Particle Physics Methods for Precision Gravitational Wave Physics

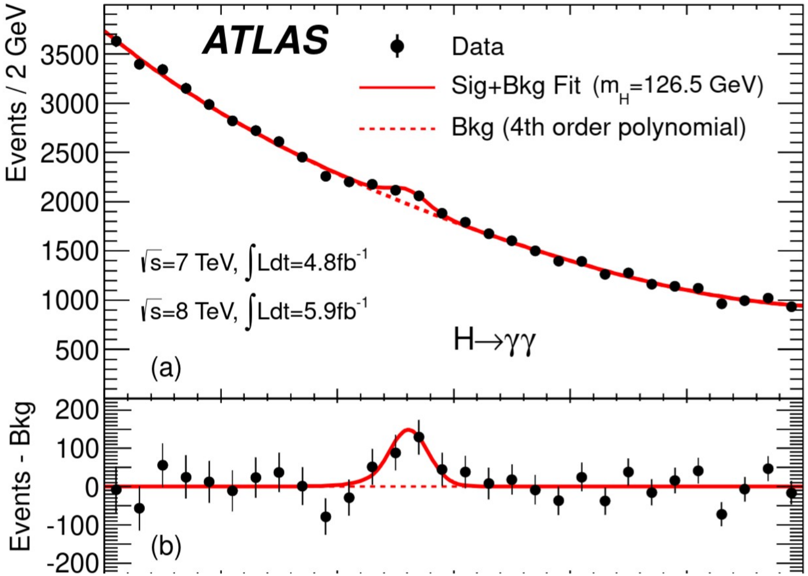
Mao Zeng, University of Edinburgh

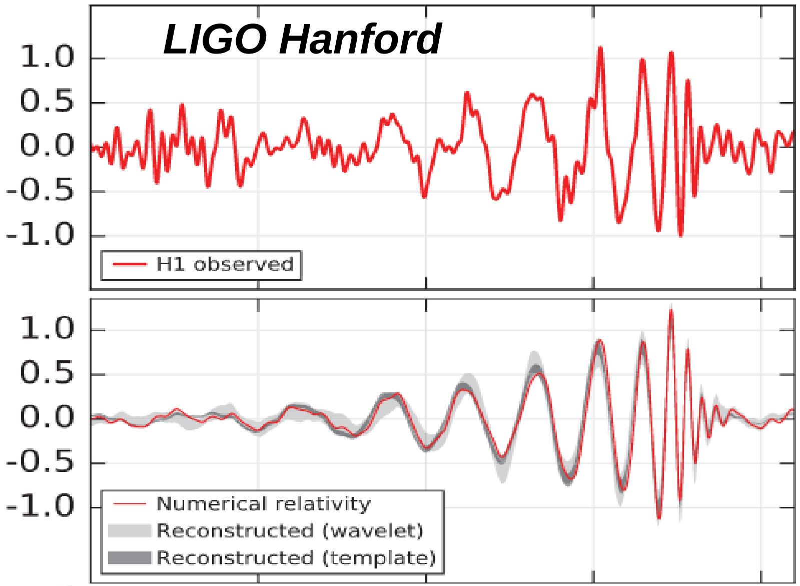
IOP Joint APP-HEPP Conference, 08 April 2026

Outline

- Background – gravitational wave physics
- How particle physics helps
  - □ Effective field theory □ Scattering amplitudes □ Feynman integrals
- Results & Comparisons
  - □ Bound orbits □ Scattering orbits
- Outlook

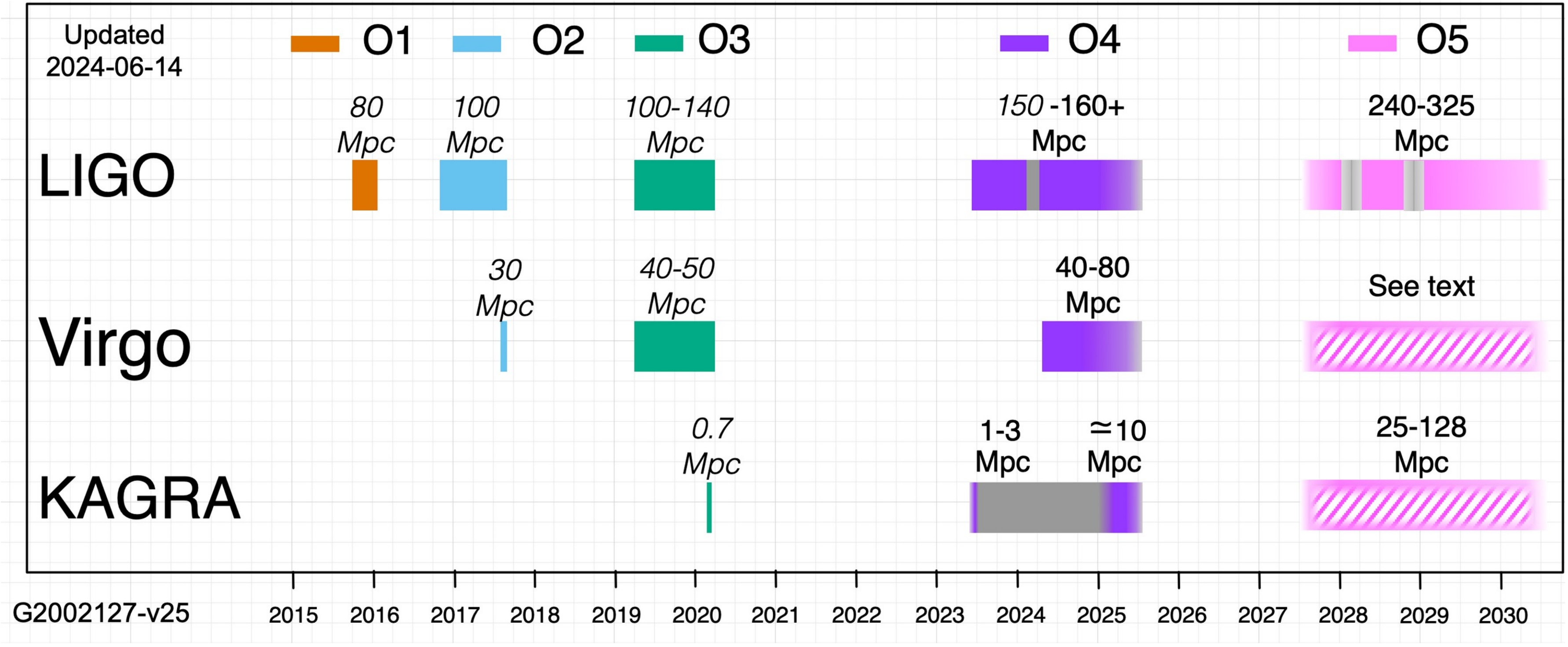
Discoveries of the 21st century





- Higgs boson (2012) & Gravitational waves (2015) – spectacular confirmations of Standard Model & General Relativity.
- This talk – transferring knowledge from **collider** to **GW** physics.

