Physical First Laws for Black Holes in de Sitter Space

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Where's quantum gravity?

To date, we don't have any direct evidence of **quantum gravity**.

- No direct evidence for string theory.
- No direct evidence for (low-scale) supersymmetry.
- No direct evidence for high-scale inflation.

Of course, also **no violations** from rigorous models of quantum gravity have been observed.

Too small to observe?

- Perhaps, this just means that all effects of quantum gravity are tied to the **Planck scale**.

$$l_p = \sqrt{\frac{\hbar G_N}{c^3}} \sim 10^{-35}\, m \quad \longleftarrow \quad \text{Too small to observe}$$

- If so, no hope to directly access quantum gravity effects.

- Luckily, there's evidence that quantum gravity does have **implications at low energies**.

Quantum gravity in the IR

Evidence includes:

- **The holographic principle**: reduction of degrees of freedom to satisfy area law.

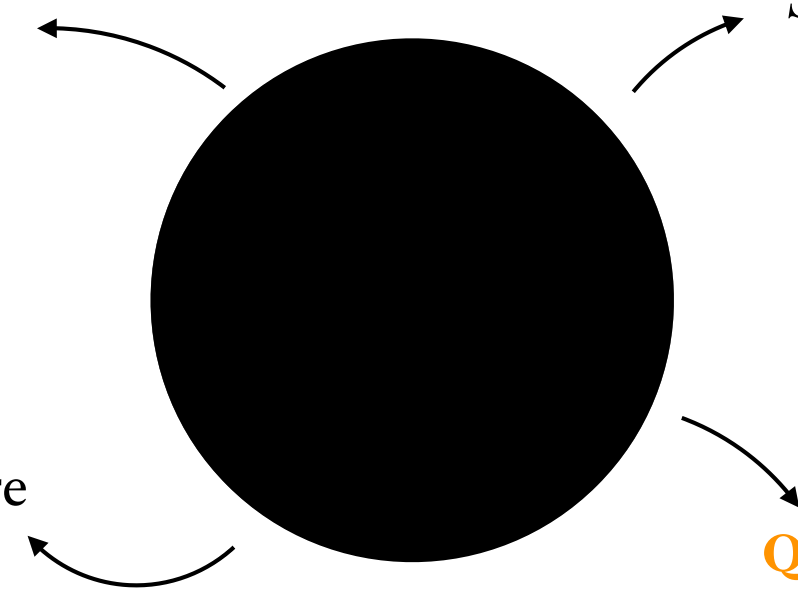
$$S = \frac{\text{Area}}{4G_N} = \log\left(\#\text{micro}\right) \longleftarrow \text{Not described by QFT!}$$

- **Black hole information paradox**: $\mathcal{O}(1)$ modification to semi-classical Hawking radiation when curvature is still small.

- **Swampland constraints**: not all effective (field) theories can be consistently coupled to quantum gravity.

Black holes as probes of QG

These different hints are all closely related to **black holes**.



My focus today will be on **charged black holes** in **de Sitter space**.

Plan for the talk

- Quantum corrections in near-extremal black holes
- New 1st laws for black holes in de Sitter space
- Applications
- Summary and future work

Quantum corrections in near-extremal black holes

Intro to extremal black holes

- Schwarzschild black holes have a horizon $r_+ \sim G_N M$ and a Hawking temperature:

$$T \sim 1/r_+$$

- When we add another conserved quantity, e.g. charge, get two horizons: $r_\pm = M \pm \sqrt{M^2 - Q^2}$

$$T \sim (r_+ - r_-)/r_+ \geq 0 \quad \longleftrightarrow \quad M \geq |Q|$$

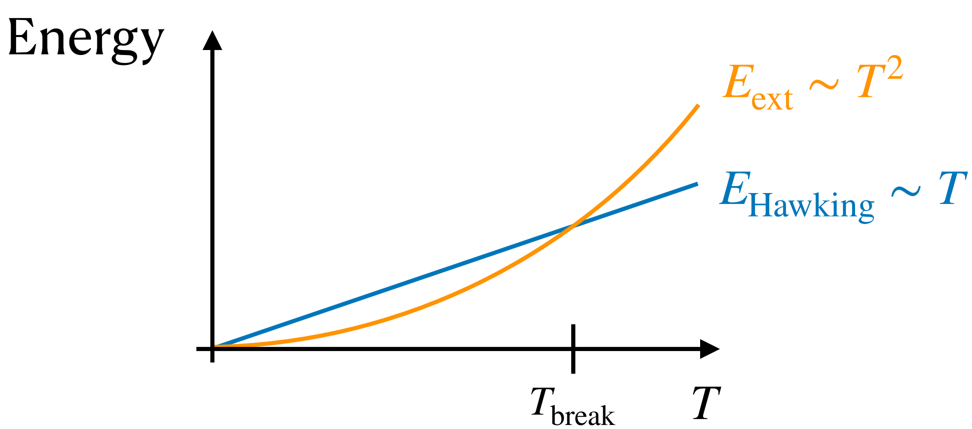
- Zero-temperature solutions are called **extremal black holes**.

Thermodynamic puzzles

Two puzzles appear near extremality. [Preskill, Schwarz, Shapere, Trivedi, Wilczek '91]

1. At $T = 0$, area is still finite: large (ground state) entropy without obvious symmetry. **Violates 3rd law of thermodynamics**.

