

# Systematic Study of Nuclear Level Density Models for Unresolved Resonance Region Evaluations using **SAMMY**

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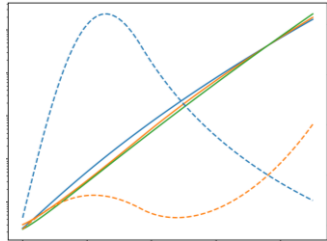


Level density data alone has not constrained LD models [1-3]

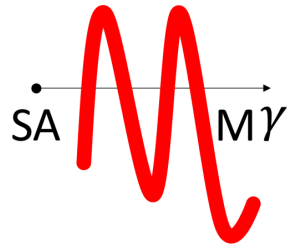
# Hypothesis

Cross section data in the unresolved resonance region (URR) could constrain LD models

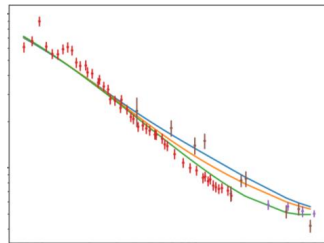
# Outline



Method of Comparing LD Models



Implementation in SAMMY



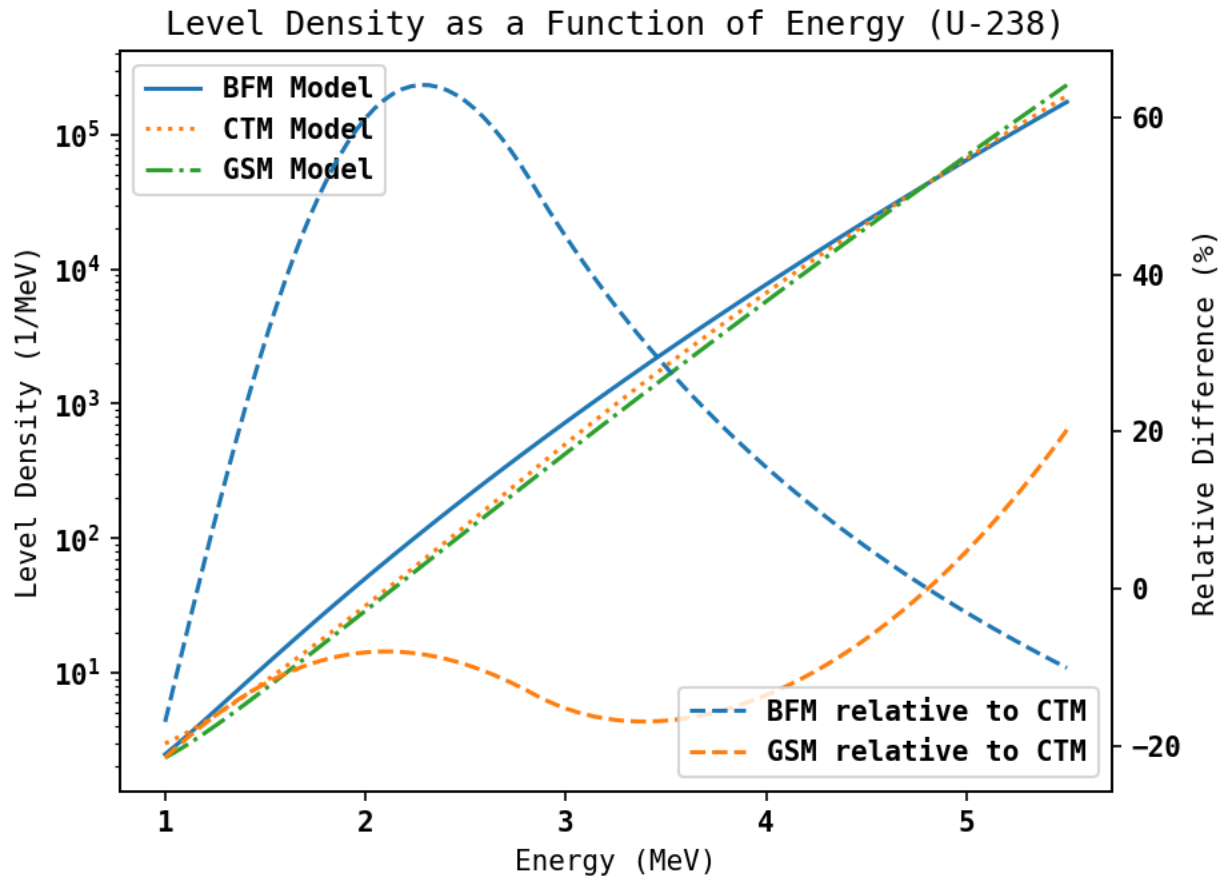
Cross Section Evaluation Results

$\rho(E)$  vs.  $\langle \Gamma_\gamma \rangle$  Conclusions and Future Work

# Comparing LD Models with XS Data

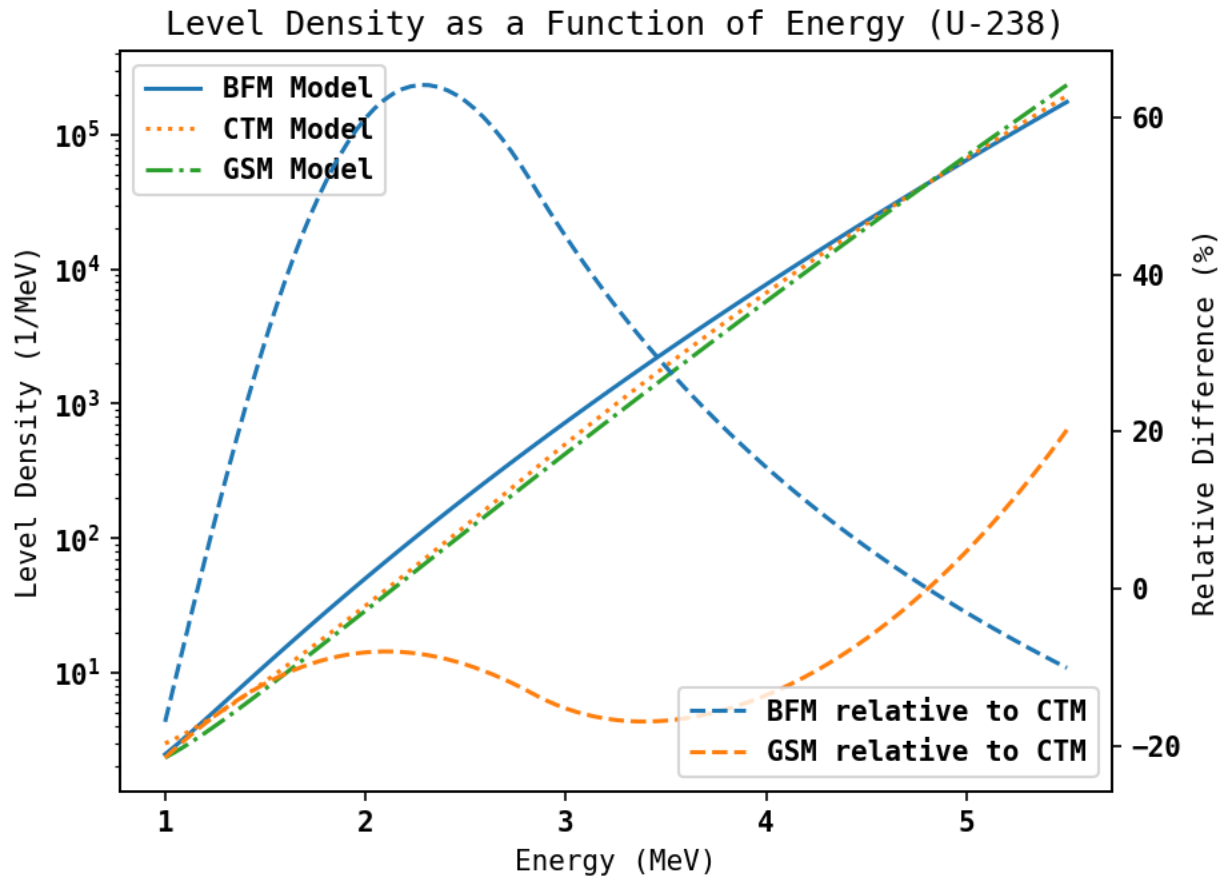
- Hauser-Feshbach Model [4]
- Fit to XS data with multiple LD models
- Determine quality of fit (chi-squared metric)

# Level Density as a Function of Energy



- Exponential spanning many orders of magnitude
- Phenomenological models agree at  $S_n$
- Differences amplified at large energies

# Level Density Models Considered



- Constant Temperature Model (CTM, also called Gilbert-Cameron Composite model) [5]
- Back-shifted Fermi Gas Model (BFM) [6]
