Efficient Free-Electron Laser Modelling Using a Lorentz-Boosted Coordinate System

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Free-electron Laser (FEL)

Demonstration of the laser wakefield based FEL.
W. Wang et al. Nature (2021)

- $\lambda \propto \frac{1}{\gamma^2}$
- Å FEL light using 10GeV electrons

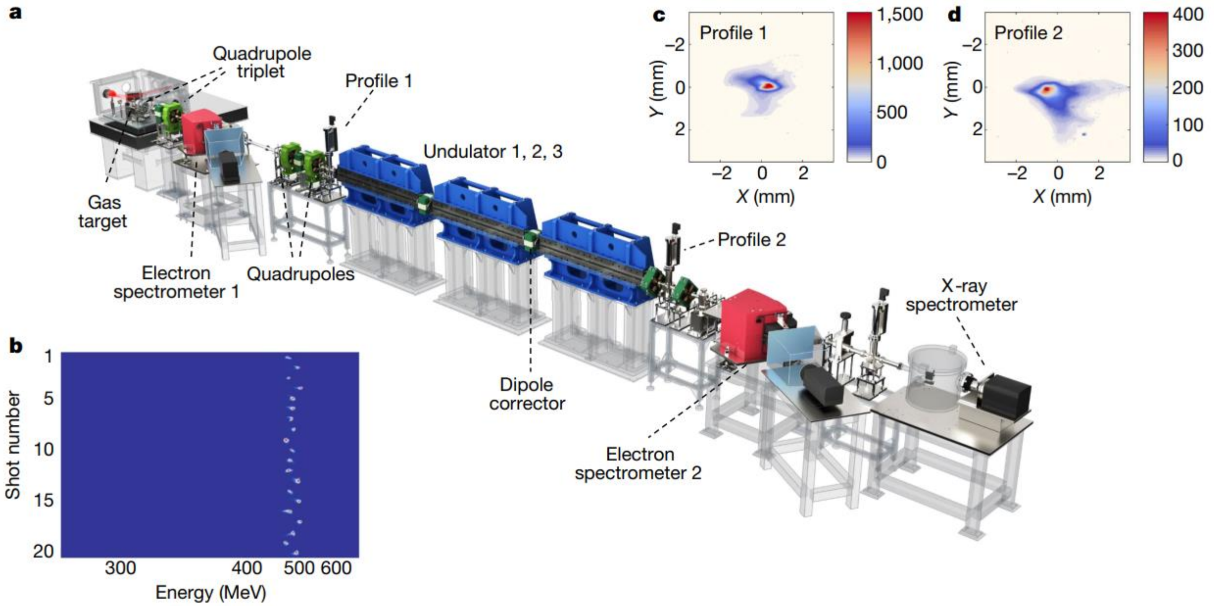
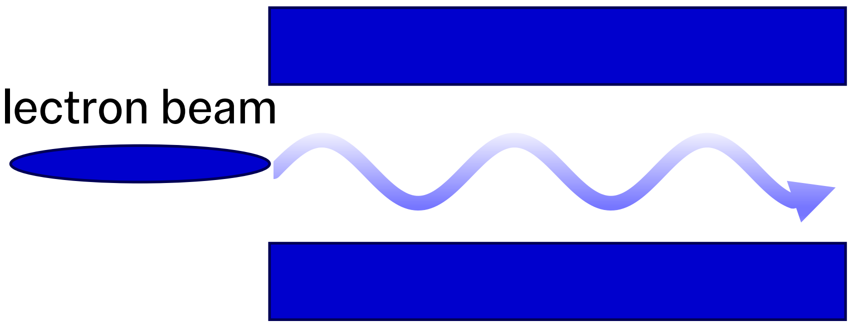


Fig. 1 | Schematic layout of LWFA-based free electron laser experiment. **a**, Undulator beamline with a total length of approximately 12 m from the gas target for the LWFA to the X-ray spectrometer. **b**, Typical spectra of electron beams from the LWFA for 20 consecutive shots. **c**, **d**, Measured transverse profiles of the electron beam at the entrance (**c**) and exit (**d**) of the undulators. The scale bars are normalized.



undulator

electron beam

Genesis FEL Code

- Time dependent code
- Solve Maxwell equations in paraxial approximation
- Solve longitudinal slices
- Doesn't fully model longitudinal expansion

S. Reiche, thesis (1999)

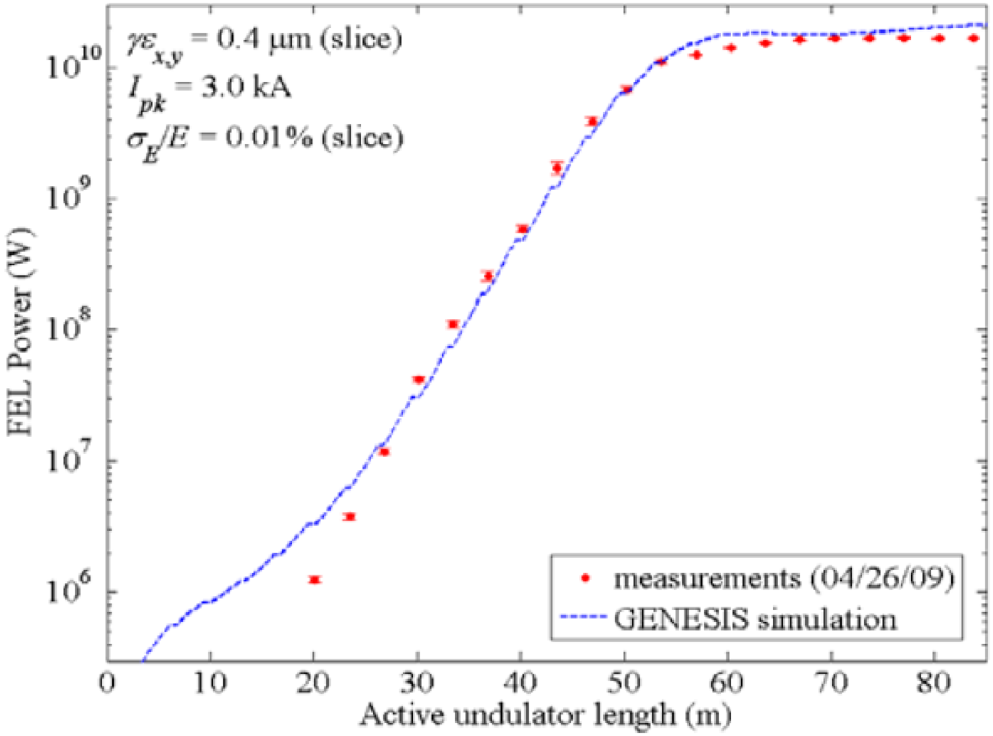
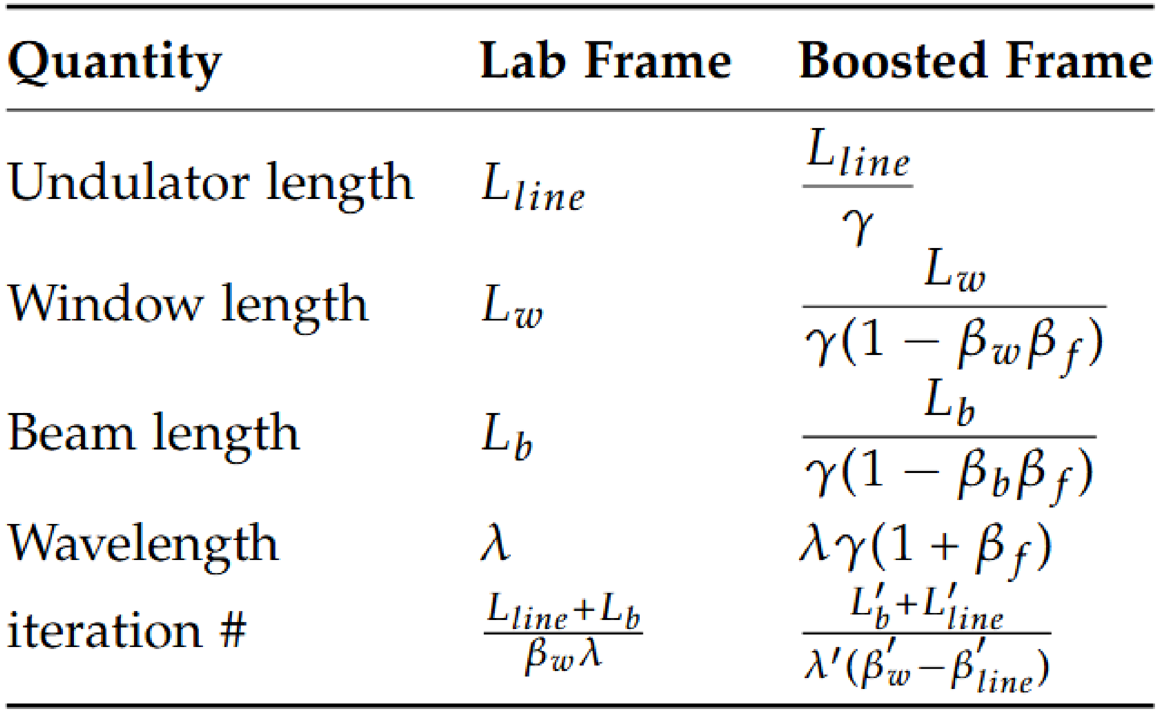


Figure 11: FEL power gain length measurement (red points) at 1.5 Å made by kicking the beam after each undulator sequentially. The measured gain length is 3.3 m and a *Genesis* simulation is overlaid in blue with e^- beam parameters shown. There are twenty-five 3.35-m long undulators installed here.

P. Emma et al. Proceedings of Particle Accelerator Conference (2009)

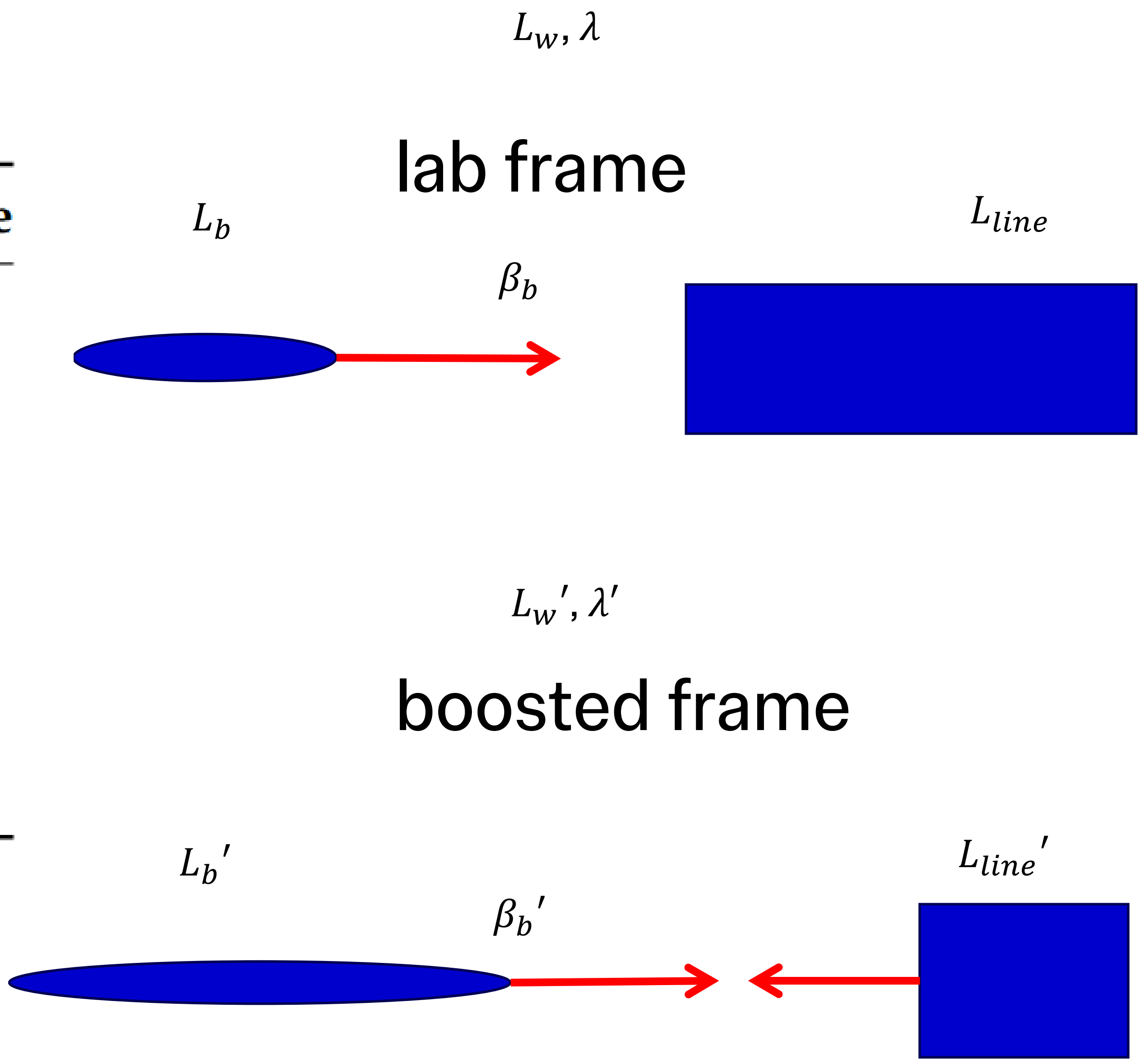
Principle of the Boosted Frame Simulations



