

Lucien Heurtier

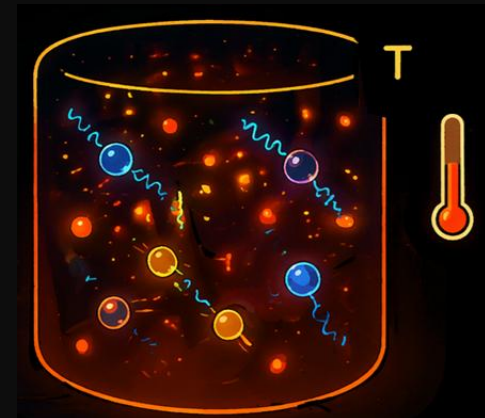
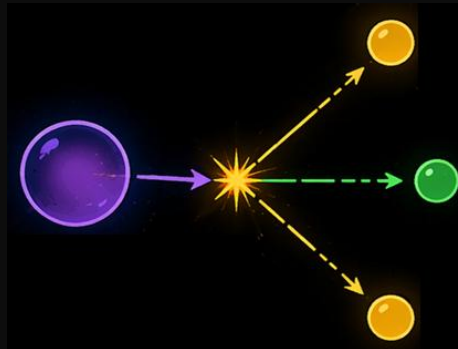
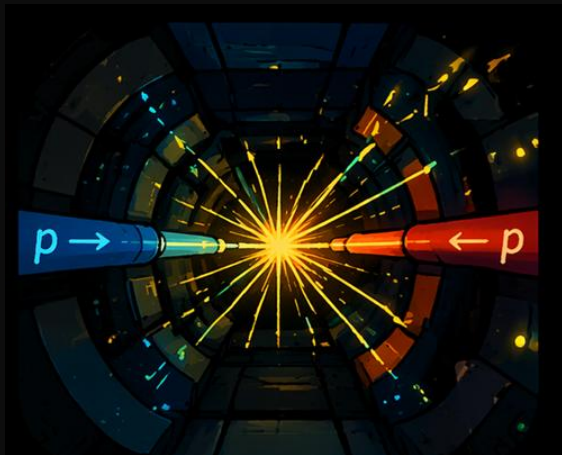
HOW SENSITIVE IS COSMIC INFLATION TO QUANTUM CORRECTIONS?

Based on ArXiv:[2511.05296](https://arxiv.org/abs/2511.05296) and [2604.XXXX](https://arxiv.org/abs/2604.XXXX)– J. Alexandre, LH, S. Pla

