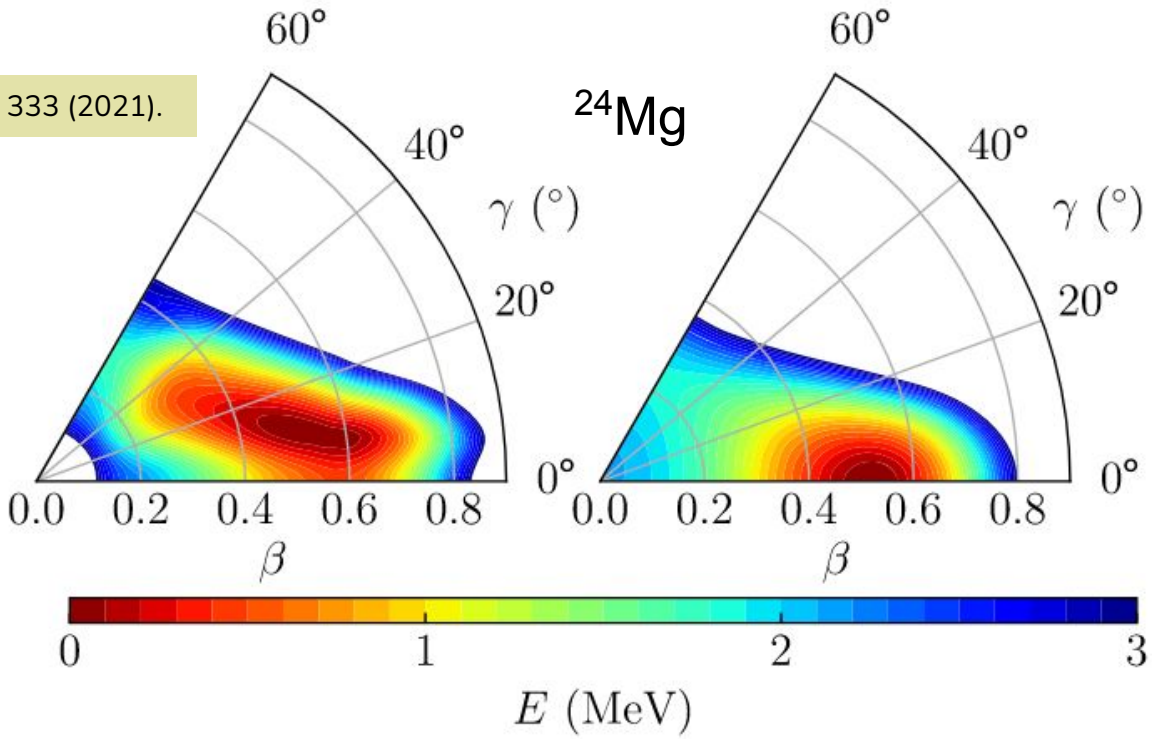
$$E_{\text{tot}} = E_{\text{Skyrme}} + E_{\text{kin}} + E_{\text{Coul}} + E_{\text{pair}} + \boxed{E_{\text{corr.}}}$$

Corrections

✓ INCLUDED

✗ EXCLUDED

G. Scamps et al., EPJA **57**, 333 (2021).

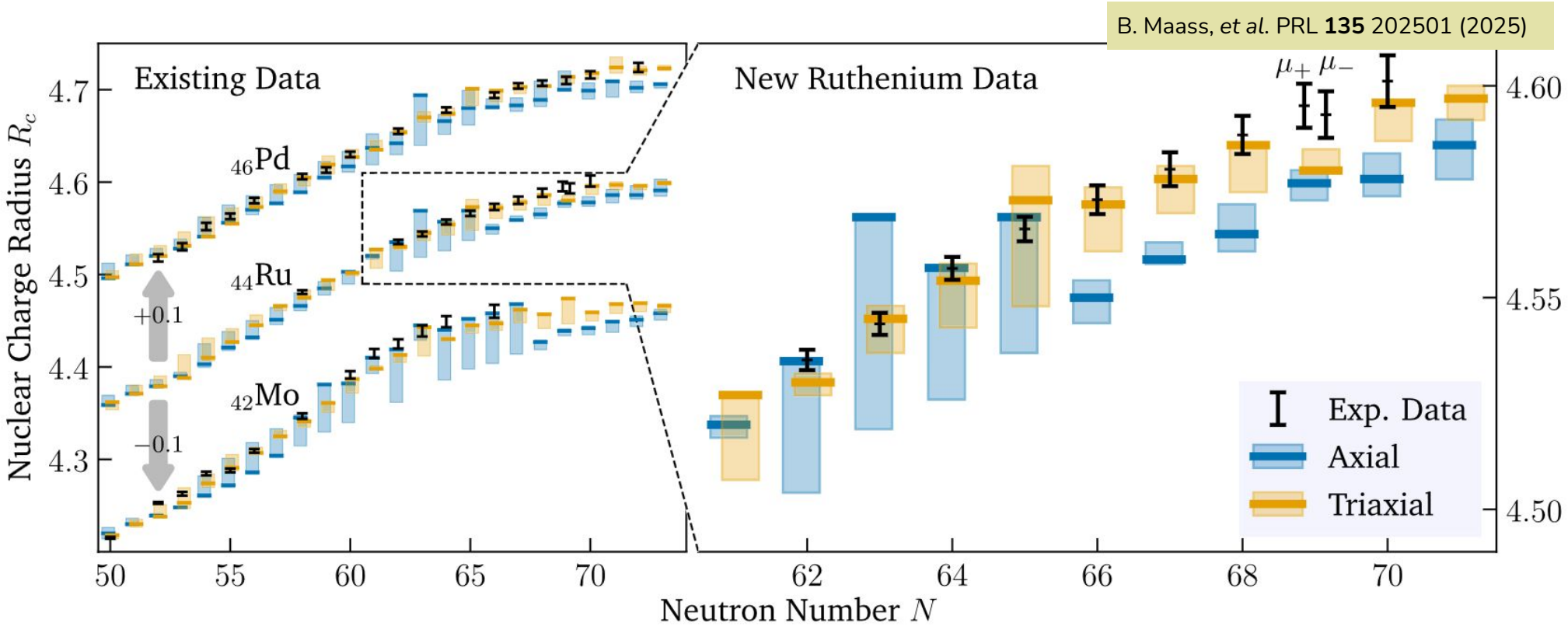


Many nuclei triaxially deformed due to inclusion of rotational correction

Impact of triaxiality on mean-squared charge radii

spread between BSkG[1,2,3,4] as proxy for model uncertainty

B. Maass, *et al.* PRL **135** 202501 (2025)



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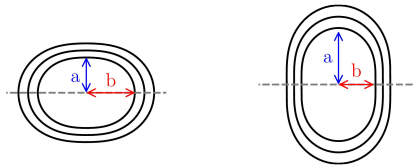

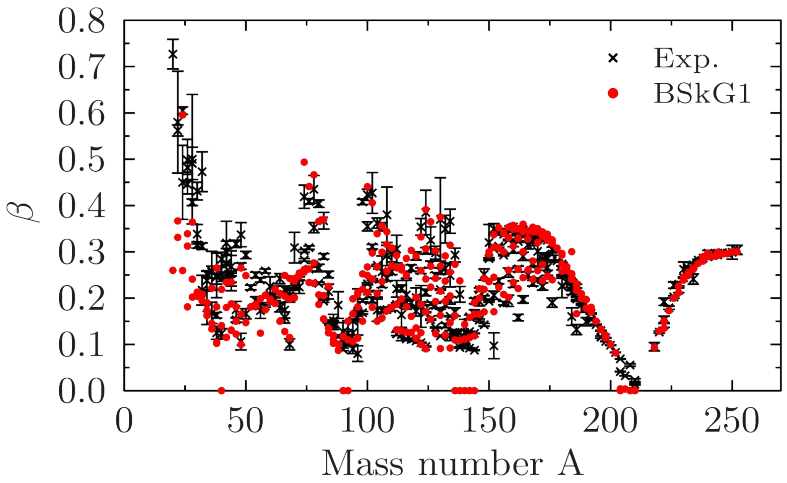

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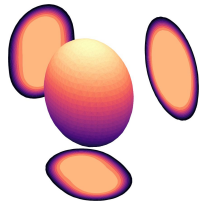
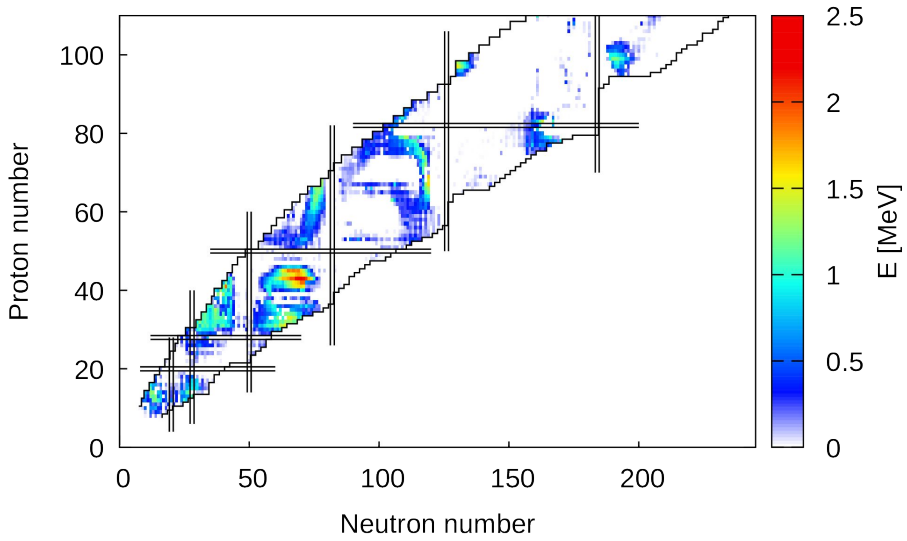
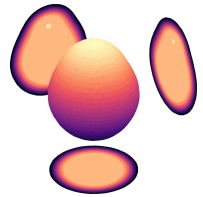
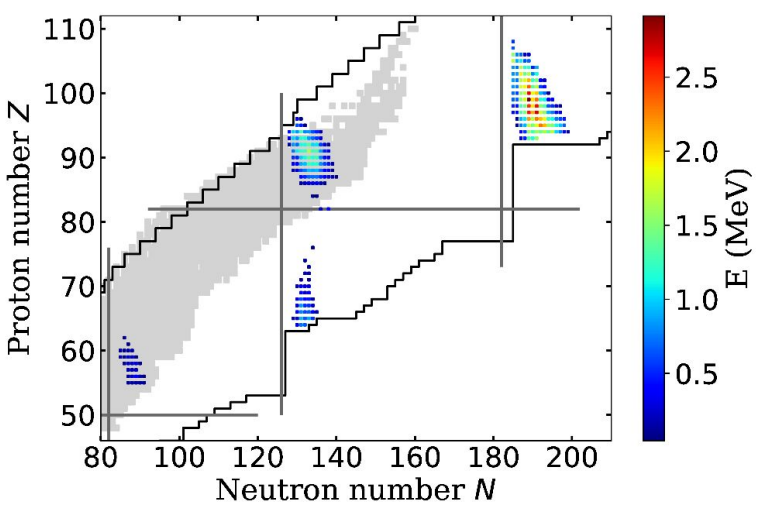




