Configuration Interaction Shell Model Studies of Photon Strength Functions

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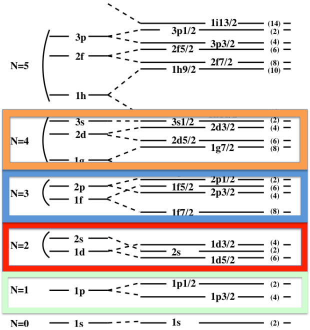
Configuration Interaction Shell Model

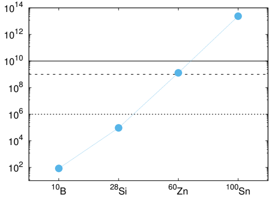
$$|\Psi_0\rangle = C_0|\Phi_0\rangle + \sum_{i\alpha} C_{i\alpha}|\Phi_{i\alpha}\rangle + C_{ij\alpha\beta}|\Phi_{ij\alpha\beta}\rangle + \cdots = \sum_{ph} C_{ph}|\Phi_{ph}\rangle$$

$$E = \langle\Psi_0|\hat{H}|\Psi_0\rangle = \sum_{pp'hh'} C^*_{p'h'}\langle\Phi_{p'h'}|\hat{H}|\Phi_{ph}\rangle C_{ph}$$



EXACT solution for a given Hamiltonian: all possible many-body states included

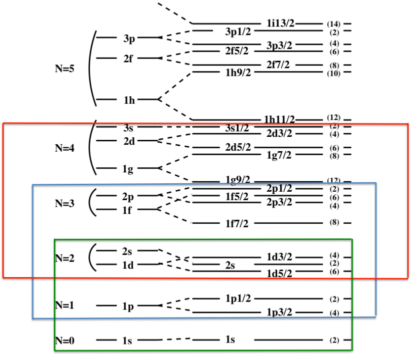
- To describe low-energy spectroscopy, $0\hbar\omega$ model spaces are often sufficient
- $0\hbar\omega$ CI-SM dimensions:



10^6: feasible on a laptop
10^9: standard for M-scheme SM codes
10^{10}: current limit

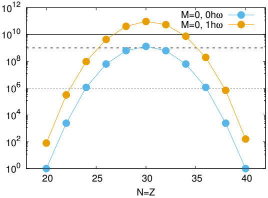
- Many empirically-adjusted SM interactions of good quality publicly available in classical model-spaces, i.e. *p*, *sd*, *pf*, *gds* shells

Configuration Interaction Shell Model



Coupling to 2p-2h and other many-body states automatically included

- To describe $E1$ transitions, $1\hbar\omega$ model spaces needed
- CI-SM dimensions in the *pf*-shell:



10^6: feasible on a laptop
10^9: standard for M-scheme SM codes
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- SM interactions of good quality rare in multi-shell valence spaces
- Few applications to photonuclear reactions

Lanczos strength function method

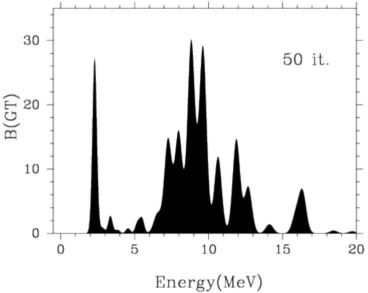
$$S = |\hat{O}|\psi_i\rangle| = \sqrt{\langle\psi_i|\hat{O}^2|\psi_i\rangle}$$

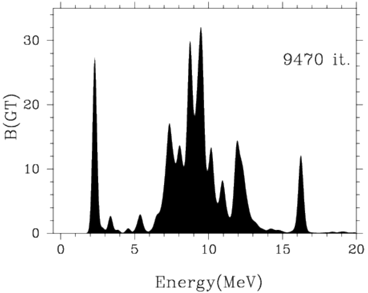
The operator $\hat{O}$ does not commute with H and $\hat{O}|\psi_i\rangle$ is not necessarily the eigenstate of the Hamiltonian. But it can be developed in the basis of energy eigenstates:

$$\hat{O}|\psi_i\rangle = \sum_f S(E_f)|E_f\rangle,$$

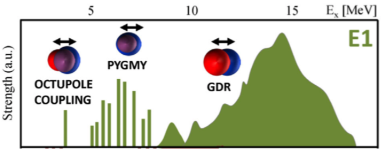
where $S(E_f) = \langle E_f|\hat{O}|\psi_i\rangle$ is called **strength function**.