Quantity	Lab Frame	Boosted Frame
Undulator length	L_{line}	$\frac{L_{line}}{\gamma}$
Window length	L_w	$\frac{L_w}{\gamma(1-\beta_w\beta_f)}$
Beam length	L_b	$\frac{L_b}{\gamma(1-\beta_b\beta_f)}$
Wavelength	λ	$\lambda\gamma(1+\beta_f)$
iteration #	$\frac{L_{line}+L_b}{\beta_w\lambda}$	$\frac{L'_b+L'_{line}}{\lambda'(\beta'_w-\beta'_{line})}$

$2\gamma^2$ speed up in beam rest frame

R. Luo et al. (in preparation), On the Numerical Behavior and Modeling Considerations of Boosted-Frame PIC Simulations



J.-L. Vay, Phys. Rev. Lett. (2007)

Extreme Photonics Applications Centre (EPAC)

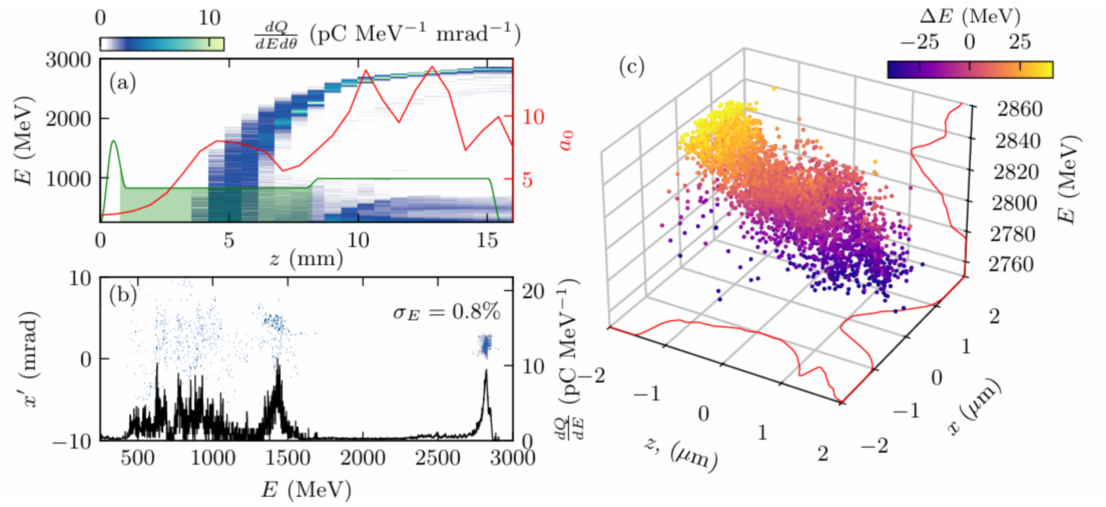


Figure 5. Lowest energy spread electron beam from the LWFA simulations. (a) Evolution of the electron spectrum and laser a_0, plotted alongside the longitudinal plasma density. Only electrons originating from the inner shells of the dopant species are shown. The green shaded area indicates the doped region. (b) Angularly resolved full electron spectrum at the end of the simulation, with the integrated lineout plotted against the right axis. (c) Phase space of electrons in the peak of the spectrum, downsampled at a 1:10 ratio for visualisation. The red lines show the integrated histograms along each axis.

Courtesy of M. P. Backhouse

Case	I [A]	Energy [MeV]	λ [nm]
Beam driven	300	93.9	830
Laser	2000	490	27
EPAC	60,000	2900	0.77

- 10Hz, 1PW, 30fs (EPAC)
- 2.8-3.1GeV monoenergetic beams
- 0.8% energy spread
- 378pC
- $\epsilon_n = 1.7 mm\ mrad$
- Self-focusing with no external guiding

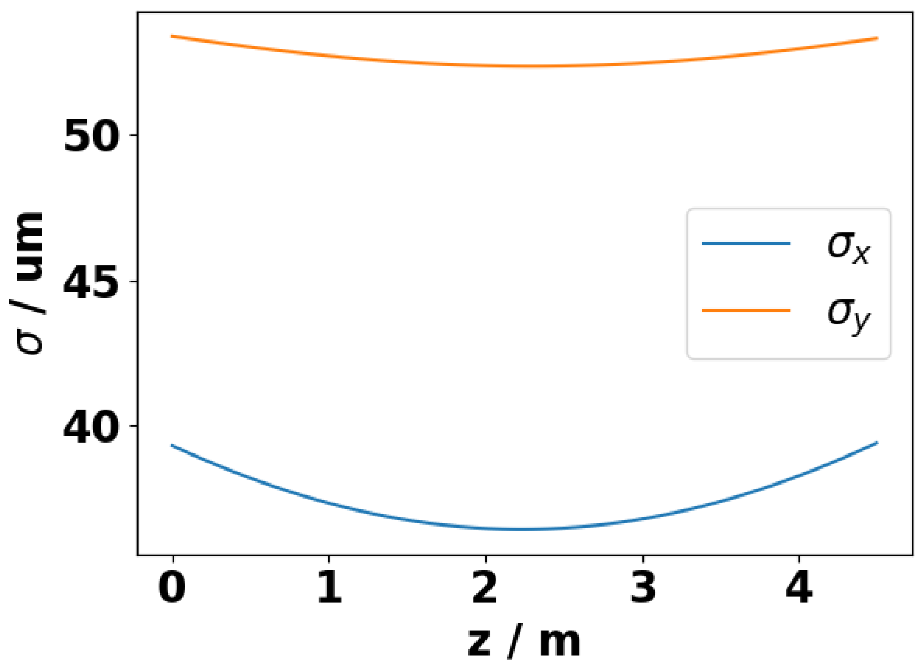
W. Wang et al. Nature (2021)
R. Pompili et al. Nature (2022)
M. P. Backhouse et al. Machine Learning: Science and Technology (submitted)

Transverse Beam Size (490MeV)

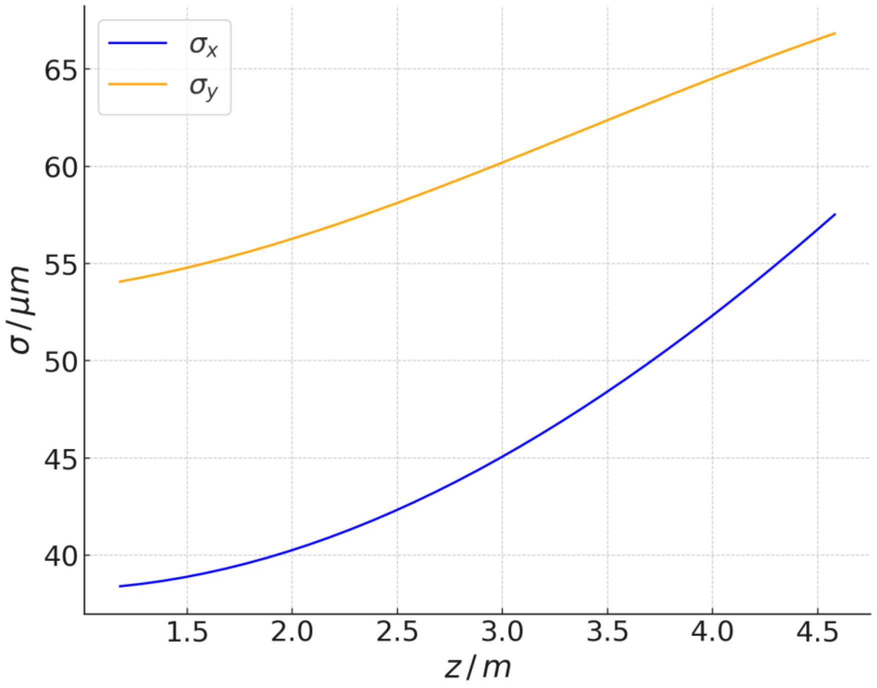
W. Wang et al. Nature (2021)

- 490MeV beam
- Lasing at 27nm
- Three 1.5m long undulator
- 2.5cm period, K=1.4
- Assumed a single undulator unit for simplicity
- Reduced computation time from months to hours