LIGO/Virgo/KAGRA timeline

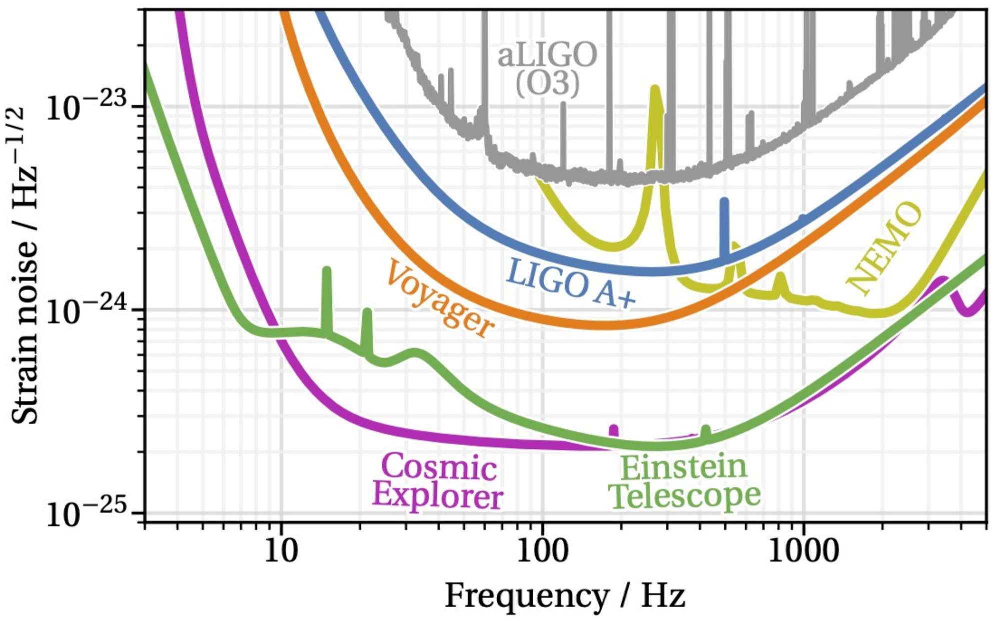


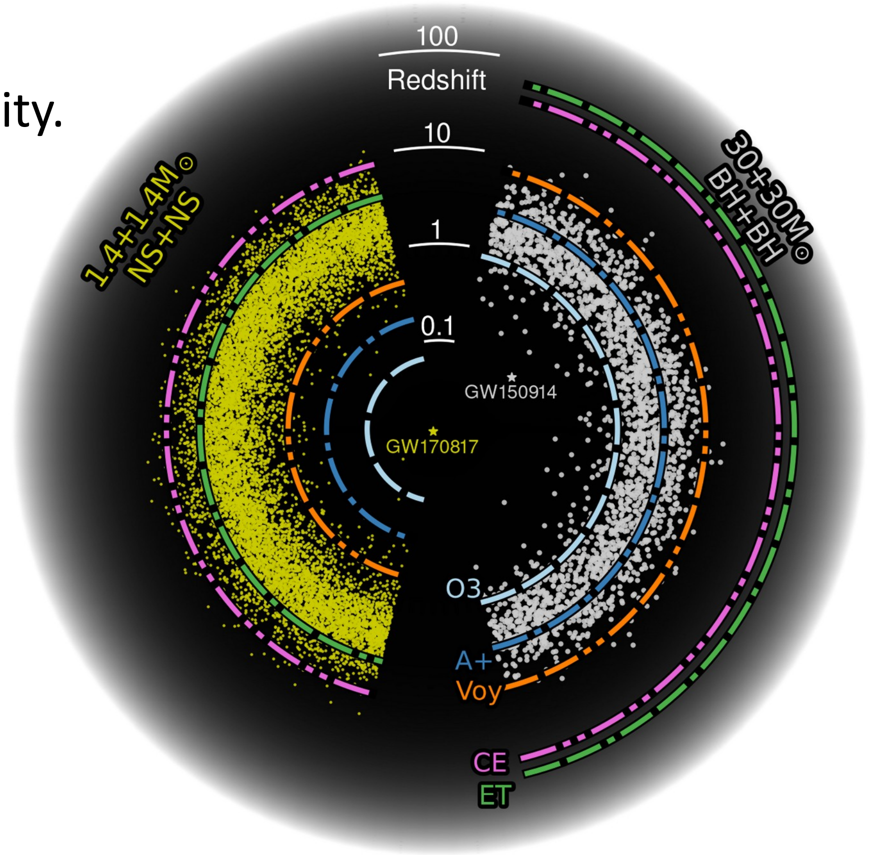
90 events combined

~ 254 candidate events!

Future detectors

- Ground: Advanced LIGO A+, Einstein Telescope, Cosmic Explorer.
  Space: LISA, TianQin, Taiji…
- Orders of magnitude increase in strain sensitivity.





https://cosmicexplorer.org/sensitivity.html

Future detectors

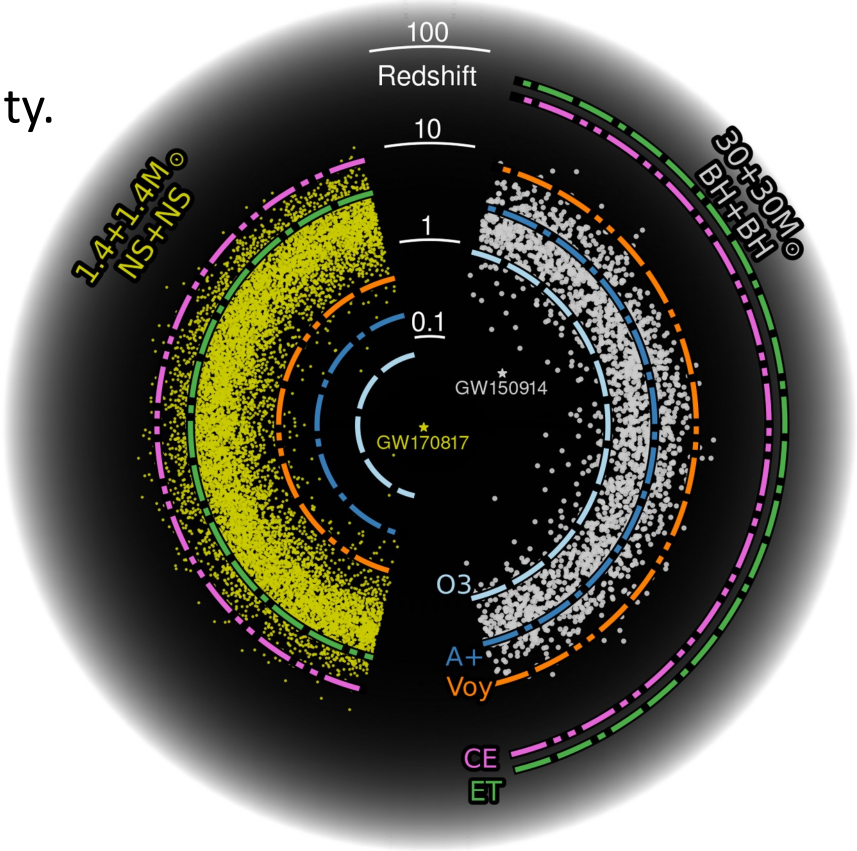
- Ground: Advanced LIGO A+, Einstein Telescope, Cosmic Explorer.
  Space: LISA, TianQin, Taiji…
- Orders of magnitude increase in strain sensitivity.

Vast improvements in theoretical modeling are needed! Perturbative calculation needed up to the 7th order in *G*.