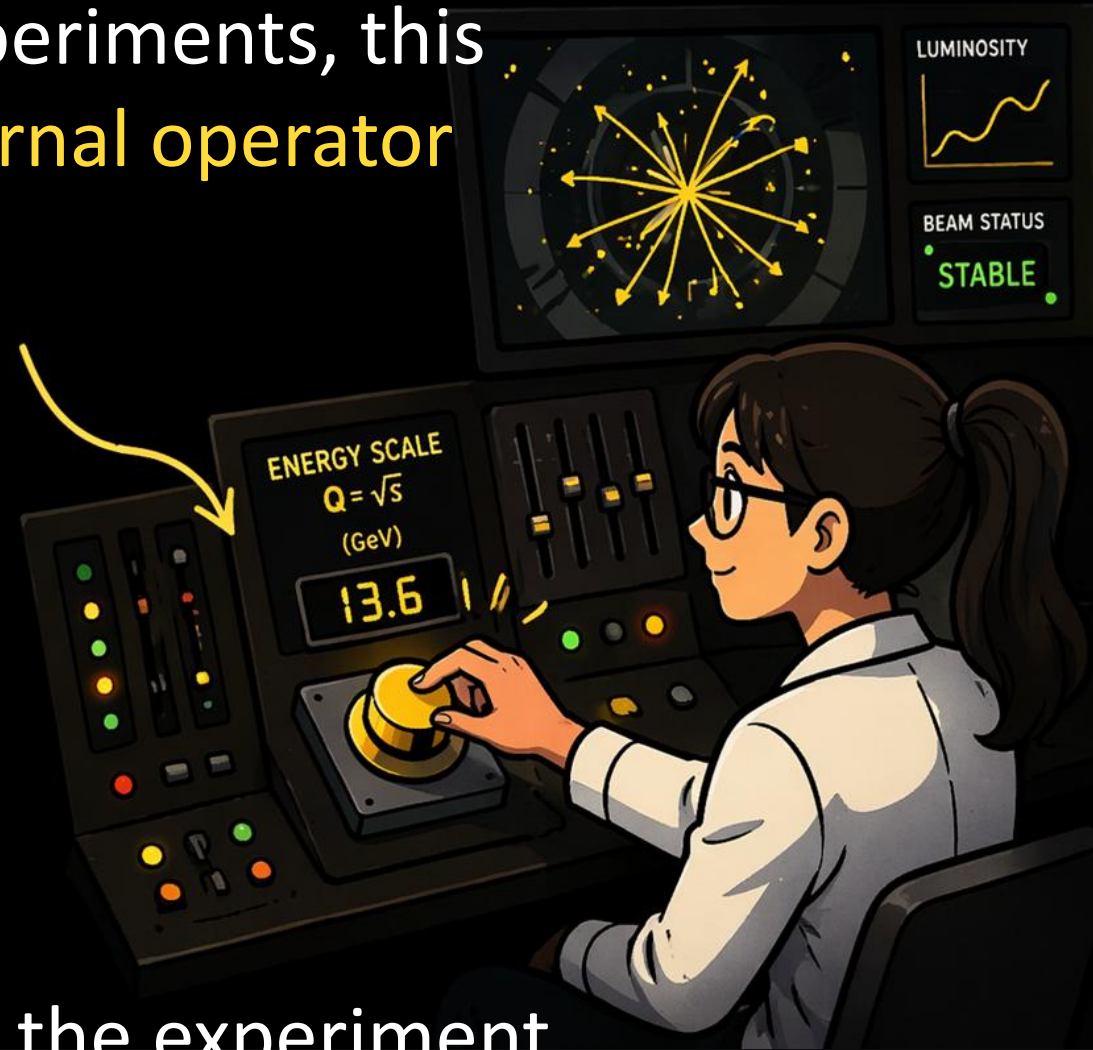
In QFT, renormalisation is crucial

Observables **independent on renormalisation scale μ** , but **dependent on the energy scale** of the system

- Collider $\longrightarrow \sqrt{s}$ (CoM energy)
- Particle decay $\longrightarrow m$ (vacuum/thermal mass)
- Thermal bath $\longrightarrow T$ (temperature)



In particle physics experiments, this scale is **set by an external operator**



What happens inside the experiment **does not affect this choice.**

... unlike in cosmology

In cosmology

$$\frac{a'(t)}{a(t)} \equiv H(t) \quad \text{Hubble scale}$$

Small-scale
fluctuations

$$\omega \gg H$$

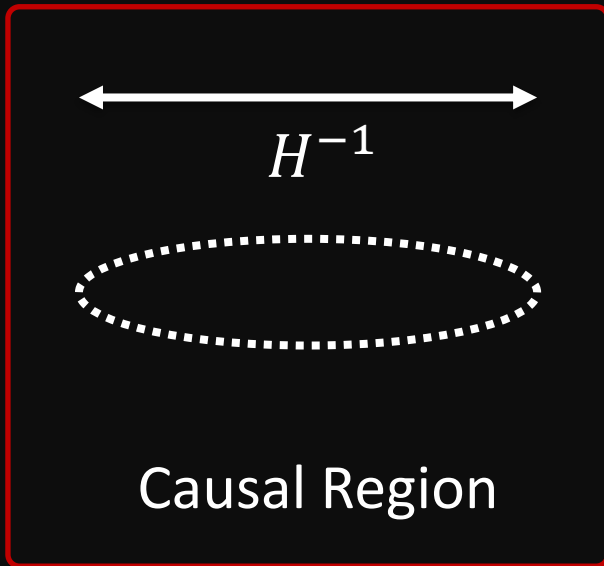
Ultraviolet
theory
(quantum)