2. Semi-classical description requires small change of temperature when emitting Hawking radiation. **Violated for small T**.



$$T_{\text{break}} \sim \frac{1}{l_p Q^3}$$

Heat capacity

- The location where these two lines cross is given by $\delta E \sim \delta T$.
- We can now use the **1st law** of thermodynamics:

$$\delta M = T_b \delta S_b + \Phi_b \delta Q$$

- Doing so, we find that the semi-classical approximation breaks down when:

$$C_Q := T_b \left(\frac{\delta S_b}{\delta T_b} \right)_Q = \left(\frac{\delta M}{\delta T_b} \right)_Q \leq 1 \qquad \longleftarrow \qquad \text{Heat capacity of } \mathcal{O}(1)$$

Resolution

- These puzzles have recently been addressed. [Turiaci, Iliesiu '20] + [follow up work]

- Computations based on the Euclidean gravitational path integral have revealed **large quantum corrections** at low temperature:

$$Z = \int DgD\phi e^{-I_E[g,\phi]} \longrightarrow \log Z = -S_0 + \log(Z_{1-\text{loop}}) + \dots$$

classical entropy quantum corrections

- Making use of near-horizon geometry these corrections can be incorporated (one-loop exact!).

$$Z_{1-\text{loop}} \sim T^{\alpha} \longleftarrow$$ Power depends on precise black hole

Consequences

- Leads to a **large deviation** of semi-classical thermodynamics at low temperature: addresses the 2nd puzzle.

- **Density of states** becomes small for low temperature: addresses the 1st puzzle. No (quasi-)stable extremal black hole anymore!

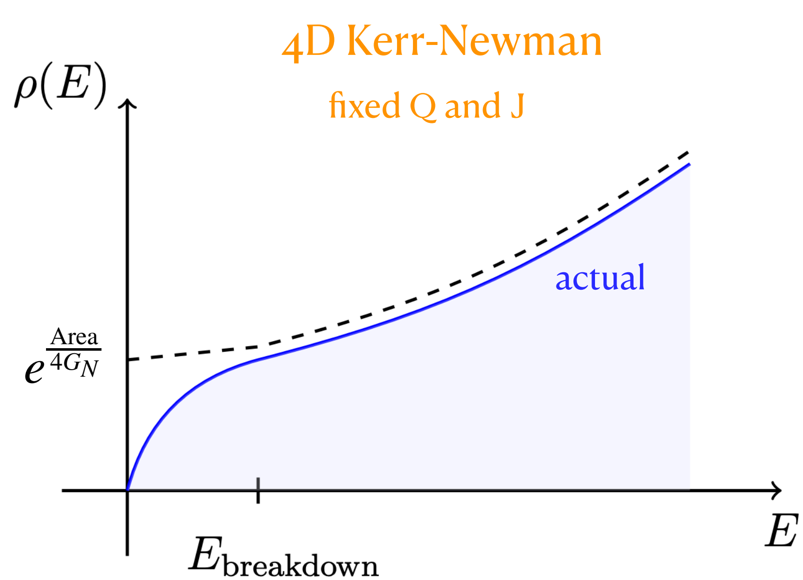


Figure adapted from: arXiv:2003.02860

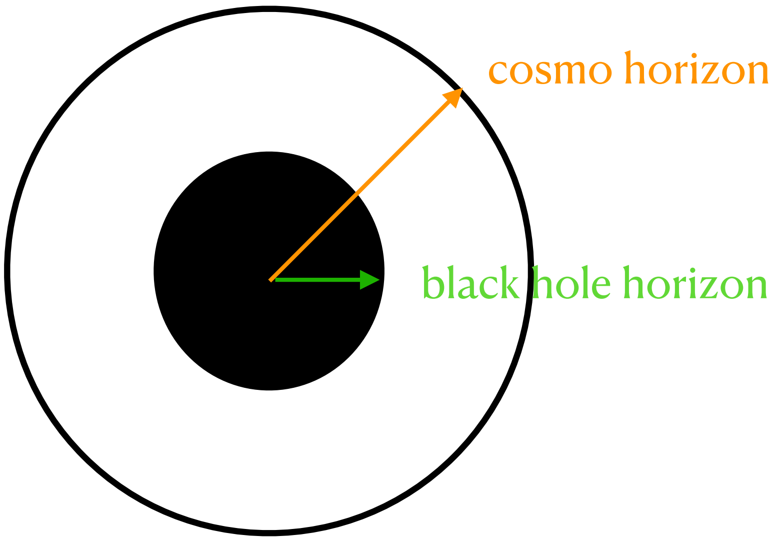
What about cosmology?

- Interesting to understand if there are other geometries for which we expect **large quantum corrections**.
- Recent works have suggested that similar corrections might be important for (black holes in) cosmological spacetimes. [Turiaci '25] [Turiaci '25] [Blacker, Castro, Sybesma, Toldo '25]
- We are specifically interested in computing heat capacity of (extremal) **black holes in de Sitter space**.
- We'll see this requires a new version of **first laws of thermodynamics**.