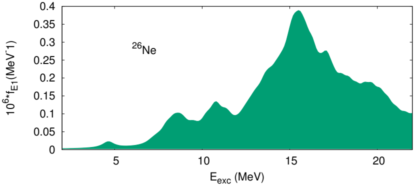
If we carry Lanczos procedure using $|O\rangle = \hat{O}|\psi_i\rangle$ as initial vector then H is diagonalized to obtain eigenvalues $|E_f\rangle$ and after N iterations we have the also the strength function:
$\tilde{S}(E_f) = \langle E_f|O\rangle = \langle E_f|\hat{O}|\psi_i\rangle$.

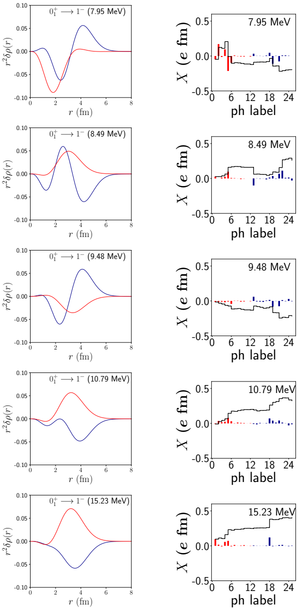




PDR in ^{26}Ne: a shell-model perspective







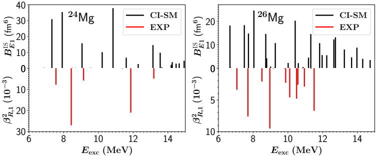
PDR characteristics:

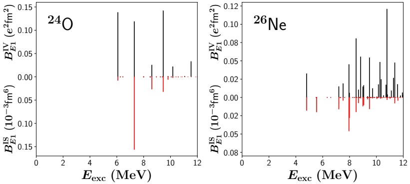
- carries a few % of EWSR
- distinct from the tail of the GDR
- moderately collective, not single-particle
- consistent with the “classic” neutron-skin oscillation picture
- of isoscalar-isovector character

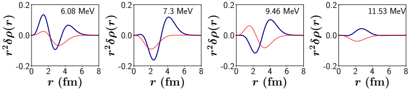
PDR in ^{26}Ne: IS vs IV excitations

$$\hat{O}^{IV}_{1\mu} = -e\frac{Z}{A}\sum_{i=1}^{N} r_i Y_{1\mu}(\hat{r}_i) + e\frac{N}{A}\sum_{i=1}^{Z} r_i Y_{1\mu}(\hat{r}_i)$$

$$\hat{O}^{IS}_{1\mu} = \sum_{i=1}^{A} r_i^3 Y_{1\mu}(\hat{r}_i)$$







Transition densities of selected states in ^{24}O

CI-SM and QRPA isoscalar strength (in fm^6) summed up to 16MeV

Nucleus	QRPA	CI-SM
^{24}Mg	162	176
^{26}Mg	230	223
^{28}Si	189	251

P. Adsley et al., Phys. Rev. C103, 044315 (2021)

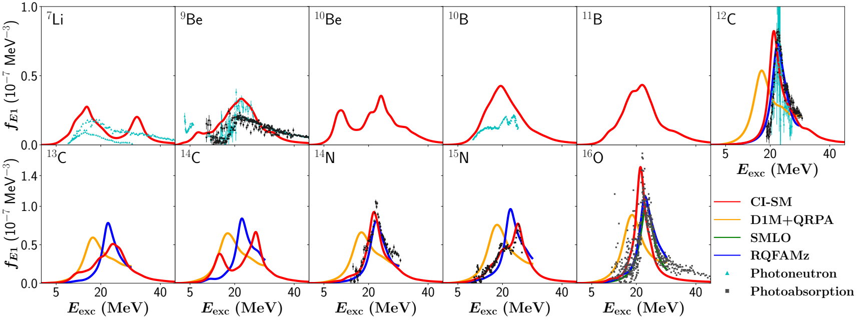
- Strong isoscalar transitions in the PDR region, no isoscalar reponse in the GDR tail
- Isospin-mixing of the PDR, isospin-splitting due to passing to the GDR tail