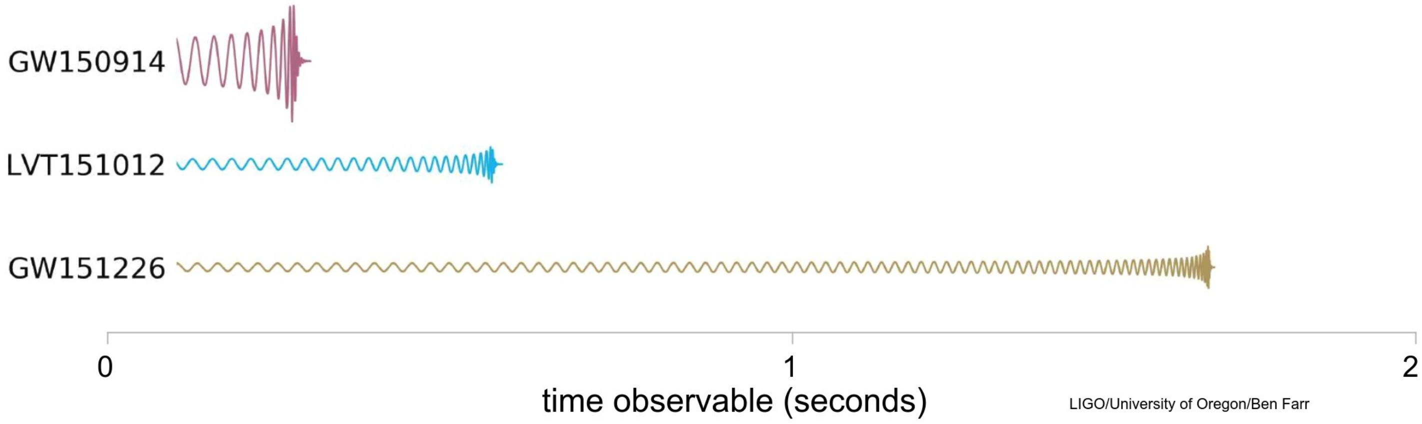
[e.g. Snowmass White paper, Buonanno, Khalil, O'Connell, Roiban, Solon, **MZ** '22]

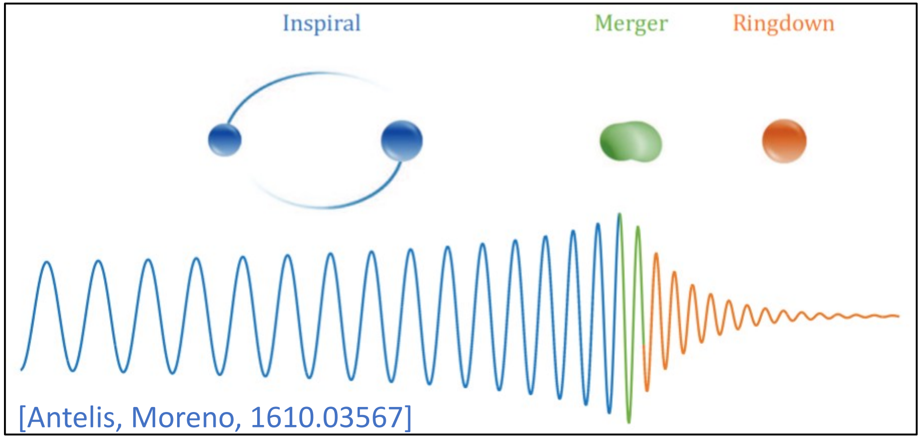
Physics Reach:

- Testing general relativity
- Stellar population, BH formation
- Extreme nuclear matter
- Hubble rate measurments



Waveform samples





- **Inspiral:** large separation, weak field
- **Merger:** strong-field, non-perturbative
- **Ringdown:** quasi-normal modes of black hole perturbation theory

Analytic and numerical tools

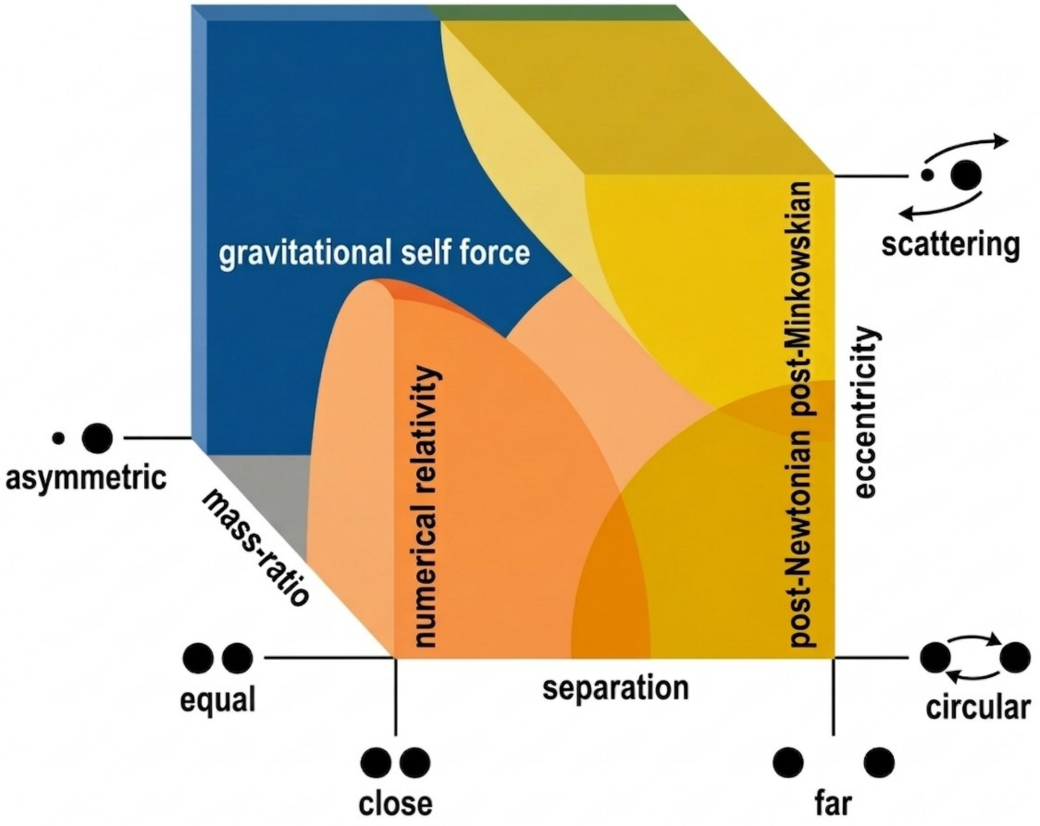
- **Numerical relativity:** non-perturbative; prohibitive in certain parameter regions