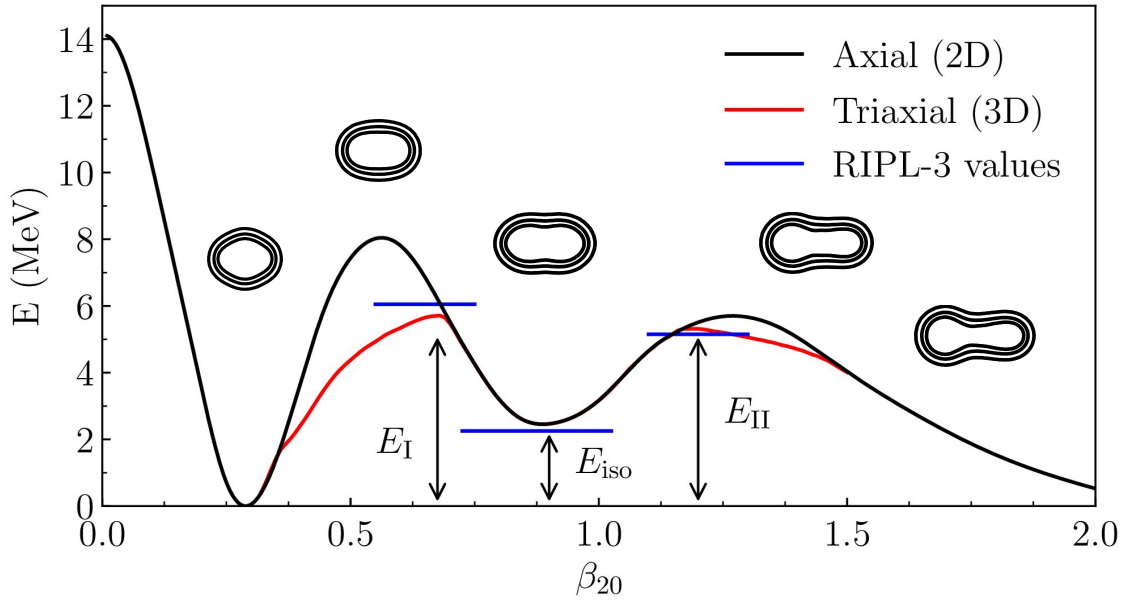
octupole deformation



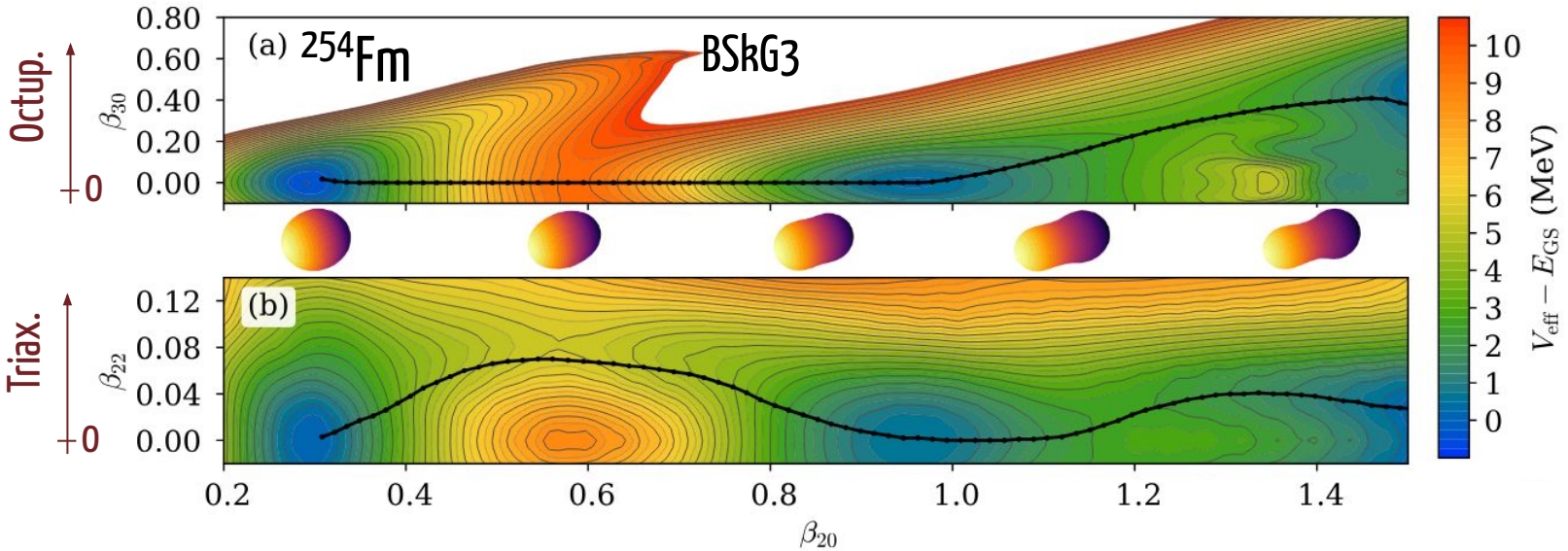


W. Ryssens et al., EPJA **59**, 96 (2023).
E. Flynn et al., PRC **105** (2022).
A. Sánchez-Fernández et al., arXiv:2508.16240

Impact of triaxiality on fission barriers

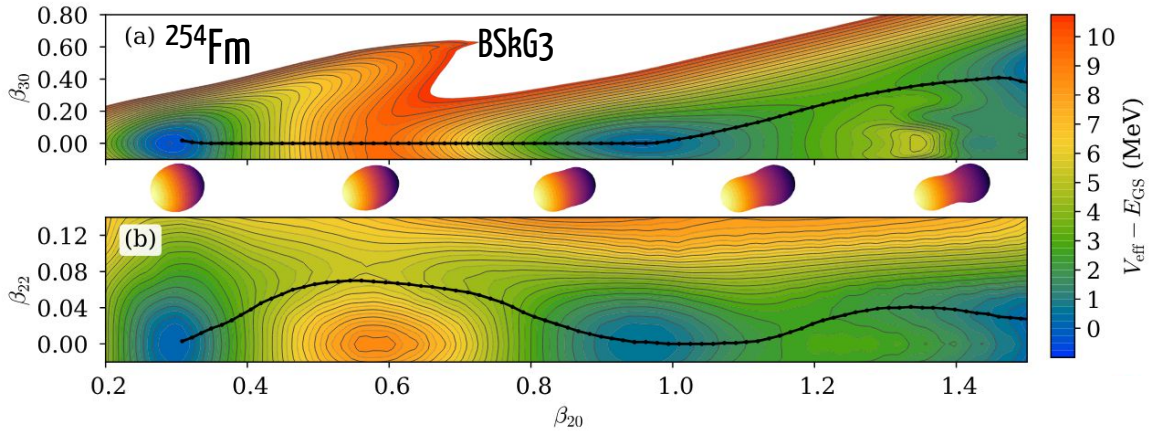


- Inner-barriers are triaxial
- Outer-barriers are triaxial + octupole deformed

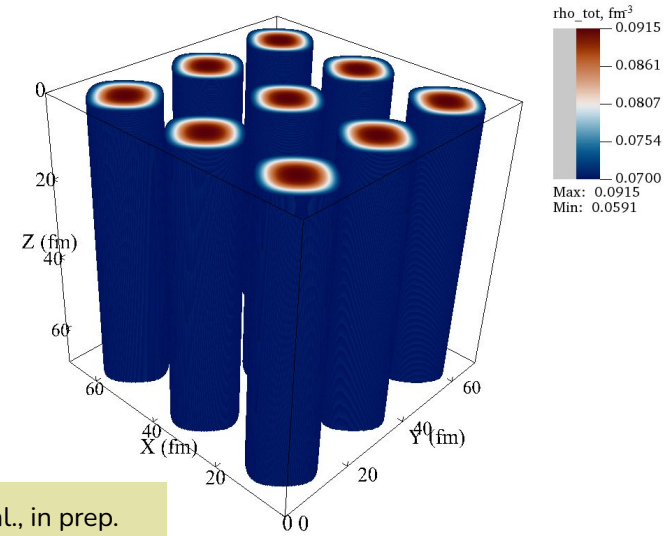


Applications which will not be discussed

Fission

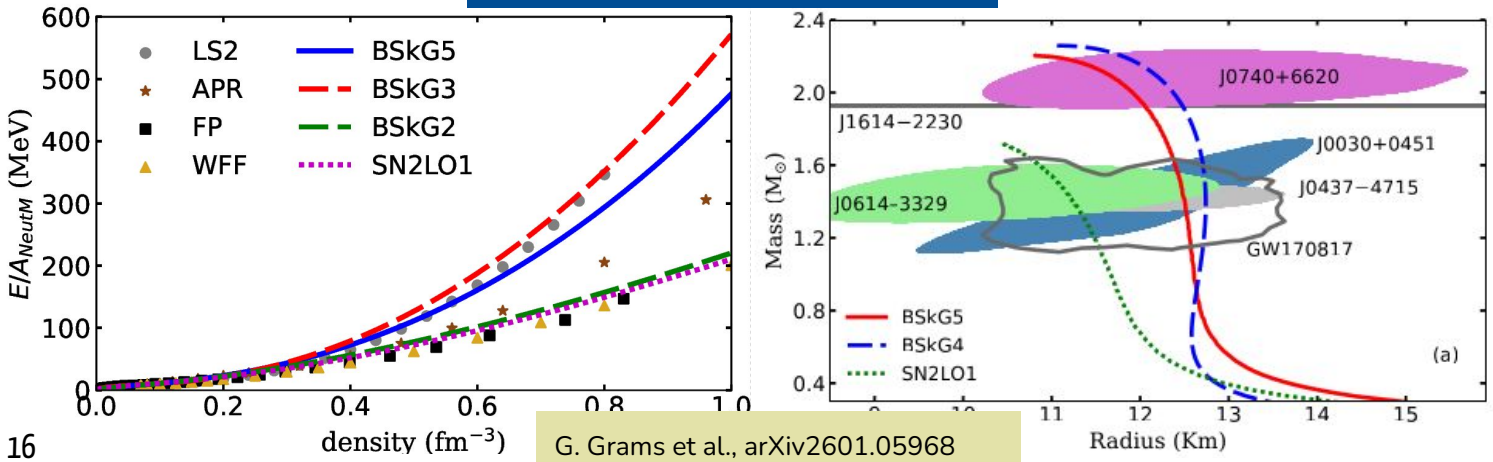


Nuclear Pasta



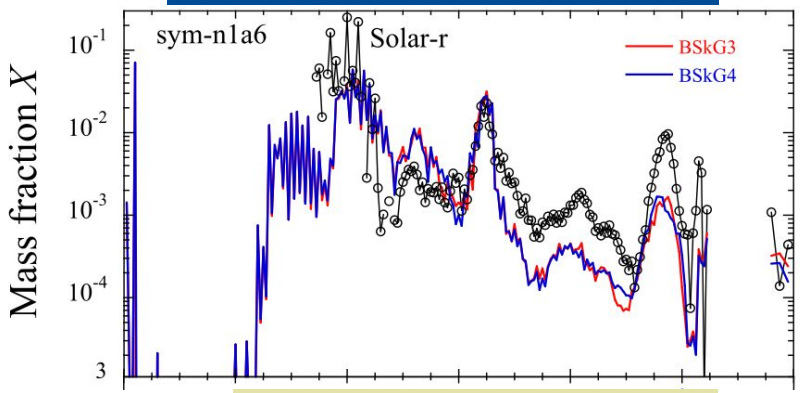
N. Shchechilin et al., in prep.