O. Le Noan and K. Sieja, to be submitted (2026)

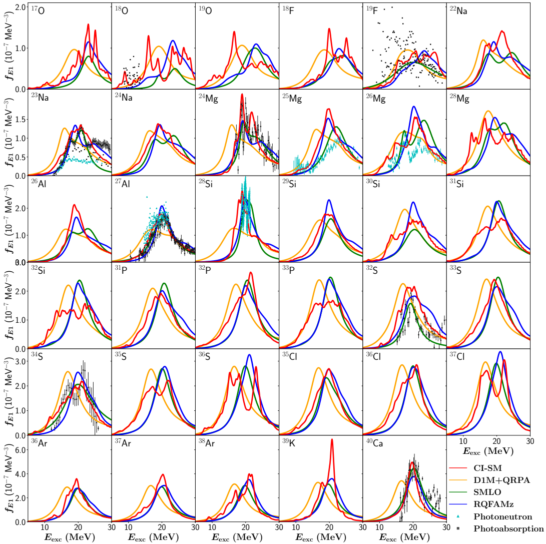
Systematics of $E1$ strength in p and sd-shell nuclei

- CI-SM study of $E1$ strength of 137 nuclei from ^{7}Li to ^{40}Ca
- use of well-established empirical interactions WBP and PSDPF
  E. K. Warburton and B. A. Brown, Phys. Rev. C 46, 923 (1992)
  M. Bouhelal et al., Nuc. Phys. A 864, 113 (2011)
- PSF available (TALYS database)

- Comparison to available experimental data and QRPA models:
  D1M+QRPA: S. Goriely, S. Hilaire, S. Péru, and K. Sieja, PRC 98, 014327 (2018)
  RQFAMz: L. Gonzalez-Miret Zaragoza, J.-P. Ebran, S. Goriely, S. Hilaire, E. Khan, and S. Péru, PRC 112, 044303 (2025)

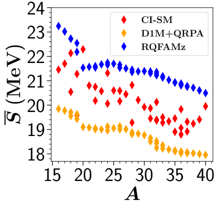


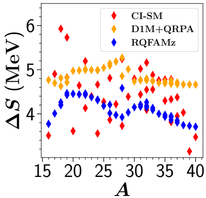
Systematics of $E1$ strength in p and sd-shell nuclei



RMS for 26 nuclei

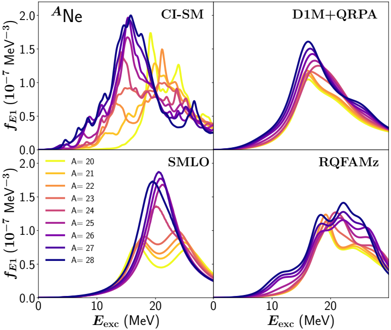
Model	$\sigma_{\bar{S}}$	$\sigma_{\Delta S}$
CI-SM	0.85	0.51
D1M + QRPA	1.71	0.49
RQFAMz	0.89	0.48

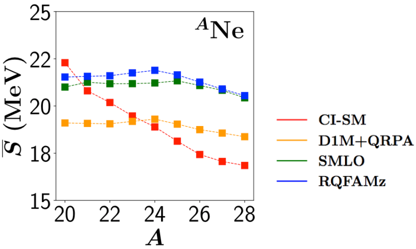




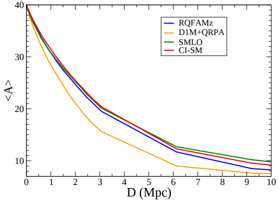
Systematics of $E1$ strength in p and sd-shell nuclei