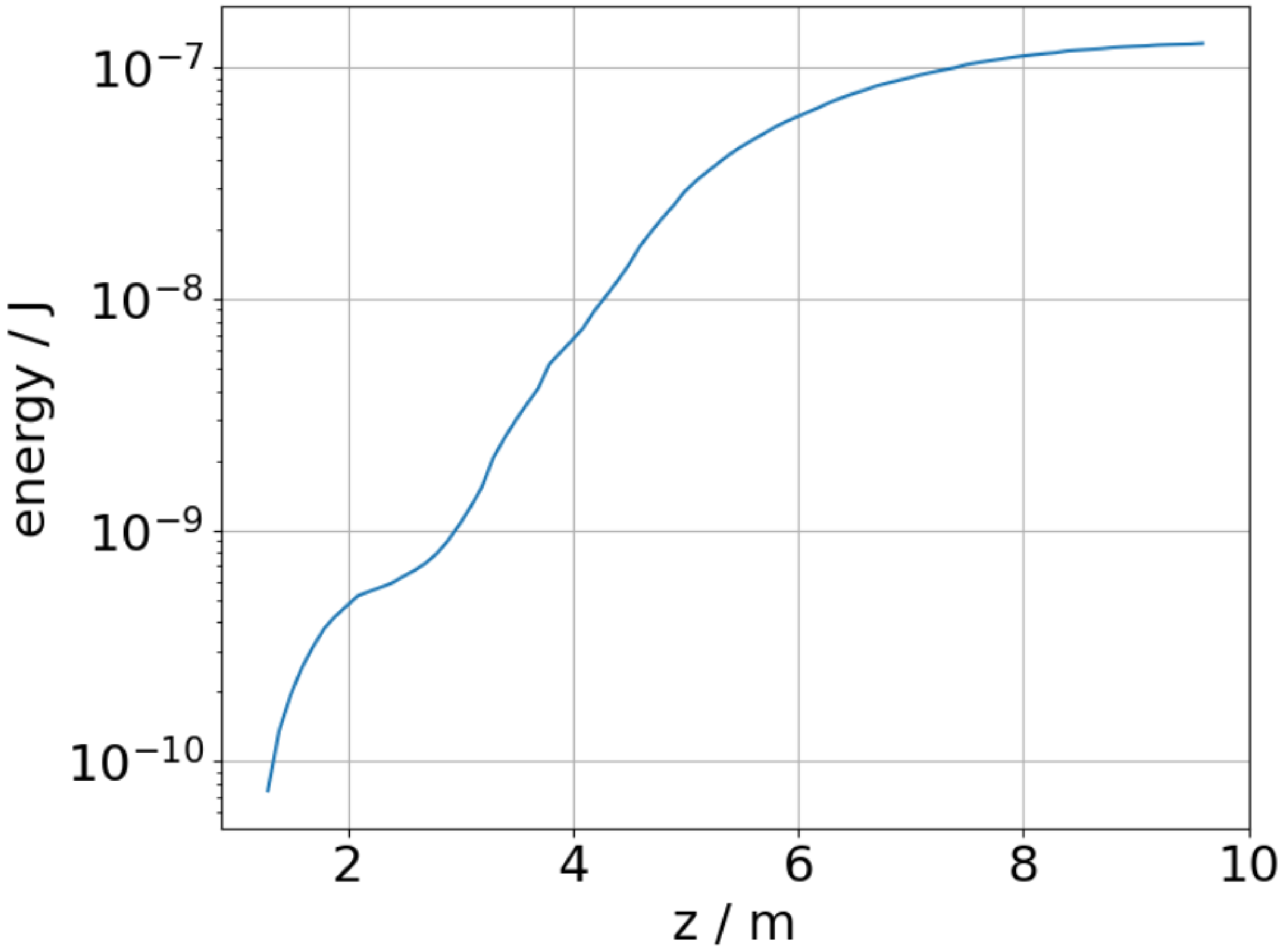
Beam matching

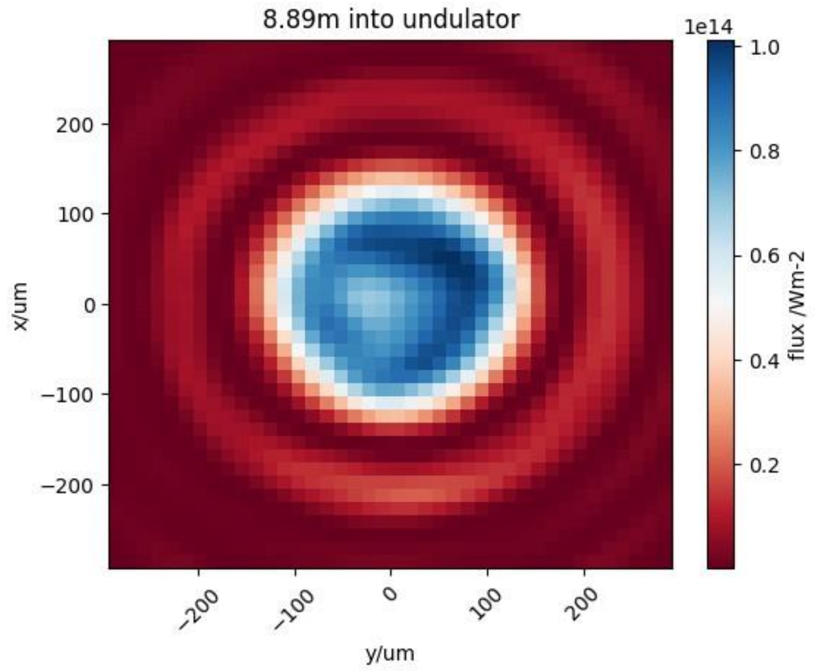


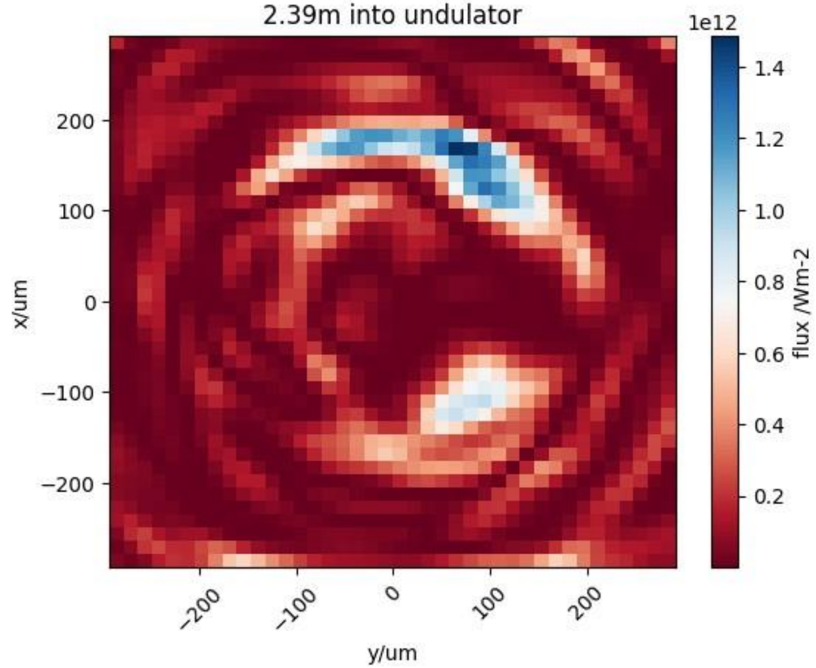
PIC



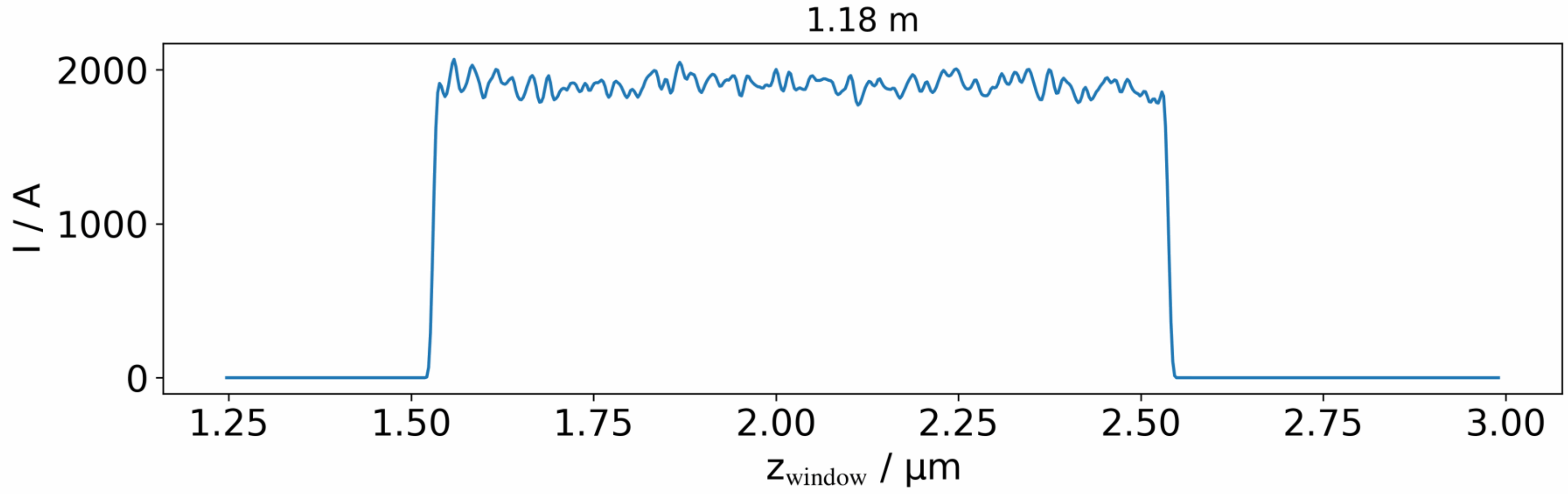
Radiation Energy







Current Profile



Space Charge

The beam envelope evolution is given by:

$$\frac{d^2R}{dz^2} + k(z)R - \frac{\varepsilon^2}{R^3} - \frac{P}{R} = 0$$

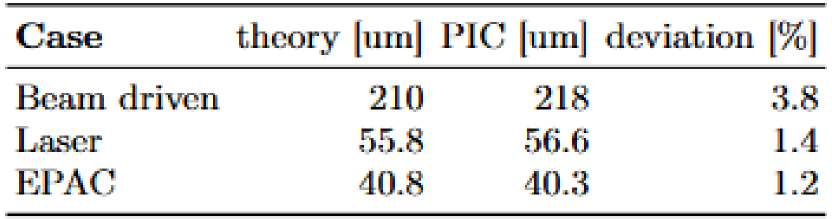
- R is the rms beam radius
- k is the external focusing strength (e.g. from undulators)
- ε is the geometric emittance
- P is the generalized prevenance: $P = \frac{2I}{I_A\beta^3\gamma^3}$
- L is the undulator length

$$\Delta\sigma \approx \frac{PL^2}{8\sigma_i}$$

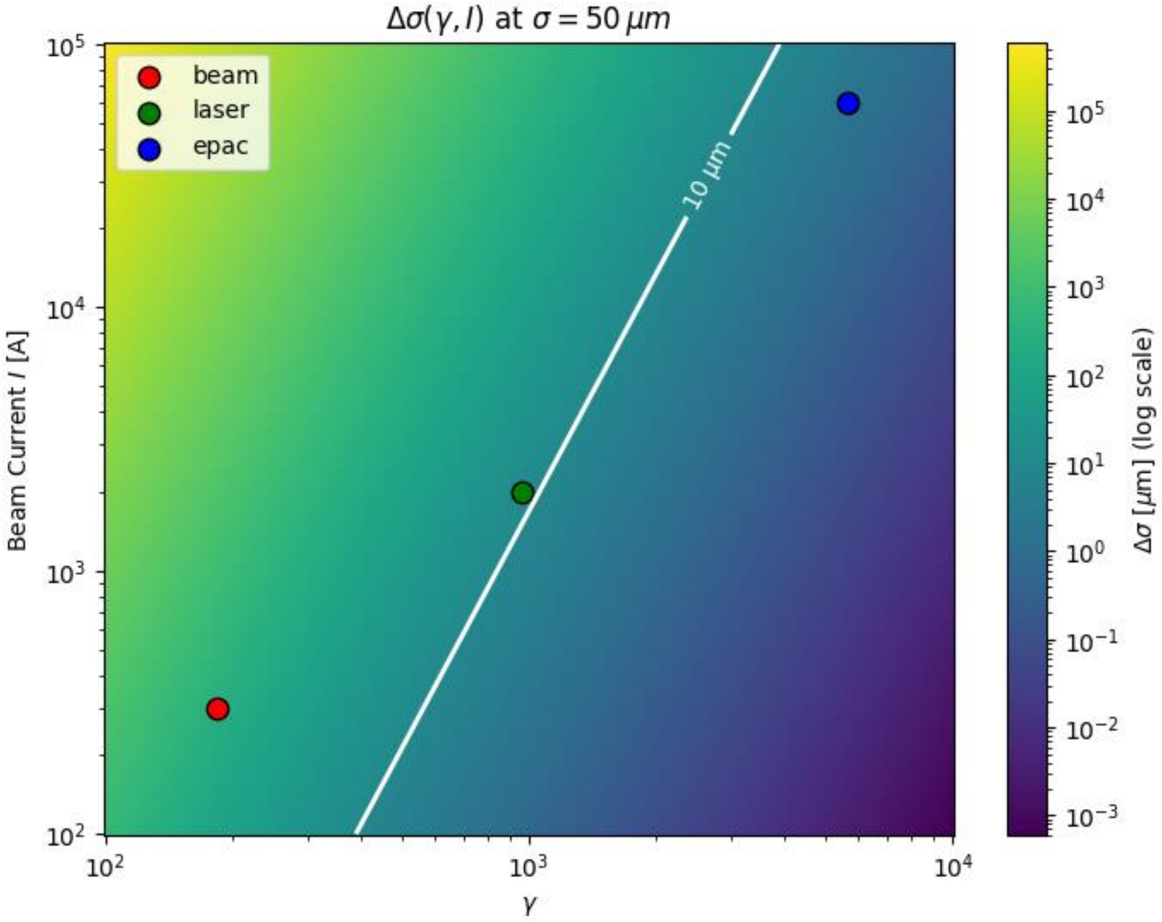
Scaling law:

- $\Delta\sigma = \frac{5.956I}{\gamma^3}$
- Assuming a 50um beam size

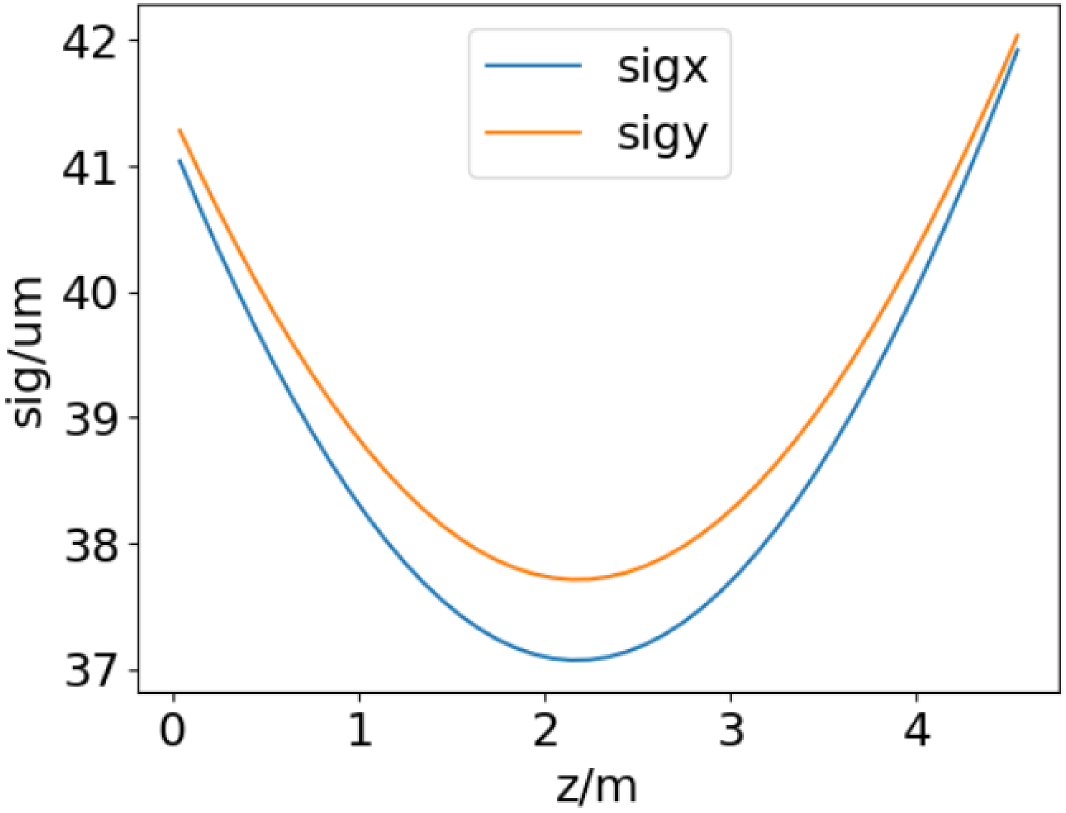
R. Luo et al. (in preparation), Efficient Free-Electron Laser Modelling Using a Lorentz-Boosted Coordinate System