- Generalized Superfluid Model (GSM) [7]
- Models also implemented in TALYS [8] and EMPIRE [9]

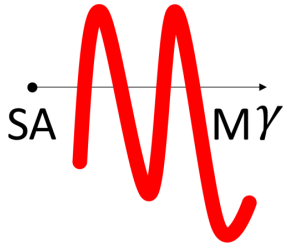
[5] A. Gilbert and A. G. W. Cameron, "A composite nuclear-level density formula with shell corrections," Canadian Journal of Physics, vol. 43, no. 8, pp. 1446–1496, 1965.

[6] M. Grossjean and H. Feldmeier, "Level density of a fermi gas with pairing interactions," Nuclear Physics A, vol. 444, no. 1, pp. 113–132, 1985.

[7] A. Ignatyuk, G. Smirenkin, and A. Tishin, "Phenomenological description of the energy dependence of the level density parameter," Sov. J. Nucl. Phys, vol. 21, p. 255, 1975.

[8] Koning, Arjan, Hilaire, Stephane, and Goriely, Stephane, "TALYS: modeling of nuclear reactions," Eur. Phys. J. A, vol. 59, no. 6, p. 131, 2023.

[9] M. Herman, R. Capote Noy, and M. Sin, "EMPIRE-3.2 malta-modular system for nuclear reaction calculations and nuclear data evaluation," 2013.



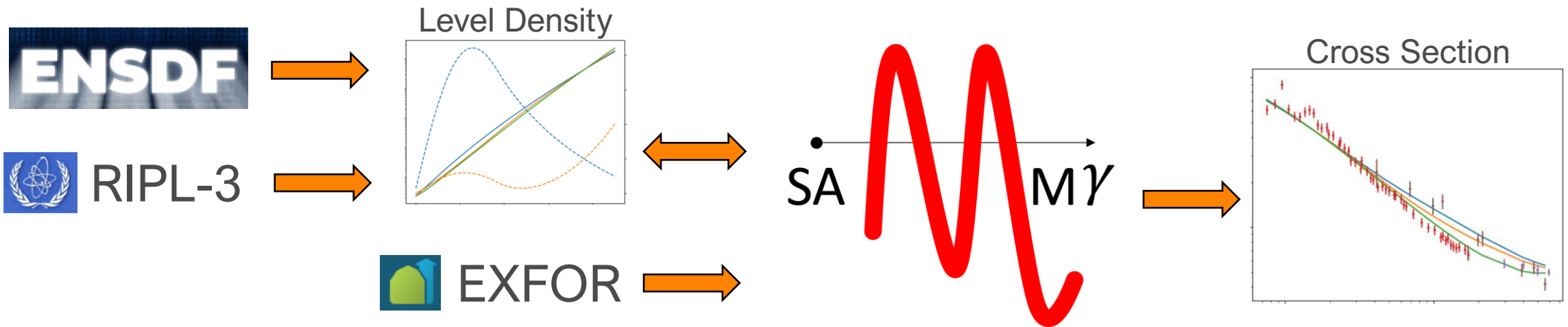
# SAMMY

- Primarily an R-Matrix code developed at ORNL [10]
- Contains a Hauser-Feshbach code by F. Fröhner [11]
- SAMMY in the URR fits average resonance parameters using a Bayesian update process
- Open source