New 1st laws for black holes in de Sitter space

Black holes in de Sitter

Charged black holes in de Sitter are described by 3 parameters: (M, Q, ℓ)

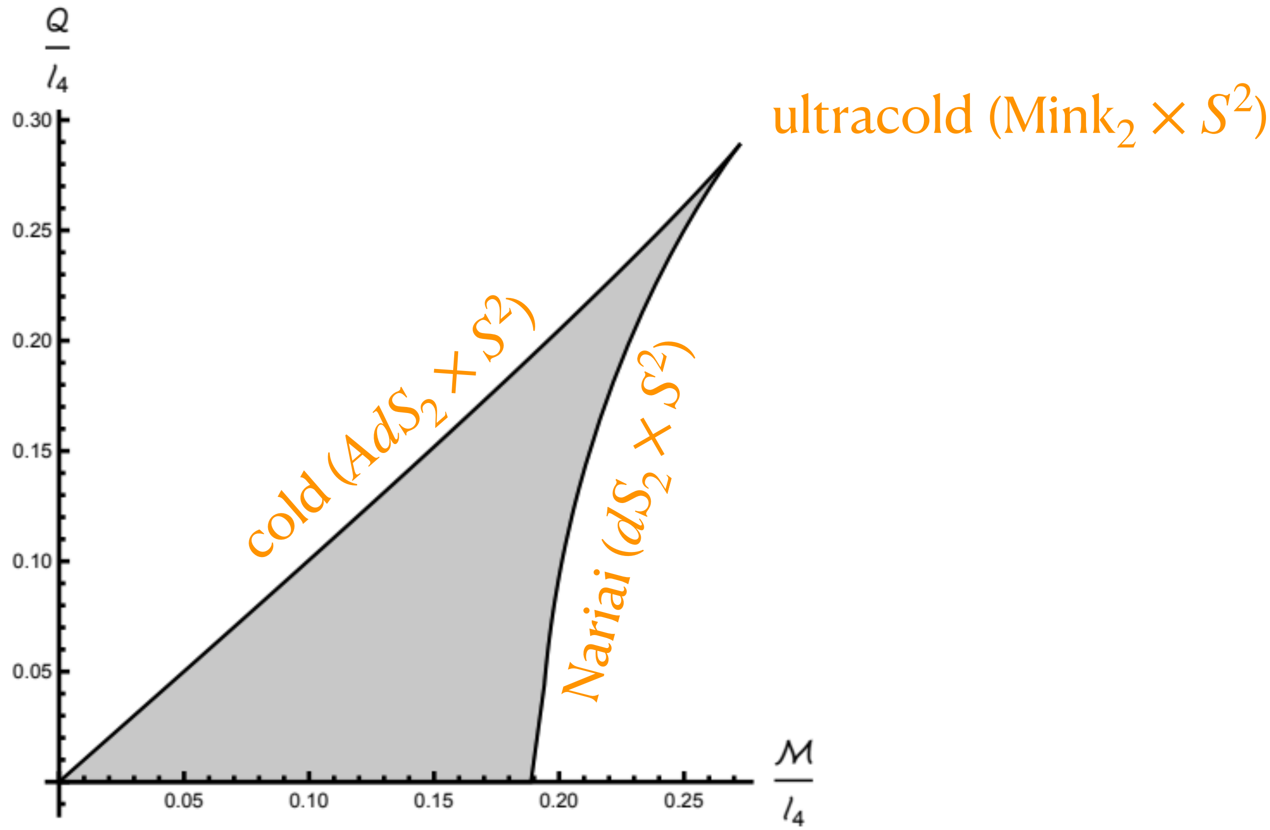


r_a : inner black hole horizon
r_b : outer black hole horizon
r_c : cosmological horizon

Radii have a complicated dependence on mass, charge and de Sitter radius. We'll focus on four dimensions.

Shark fin

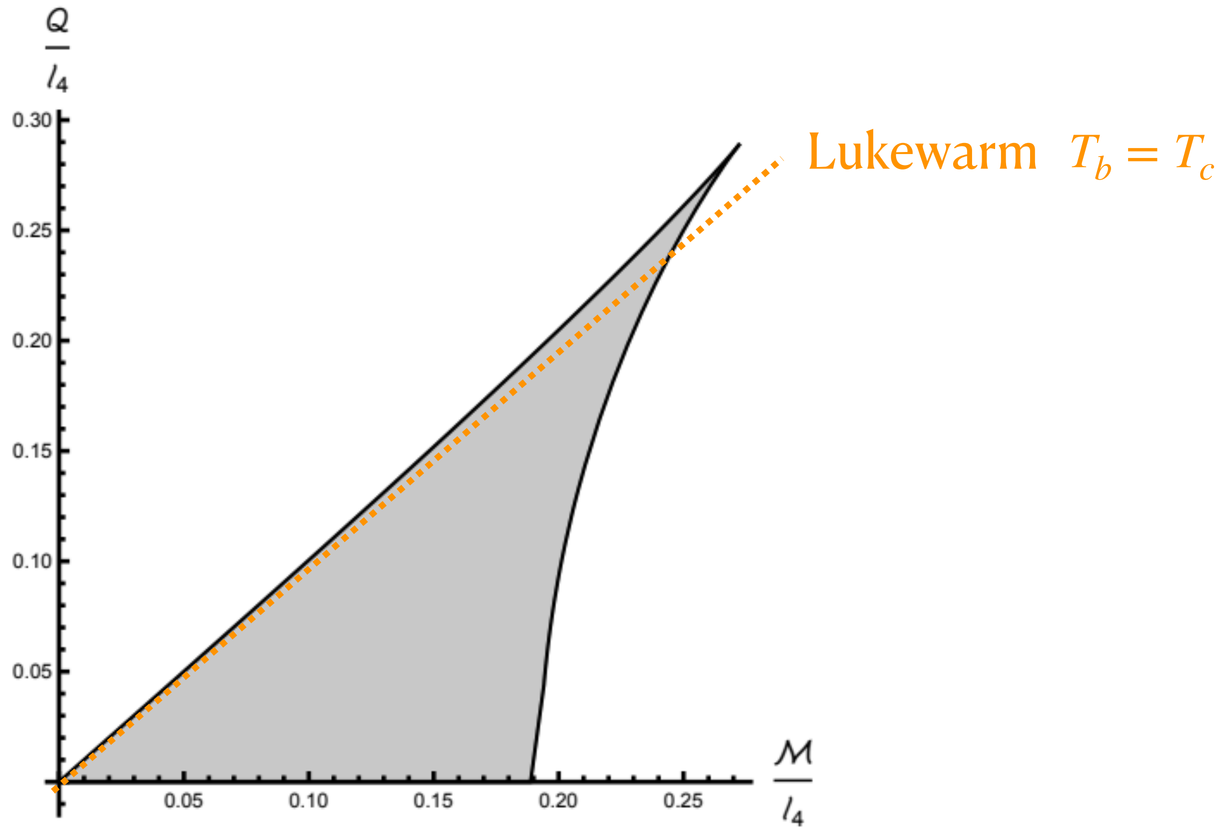
The phase space of regular solutions forms a “shark fin”.