Neutron star Equation of State (EoS)



G. Grams et al., arXiv2601.05968

r-process simulations



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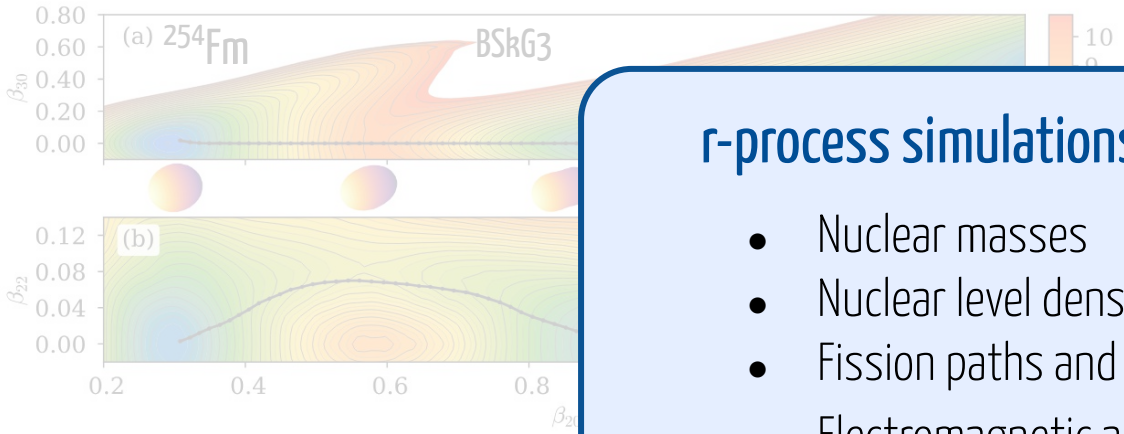

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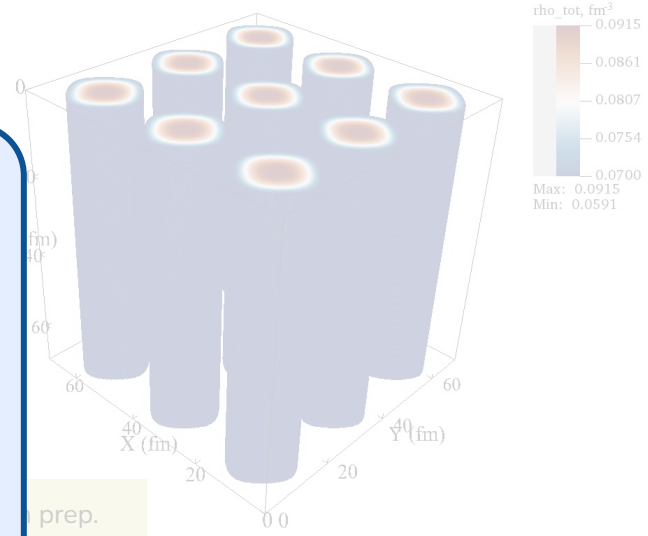

r-process simulations require **a lot** of data

- Nuclear masses
- Nuclear level densities
- Fission paths and fragment yields
- Electromagnetic and weak strength functions

for ALL nuclei

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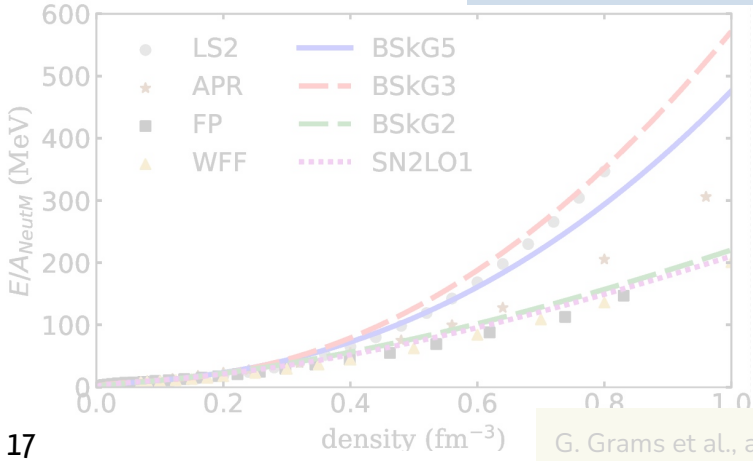

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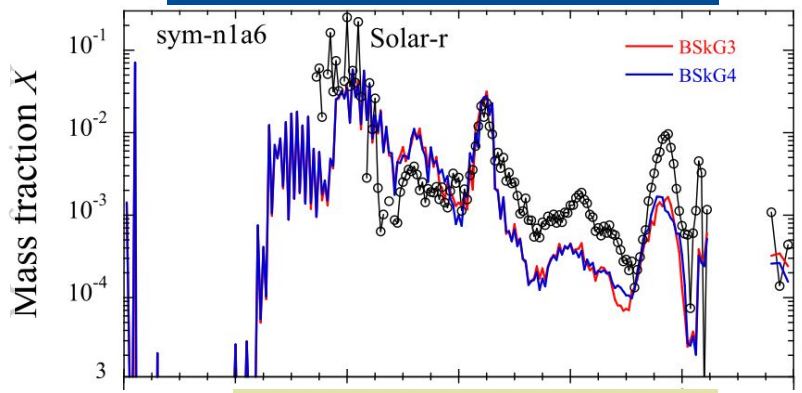

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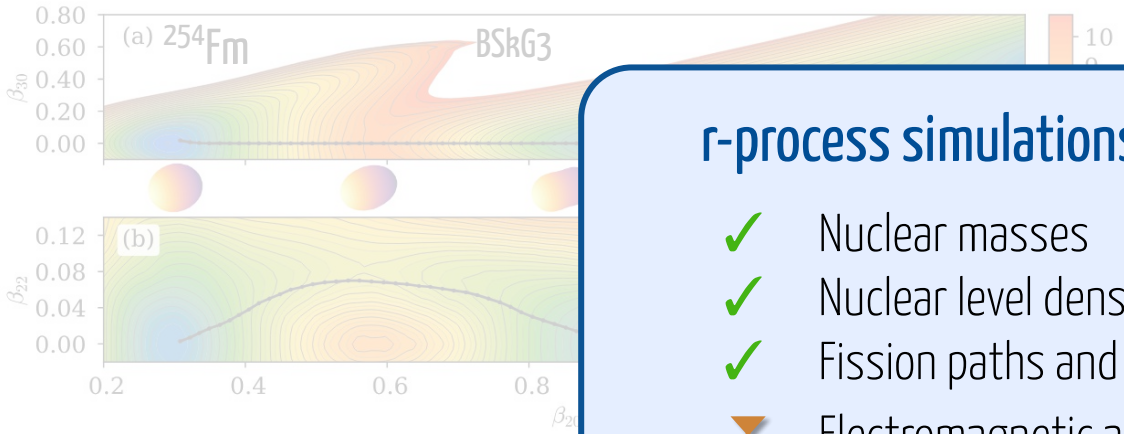

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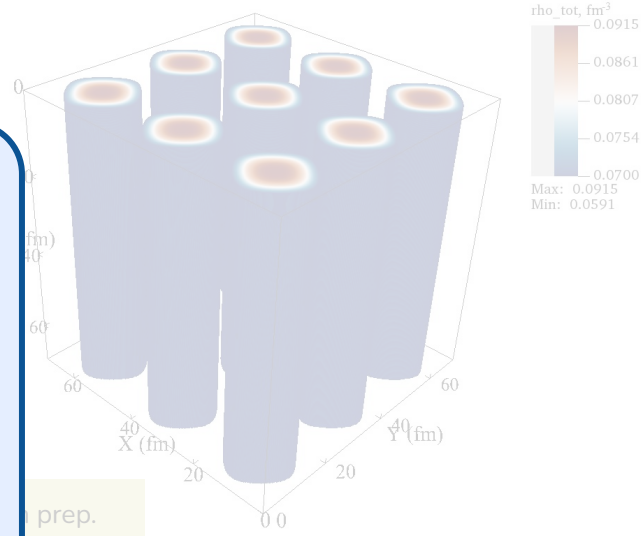

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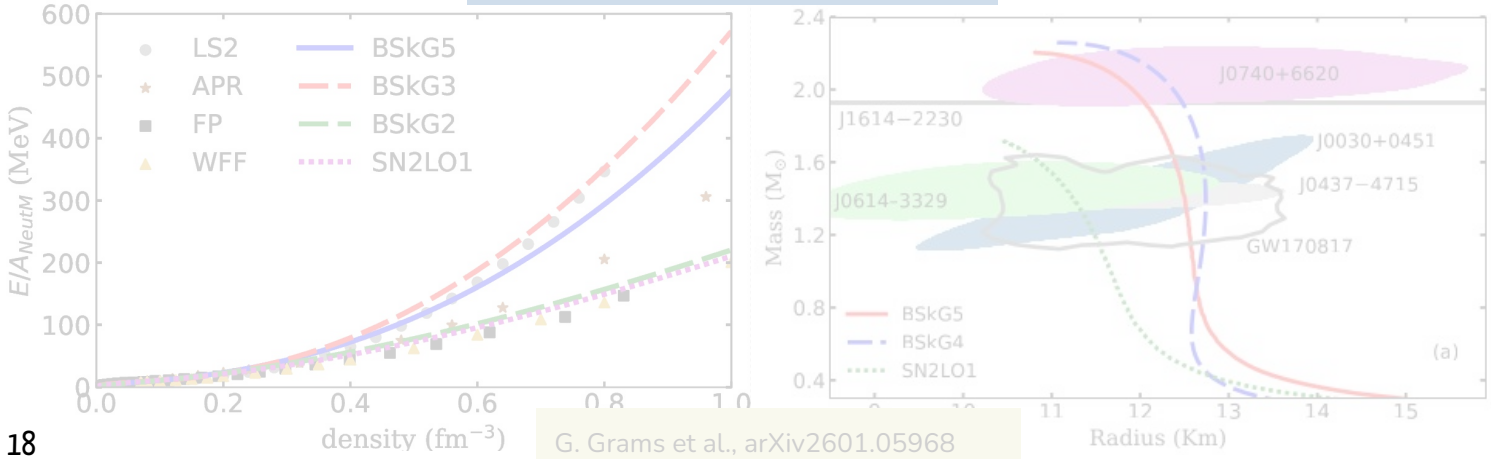