# LD Models in SAMMY

- Previously used CTM with no uncertainty analysis
- LD models are now a user choice
- LD parameters are fit to level density and cross section data
- Only partial cross sections are impacted

# Data Sources and Processing



[12] M. Bhat, "Evaluated nuclear structure data file (ENSDF)," in Nuclear Data for Science and Technology: Proceedings of an International Conference, held at the Forschungszentrum Julich, Fed. Rep. of Germany, 13–17 May 1991. Springer, 1992, pp. 817–821.

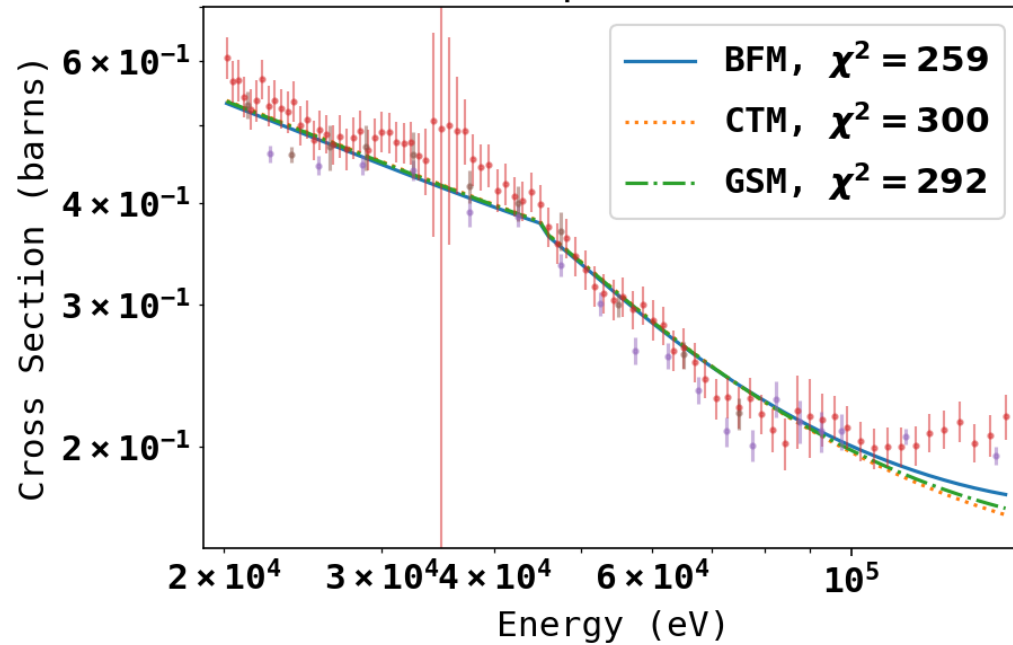
[13] R. e. a. Capote, "RIPL - reference input parameter library for calculation of nuclear reactions and nuclear data evaluations," Nucl. Data Sheets 110, 2009.

[14] V. V. Zerkov and B. Pritychenko, "The experimental nuclear reaction data (EXFOR): Extended computer database and web retrieval system," Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 888, pp. 31–43, 2018.