There are 3 notions of **“extremality”**.

Lukewarm line

There is one additional special class of solutions: lukewarm black holes.



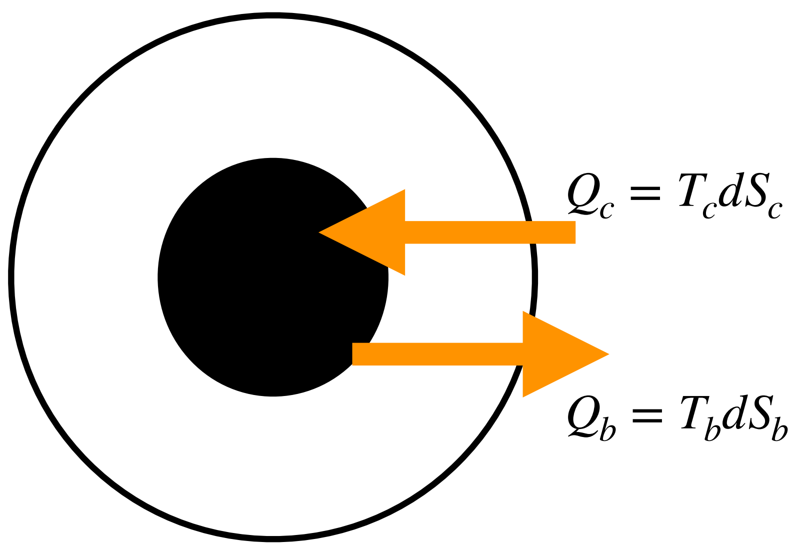
These are thermal equilibrium solutions.

Defining thermodynamics

Defining 1st laws for dS black holes is complicated due to:

1. Absence of a global timelike **Killing vector**.

2. Existence of multiple horizons with their (generally) **distinct temperatures**.



Defining temperature

- Consider a spherically symmetric metric:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

- To define temperature of horizon at $f(r_0) = 0$, use **timelike Killing vector** $\xi = \gamma \partial_t$. This is the 4-velocity at $r =$ const.

Local acceleration: $a^\mu = (\xi^\nu \nabla_\nu \xi^\mu)/(\xi^\rho \xi_\rho)$ ⟵ independent of γ, diverges at horizon

Surface gravity: $\kappa = \lim_{r \to r_0} \sqrt{(a^\mu a_\mu)(\xi^\nu \xi_\nu)}$ ⟵ dependent on γ, finite at horizon

$= -f(r_0)$, redshift factor

Interpretation of surface gravity

In general, the **redshift factor** between two locations is:

$$\frac{\lambda_0}{\lambda_{\mathcal{O}}} = \sqrt{\frac{f(r_0)}{f(r_{\mathcal{O}})}}$$

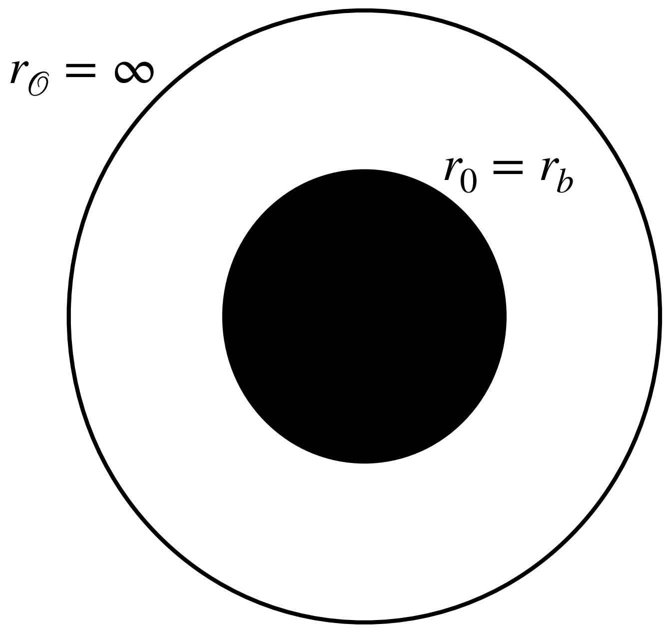
Surface gravity is an acceleration at r_0 expressed in frame of observer at $r_{\mathcal{O}}$. Location given by choice of $\gamma^2 = 1/f(r_{\mathcal{O}})$.