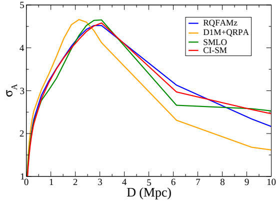
Predictions in neutron-rich nuclei





Application to UHECR propagation

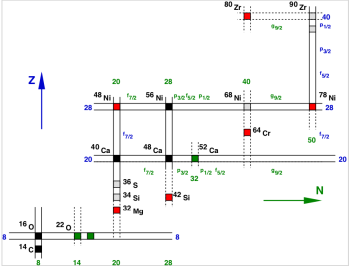


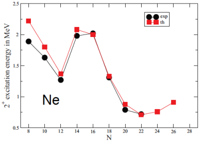


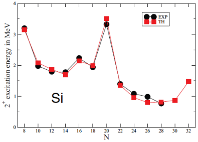
O. Le Noan, E. Khan, S. Goriely and K. Sieja, Phys. Rev. C113 (2026) 044319

Systematics of $E1$ strength: islands of inversion

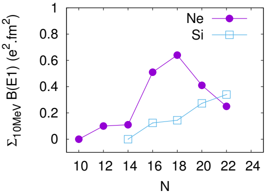
Predictions in neutron-rich nuclei

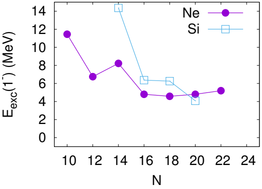






$N = 20$ shell closure washed out in Ne, but not in Si

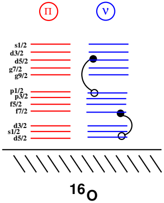


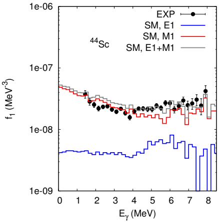


E1 strength moves up in the island of inversion

Work in progress: PDR (?) in deformed nuclei

$E1$ strength in pf-shell nuclei: work in progress

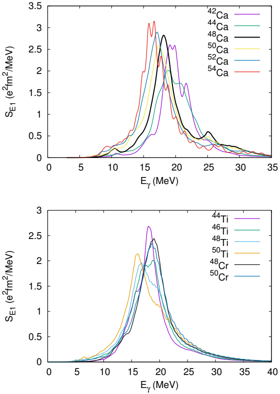




K. Sieja, PRL119 (2017) 052502

- *fp*-calculations for positive parity states
- $1\hbar\omega$ calculations for negative parity states
- Interaction: vlowk + empirical corrections

Preliminary results



- Systematics of low-energy, abnormal-parity excitations in the mass region A=40-80 (improvement of effective Hamiltonian)
- Systematics of PSF for nuclei up to $A \sim 50$ (renormalization of the effective operator)
- Analysis of PDR modes in the Ca chain

$M1$ strength in valence-spaces: FT-QRPA vs CI-SM

- FT-QRPA thermal excited states $|\mu\rangle \equiv \Gamma^{\dagger}_{\mu}|\Phi(T)\rangle$ are parametrized by finite temperature amplitudes as

$$\Gamma^{\dagger}_{\mu} \equiv \frac{1}{2}\sum_{ij}\left[P^{\mu}_{ij}\beta^{\dagger}_{i}\beta_{j} + X^{\mu}_{ij}\beta^{\dagger}_{i}\beta^{\dagger}_{j} - Y^{\mu}_{ij}\beta_{j}\beta_{i} - Q^{\mu}_{ij}\beta_{j}\beta^{\dagger}_{i}\right]$$

The strength function reads

$$S_F(\omega) = -\frac{1}{\pi(1-e^{-\omega/k_BT})}\mathrm{Im}\chi_F(\omega)$$

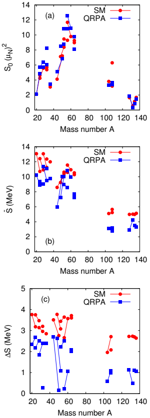
Absorption ($\omega > 0$) Deexcitation ($\omega < 0$)

- different model spaces and SM interactions
- codes HF-SHELL-v2 & PAN@CEA

Approximations of QRPA:

1. Limitation to 2qp excitations
   no explicit p-n correlations, no restoration of broken symmetries
2. Quasi-Boson Approximation
3. Linear response