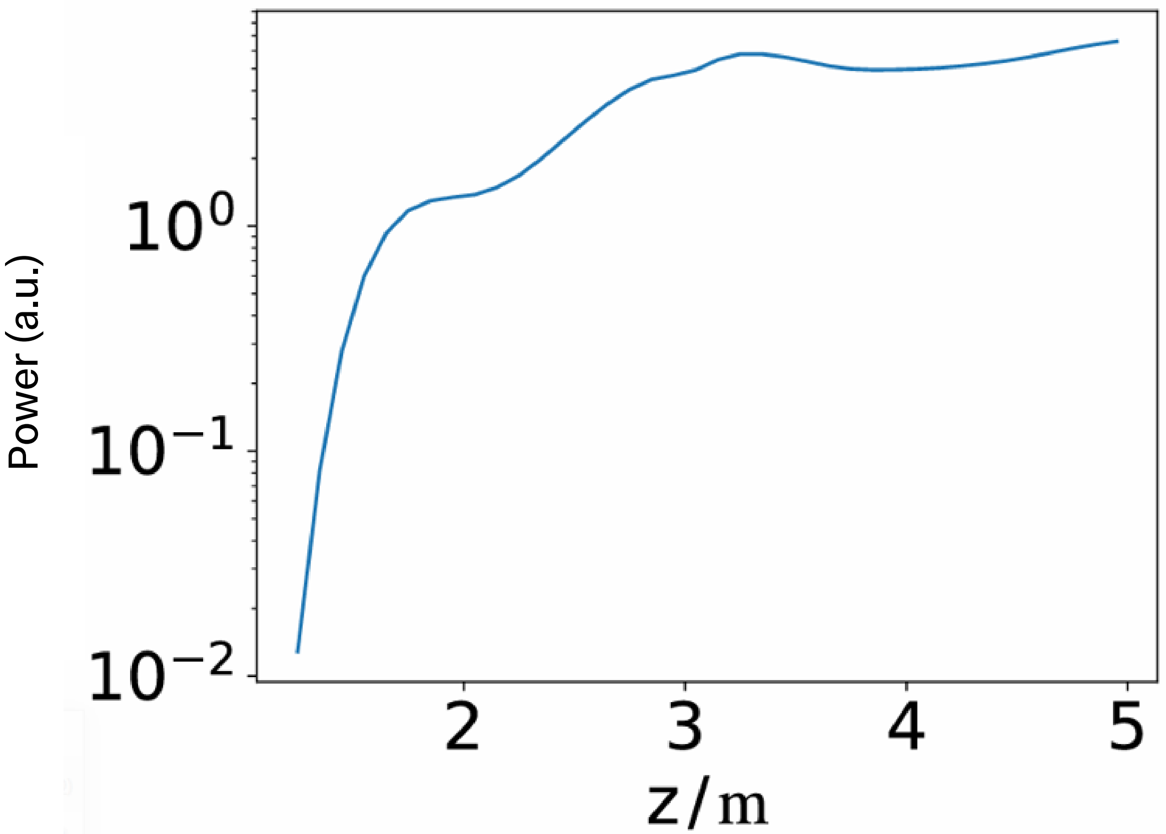


Case	theory [um]	PIC [um]	deviation [%]
Beam driven	210	218	3.8
Laser	55.8	56.6	1.4
EPAC	40.8	40.3	1.2



EPAC





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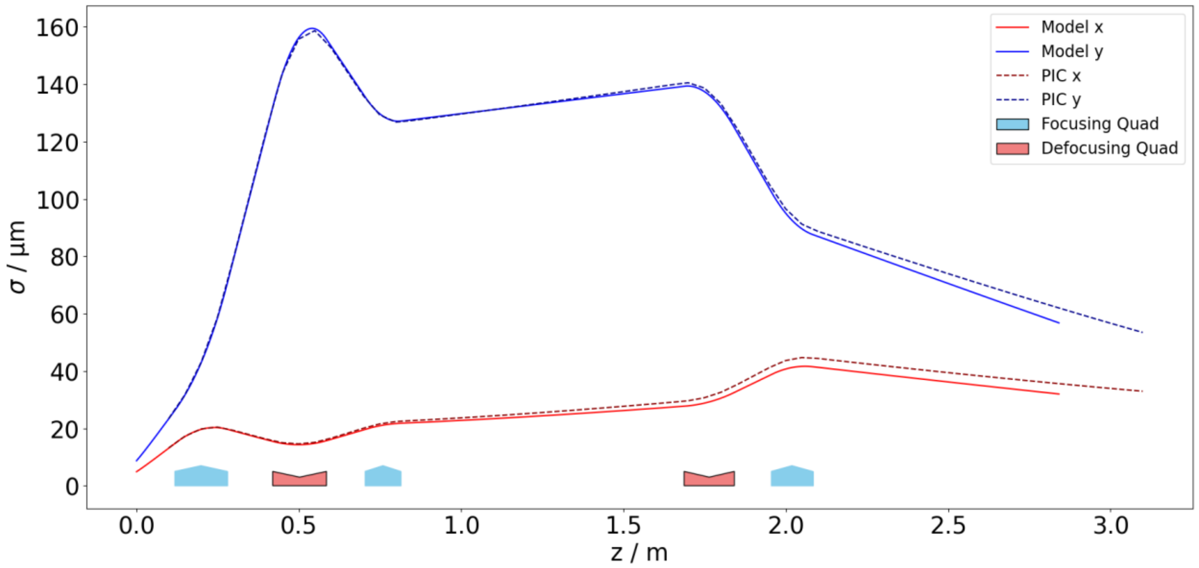
Transport (PIC vs Twiss)

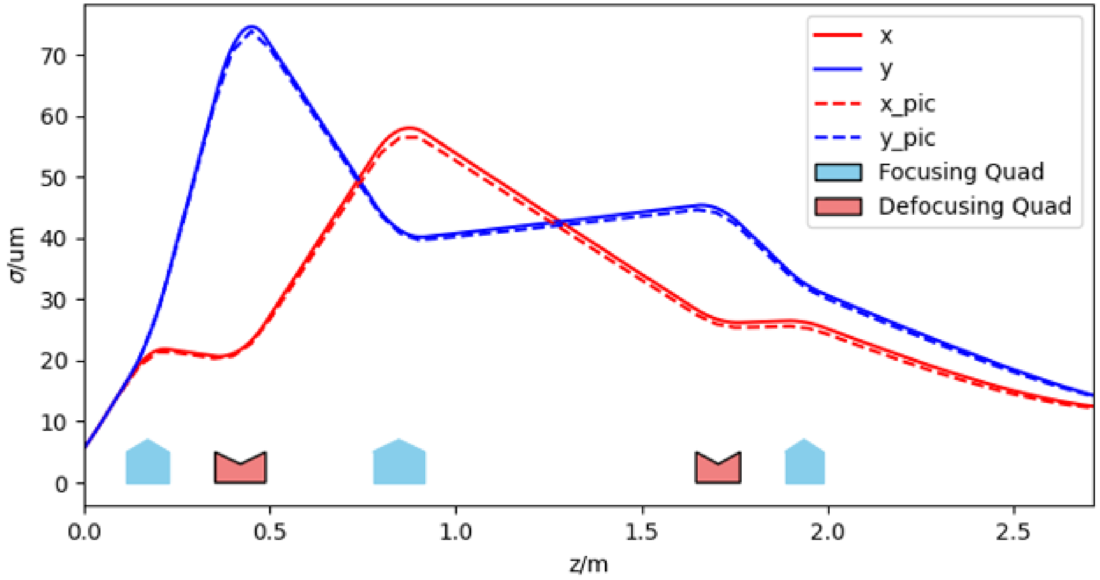
- $\alpha_x = \alpha_y = -1.5$
- $\beta_x = \beta_y = 0.1m$
- 1% energy spread
- Sub-minute computation time
- Bayesian optimisation integrated into PIC
- Obtained a 1.6m configuration for EPAC, minimising beamline length

R. Luo et al. (in preparation), Integrated Particle-in-Cell Simulation and Bayesian Optimization for Transport Beamline Design

490MeV



2.9GeV



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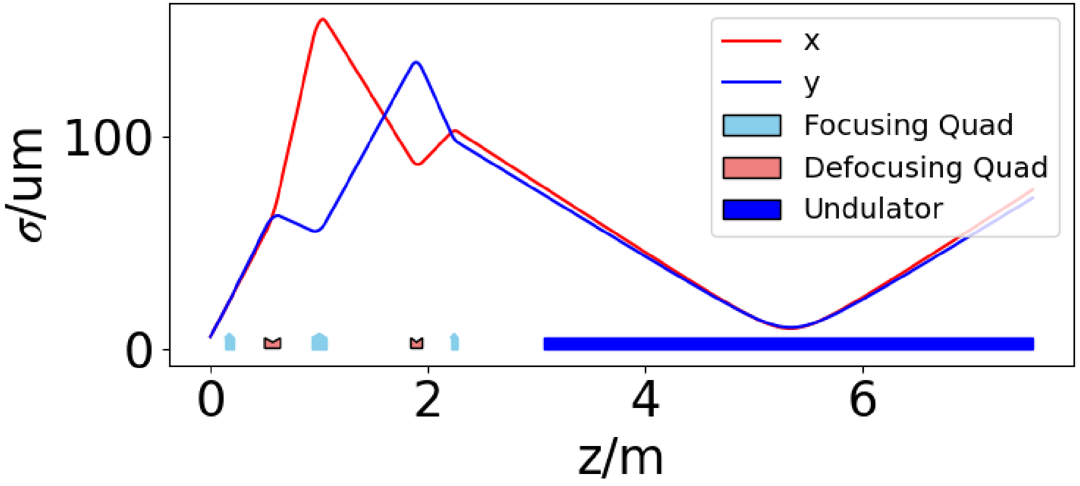
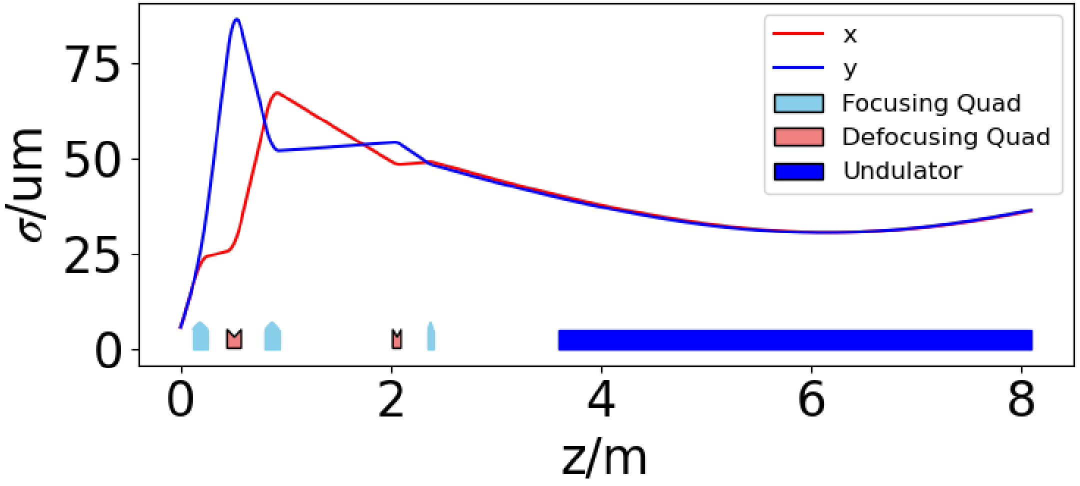
EPAC Transport

- Typical field gradient 50-300T/m (k=30m^{-2} for 300T/m)
- 20-150mm effective length

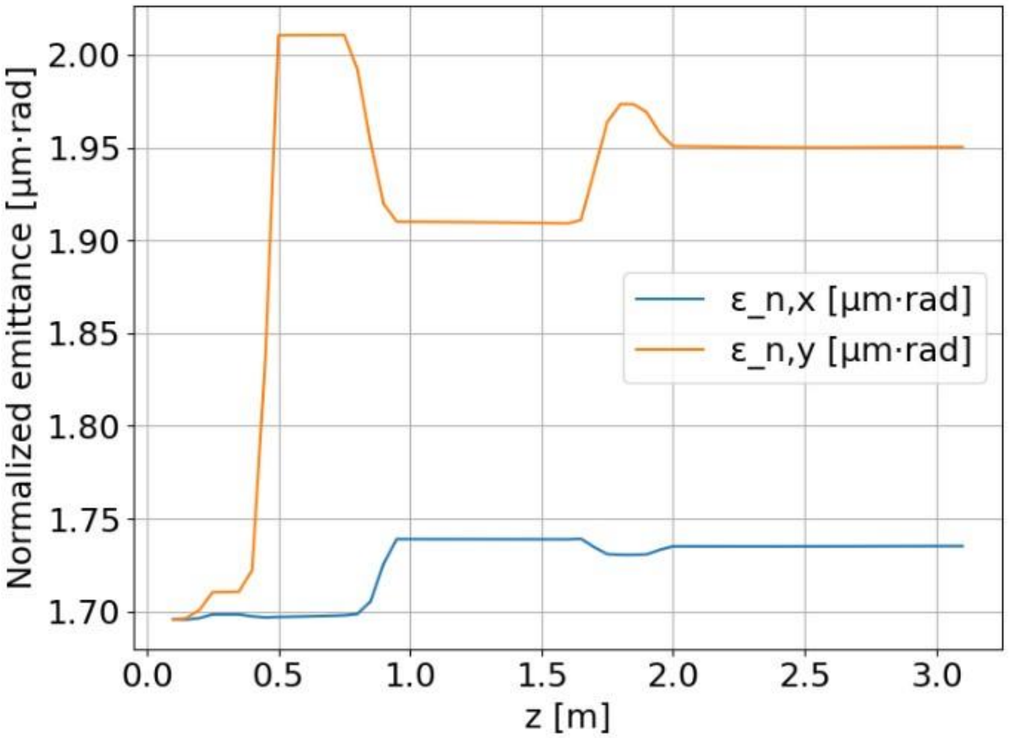
Peak gradient of 150T/m



Peak gradient of 300T/m, $\sigma^* = 30um$



EPAC Transport ($\frac{\Delta E}{E} = 1\%$)



$$\Delta\epsilon_n = \frac{\epsilon_n}{2}(kL\beta\sigma_\delta)^2$$

The emittance increase is given by:

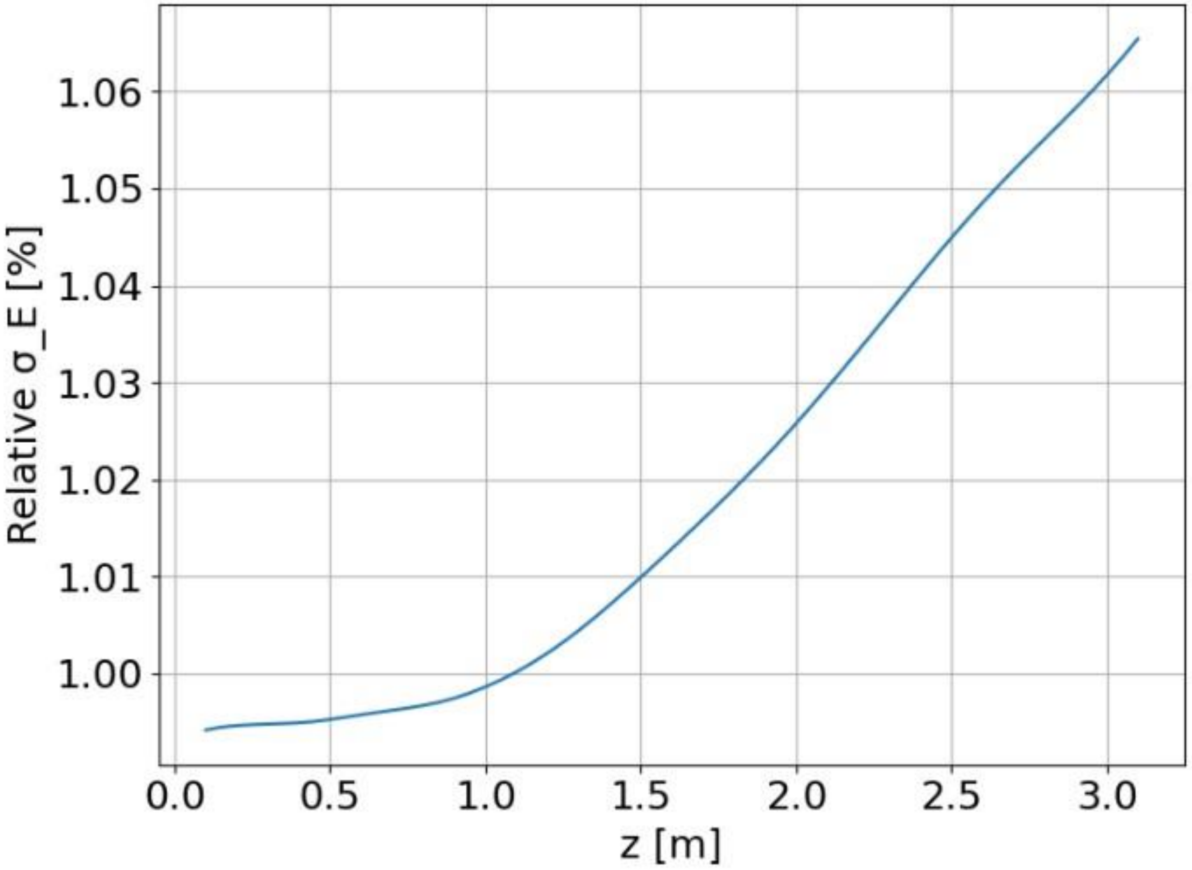
$$\Delta\epsilon_n = \frac{\epsilon_n}{2}(kL\beta\sigma_\delta)^2$$

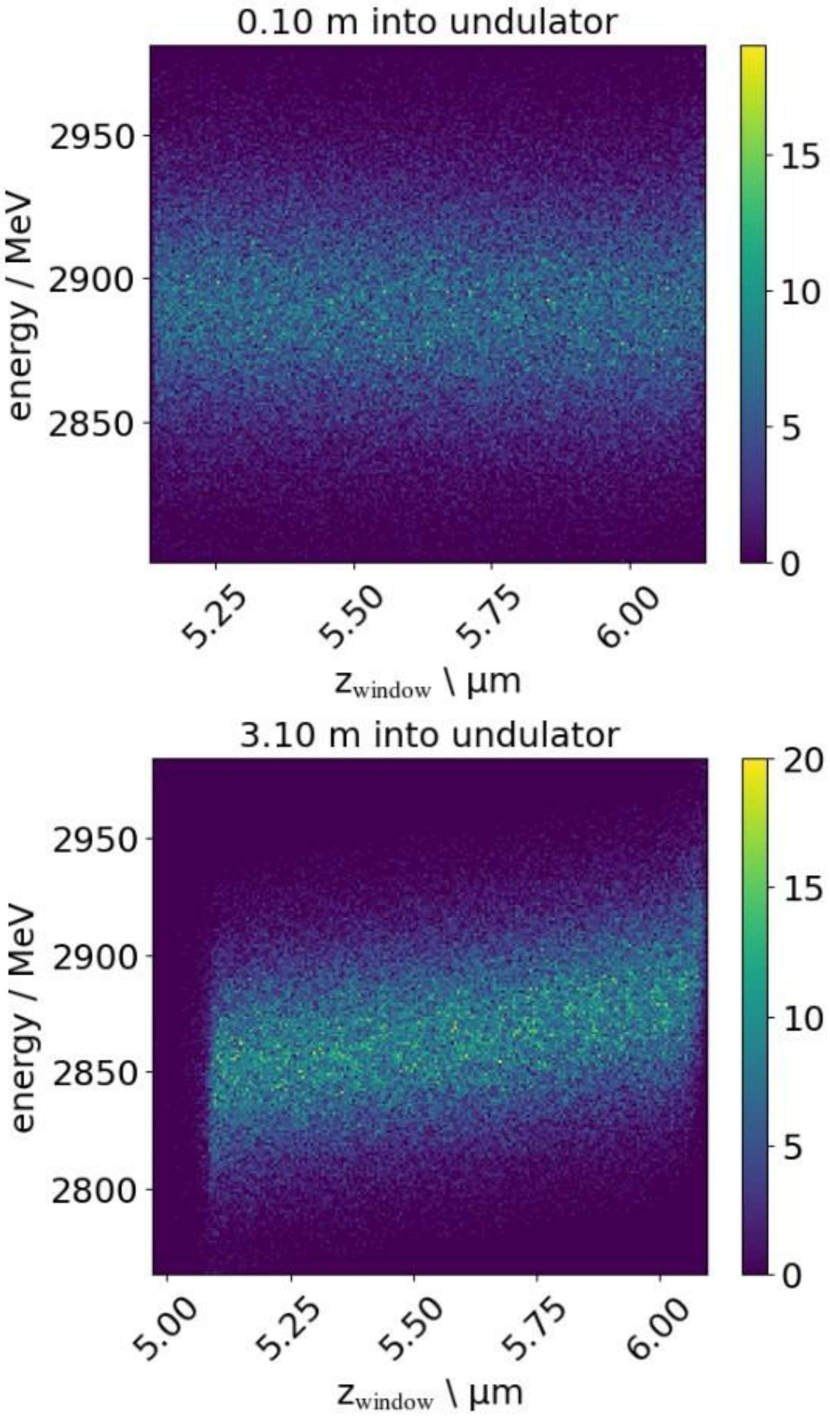
- k is the quadrupole strength (m^{-2})
- L is the quadrupole length
- σ_δ is the energy spread
- ϵ_n is the normalised emittance

Comprise between emittance preservation and minimising beamline length

W. Wang et al. Nature (2021)

Energy Spread





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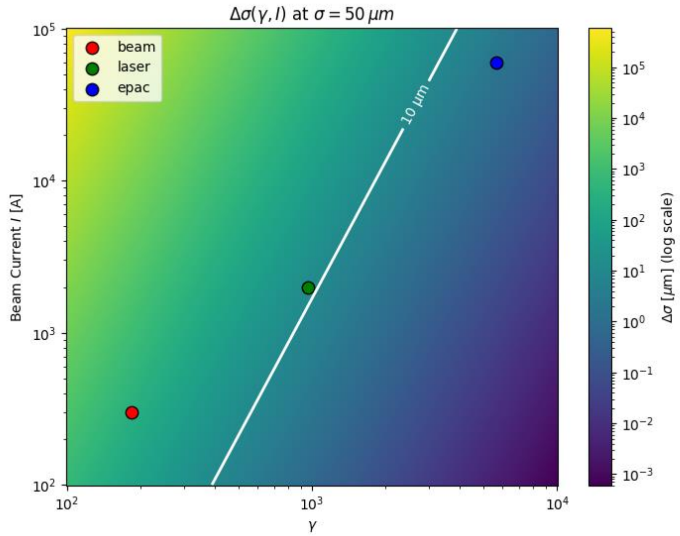
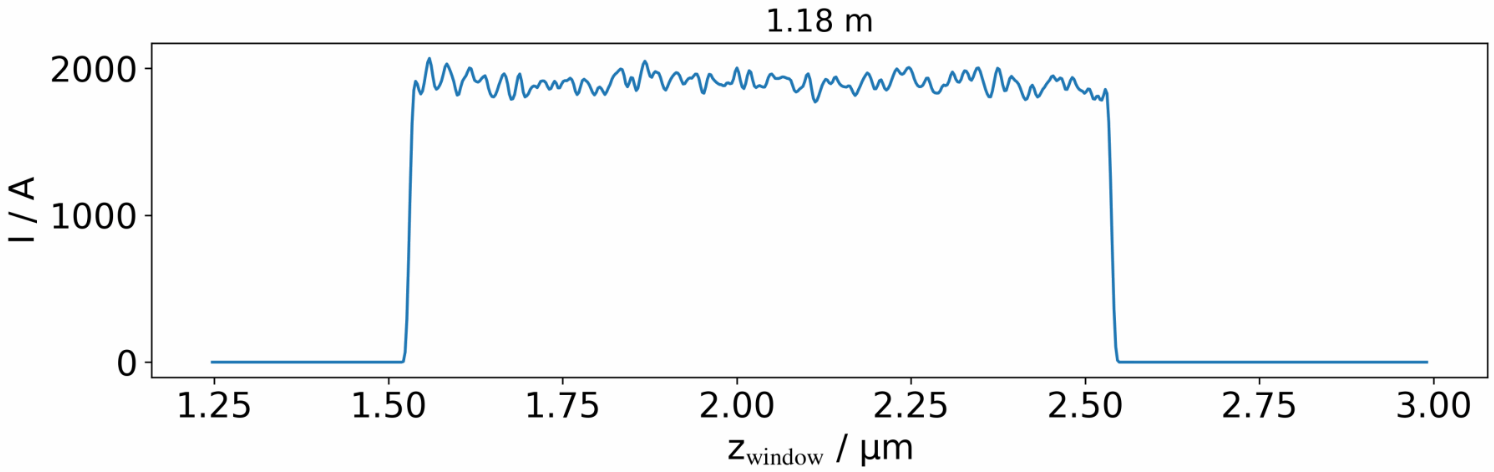
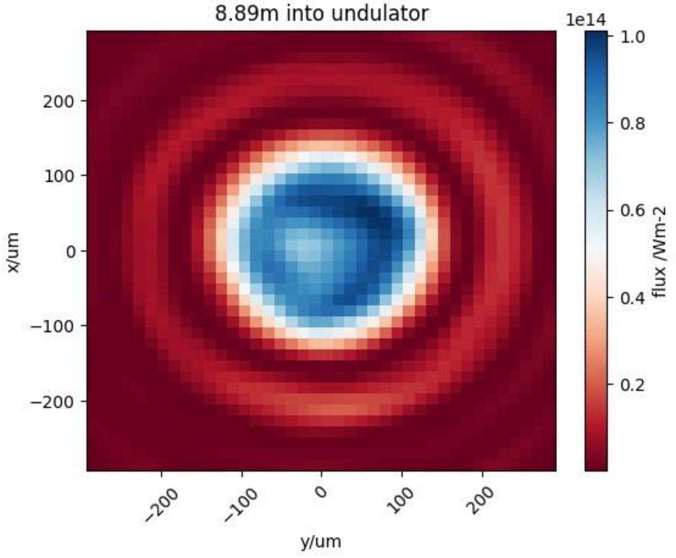
Conclusion

- Space charge effects and energy modulation are significant for wakefield based FELs
- Boosted frame simulation fully captures the dynamics
- Boosted frame simulation offers $2\gamma^2$ speed up
- Bayesian optimisation coupled with PIC for beamline design







References

[1] W. Wang et al. Free-electron lasing at 27 nanometres based on laser wakefield accelerator. 2021

[2] R. Pompili, et al. Free-electron lasing with compact beam-driven plasma wakefield accelerator. 2022.

[3] S. Reiche, GENESIS 1.3: a fully 3D time-dependent FEL simulation code

[4] P. Emma, et al. FIRST LASING OF THE LCLS X-RAY FEL AT 1.5 Å. 2009.