Normalization picks an observer!

Examples

Asymptotically Flat Black Hole

$$f(r) = 1 - \frac{2G_N M}{r}$$

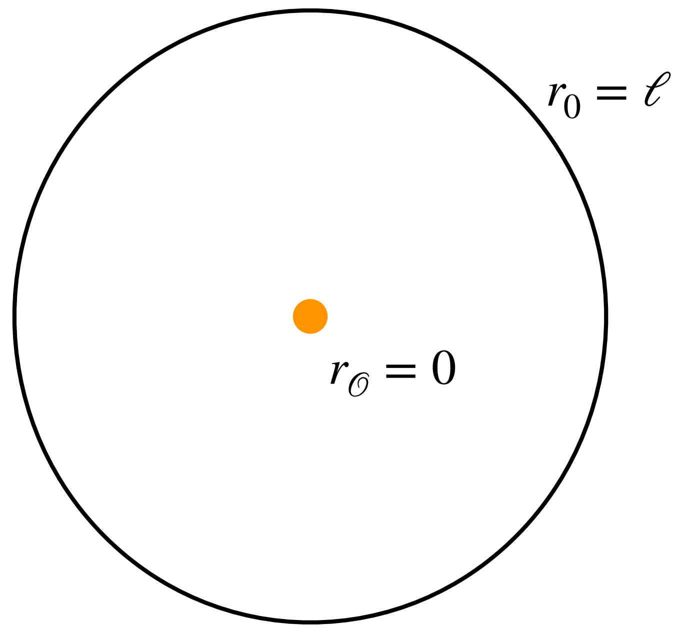


$$f(r_{\mathcal{O}}) = \gamma = 1$$

$$\xi = \partial_t$$

Empty de Sitter Space

$$f(r) = 1 - \frac{r^2}{\ell^2}$$

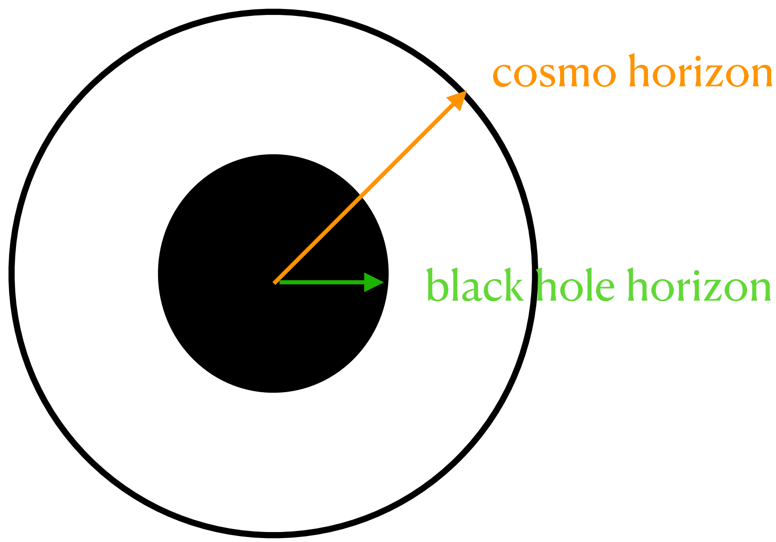


$$f(r_{\mathcal{O}}) = \gamma = 1$$

$$\xi = \partial_t$$

Black holes in de Sitter

If we again take $\xi^{\mu}\xi_{\mu} = \pm 1$, corresponds to $r_{\mathcal{O}} < r_a$ or $r_{\mathcal{O}} > r_c$.



This is not a natural place for an observer!

Consequences

- Previous **first laws** were derived using $\xi = \partial_t$. [Dolan, Kastor, Kubiznak, Mann, Traschen '13]

$$dM = +\, T_b dS_b + \Phi_b dQ$$

$$dM = -\, T_c dS_c + \Phi_c dQ$$

- This expresses the temperature as the acceleration with respect to an **accelerating trajectory** outside of the static region $r_b \leq r \leq r_c$.

- In the Nariai limit $(r_c - r_b) \to 0$ temperature goes to zero. At odds with **near-horizon geometry** $\mathrm{dS}_2 \times S^2$.

Resolution

- The resolution is to **adapt the normalization** to a more natural observer.

- For $r_b \leq r_{\mathcal{O}} \leq r_c$ there is a unique observer that is **non-accelerating** and can observe **both horizons**.

- This observer is a solution to: $f'(r) = 0$. [Bousso, Hawking '96]

$$r^4 - \frac{1}{2} r \left(r_a + r_b\right) \left(r_a + r_c\right) \left(r_b + r_c\right) + r_a r_b r_c \left(r_a + r_b + r_c\right) = 0$$