- **Post-Newtonian** expansion: large separation & slow motion

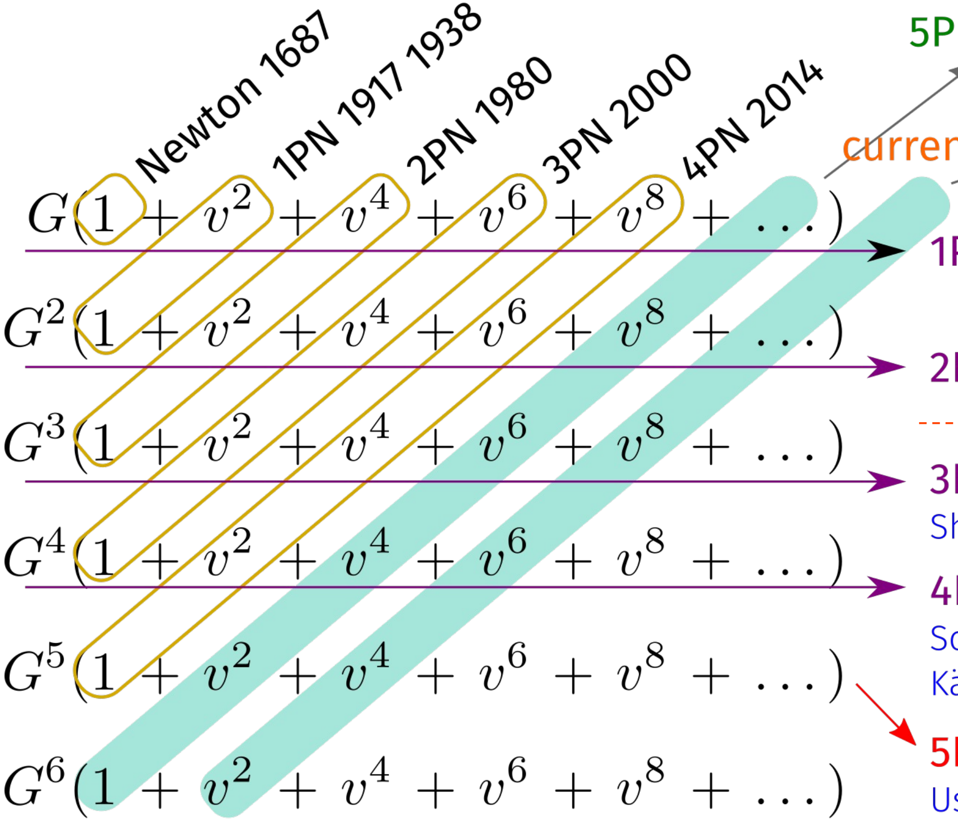
- **Post-Minkowskian** expansion: large separation & fast, possibly relativistic motion – **QFT connection!**

- **Self-force** expansion: extreme-mass ratio binaries (EMRIs).

(credit: Ana Carvalho)



QFT methods changed the field



[adapted from Mikhail Solon's slide]

5PN: distinguish BH / NS

6PN: LISA, ET...

current frontier; potential region done

1PM

2PM [Westpfahl 1985]

30-year hiatus, before arrival of QFT methods

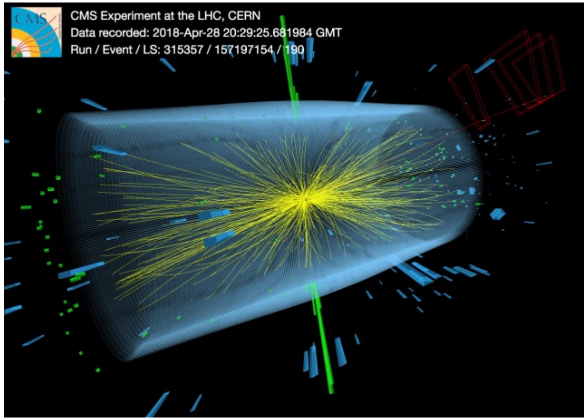
3PM [Bern, Cheung, Roiban, Shen, Solon, **MZ**, '19]

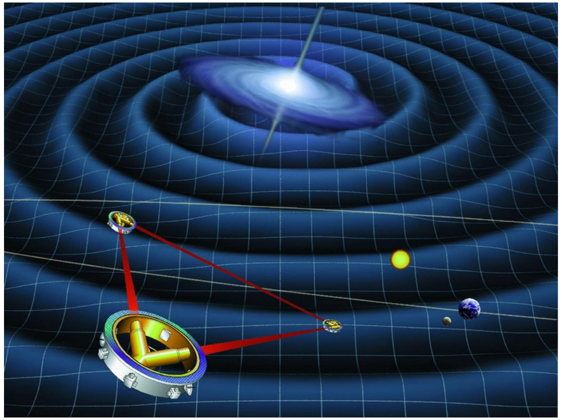
4PM [Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, **MZ**, '21. Dlapa, Kälin, Liu, Porto, '21. Dlapa, Kälin, Liu, Neef, Porto, '22.]

5PM [Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch, ’24. Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, Sauer, Usovtisch, ’24, ’26. Bern, Herrmqann,Roiban, Ruf, Smirnov, Smith, **MZ**, '25...]

Why Collider / QFT methods for gravity?

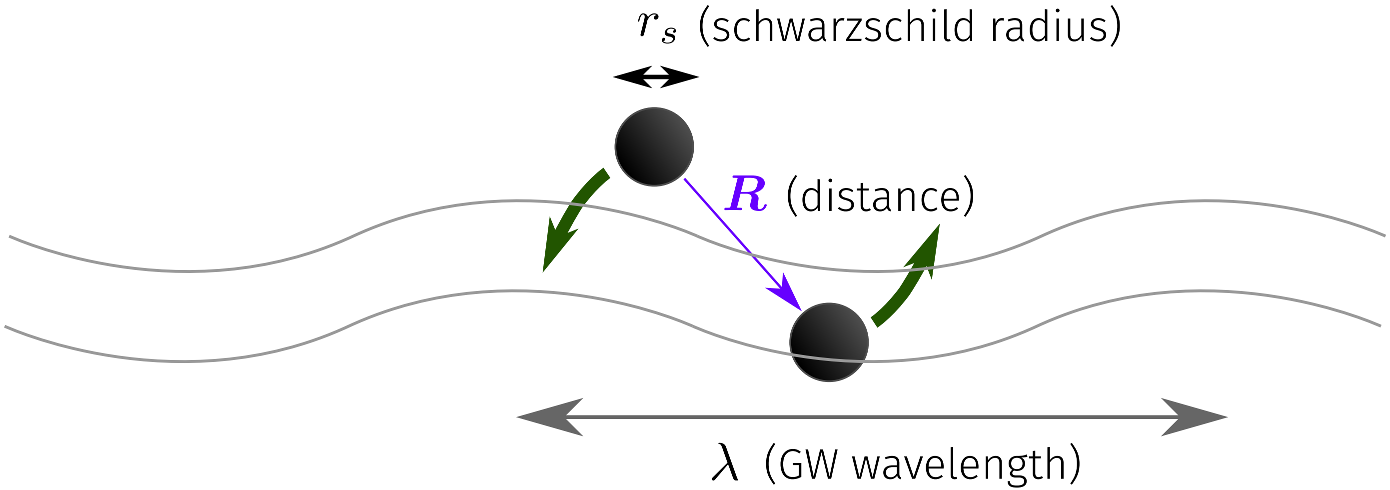
- Power of effective field theory for multi-scale problems.
- Efficiency of modern scattering amplitude methods.
- Importing Feynman integral techniques from decades of collider physics.