[5] J.-L. Vay. Noninvariance of Space- and Time-Scale Ranges under a Lorentz Transformation and the Implications for the Study of Relativistic Interactions. 2007

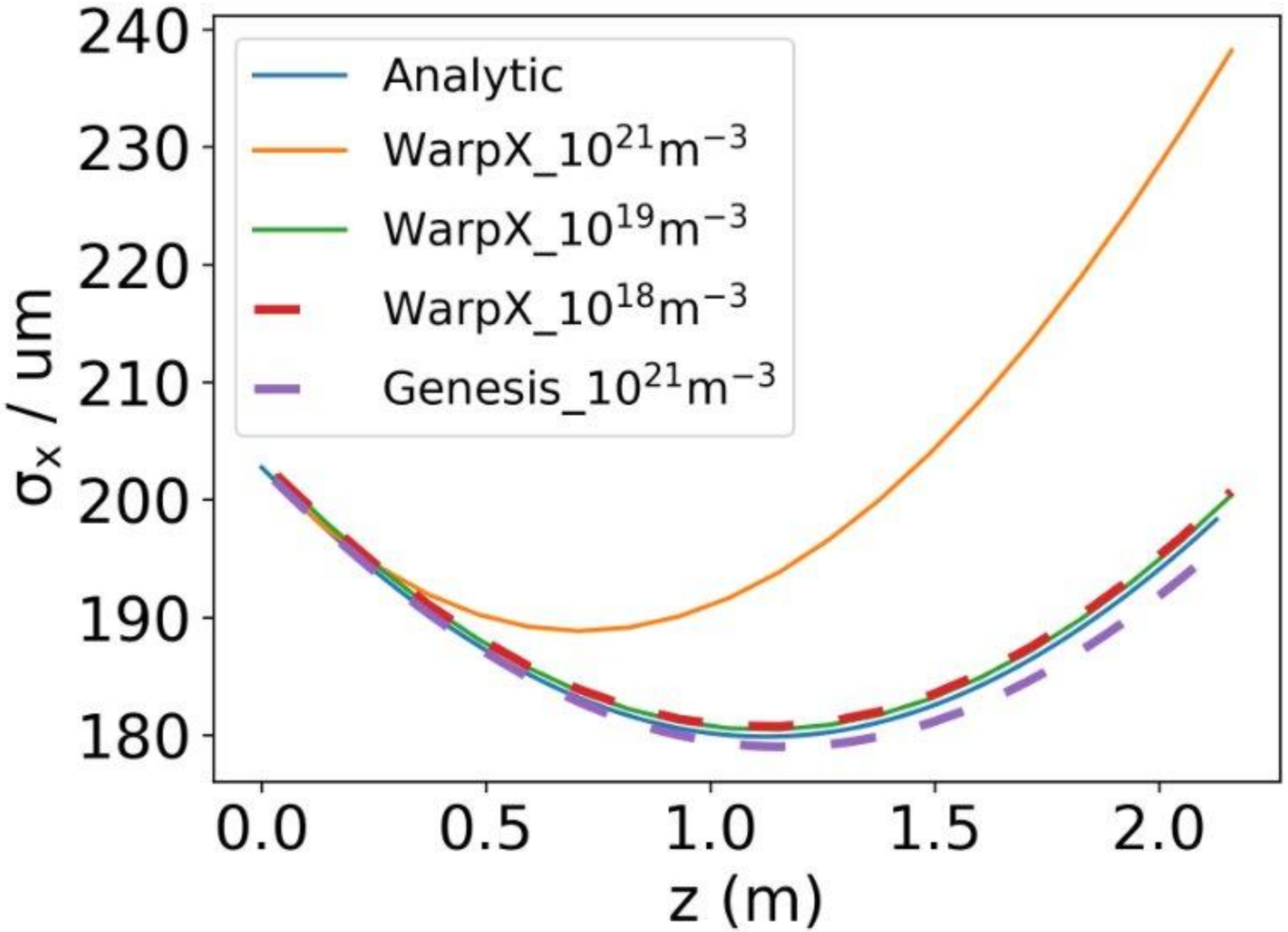
[6] J.-L. Vay, et al. Warp-X: a new exascale computing platform for beam-plasma simulations. 2018

[7] M. P. Backhouse, and Z. Najmudin. Machine Learning: Science and Technology (submitted)

IMPERIAL

Thank you

Motion along Electron Wiggling Direction (100MeV)

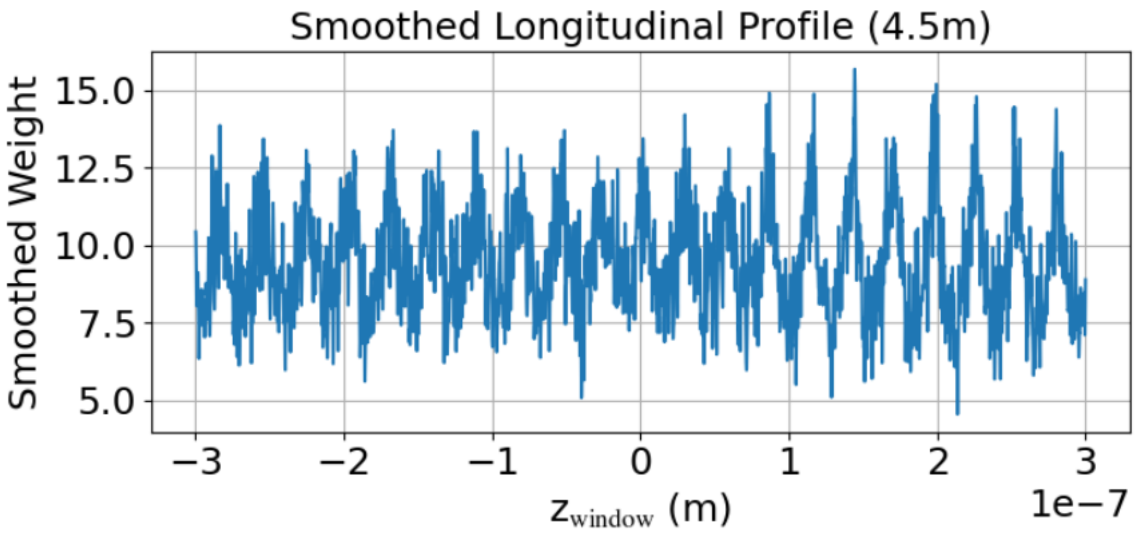


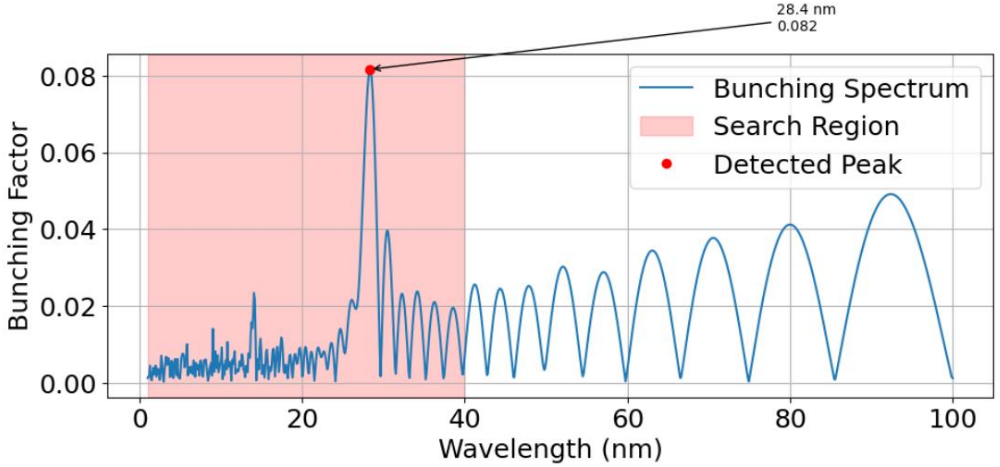
$$\frac{d^2a}{dz^2} = -K(z)a + \frac{\varepsilon^2}{a^3} + \frac{2Q}{a+b},$$

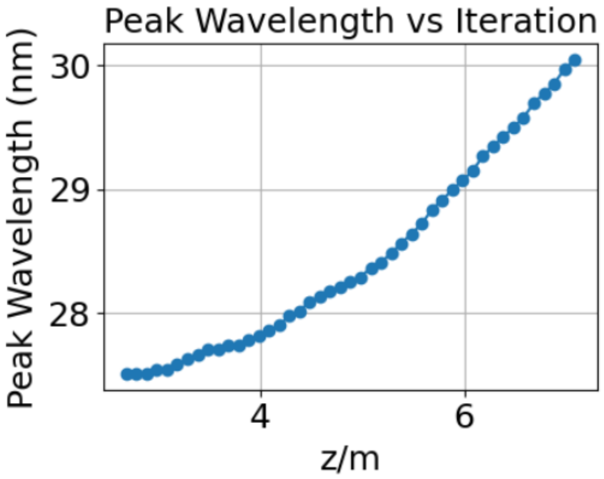
$$\frac{d^2b}{dz^2} = +K(z)b + \frac{\varepsilon^2}{b^3} + \frac{2Q}{a+b}.$$

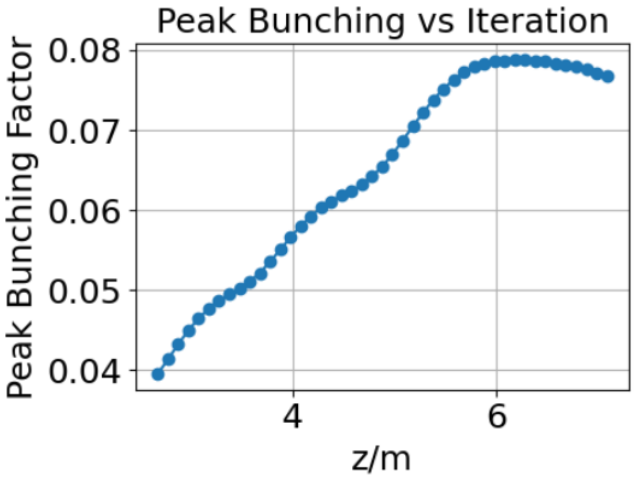
- Space charge term has a large contribution at high densities
- Transfer matrices only model the effect of the beam optics

Bunching


Smoothed Longitudinal Profile (4.5m)




Peak Wavelength vs Iteration


Peak Bunching vs Iteration

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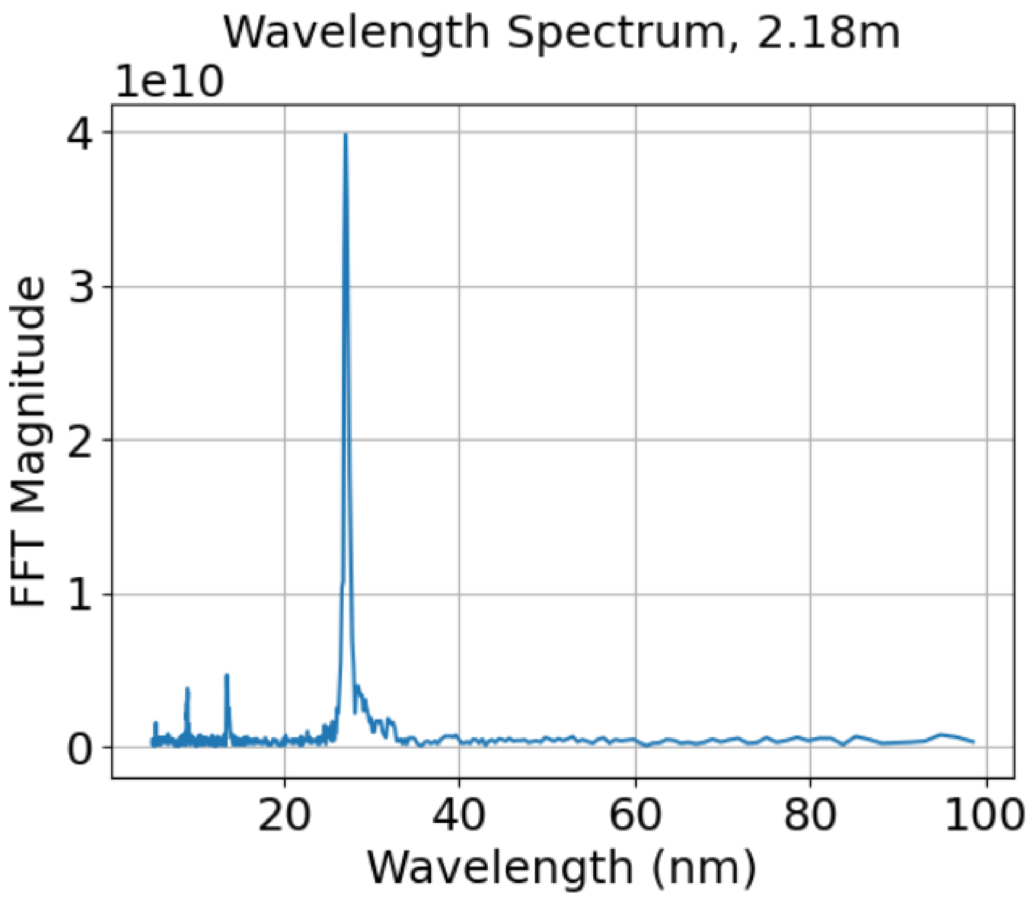
Radiation Wavelength