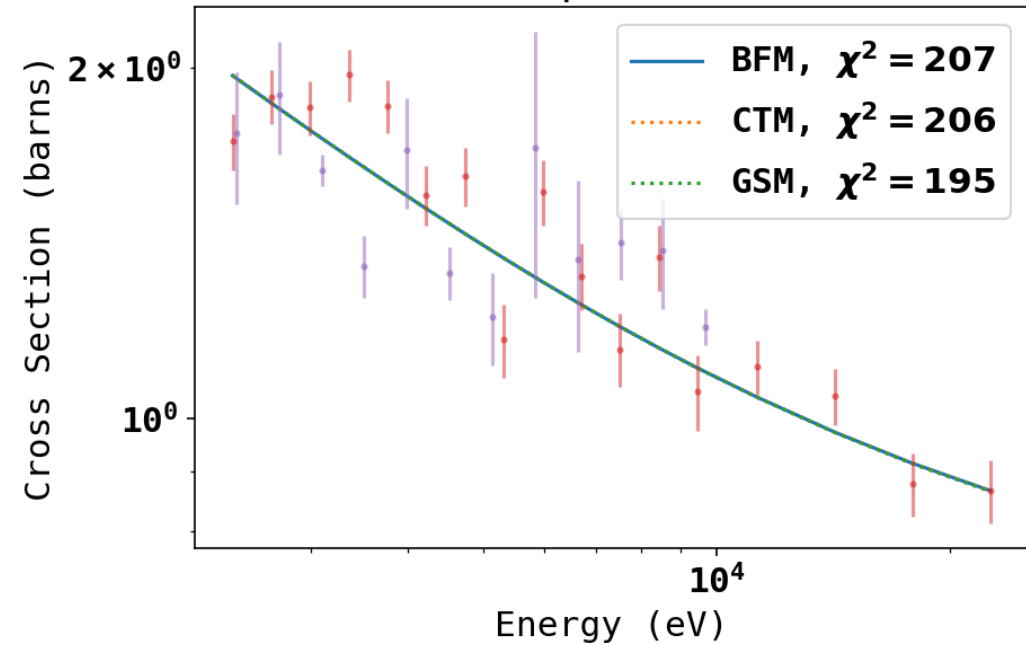
# Initial Results

# Results for U-238 and U-235

U-238 Capture Cross Section  
for Multiple LD Models

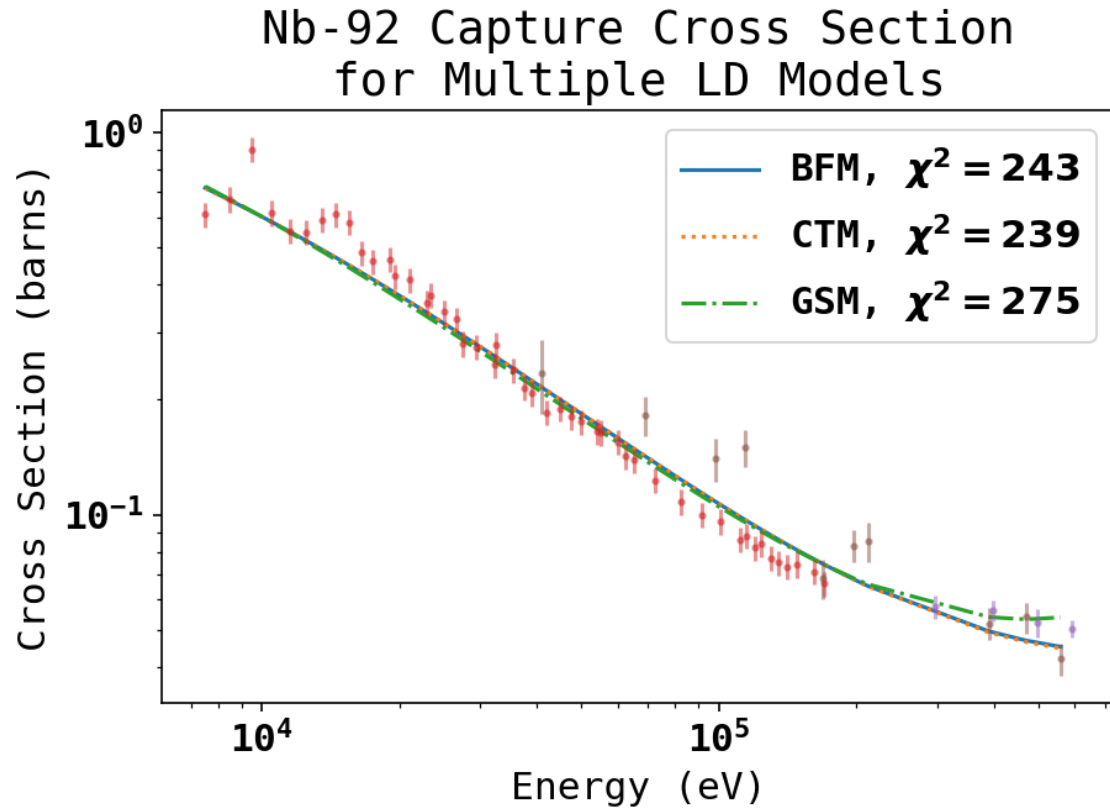


U-235 Capture Cross Section  
for Multiple LD Models



Level density models do not impact cross section after fitting

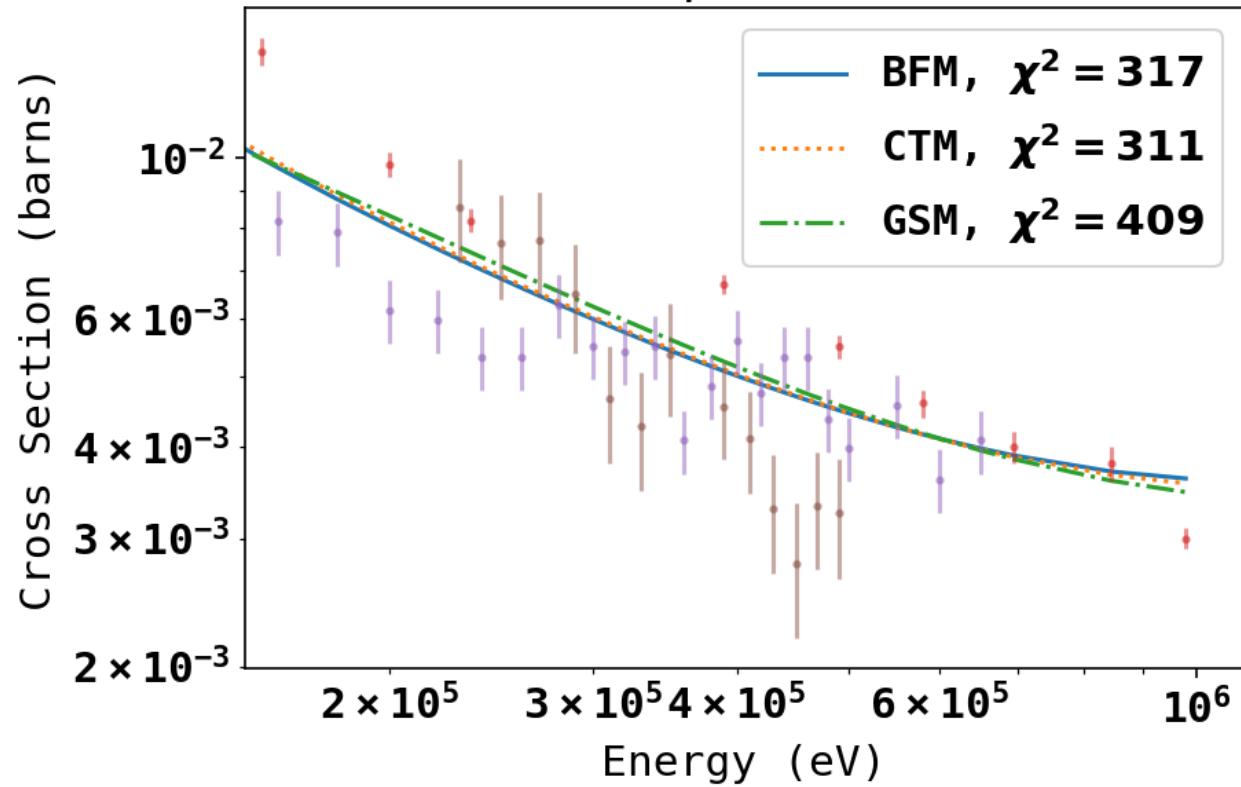
# Nb-93 Results



- Large URR
- Discrepancy with resolved region parameters