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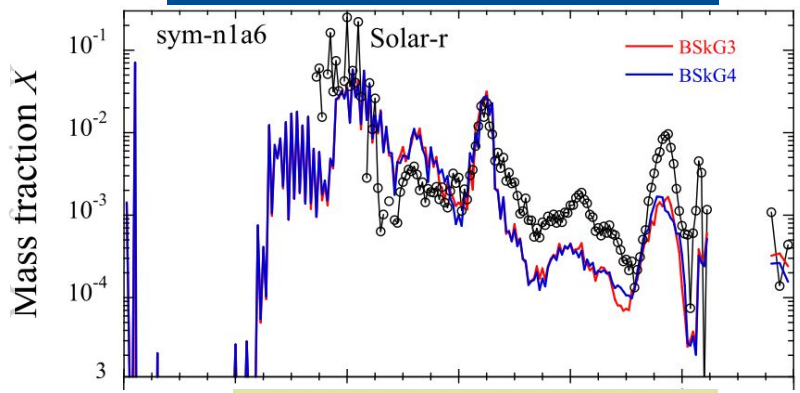

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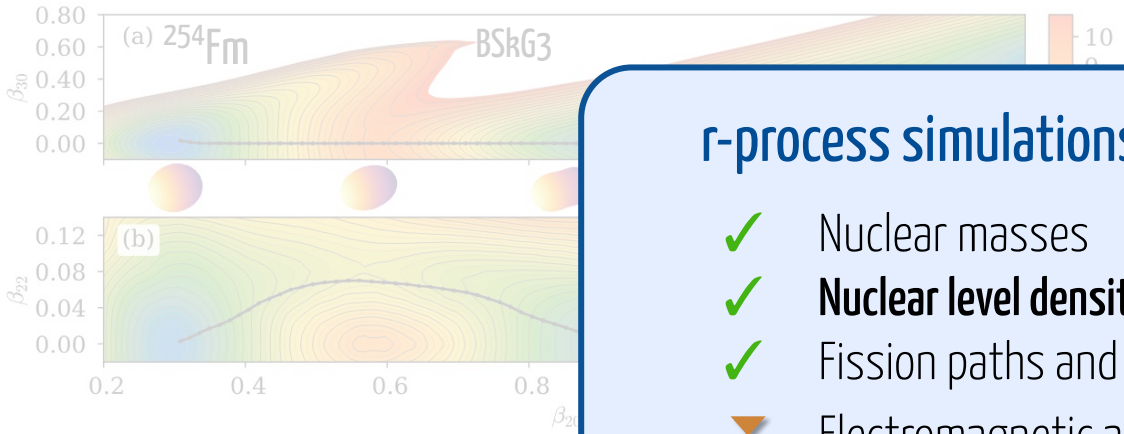

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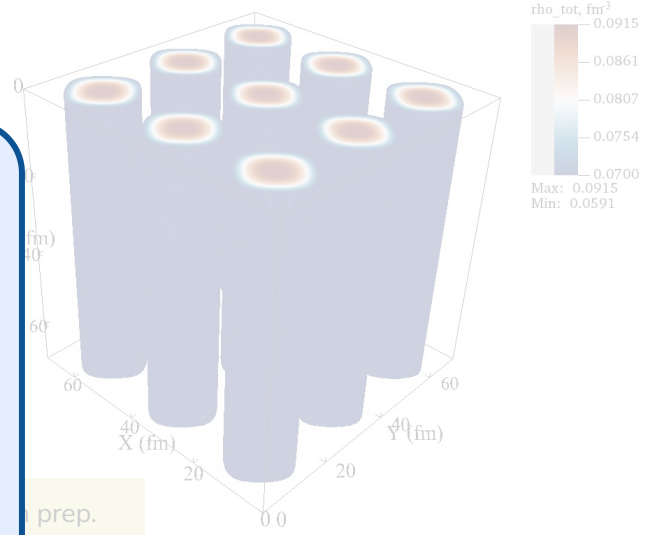

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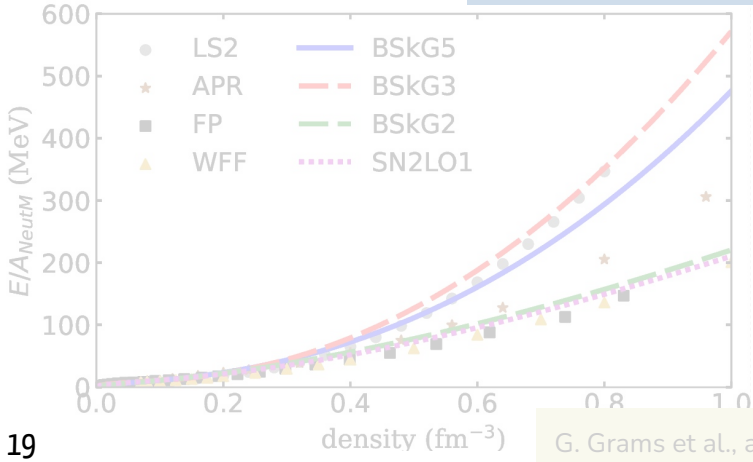

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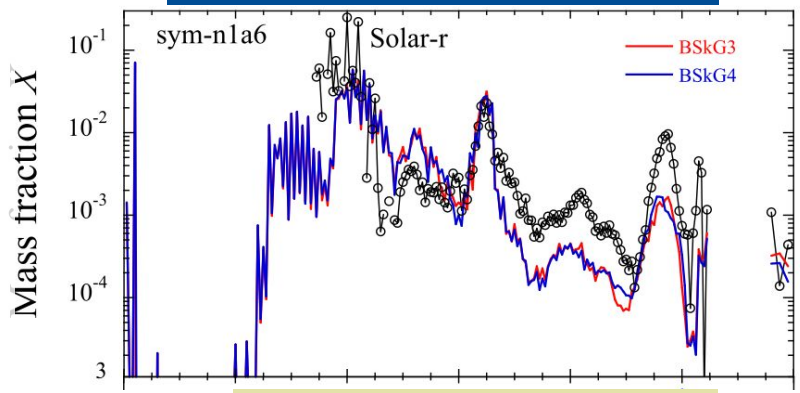


G. Grams et al., arXiv2601.05968

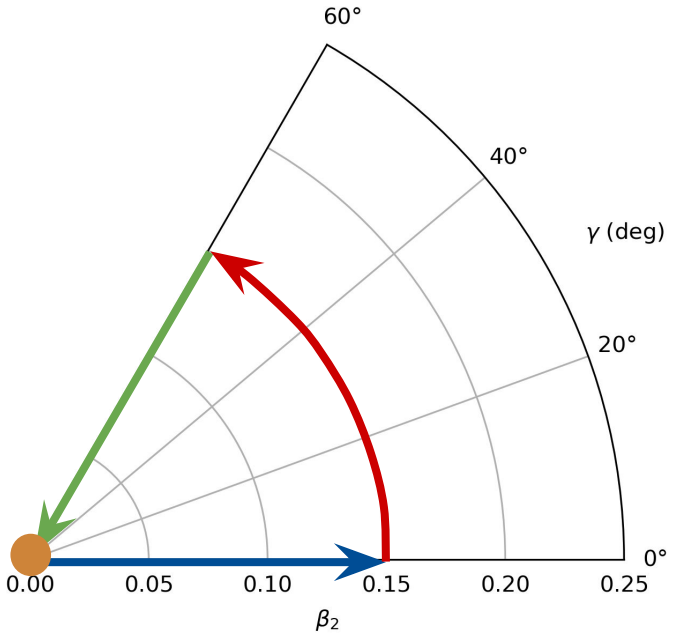
S. Goriely, W. Ryssens, S. Hilaire, A.J. Koning, PRC **113** 014320 (2026)

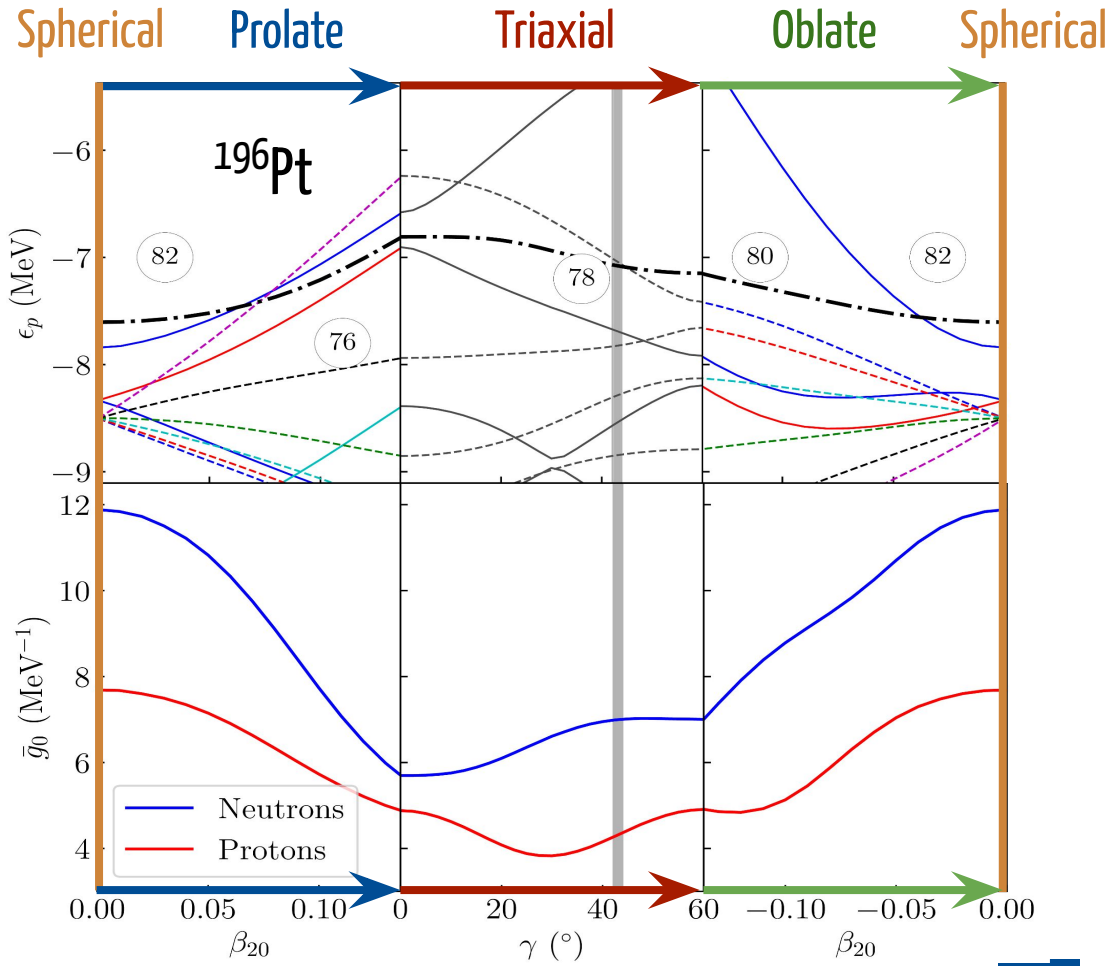
Impact of triaxiality on nuclear level densities