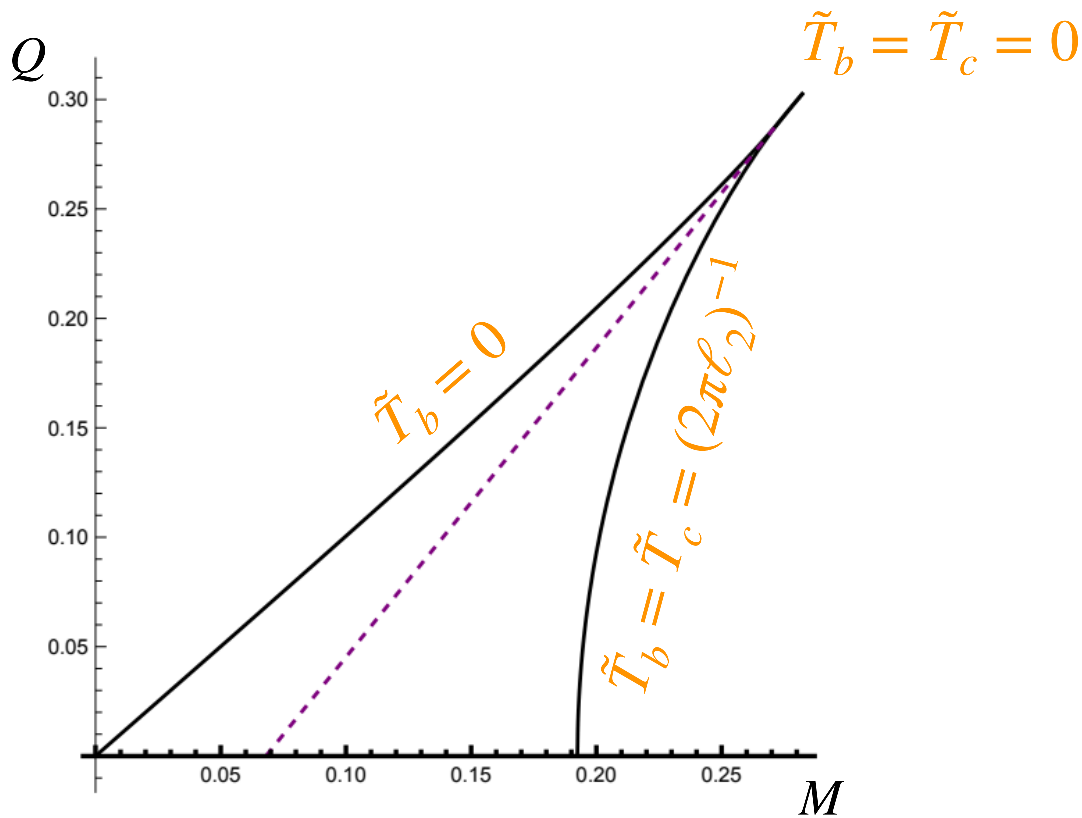
Innocent quartic polynomial?

General solution:

$$
\begin{aligned}
\texttt{rgeo} = {} & \frac{1}{6}\left(\sqrt{\left(\frac{36\times 2^{1/3}\,ra\,rb\,rc\,(ra+rb+rc)}{\left(\frac{27}{4}(ra+rb)^2(ra+rc)^2(rb+rc)^2+\sqrt{\frac{729}{16}(ra+rb)^4(ra+rc)^4(rb+rc)^4-6912\,ra^3\,rb^3\,rc^3\,(ra+rb+rc)^3}\right)^{1/3}}+\right.}\right. \\
& \left.\frac{3\left(\frac{27}{4}(ra+rb)^2(ra+rc)^2(rb+rc)^2+\sqrt{\frac{729}{16}(ra+rb)^4(ra+rc)^4(rb+rc)^4-6912\,ra^3\,rb^3\,rc^3\,(ra+rb+rc)^3}\right)^{1/3}}{2^{1/3}}\right)+ \\
& 3\sqrt{\left(-\frac{4\times 2^{1/3}\,ra\,rb\,rc\,(ra+rb+rc)}{\left(\frac{27}{4}(ra+rb)^2(ra+rc)^2(rb+rc)^2+\sqrt{\frac{729}{16}(ra+rb)^4(ra+rc)^4(rb+rc)^4-6912\,ra^3\,rb^3\,rc^3\,(ra+rb+rc)^3}\right)^{1/3}}-\right.} \\
& \frac{\left(\frac{27}{4}(ra+rb)^2(ra+rc)^2(rb+rc)^2+\sqrt{\frac{729}{16}(ra+rb)^4(ra+rc)^4(rb+rc)^4-6912\,ra^3\,rb^3\,rc^3\,(ra+rb+rc)^3}\right)^{1/3}}{3\times 2^{1/3}}+ \\
& \left(\sqrt{3}\,(ra+rb)\,(ra+rc)\,(rb+rc)\right)\Big/ \\
& \left(\sqrt{\left(\frac{12\times 2^{1/3}\,ra\,rb\,rc\,(ra+rb+rc)}{\left(\frac{27}{4}(ra+rb)^2(ra+rc)^2(rb+rc)^2+\sqrt{\frac{729}{16}(ra+rb)^4(ra+rc)^4(rb+rc)^4-6912\,ra^3\,rb^3\,rc^3\,(ra+rb+rc)^3}\right)^{1/3}}+\right.}\right. \\
& \left.\left.\left.\frac{\left(\frac{27}{4}(ra+rb)^2(ra+rc)^2(rb+rc)^2+\sqrt{\frac{729}{16}(ra+rb)^4(ra+rc)^4(rb+rc)^4-6912\,ra^3\,rb^3\,rc^3\,(ra+rb+rc)^3}\right)^{1/3}}{2^{1/3}}\right)\right)\right)
\end{aligned}
$$

Physical temperatures

In this more **physical normalization**, find following behavior in extremal limits.



[LA, Lin, van der Schaar, Shiu, Sybesma '25]

York boundary

- In order to derive new 1st laws, useful to introduce a **York boundary** at $r_{\mathcal{O}}$.