# Mn-55 Results

Mn-55 Capture Cross Section  
for Multiple LD Models

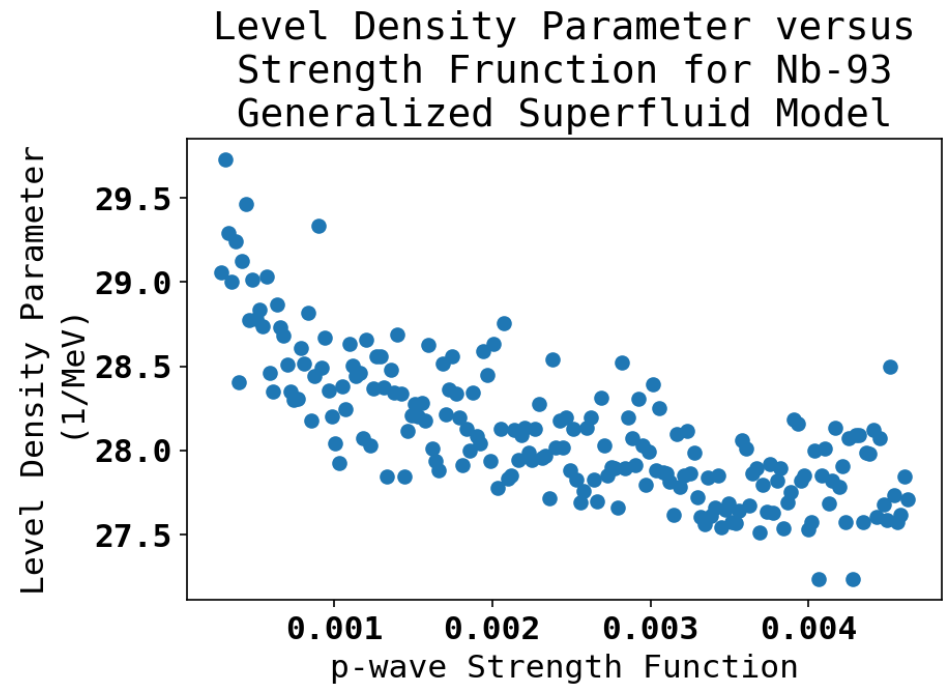


# Conclusions and Future Studies

- Cross sections are very similar between models
- Study over wide range of isotopes could reveal best LD model

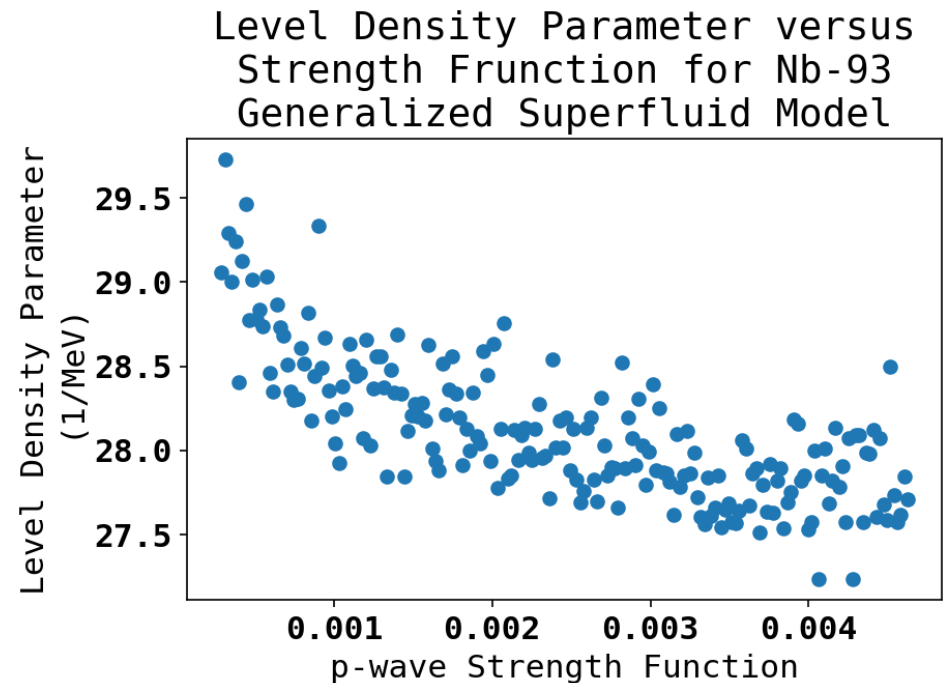
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- Cross section can provide a simultaneous constrain on Hauser-Feshbach parameters