BSkG3 NLDs built from **HFB** mean-field spectrum + **combinatorial** method to account for collective states

Triaxiality leads to **two competing effects** on NLDs

↘ Sparser single-particle level densities **decrease** NLDs





S. Goriely, W. Ryssens, S. Hilaire, A.J. Koning, PRC **113** 014320 (2026)

Impact of triaxiality on nuclear level densities

BSkG3 NLDs built from **HFB** mean-field spectrum + **combinatorial** method to account for collective states

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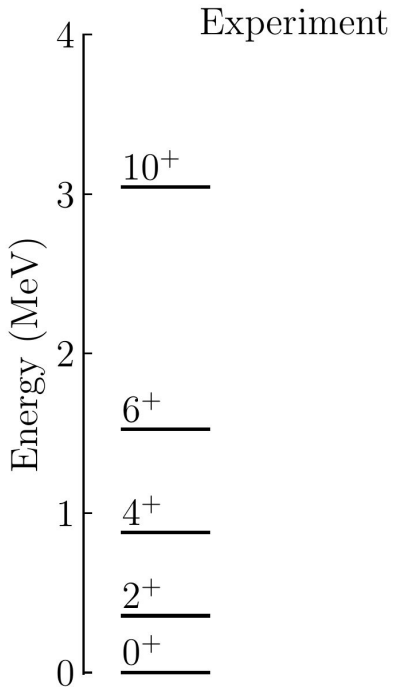

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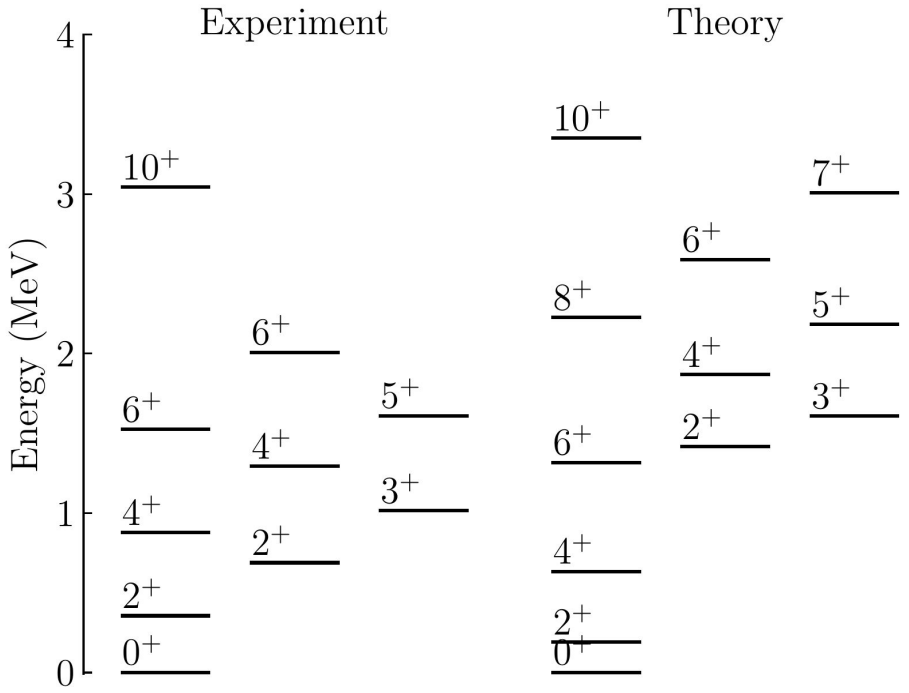

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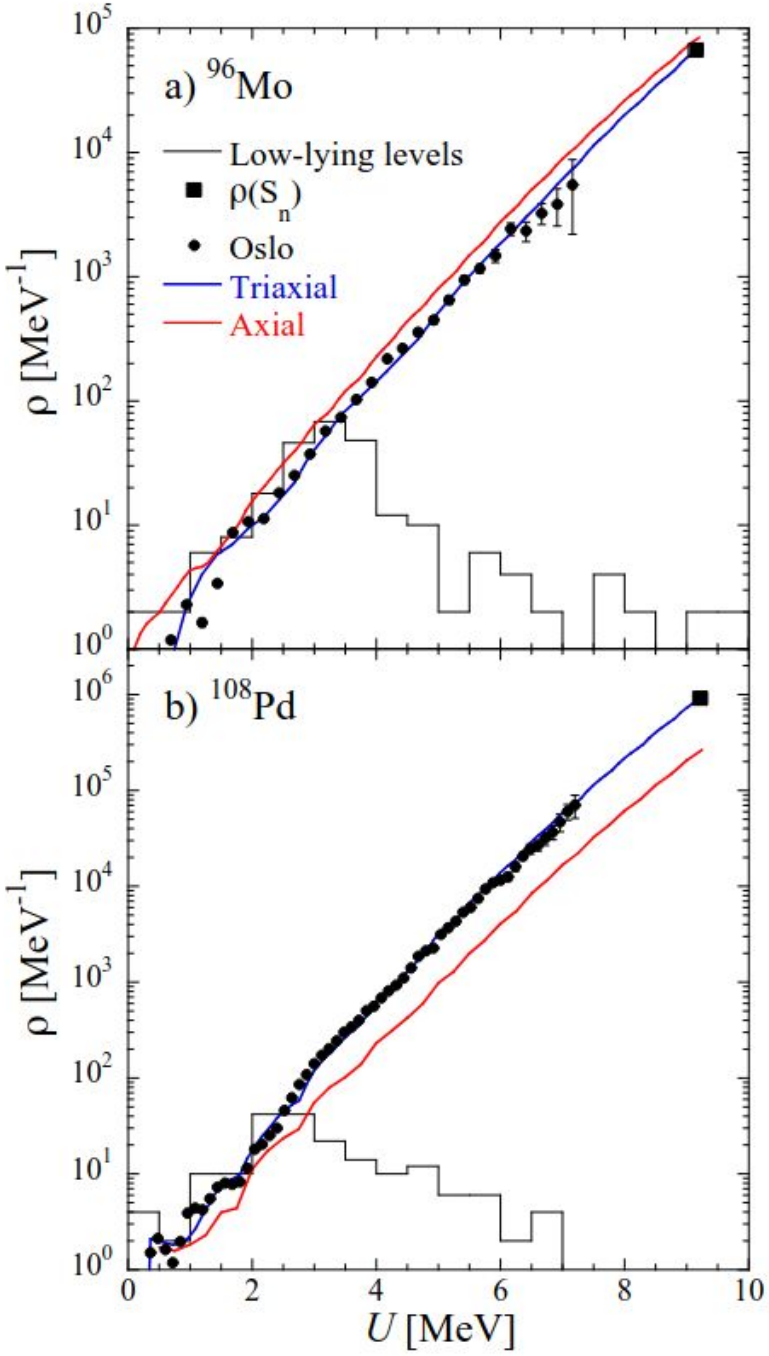
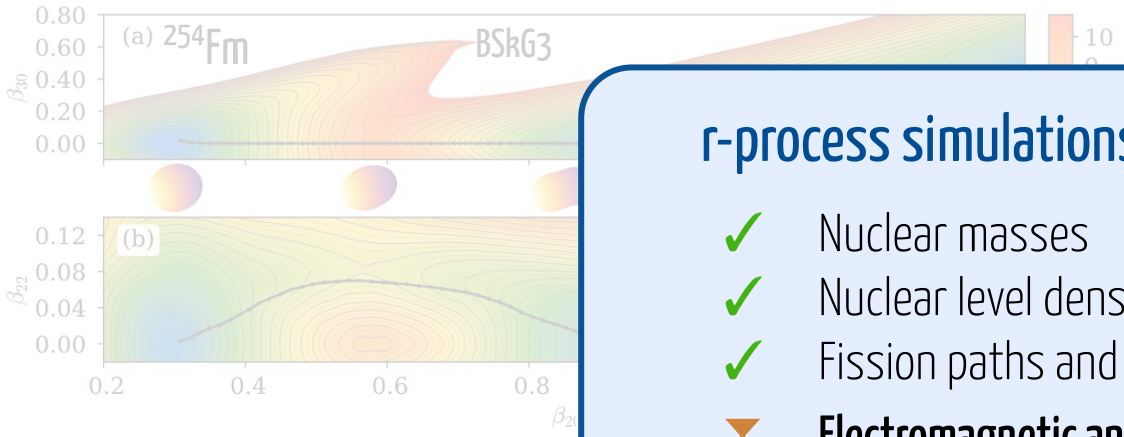
Tables with BSkG3 NLDs $\rho(U, J, \pi)$ for 8500 nuclei available on BRUSLIB & TALYS

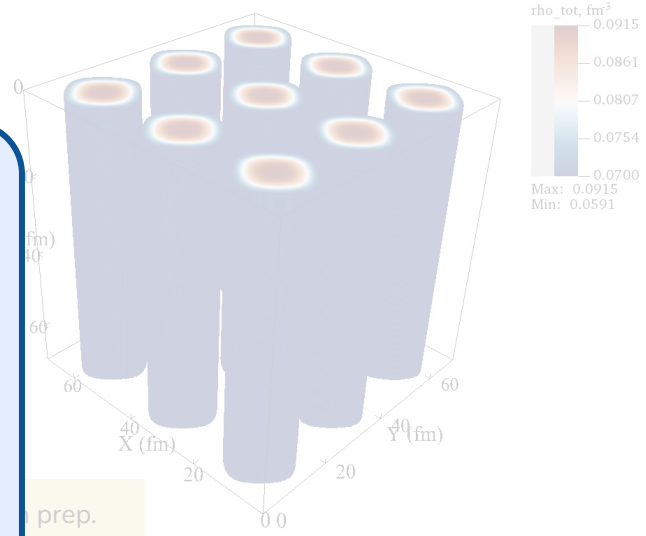


Applications which will not be discussed

Fission



Nuclear Pasta



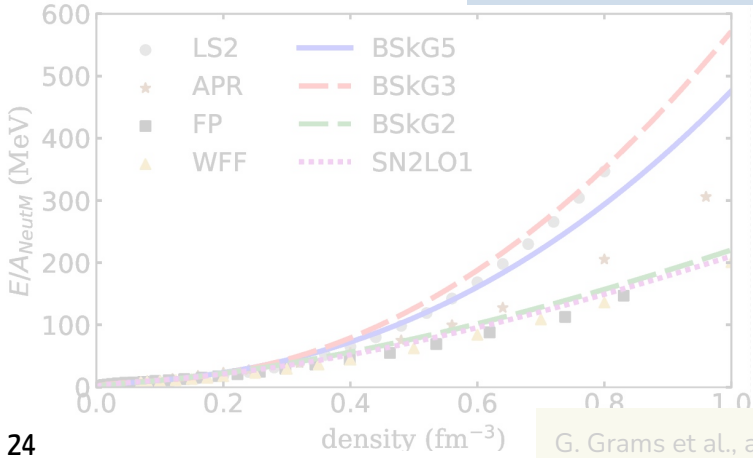
prep.