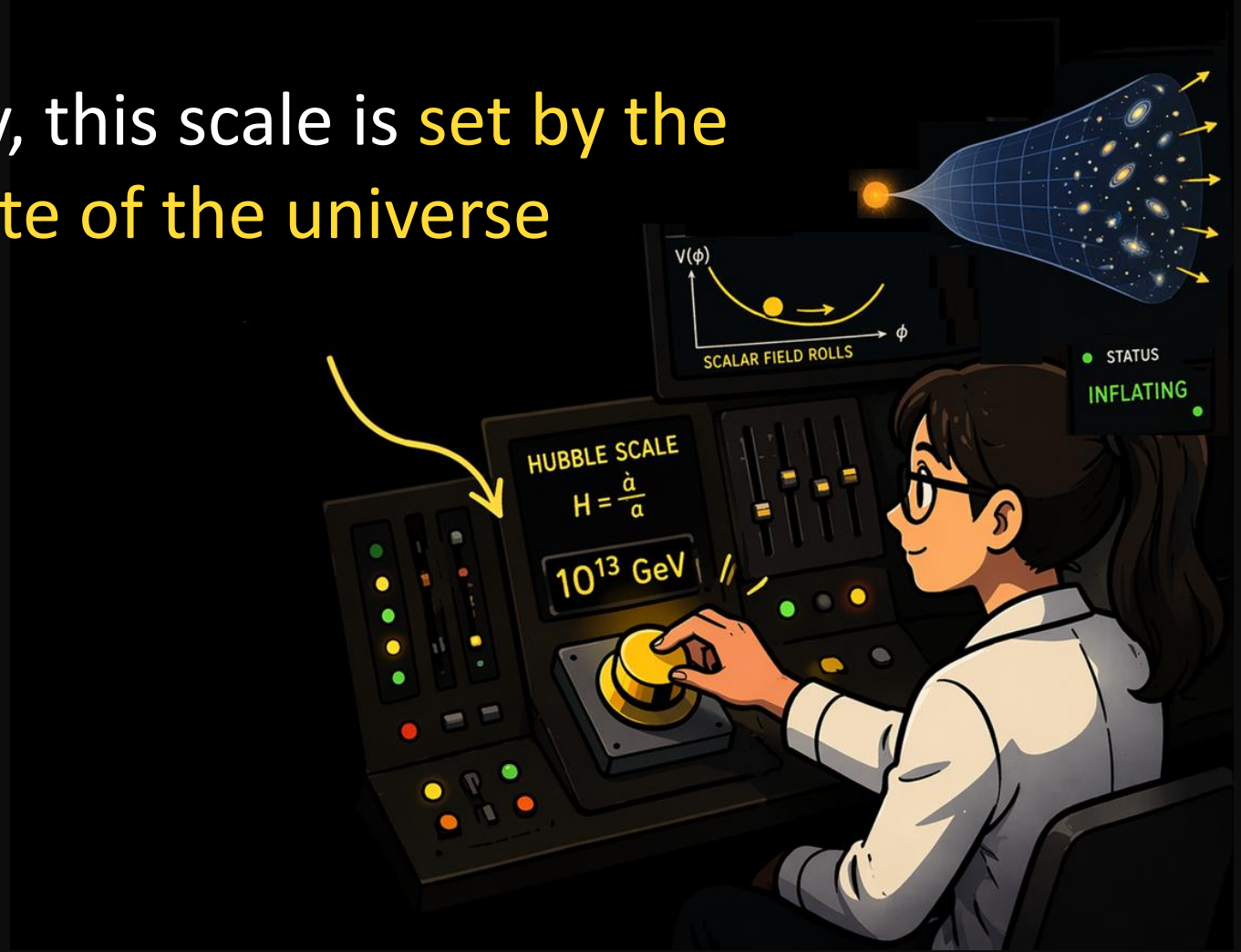
Large-scale
fluctuations

$$\omega \ll H$$

Infrared
Theory
(classical)