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SAMMY is available:

<https://code.ornl.gov/RNSD/SAMMY>

**Questions?**

# References

- [1] S. Goriely, A.-C. Larsen, and D. Mucher, “Comprehensive test of nuclear level density models,” *Physical Review C*, vol. 106, no. 4, p. 044315, 2022.
- [2] A. Koning, S. Hilaire, and S. Goriely, “Global and local level density models,” *Nuclear Physics A*, vol. 810, no. 1, pp. 13–76, 2008. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0375947408005903>
- [3] A. V. Voinov, S. M. Grimes, C. R. Brune, M. J. Hornish, T. N. Massey, and A. Salas, “Test of nuclear level density inputs for hauser-feshbach model calculations,” *Phys. Rev. C*, vol. 76, p. 044602, Oct 2007. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevC.76.04460>
- [4] W. Hauser and H. Feshbach, “The inelastic scattering of neutrons,” *Phys. Rev.*, vol. 87, pp. 366–373, Jul 1952. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRev.87.366>
- [5] A. Gilbert and A. G. W. Cameron, “A composite nuclear-level density formula with shell corrections,” *Canadian Journal of Physics*, vol. 43, no. 8, pp. 1446–1496, 1965.
- [6] M. Grossjean and H. Feldmeier, “Level density of a fermi gas with pairing interactions,” *Nuclear Physics A*, vol. 444, no. 1, pp. 113–132, 1985. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/0375947485902945>
- [7] A. Ignatyuk, G. Smirenkin, and A. Tishin, “Phenomenological description of the energy dependence of the level density parameter,” *Sov. J. Nucl. Phys*, vol. 21, p. 255, 1975.
- [8] Koning, Arjan, Hilaire, Stephane, and Goriely, Stephane, “TALYS: modeling of nuclear reactions,” *Eur. Phys. J. A*, vol. 59, no. 6, p. 131, 2023. [Online]. Available: <https://doi.org/10.1140/epja/s10050-023-01034-3>
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- [11] F. H. Frohner, “Evaluation of the unresolved resonance range of u238,” *Nuclear Science and Engineering*, vol. 103, no. 2, pp. 119–128, 1989. [Online]. Available: <https://doi.org/10.13182/NSE89-A28501>
- [12] M. Bhat, “Evaluated nuclear structure data file (ENSDF),” in *Nuclear Data for Science and Technology: Proceedings of an International Conference*, held at the Forschungszentrum Julich, Fed. Rep. of Germany, 13–17 May 1991. Springer, 1992, pp. 817–821.
- [13] R. e. a. Capote, “Ripl - reference input parameter library for calculation of nuclear reactions and nuclear data evaluations,” *Nucl. Data Sheets* 110, 2009.
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# Data Sources

EXFOR Subentry  
Numbers:

U-235

14149003 M. Jandel  
10249005 R. B. Perez  
23453002 S. Amaducci  
40616004 T. A. Mostovaya  
12877004 L. W. Weston  
23294003 C. Paradela

U-238

23234002 F. Mingrone  
21334003 M. C. Moxon  
23364004 T. Wright

Nb-93

11329023 H. J. Gibbons  
40452003 Yu. Ya. Stavisskii  
12670012 W. P. Poenitz

Mn-55

10752004 J. B. Garg  
20572006 C. Le Rigoleur  
40642002 Ju. Ya.  
Stavisskii

**Extra Slides**

# Level Density as a Parameter: Hauser-Feshbach

$$\langle \sigma_{a,b} \rangle = \frac{\pi g_a}{k_a^2} \frac{T_a T_b}{\sum_c T_c} \int_0^\infty \exp\left(-t \frac{\sum_\gamma T_\gamma}{\sum_c T_c}\right) \prod_{c \notin \gamma} \left(1 + \frac{2t T_c}{\sum_c T_c}\right)^{\frac{-\nu_c}{2 - \delta_{ac} - \delta_{bc}}} dt.$$

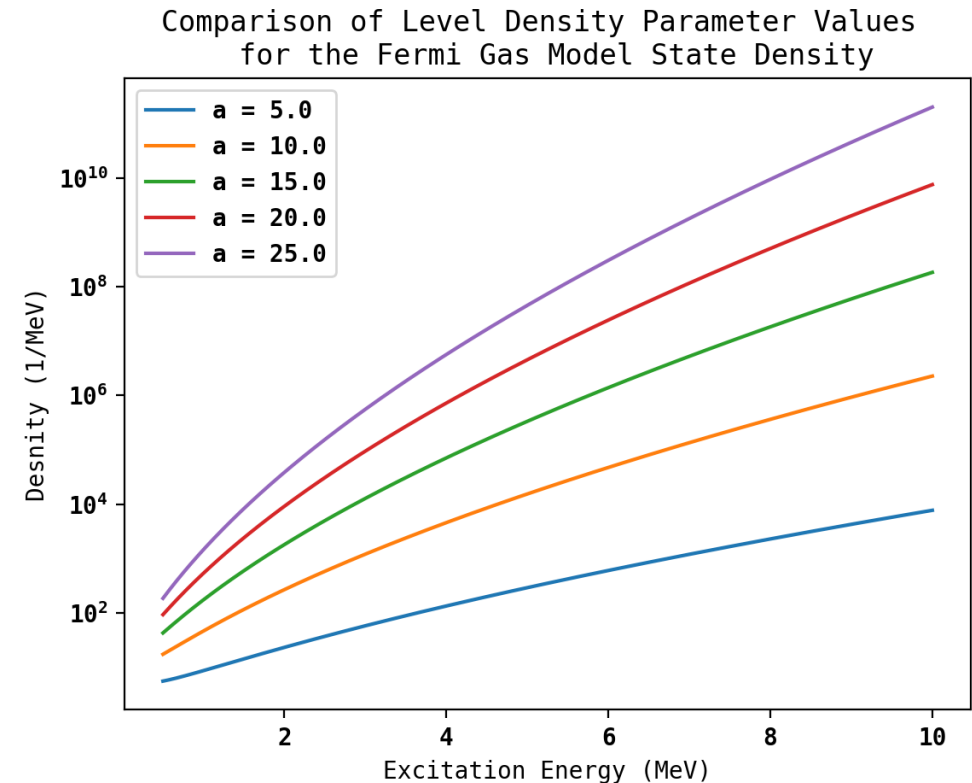
$$T_c = 2\pi \frac{\langle \Gamma_c \rangle}{D_J} = 2\pi \langle \Gamma_c \rangle \rho_J.$$