r-process simulations require **a lot** of data

- ✓ Nuclear masses
- ✓ Nuclear level densities
- ✓ Fission paths and fragment yields
- ⧗ **Electromagnetic and weak strength functions**

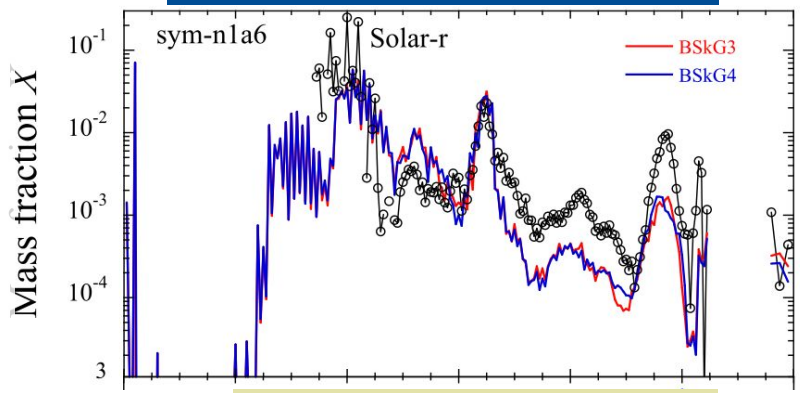
for ALL nuclei

Neutron Equation of State (EoS)



G. Grams et al., arXiv2601.05968

r-process simulations



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Nuclear linear response

1) Solve mean-field problem for ground-state density ρ_0

2) Add a harmonic external perturbation $F(\omega)$, e.g. E0, E1, ..., M1, M2, ...

E.g. isoscalar quadrupole (E2)

$$E[\rho] \to E[\rho_0] + \frac{\partial E[\rho]}{\partial \rho} \cdot \delta\rho(\omega) + F(\omega)$$

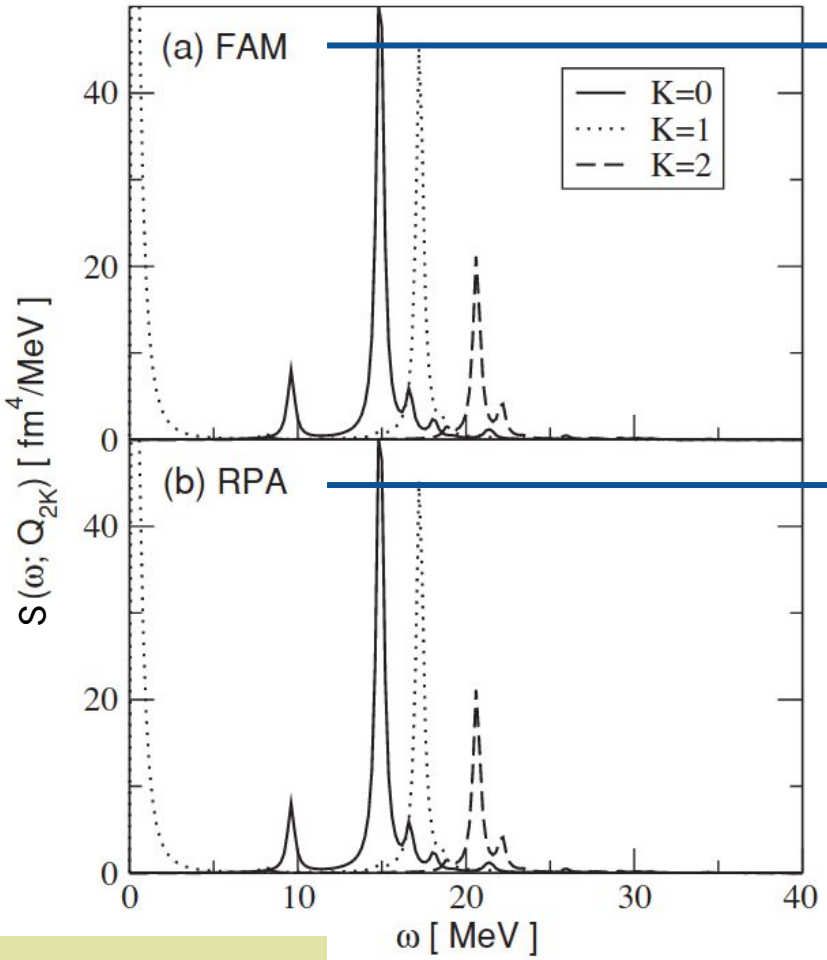
Static mean-field solution

Harmonic oscillations around ρ_0
→ in terms of **1-particle 1-hole excitations**

⇒ Response quantified by the **strength function** $S(\omega, F) = \mathrm{Tr}(F(\omega)\ \delta\rho(\omega))$

Nuclear linear response

Example: isoscalar quadrupole response of ^{20}Ne



Recently implemented in our 3D solver

Finite amplitude method

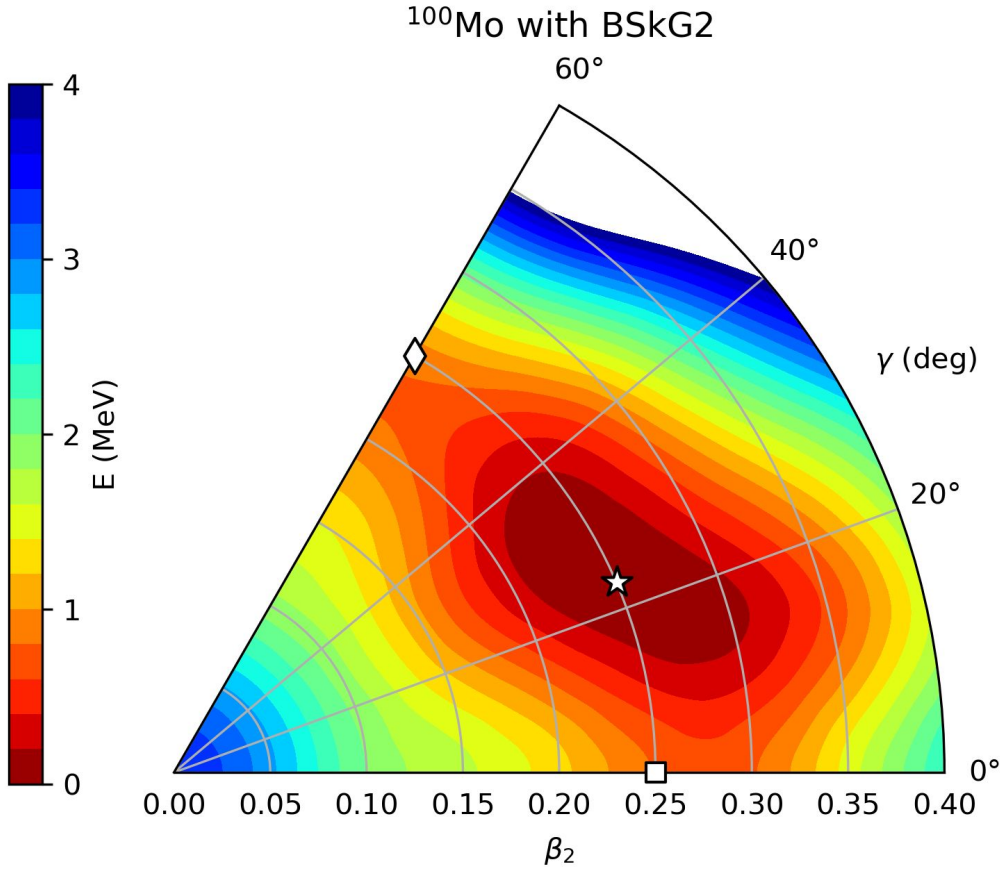
More affordable

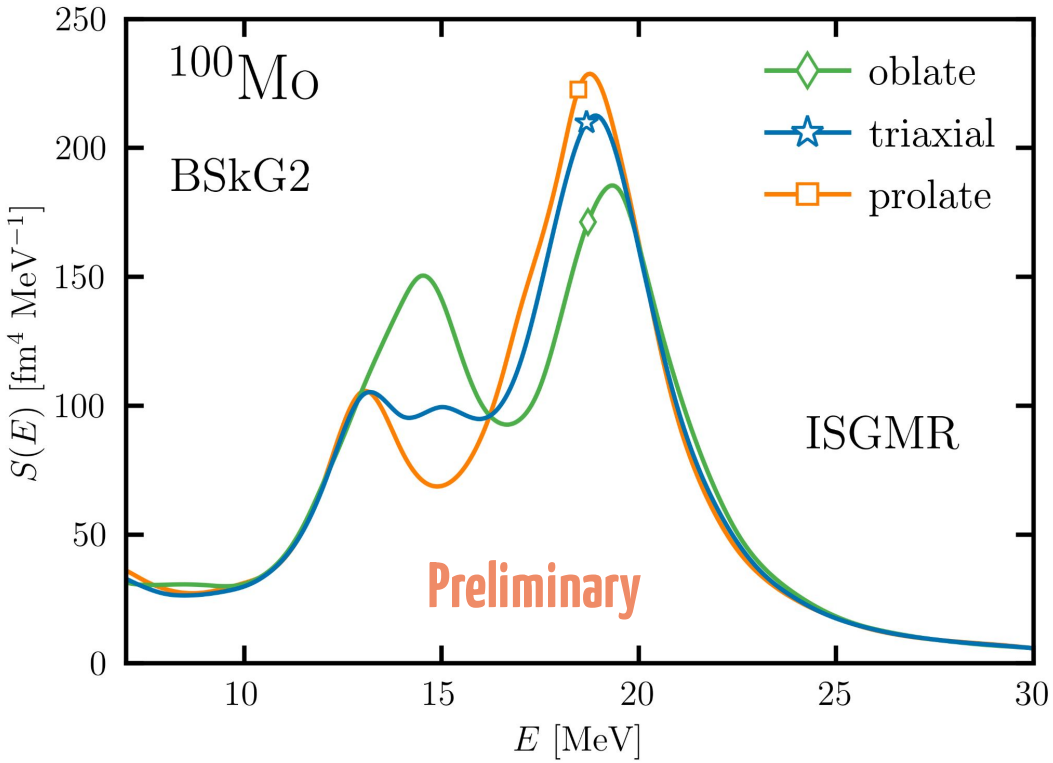
Numerically costly

Random phase approximation

Adapted from Nakatsukasa et al. PRC **76** (2007)