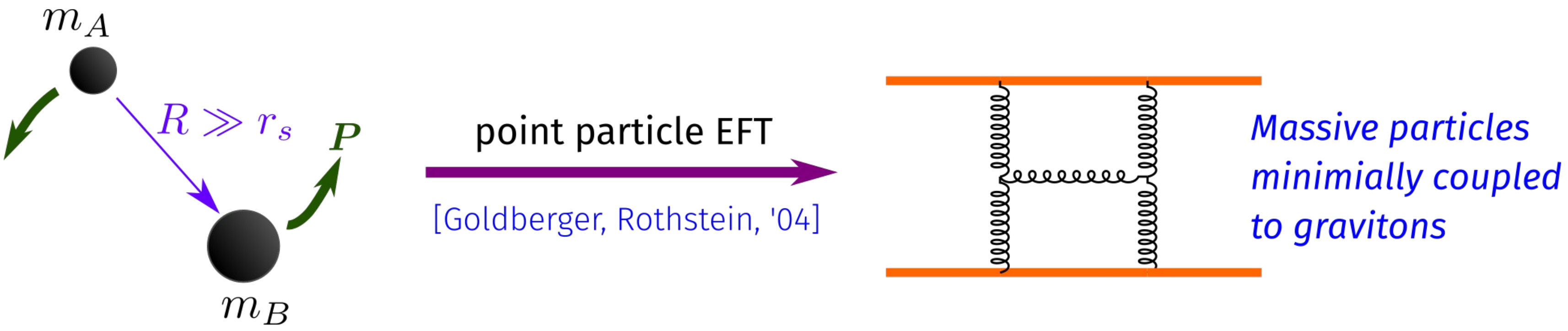
Scale hierarchy in binary problem

- During inspiral phase, $R_s \leq R \leq \lambda$



- Use point-particle effective field theory [Goldberger, Rothstein, '04; Cheung, Rothstein, Solon, '18; Damgaard, Haddad, Helset, '19; Kalin, Porto, '20]. EFT expansion captures the multi-pole expansion.

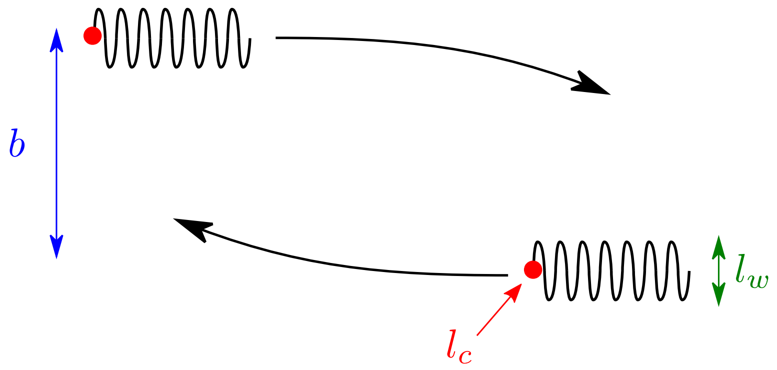
Point-particle effective field theory



- Finite-size effects (tidal deformations): higher-dimensional operators, e.g. $\sim R^2(\partial^2\phi)^2$.
- Systematic EFT expansion in r_s / R, similar to Q / M_W expansion in Fermi's 4-particle EFT for weak interactions.
- Leading finite-size coefficient, "Love Number", vanishes for BHs.

Classical from Quantum Amplitudes

- Taking classical limit is surprisingly nontrivial, but several formalisms exist, e.g. [Kosower, Maybee, O'Connell, '18], a collider-like observable formalism.



- For any observable $\mathcal{O}$, e.g. the momentum, change after scattering:

$$\Delta O = \langle \mathrm{out}|\mathcal{O}|\mathrm{out}\rangle - \langle \mathrm{in}|\mathcal{O}|\mathrm{in}\rangle = \langle \mathrm{in}|(S^\dagger \mathcal{O} S - \mathcal{O})|\mathrm{in}\rangle$$

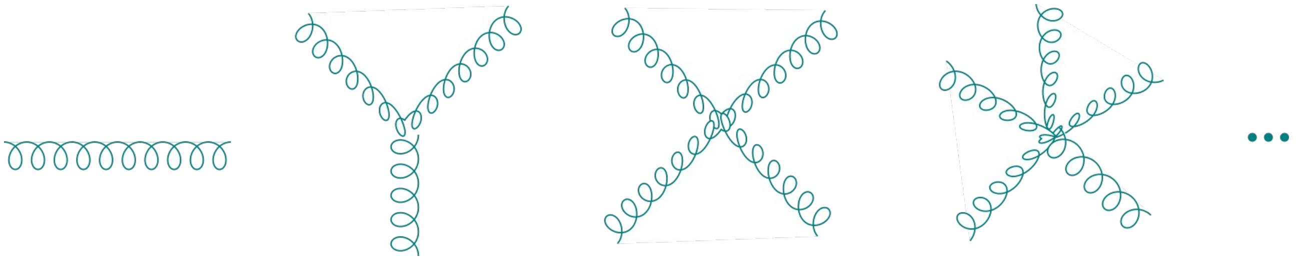
- Obtains clean, gauge-invariant observables for scattering orbits.

Efficient Amplitude Calculations

- Quantization of GR starts from the Einstein-Hilbert action

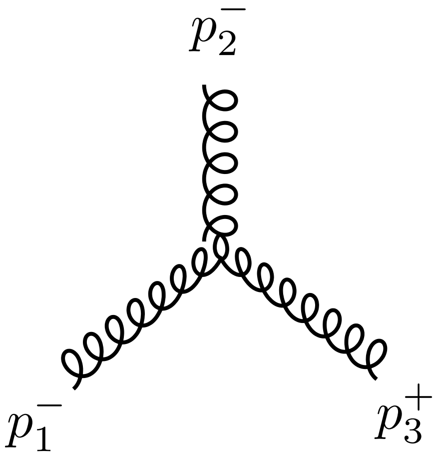
$$S_{\mathrm{EH}} = \frac{1}{2\kappa^2}\int d^dx\sqrt{-g}R, \qquad \kappa \equiv \sqrt{8\pi G}$$

- Non-linear theory of spin-2 fields. Very complicated Feynman rules!



- 3-point vertex already has ~100 terms.

Simplification: Gravity = (Gauge Theory)^2



Yang-Mills — *Gravity*

$$\mathcal{A}_3(1^-2^-3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}, \quad \mathcal{M}_3(1^-2^-3^+) = \frac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 31\rangle^2}$$

square!

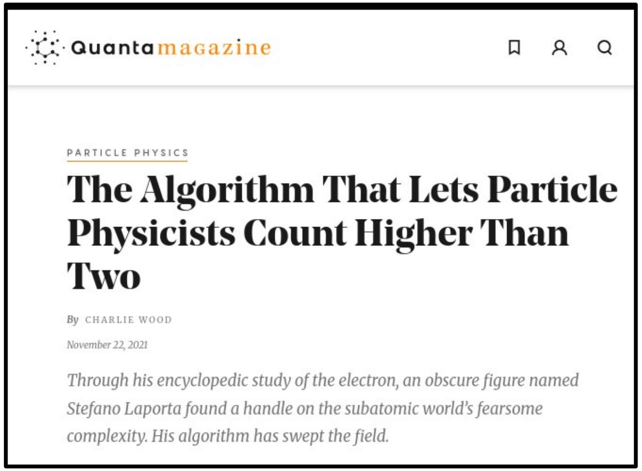
- Beyond 3 points - early work: **Kawai-Tye-Lewellen relations** in string theory
- Modern variant: color-kinematic duality, **“double copy”** [Bern, Carrasco, Johansson ’08, ’10]
- Very efficient in generating *integrand* of amplitudes; still need to integrate!

How to calculate a million Feynman integrals

- **Integration-by-parts** (IBP) reduction: [Tkachov '81. Chetrynkin, Tkachov '81] use linear relations to express all integrals of a given family in terms of a handful of **master integrals.**

- Systematized by the algorithm of [Laporta, '01] in the effort to calculate electron *g*-2 to 4 loops.

- Refined by 25 years of collider physics applications.