Level density as a function of energy.

# Fermi Gas Model of the Nucleus

$$\omega(E_x) = \frac{\sqrt{\pi} e^{2\sqrt{aE_x}}}{12a^{\frac{1}{4}} E_x^{\frac{5}{4}}}$$

- Models the nucleus as an ideal gas of two fermion species.
- Nucleons do not interact but exist in a potential well.
- Most valid at high excitation energies (> 5MeV).



# Spin Cutoff Parameter

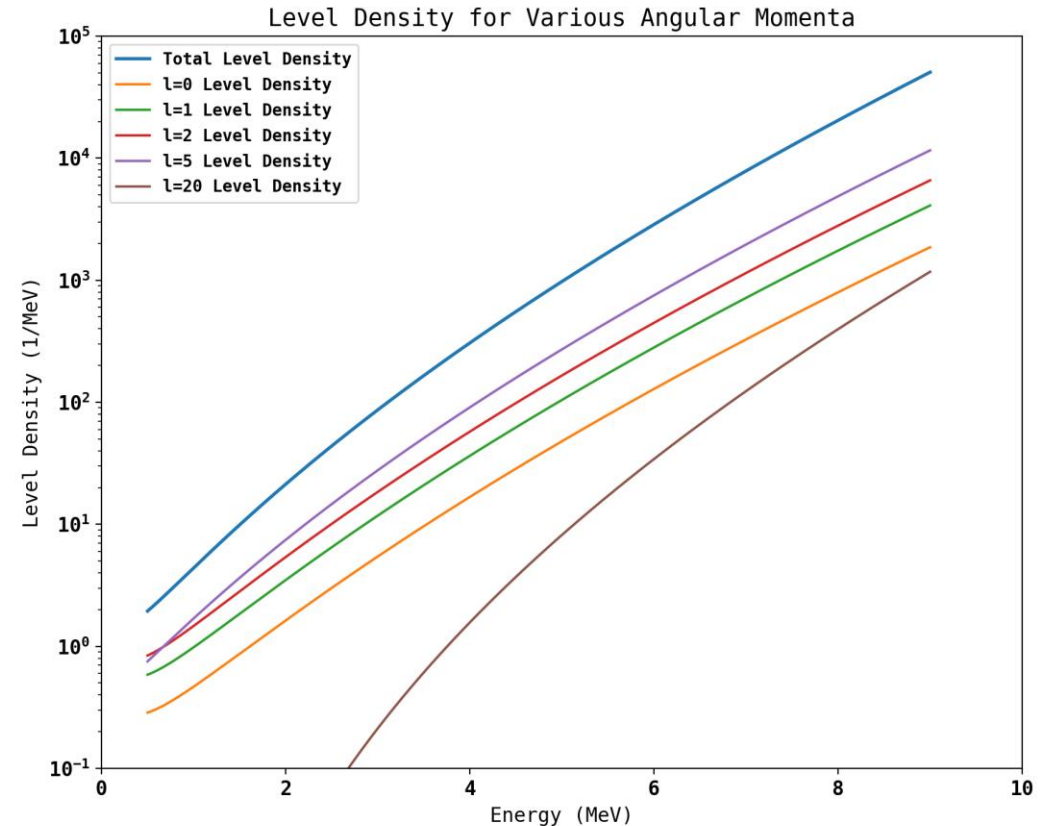
$$\rho_{Fermi}(E_x) = \frac{\omega(E_x)}{\sigma\sqrt{2\pi}} = \frac{1}{12\sqrt{2}} \frac{e^{2\sqrt{a}E_x}}{\sigma a^{\frac{1}{4}} E_x^{\frac{5}{4}}}.$$

- State density is degenerate in the magnetic quantum number.
- Assume a Gaussian distribution of angular momentum,  $\sigma$  is the standard deviation.

# Angular Momentum Distribution

$$\frac{\rho_J}{\rho_{tot}} = e^{\frac{-J^2}{2\sigma^2}} - e^{\frac{-(J+1)^2}{2\sigma^2}}$$

- The spin cutoff parameter also determines L-dependent level density.



# Back-Shifted Fermi Gas Model

$$\rho_{BFM}(E_x) = \frac{1}{12\sqrt{2}} \frac{e^{2\sqrt{a}U_{BFM}}}{\sigma a^{\frac{1}{4}} U_{BFM}^{\frac{5}{4}}}$$

$$U_{BFM} = E_x - \Delta - \delta$$

- Experiments tell us available energy is limited by a pairing energy.
- Pairing energy chosen as fit variable.