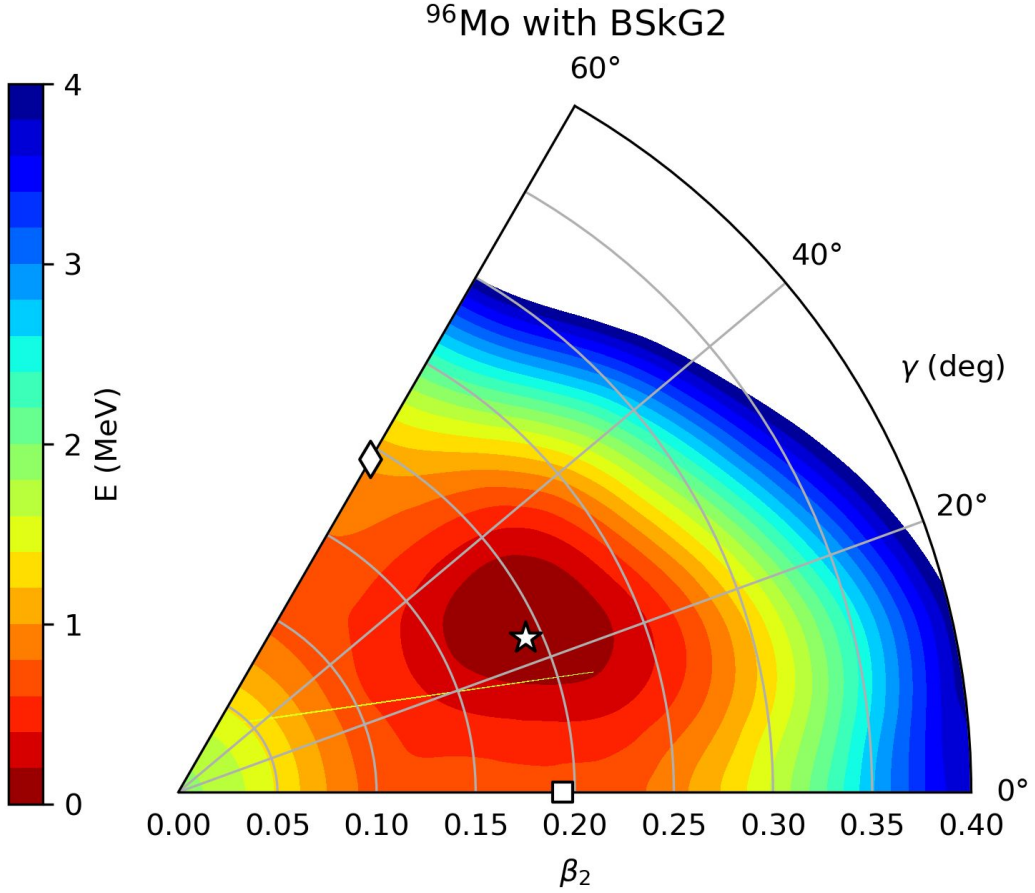
Impact of triaxiality on isoscalar giant monopole resonance (ISGMR)

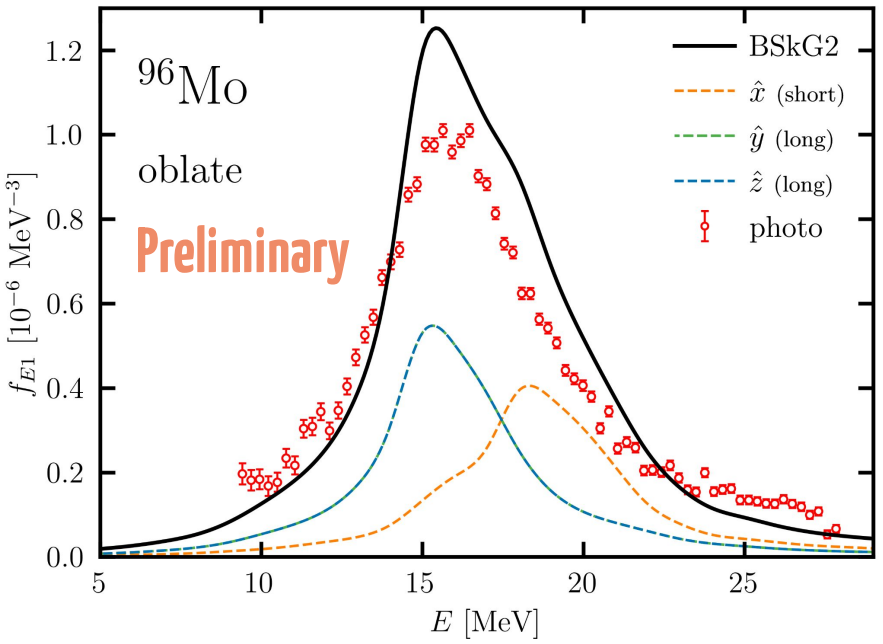


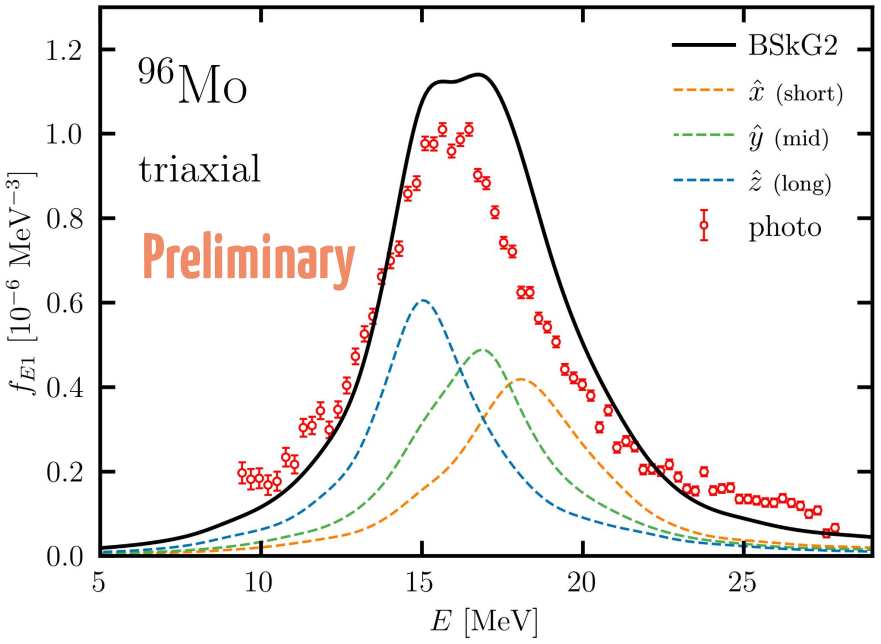


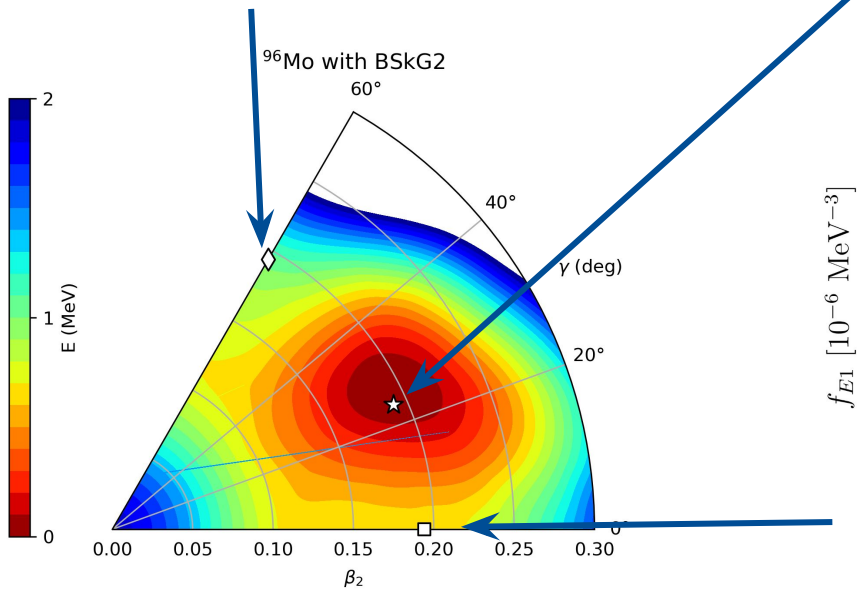
Appearance of a third peak, in accordance with Washiyama et al PRC 109 (2024)

Impact of triaxiality on isovector giant dipole resonance (IVGDR)




^{96}Mo
oblate
Preliminary
BSkG2
$\hat{x}$ (short)
$\hat{y}$ (long)
$\hat{z}$ (long)
photo
f_{E1} [10^{-6} MeV^{-3}]
E [MeV]


^{96}Mo
triaxial
Preliminary
BSkG2
$\hat{x}$ (short)
$\hat{y}$ (mid)
$\hat{z}$ (long)
photo
f_{E1} [10^{-6} MeV^{-3}]
E [MeV]


^{96}Mo with BSkG2
60°
40°
20°
0°
γ (deg)
E (MeV)
β_2

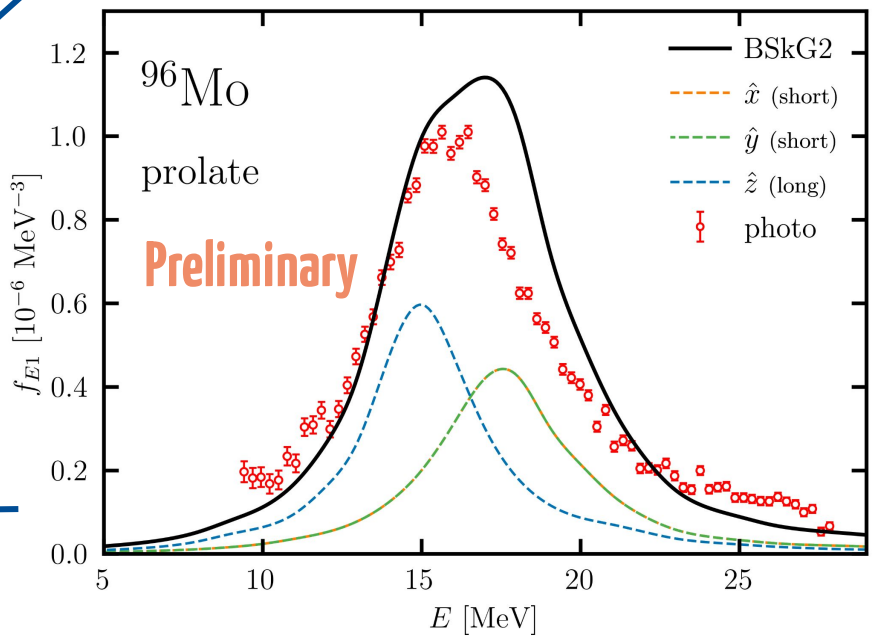

^{96}Mo
prolate
Preliminary
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photo
f_{E1} [10^{-6} MeV^{-3}]
E [MeV]