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Central Wavelength: 27.03 nm
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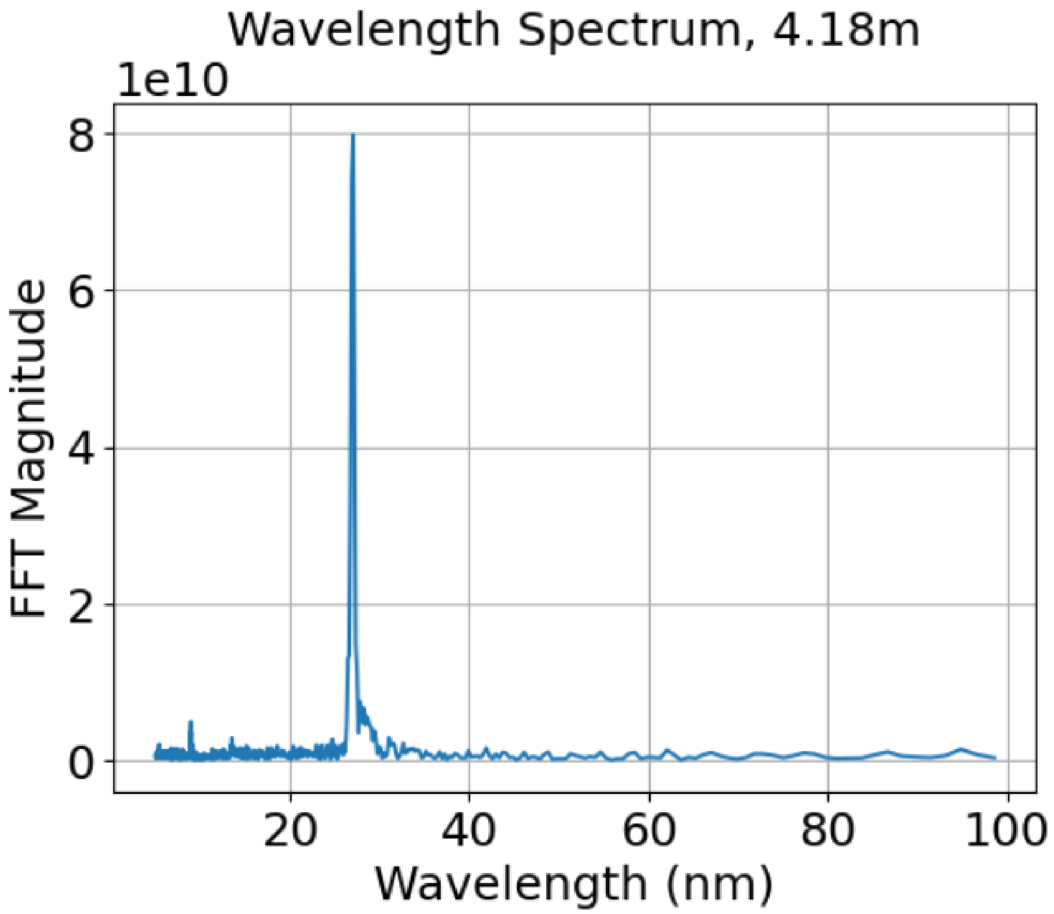

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Central Wavelength: 27.03 nm
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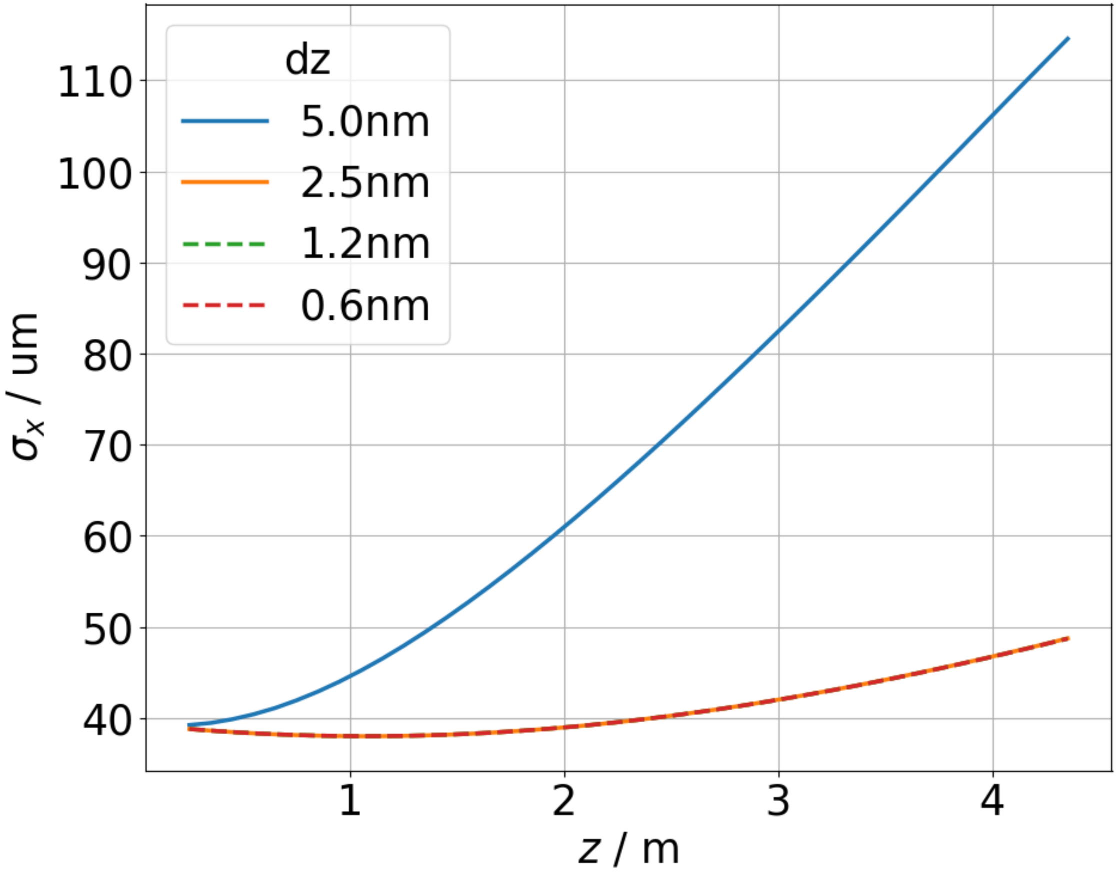
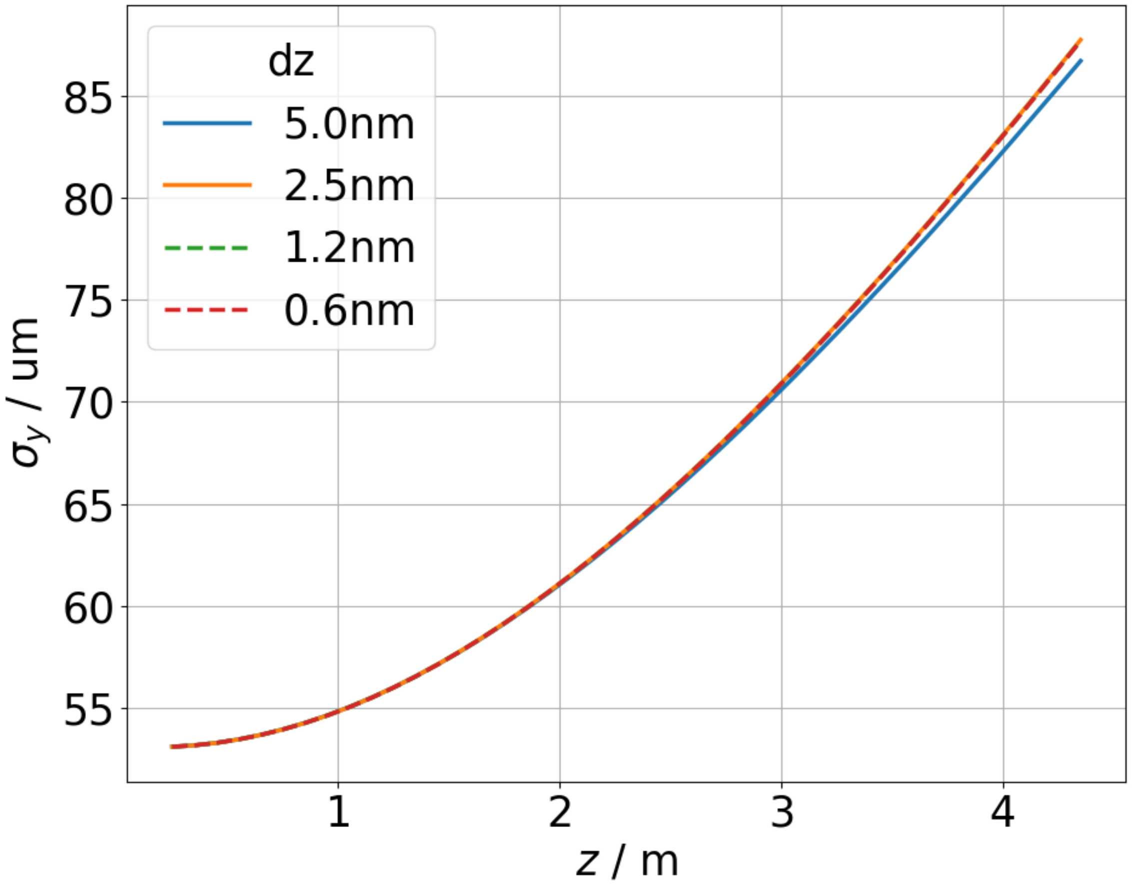
Convergence Scan (490MeV)





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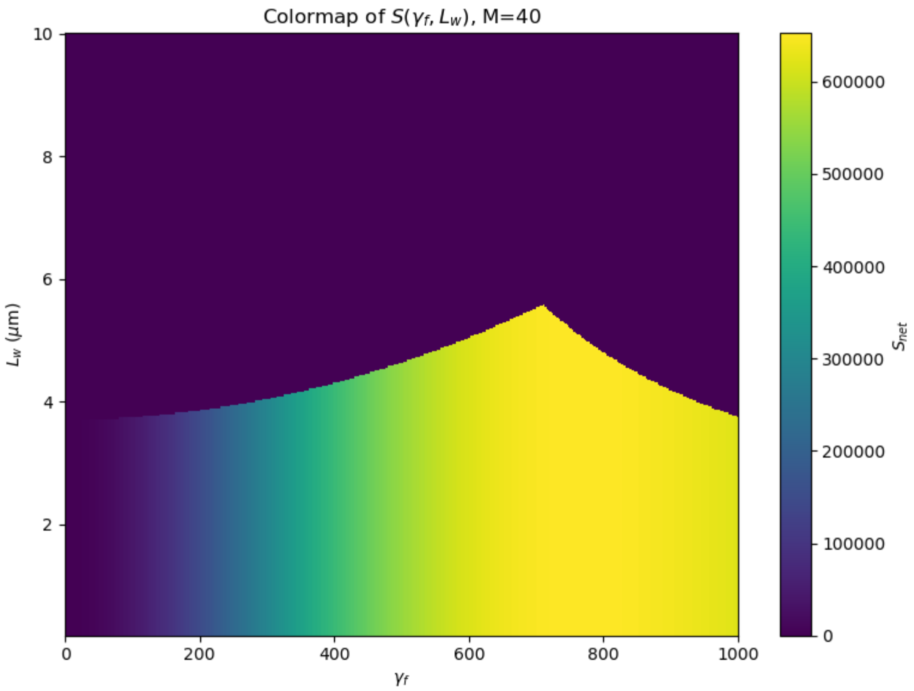
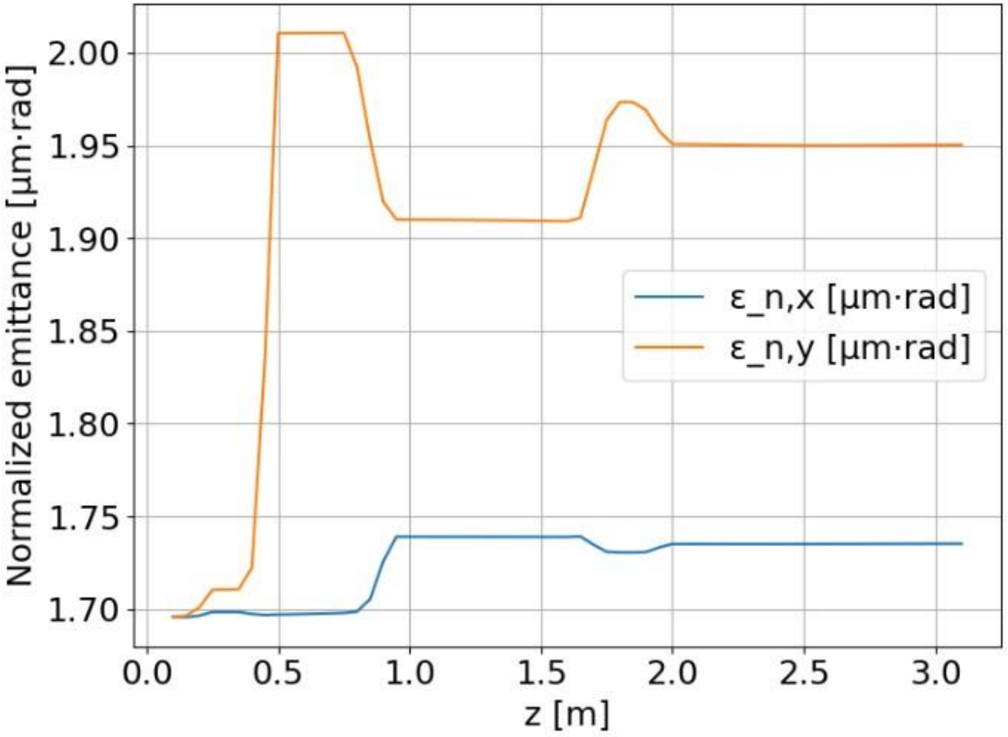
Optimal Frame and Speed Up