Quanta magazine

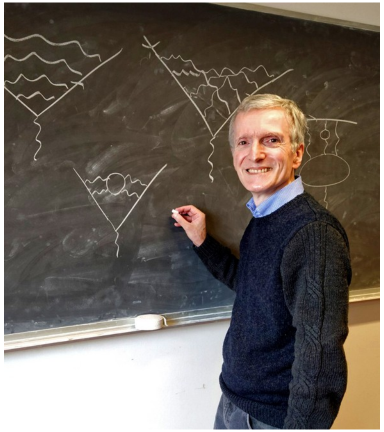
PARTICLE PHYSICS

The Algorithm That Lets Particle Physicists Count Higher Than Two

By CHARLIE WOOD

November 22, 2021

Through his encyclopedic study of the electron, an obscure figure named Stefano Laporta found a handle on the subatomic world's fearsome complexity. His algorithm has swept the field.

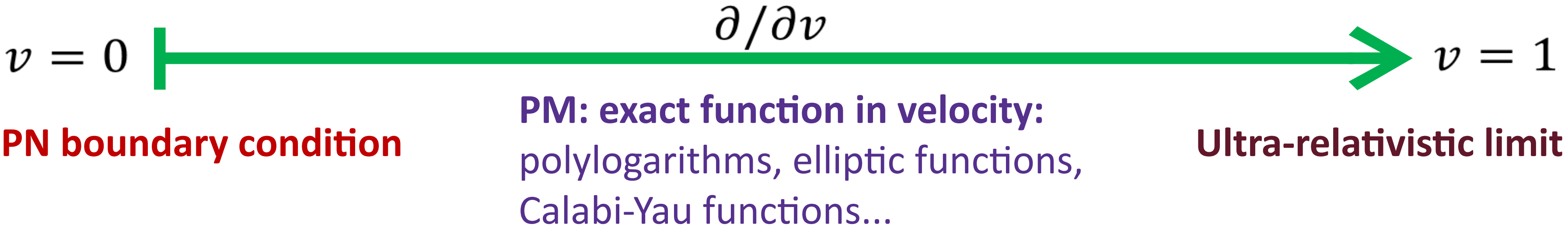


Master integrals from differential equations

- Solve homogeneous ODEs of master integrals w.r.t. kinematic invariants. *Essential in modern collider calculations.*

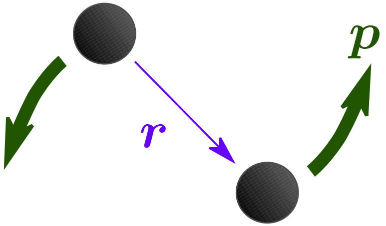
  [Kotikov ’91. Bern, Dixon, Kosower, ’92. Gehrmann, Remiddi, ’99. Henn, ’13]

$$\frac{\partial}{\partial v}\vec{I} = A \cdot \vec{I}$$



- Promotes post-Newtonian expansions to post-Minkowskian results. First gravity application in [Parra-Martinez, Ruf, **MZ**, ’20], widely adopted since then.

3PM / 2-loop conservative dynamics



[Bern, Cheung, Roiban, Shen, Solon, **MZ**, '19, *Phys.Rev.Lett.*, Editor's Suggestion]

$$H^{3\mathrm{PM}}(p,r) = \sqrt{p^2+m_1^2} + \sqrt{p^2+m_2^2} + V^{3\mathrm{PM}}(p,r),$$

$$V^{3\mathrm{PM}}(p,r) = \left(\frac{G}{|r|}\right) c_1(p^2) + \left(\frac{G}{|r|}\right)^2 c_2(p^2) + \left(\frac{G}{|r|}\right)^3 c_3(p^2)\,.$$

Westpfahl, '85 — Our new result

$$m = m_1 + m_2,\ \nu = \frac{m_1 m_2}{m^2},\ E = E_1 + E_2,\ \xi = \frac{E_1 E_2}{E^2},\ \gamma = \frac{E}{m},\ \sigma = \frac{p_1\cdot p_2}{m_1 m_2}$$

$$\mathbf{c_1} = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \quad \mathbf{c_2} = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$\mathbf{c_3} = \frac{\nu^2 m^4}{\gamma^2\xi}\left[\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right.$$
$$- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3}$$
$$\left.- \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right].$$

4PM / 3-loop conservative dynamics

[Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, **MZ**, ’21, *Phys.Rev.Lett.*]

$$\mathcal{M}_4^{\rm cons} = G^4 M^7 \nu^2 |\boldsymbol{q}| \pi^2 \left[\mathcal{M}_4^{\rm probe} + \nu \left(4\mathcal{M}_4^{\rm tail} \log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \text{Iterations}$$

$p_\infty \equiv \sqrt{(u_1 \cdot u_2)^2 - 1}$

$$\mathcal{M}_4^{\rm probe} = -\frac{35\left(1-18\sigma^2+33\sigma^4\right)}{8\left(\sigma^2-1\right)}\,, \quad \mathcal{M}_4^{\rm tail} = r_1 + r_2 \log\left(\tfrac{\sigma+1}{2}\right) + r_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\,,$$

$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\, K\left(\frac{\sigma-1}{\sigma+1}\right) E\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\, K^2\left(\frac{\sigma-1}{\sigma+1}\right) + r_7\, E^2\left(\frac{\sigma-1}{\sigma+1}\right)\,,$$

$$\mathcal{M}_4^{\rm rem} = r_8 + r_9 \log\left(\frac{\sigma+1}{2}\right) + r_{10}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\frac{\sigma+1}{2}\right) + r_{13}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\frac{\sigma+1}{2}\right)$$
$$+ r_{14}\frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1} + r_{15}\operatorname{Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\operatorname{Li}_2\left(\frac{1-\sigma}{1+\sigma}\right) + r_{17}\frac{1}{\sqrt{\sigma^2-1}}\left[\operatorname{Li}_2\left(-\sqrt{\frac{\sigma-1}{\sigma+1}}\right) - \operatorname{Li}_2\left(\sqrt{\frac{\sigma-1}{\sigma+1}}\right)\right]$$