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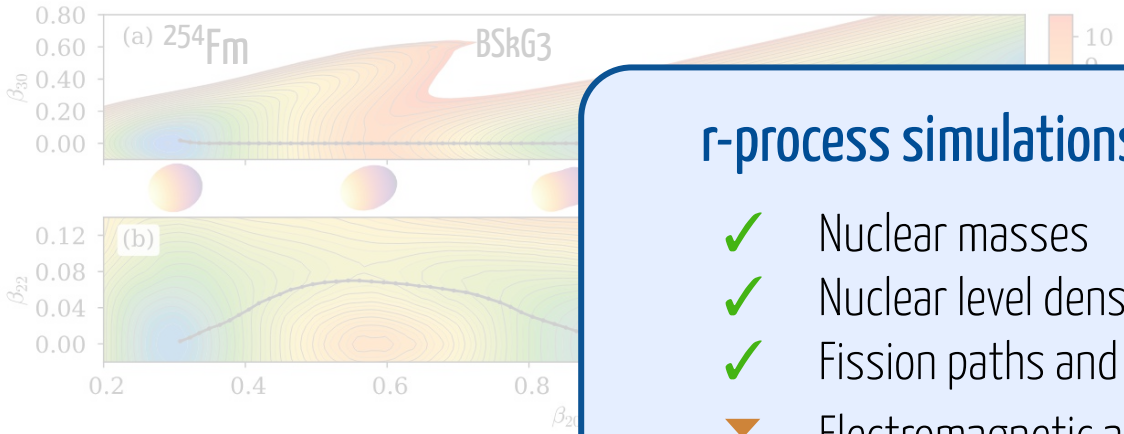
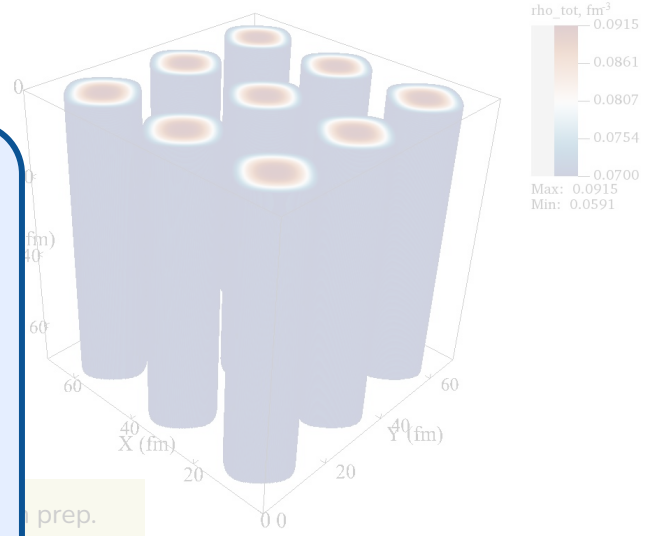


Nuclear Pasta



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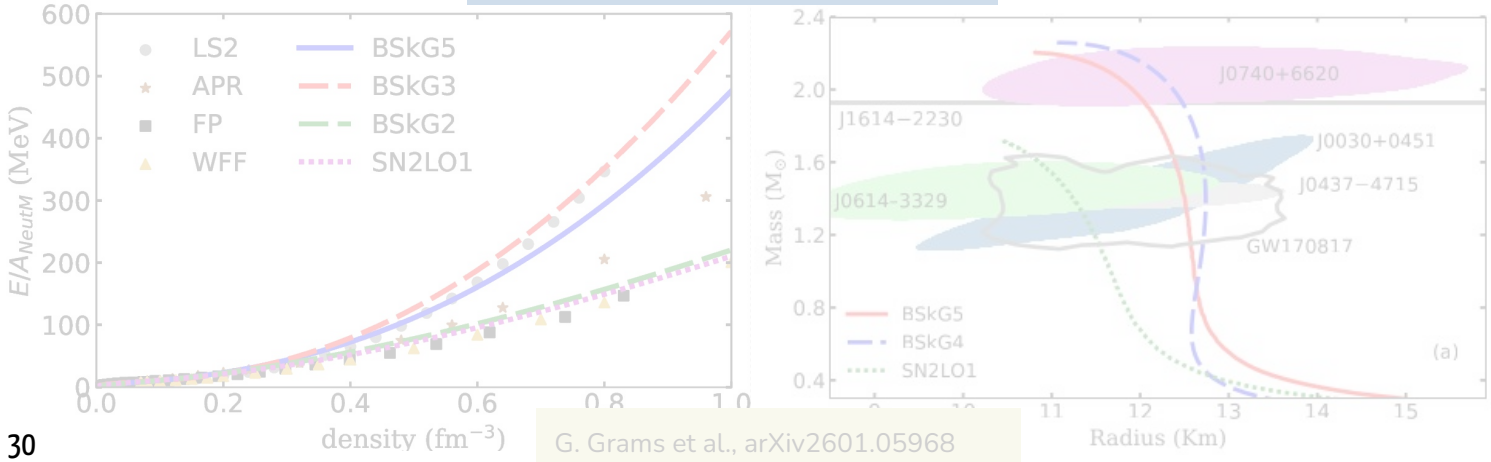
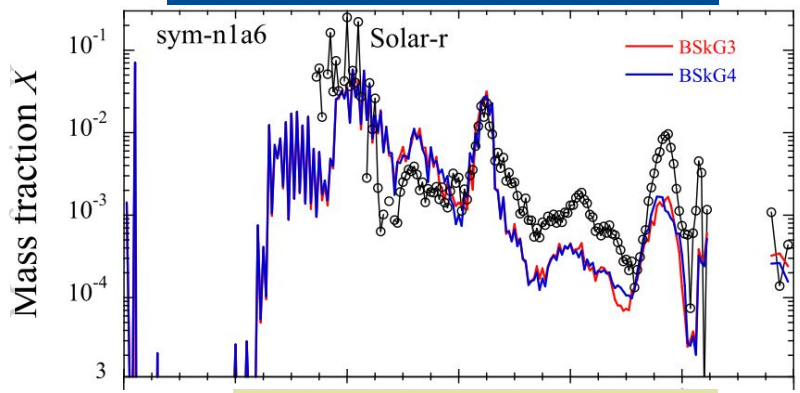


G. Grams et al., arXiv2601.05968

r-process simulations



G. Grams et al., arXiv2601.05968

Upscaling FAM via reduced order model

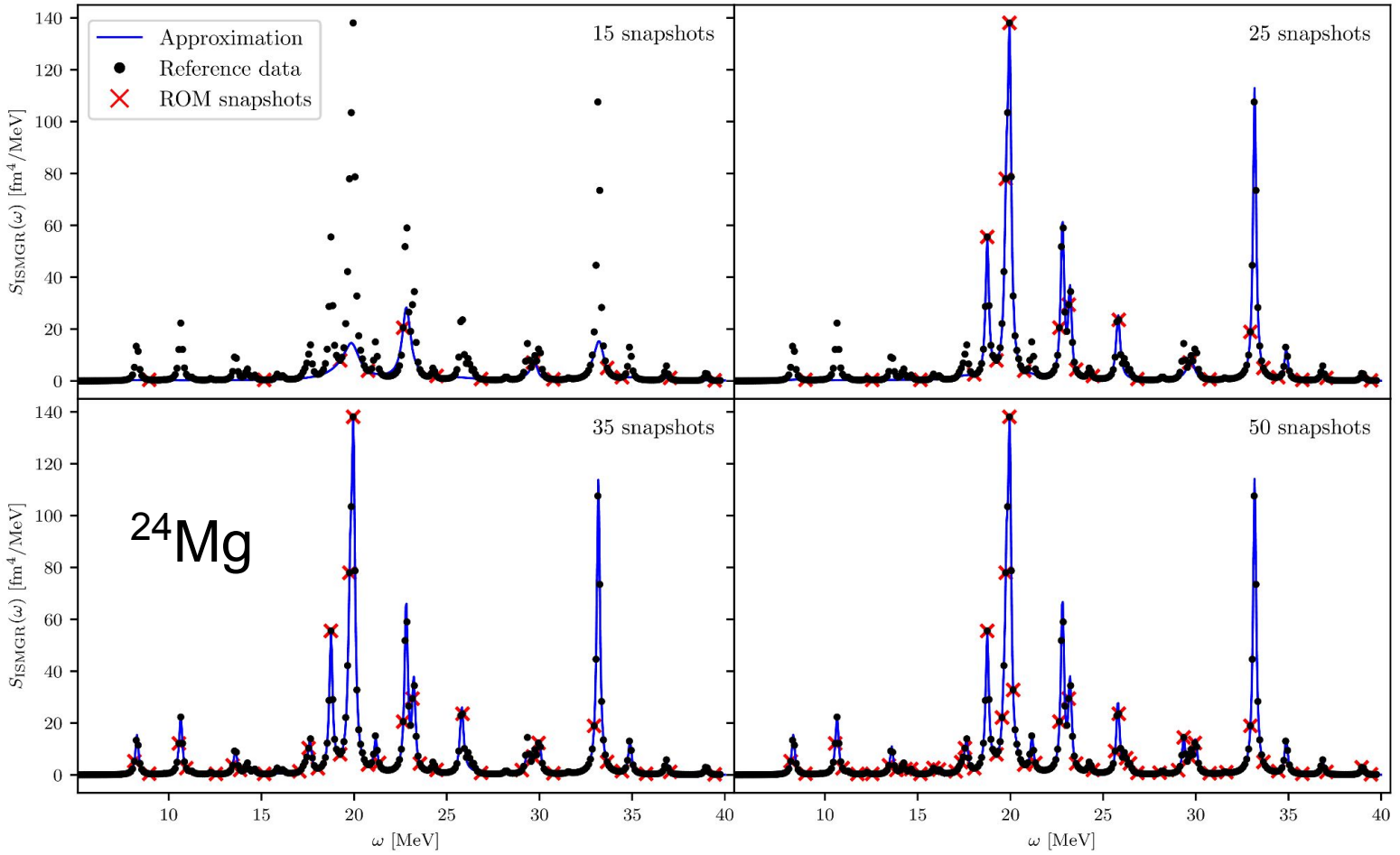
Work of master's student **Emma Vancayseele**

Solve FAM for few snapshot frequencies ω_i

$\longrightarrow$ Interpolate X and Y amplitude for any ω

$\longrightarrow$ Efficient interpolation of strength function

+ Extrapolations in complex frequency plane



Global evaluation of E1 strength in the near future !

Thanks to **Wouter Ryssens** for several slides

Conclusion

BSkG provides **large-scale**, **microscopic** model of nuclear structure

- **Large-scale** : **thousands** of nuclei and **many observable**
- **Microscopic** : simple wave functions yet complex **symmetry breaking** (triaxial, octupole, time-reversal)
- with accurate bulk predictions of **masses, radii, fission properties**
- Recently extended to **nuclear level densities** and **photo strength functions**

Triaxiality leaves its fingerprints on **many** nuclear observables : radii, fission barriers, NLDs, PSFs

Outlook

- **Global evaluation of E1 strength** with BSkG under way!
- Implementation of **magnetic** operators
- Investigation of **scissor modes** with triaxial+octupole deformation

Thanks to all my collaborators

N. Chamel
L. González-Mirét-Zaragoza
S. Goriely
G. Grams
A. Sánchez-Fernández
N. Shchechilin
E. Schroeder
W. Ryssens

M. Bender

E. Vancayseele

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Computational resources