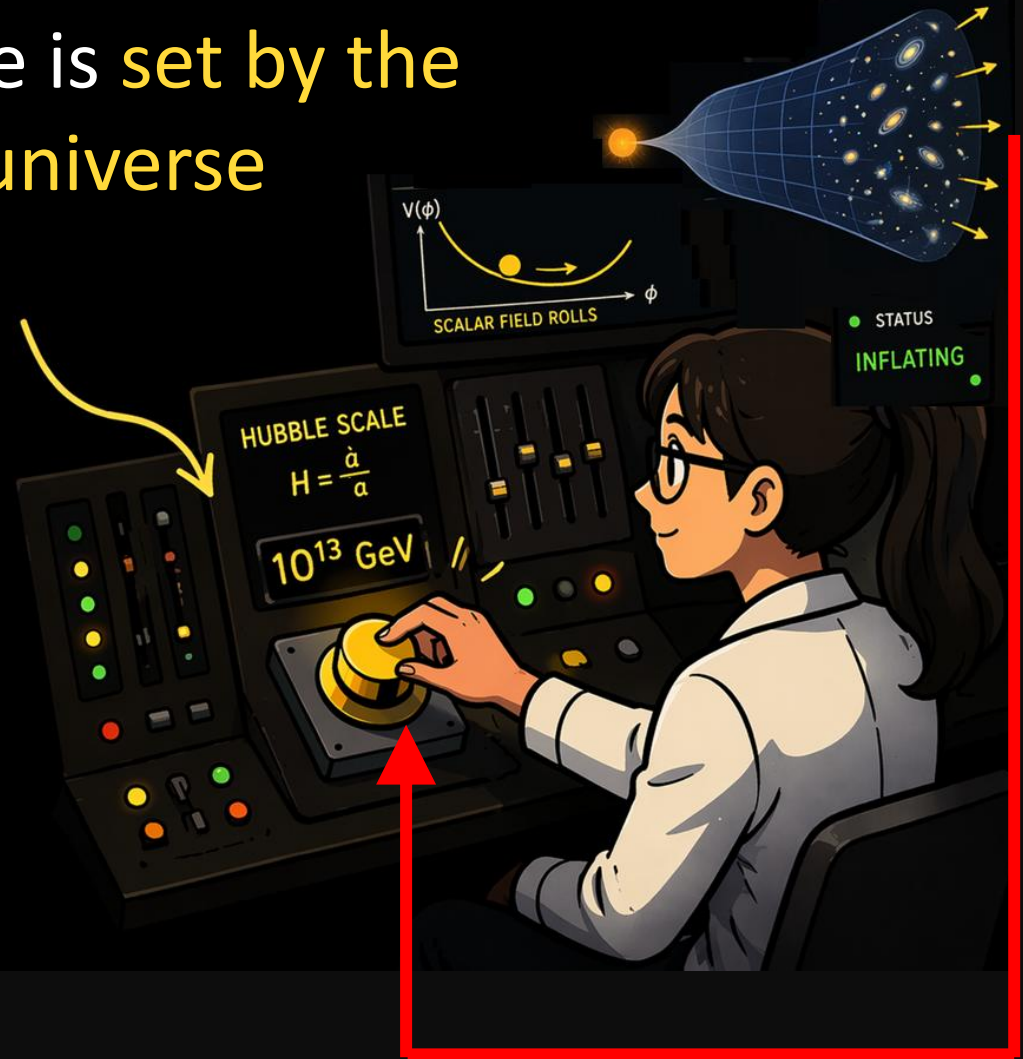
In cosmology, this scale is set by the expansion rate of the universe