complete elliptic integrals of the 1st & 2nd kind

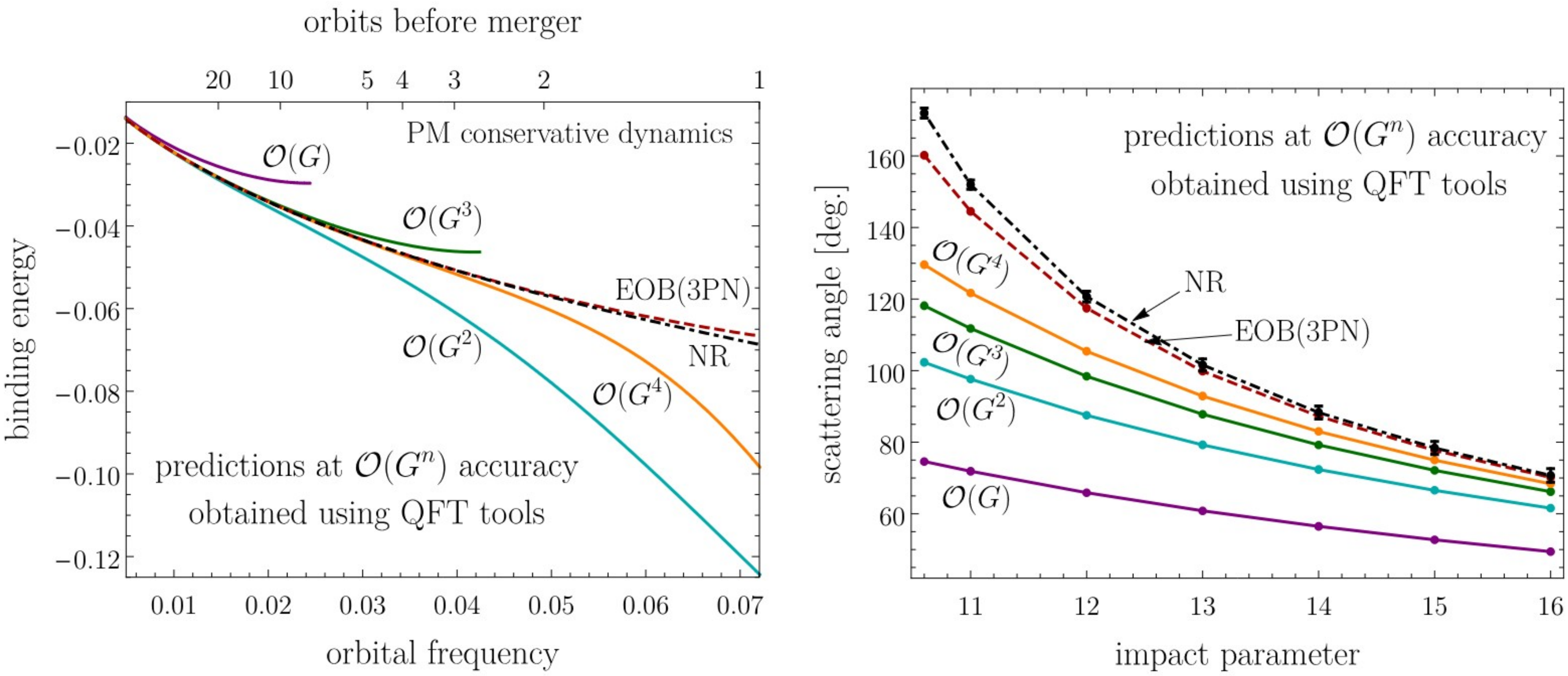
polylogarithms up to transcendental weight 2

Rational functions:

$$r_1 = \frac{1151 - 3336\sigma + 3148\sigma^2 - 912\sigma^3 + 339\sigma^4 - 552\sigma^5 + 210\sigma^6}{12(\sigma^2-1)}\,, \quad r_2 = \frac{1}{2}\left(5 - 76\sigma + 150\sigma^2 - 60\sigma^3 - 35\sigma^4\right)\,, \quad \ldots$$

Comparing with numerical relativity

[Snowmass White paper, Buonanno, Khalil, O’Connell, Roiban, Solon, **MZ** ’22]

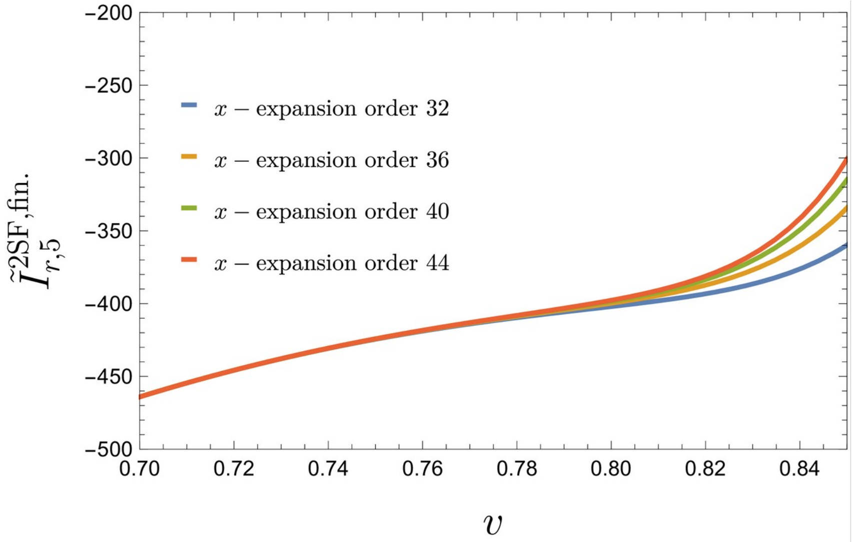


5PM / 4-loop conservative dynamics

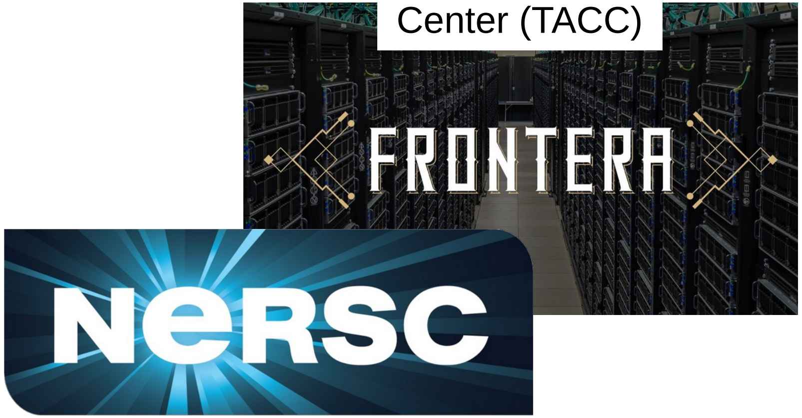
[Bern, Herrmann, Roiban, Ruf, Smirnov, Smith, **MZ**, 2025]

...

- Cutting-edge problem. Integration-by-parts reduction consumed ~ 1M CPU core hours, with private version of FIRE 7 [Smirnov, **MZ**, '25]



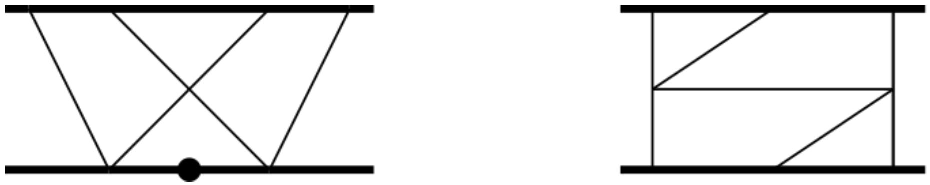
Texas Advanced Computing Center (TACC)



4 loops / 5PM: new special functions

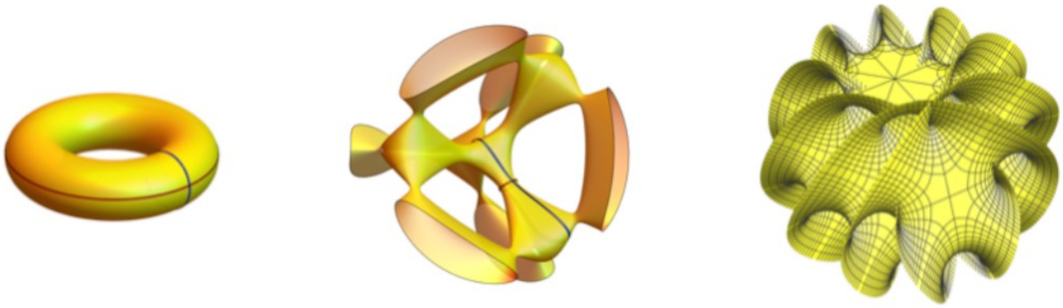
- Calabi-Yau integrals [Bourjaily, Brown, Bönisch, Cao, Duhr, Durh, Fischbach, He, Klemm, Loebbert, Maggio, McLeod, Nega, Porkert.Lairez, Pögel, Sauer, Schnetz, Sohnle, Tancredi, Tang, Vanhove, Wagner, Wang, Weinzierl, Wilhelm, von Hippel…] appear, from the geometry of the Feynman integrals and Picard-Fuchs operators in the differential equations.