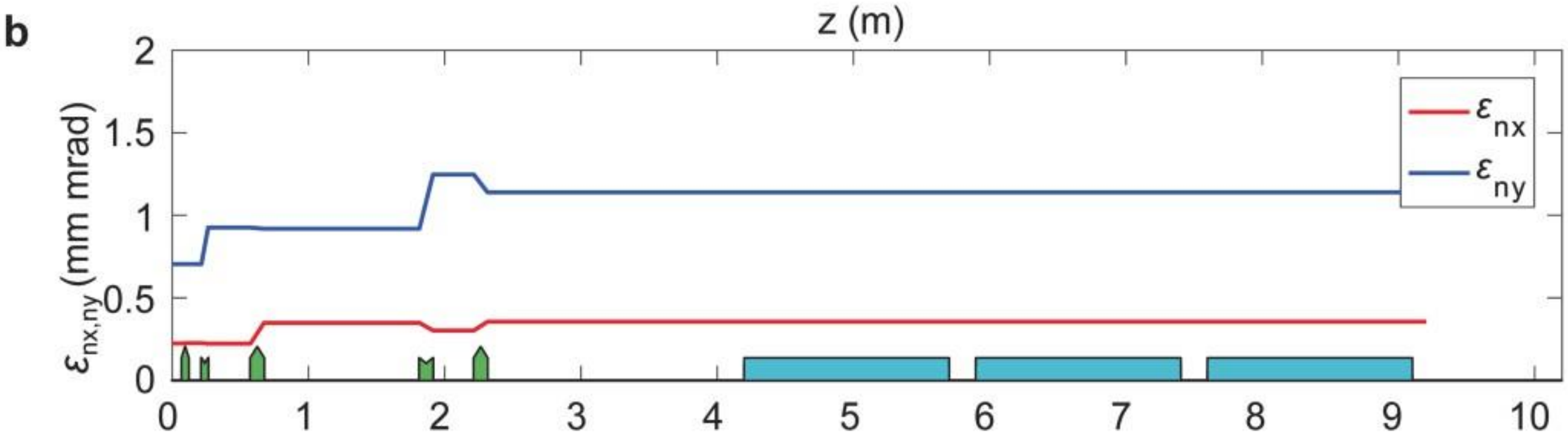
- Assuming $\beta_f \sim \beta_w$, lab iterations / boost iterations is given by: $2\gamma^2$
- Less time iterations, harder to resolve undulator period
- Limit longitudinal cell # to 6000 with $\mathrm{dz}' = \min(\lambda_{und}', \lambda_{rad}')/40$
- If there is no hardware limit, the optimal frame is the beam rest frame:

$$\gamma_{parallel} = \gamma/\sqrt{1+K^2/2}$$



EPAC Transport ($\frac{\Delta E}{E} = 1\%$)

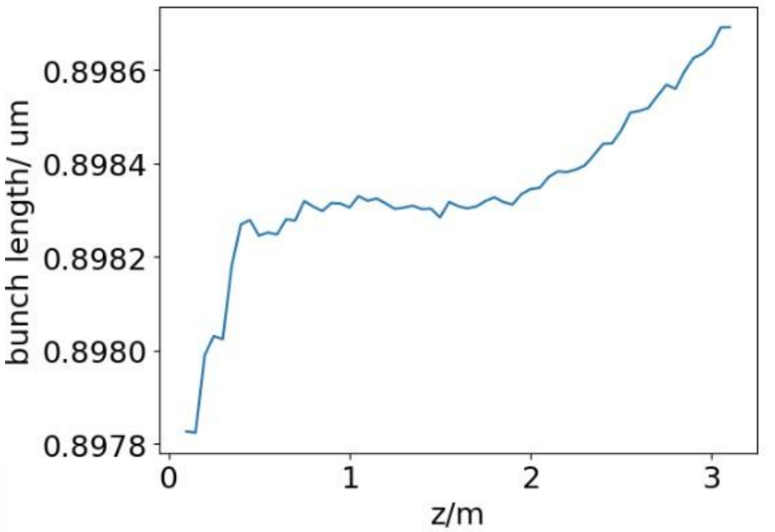




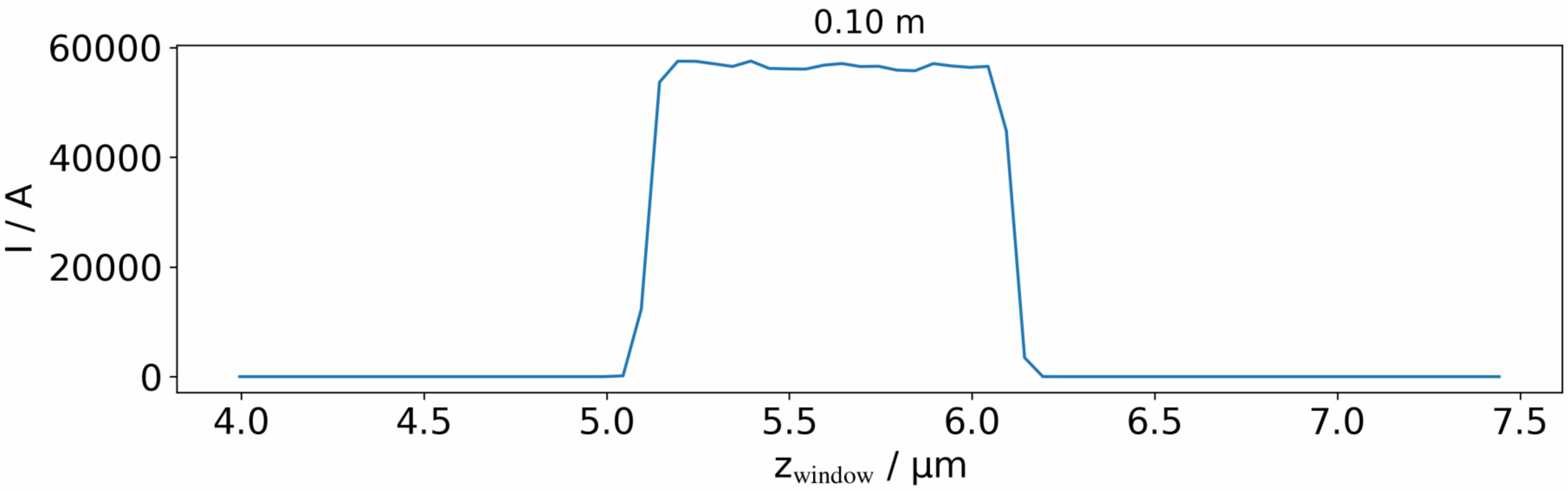
W. Wang et al. Nature (2021)

EPAC Transport ($\frac{\Delta E}{E} = 1\%$)

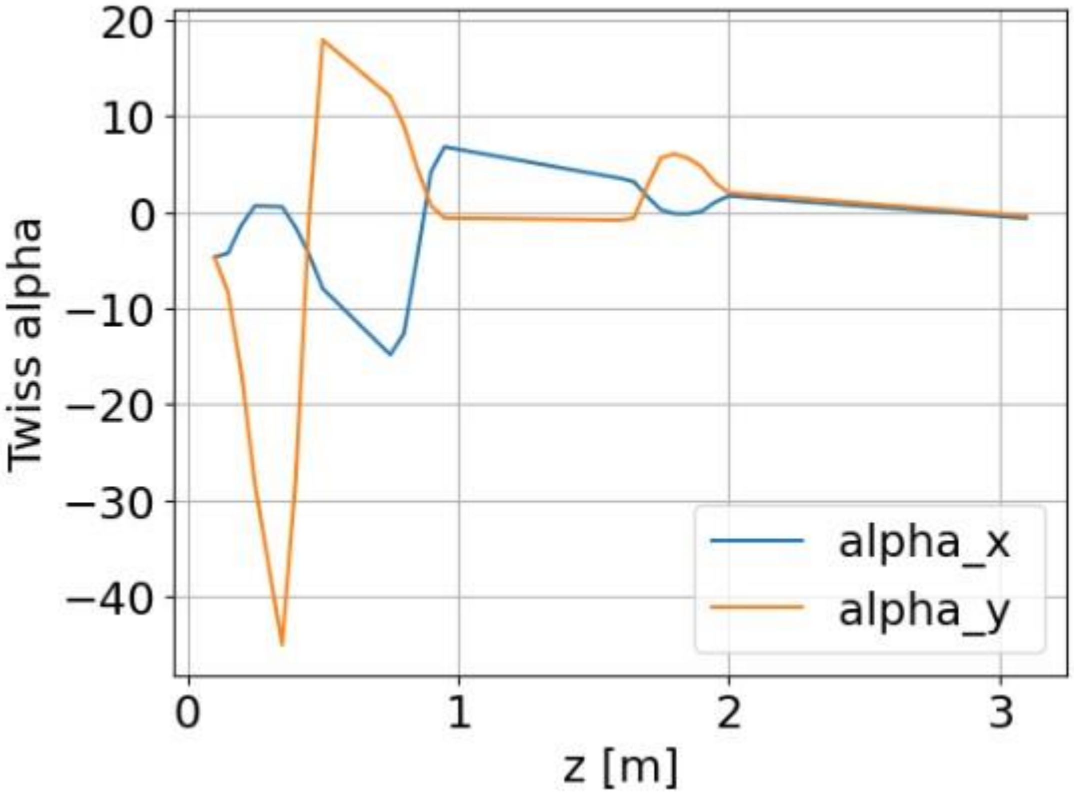
90% charge

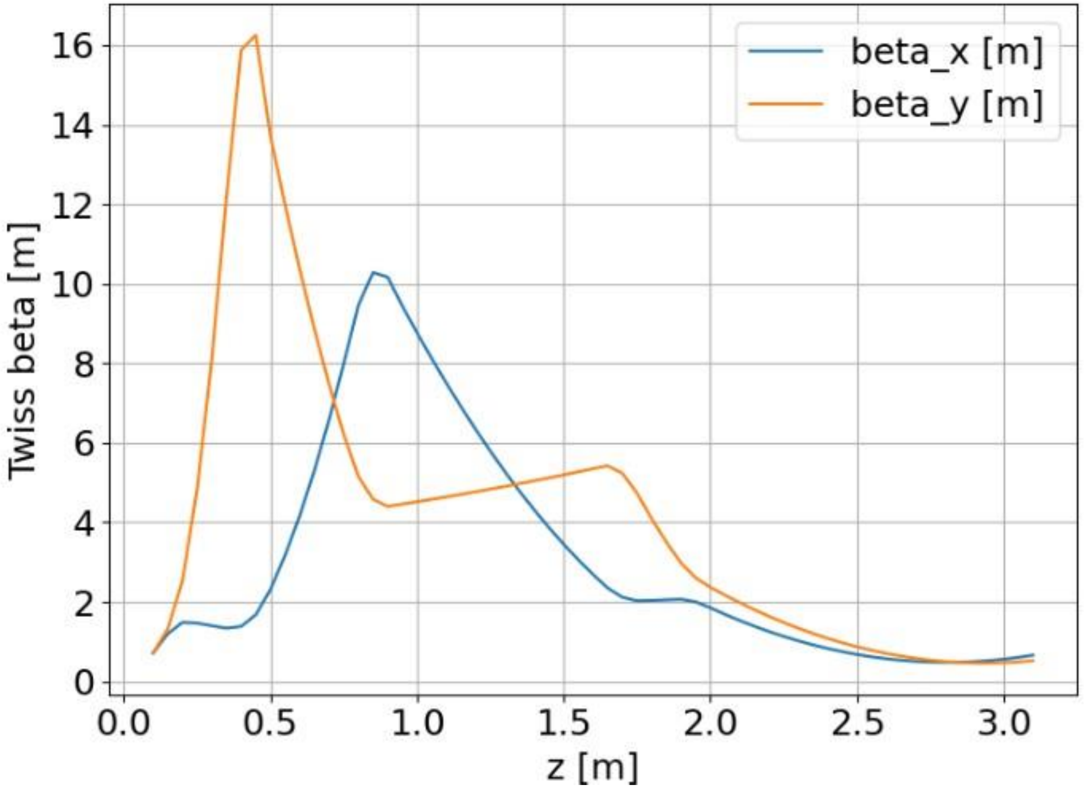


- $1nm$ $growth << dz$=$50nm$
- $E = 2900MeV$
- $\Delta E = 29MeV$
- $E_{max} = 2929MeV, E_{min} = 2871MeV$
- $2nm$ $path$ $difference$ $after$ $3m$



Epac Twiss





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FEL Beamlines

Case	I [A]	Energy [MeV]	λ [nm]
Beam driven	300	93.9	830
Laser	2000	490	27
EPAC	60,000	2900	0.77

- $\frac{\lambda}{20}$ Boosted frame simulation in beam rest frame
- WarpX simulation takes a few hours on RTX6000 to simulate several meters