M. Frosini, W. Ryssens and K. Sieja, Phys. Rev. C110 (2024) 014307



- Too low centroids (lack of correlations)
- Too narrow distributions (2qp only)

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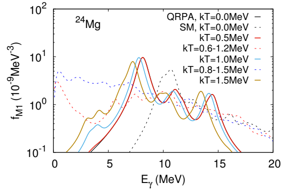
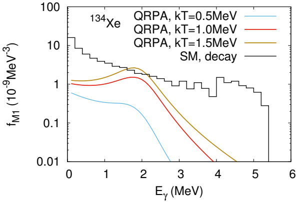




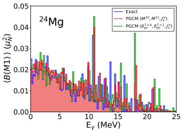
- Too low centroids (lack of correlations)
- Too narrow distributions (2qp only)
- No LEE (too low level density)

$M1$ strength in valence-spaces: PGCM vs CI-SM

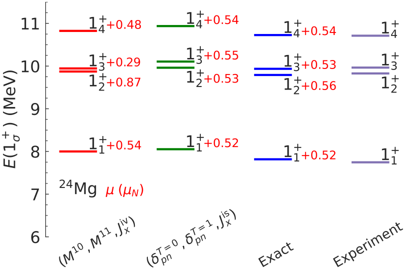
- PGCM uses a set of nonorthogonal, symmetry-projected HFB states:

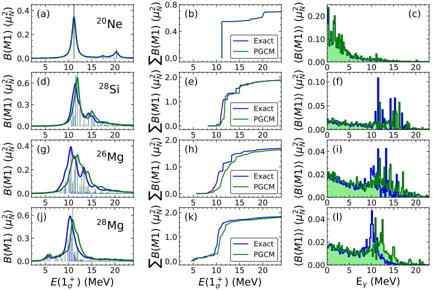
$$|\Psi_\sigma^{JMNZ\pi}\rangle = \sum_{qK} f_{\sigma;qK}^{JMNZ\pi} P_{MK}^J P^N P^Z P^\pi |\phi(q)\rangle$$

  The Hamiltonian is diagonalized within this subspace by solving Hill-Wheeler-Griffin equation.
- sd-shell model space with USDb interaction
- two different sets of generating coordinates
- codes TAURUS & PAN@CEA



^{24}Mg	$J=0^+$	$J=1^+$
M=0	$2.8\cdot10^4$	$2.8\cdot10^4$
J-coupled	1161	3096
PGCM	320	890





S. Bofos, J. Martinez-Larraz, B. Bally, T. Duguet, M. Frosini, T. Rodriguez and K. Sieja, PRC112 (2025) 064312

Conclusions & Perspectives

- CI-SM provides a good quality dipole PSF in addition to its “traditional” applications in low-energy spectroscopy:
  1. Systematics of PSF in light nuclei (up to ^{40}Ca) available
  2. Short-term perspective: $E1$ in low-mass *pf*-shell nuclei and study of PDR in Ca isotopes
  3. Mid-term perspective: remaining *pf*-shell nuclei using importance-truncated CI-SM methods

- Other methods of solution of the many-body problem (PGCM) can be applied in valence spaces with CI-SM Hamiltonians:
  1. Short-term perspective: predictions of $M1$ strength in mid-mass and heavy nuclei (PGCM in valence spaces)

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B. Bally, J. Martinez-Larraz (GSI), T. Rodriguez (Universidad Sevilla)
E. Khan (IJCLab)
W. Ryssens (ULB Brussels)

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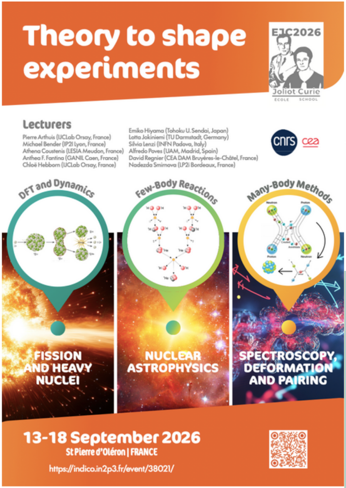
Ecole Joliot-Curie 2026: Theory to shape experiments



Pre-registration open till 31th May

https://indico.in2p3.fr/event/38021/