$$\mathcal{L} = \left(x\frac{d}{dx} - 1\right)^4 - x^4\left(x\frac{d}{dx} + 1\right)^4$$



Picture: Frellesvig, Morales, Wilhelm, arXiv:2505.10274

- Appears in dissipative dynamics [Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, Sauer, Usovitsch, *Nature* '24] but cancels in conservative case [Bern, Herrmann, Roiban, Ruf, Smirnov, Smith, **MZ**, 2025]



Torus $(n = 1)$ K3 $(n = 2)$ CY3 $(n = 3)$

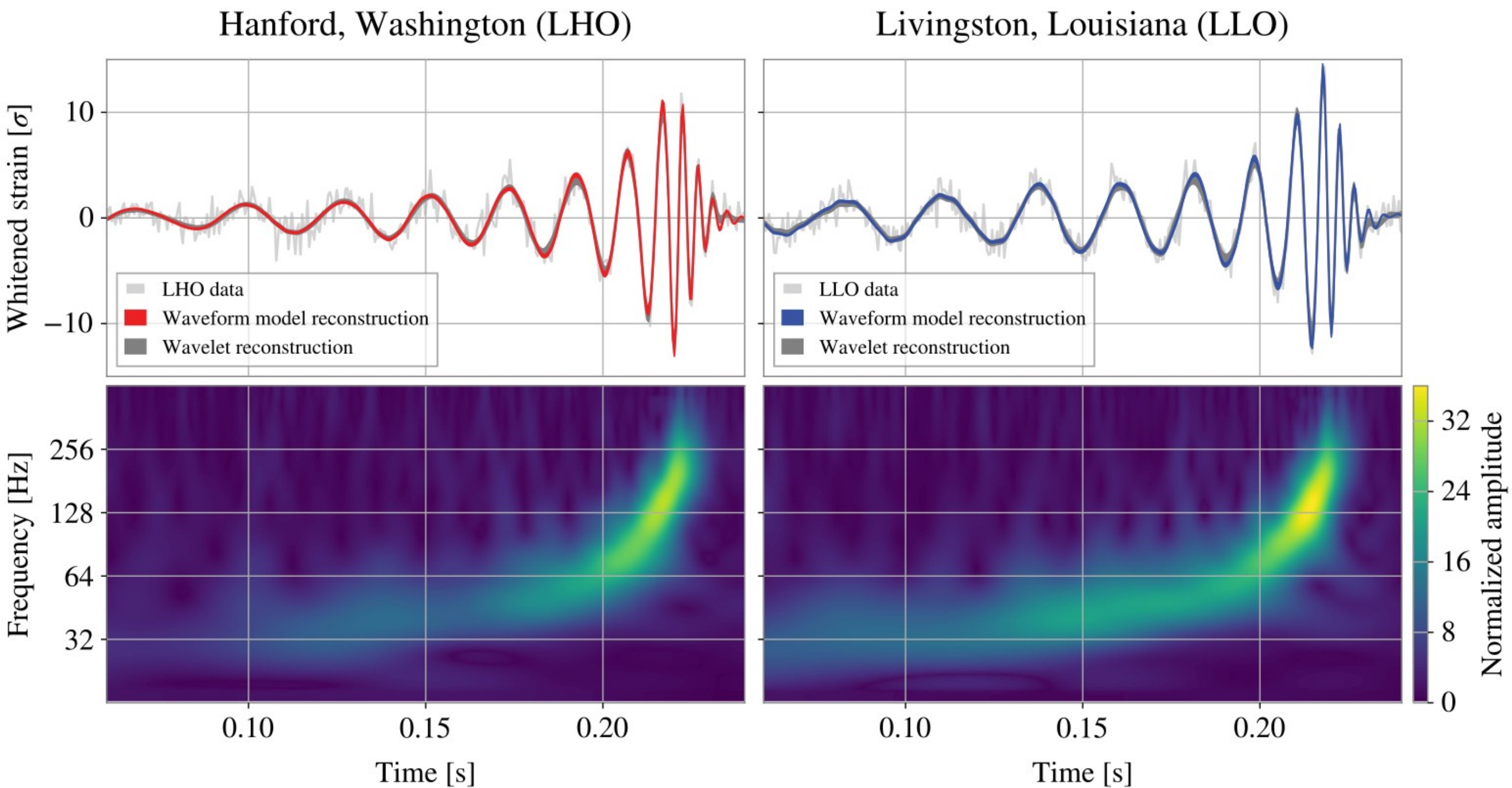
Outlook

- **Relativistic QFT & Scattering Amplitudes** are becoming an established, full-fledged toolkit for precision modeling of binary dynamics
- **Particle physics methods** applied to classical gravitational physics.
- Revolutionized **post-Minkowskian expansion**; complementary to post-Newtonian expansion, gravitational self-force, numerical relativity, and semi-analytic models
- **Analytic insights** – iterated integrals (polylogarithms, elliptic functions, Calabi-Yau functions, etc.), high-energy limits, hidden symmetries…
- The QFT framework is **flexible:** allows studying spin effects, finite-size effects, environmental effects… Growing and exciting explorations ahead!

Backup slides

Sample O4 signal

GW250114: signal-to-noise ratio ~ 80. Unprecedented test of Hawking area law.



Post-Newtonian (PN) expansion

- Joint expansion in v^2 and GM/R, locked together by Virial's theorem.

$$m = m_A + m_B, \quad \nu = \mu/m$$
$$\mu = m_A m_B / m$$

$\mathcal{O}(v^2)$ $\mathcal{O}(G)$

$$\frac{H}{\mu} = \boxed{\frac{v^2}{2} - \frac{Gm}{R}} + H_{1\mathrm{PN}} + H_{2\mathrm{PN}} + H_{3\mathrm{PN}} + H_{4\mathrm{PN}} \ldots$$

0PN, Newton

(1980) (2000) (2014)

$$\mathcal{O}(v^4) + \mathcal{O}(Gv^2) + \mathcal{O}(G^2)$$

$$+\frac{1}{c^2}\left\{-\frac{v^4}{8} + \frac{3\nu v^4}{8} + \frac{Gm}{R}\left(-\frac{v_R^2 \nu}{2} - \frac{3v^2}{2} - \frac{\nu v^2}{2}\right) + \frac{G^2 m^2}{2R^2}\right\}$$

1PN, Lorentz, Droste, 1917; Einstein, Infeld, Hoffman, 1938

Post-Minkowskian (PM) expansion

- Expansion in coupling constant *G*, **exact velocity dependence.**
  [Bertotti, Kerr, Plebanski, Portilla, Westpfahl, Gollder, Bel, Damour, Derulle, Ibanez, Martin, Ledvinka, Schaefer, Bicak…]

- Most accurate PM scattering angle *(unbound hyperbolic-like orbit)* until 2019
  [Westpfahl, '85]

Scattering angle of two black holes, as function of $m_1, m_2, b, \sigma \equiv \hat{p}_1 \cdot \hat{p}_2$.

$$\theta = \frac{4G(m_1+m_2)}{b}\frac{2\sigma^2-1}{2(\sigma^2-1)} + \frac{3\pi}{4}\frac{G^2(m_1+m_2)^2}{b^2}\frac{5\sigma^2-1}{\sigma^2-1} + \mathcal{O}(G^3)$$