- Doing so, there is a standard procedure to derive 1st laws.
  [Banihashemi, Jacobson, Svesko, Visser '22]

$$\delta E = +\tilde{T}_b \delta S_b + (\tilde{\Phi}_b - \tilde{\Phi}_{\mathcal{O}})\delta Q - \tilde{P}_{\mathcal{O}} \delta A_{\mathcal{O}}$$

$$\delta E = -\tilde{T}_c \delta S_c + (\tilde{\Phi}_c - \tilde{\Phi}_{\mathcal{O}})\delta Q - \tilde{P}_{\mathcal{O}} \delta A_{\mathcal{O}}$$

Brown-York energy

Pressure at $r_{\mathcal{O}}$

[LA, Lin, van der Schaar, Shiu, Sybesma '25]

Final 1st laws

- Importance difference with standard treatment of York boundary is that variations $\delta r_{\mathcal{O}}$ are **not independent**.

- Because $r_{\mathcal{O}}$ is determined by $f'(r_{\mathcal{O}}) = 0$, only **two independent variations** are $(\delta M, \delta Q)$.

Defining: $\delta \tilde{M} = \dfrac{\delta M}{\sqrt{f(r_{\mathcal{O}})}}$ $\longrightarrow$

$$\delta \tilde{M} = + \tilde{T}_b \delta S_b + \tilde{\Phi}_b \delta Q$$

$$\delta \tilde{M} = - \tilde{T}_c \delta S_c + \tilde{\Phi}_c \delta Q$$

- New 1st laws in terms of **redshifted mass**.

Applications

When does this matter?

- For some thermodynamic quantities, the normalization drops out. For instance:

$$\delta\tilde{M} = +\,\tilde{T}_b\delta S_b + \tilde{\Phi}_b\delta Q$$

$$\delta\tilde{M} = -\,\tilde{T}_c\delta S_c + \tilde{\Phi}_c\delta Q \quad -$$

$$0 = T_b\delta S_b + T_c\delta S_c + (\Phi_b - \Phi_c)\delta Q$$

- Because $r_{\mathcal{O}}$ depends on mass and charge, normalization will be important **when taking derivatives**.

Heat capacity

- Remember we are interested in understanding **quantum corrections** signaled by a vanishing heat capacity.

- It is clear that in general:

$$\underbrace{T_b \left(\frac{\partial S_b}{\partial T_b}\right)_Q}_{C_b^Q} \neq \underbrace{\tilde{T}_b \left(\frac{\partial S_b}{\partial \tilde{T}_b}\right)_Q}_{\tilde{C}_b^Q}$$

- Let's see the difference.

Het capacity

- General expressions complicated, look at **extremal solutions**.

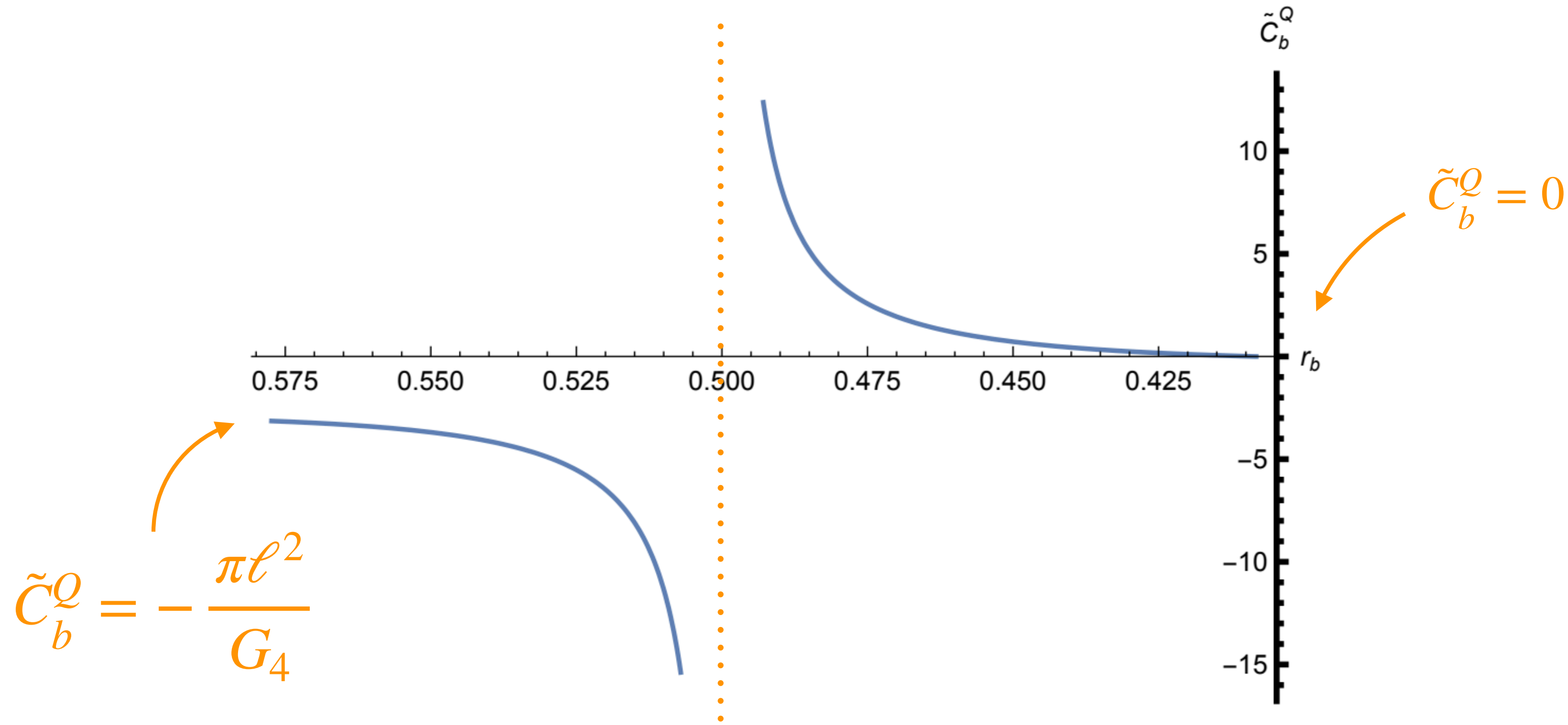
	C_b^Q	$\tilde{C}_b^Q$
Nariai:	0	$-\frac{\pi r_b^2}{G_4}\left(\frac{6r_b^2-\ell_4^2}{4r_b^2-\ell_4^2}\right)$
Cold:	0	0
Ultracold:	0	0
Nariai $\cap$ Lukewarm:	0	$\pm\infty$