# Constant Temperature Model (Gilbert-Cameron)

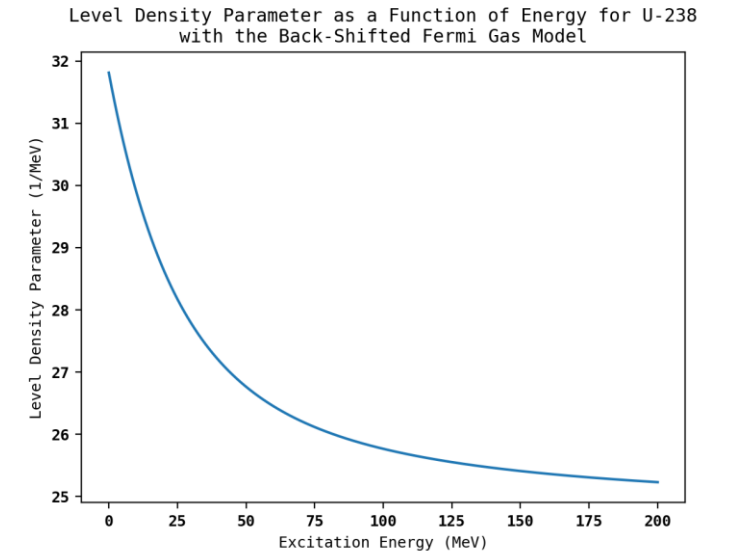
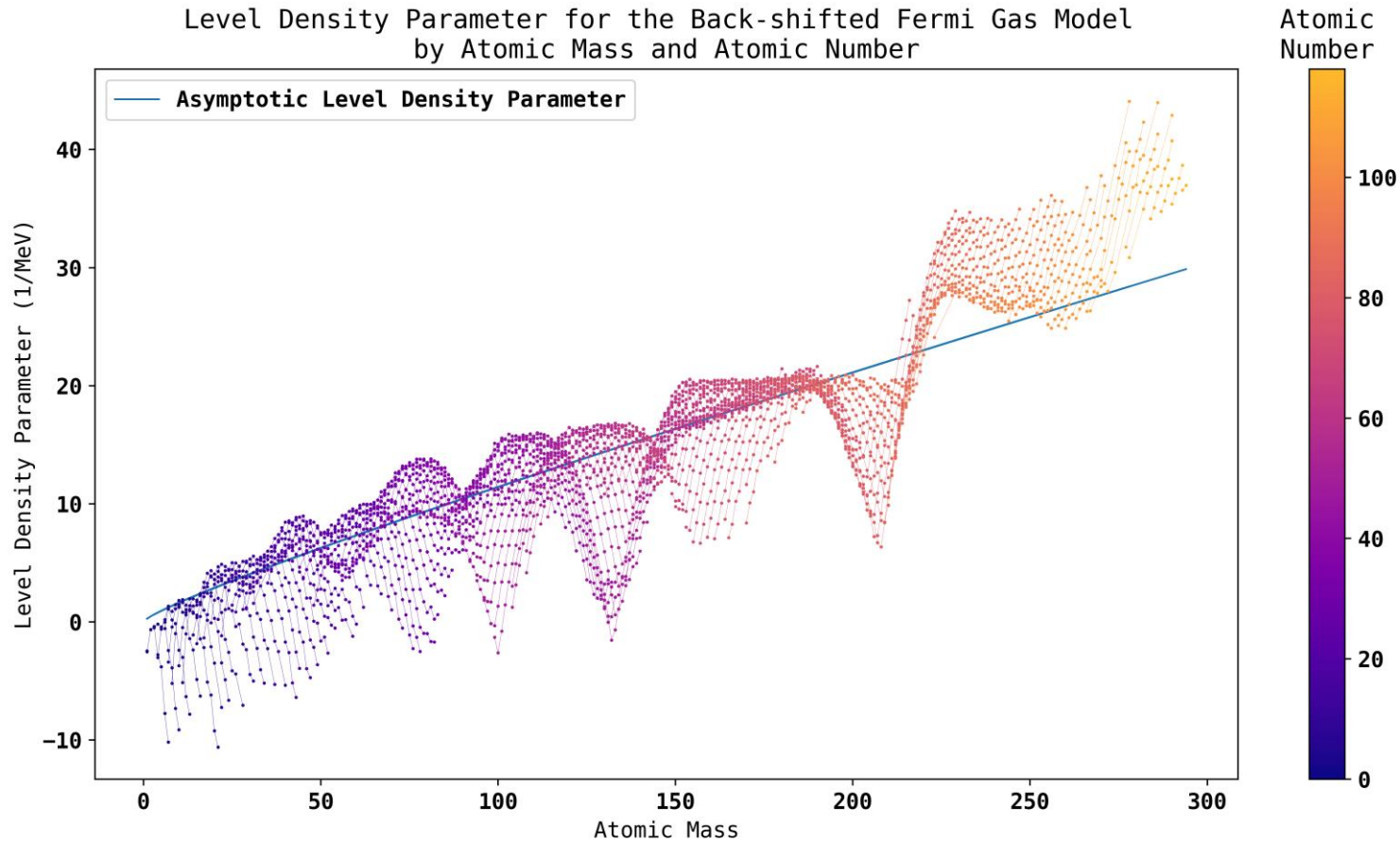
$$\rho_{CTM}(E_x) = \begin{cases} \rho_{const}(U_x) & E_x \leq E_m \\ \rho_{Fermi}(U_x) & E_x > E_m \end{cases}$$

$$\rho_{const}(U_x) = \rho_{Fermi}(U_m) \exp\left(\frac{U_x - U_m}{T}\right)$$

$$U_x \equiv E_x - \Delta, U_m \equiv E_m - \Delta$$

- Divides energies into two regions.
- High energy: Fermi gas
- Low energy: empirical formula

# Level Density Parameter Details



$$a(U) = \tilde{a} \left( 1 + dW \frac{1 - e^{-\gamma U}}{U} \right).$$