- **No reason to expect large corrections** along Nariai in general!

- Take a closer look at the Nariai heat capacity.

Nariai Heat Capacity

Heat capacity along Nariai line:



Uncharged Nariai

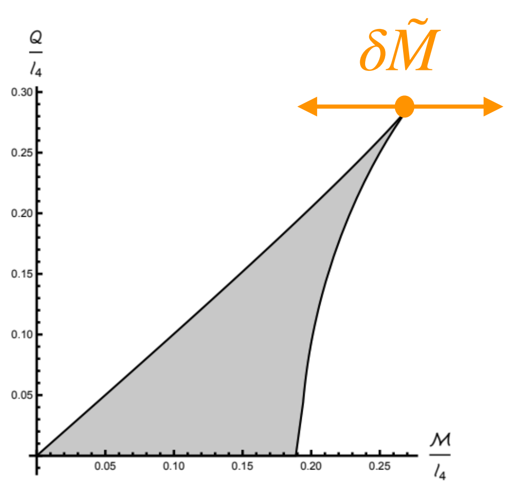
Nariai ∩ Lukewarm

Ultracold

Other ensembles

- Up to now, we worked in a **fixed charge ensemble**.
- This leads to two problems:

    1. **Diverging heat capacity** at intersection of lukewarm and Nariai.
    2. Variations away from the ultracold point **exit the sharkfin**.



Fixed charge-to-mass ratio

- Both problems are resolved by working at fixed charge-to-mass ratio $z = \delta Q/\delta M$.

$$\tilde{T}_b \delta S_b = (1 - z\Phi_b)\delta \tilde{M}$$

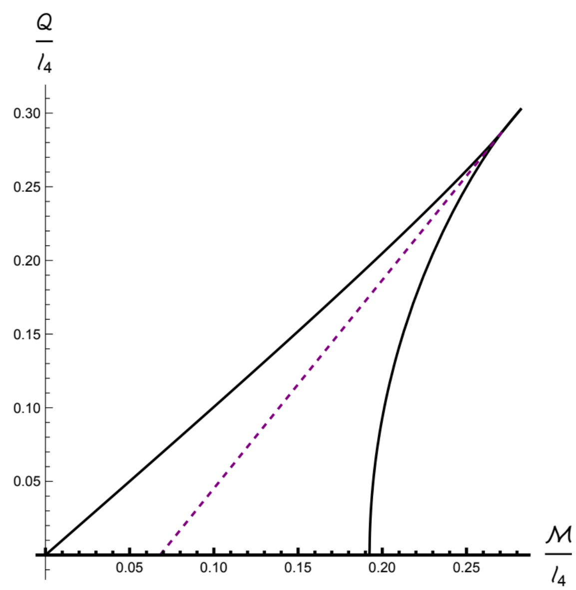
- At intersection of lukewarm and Nariai, there is a **unique charge-to-mass ratio** for which the heat capacity remains finite.

- Corresponds the vanishing heat flow: $z = \Phi_b^{-1} = 4\sqrt{\pi G_4}$.

Variations away from ultracold

- For variations away from the ultracold point, there is a unique charge-to-mass ratio.

- Again, corresponds to the point where heat flow vanishes:
  $z = \Phi_b^{-1} = 2\sqrt{2\pi G_4}$



Heat capacity at fixed z still vanishes!

Interpretation

- Would be interesting to have a **microscopic understanding** of the behavior of heat capacity.
- Divergences can be associated with **phase transitions**.
- Does this tell us about the microscopic **degrees of freedom**?
- We now have a prediction where **quantum corrections** should become large.

Summary

- We derived **new 1st laws of thermodynamics** for four-dimensional charged de Sitter black holes.
- Using those, we computed the **heat capacity** of different extremal solutions.
- Heat capacity vanishes for **cold black holes** and **ultracold Nariai black holes**.
- Gives predictions where **quantum corrections** should become large!

Future directions

- Relate our results to recent computations of one-loop determinants for de Sitter black holes.
- Not a direct comparison due to:
  - **Lorentzian vs. Euclidean**.
  - **Phase of the partition function**.
- Also interesting to compare to **microscopic models** (DSSYK/dS $_3$?)

Thank you!