In cosmology, this scale is set by the expansion rate of the universe

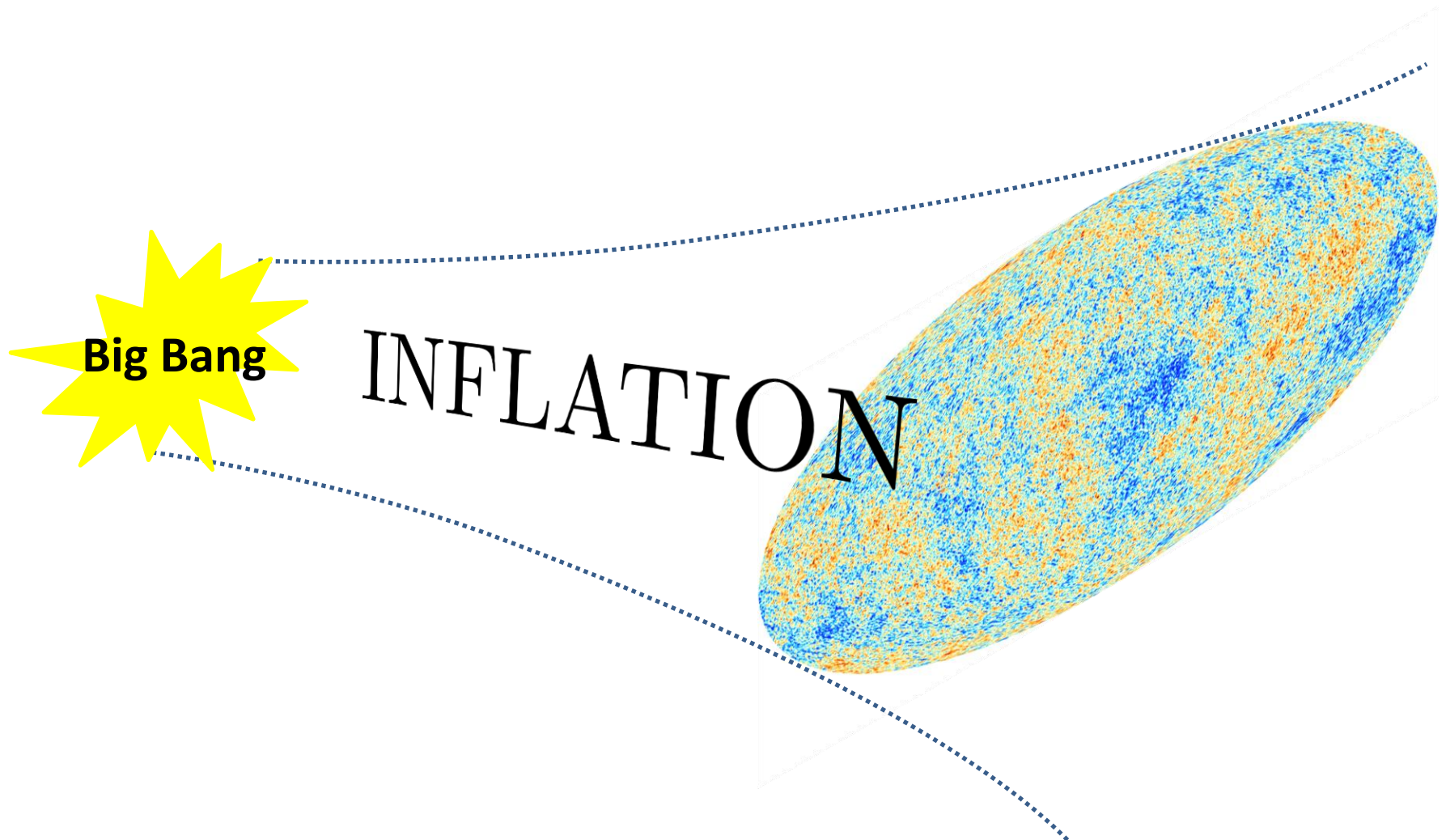
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

Friedmann equations



What happens inside the experiment
perturbs the operator!

Cosmic Inflation

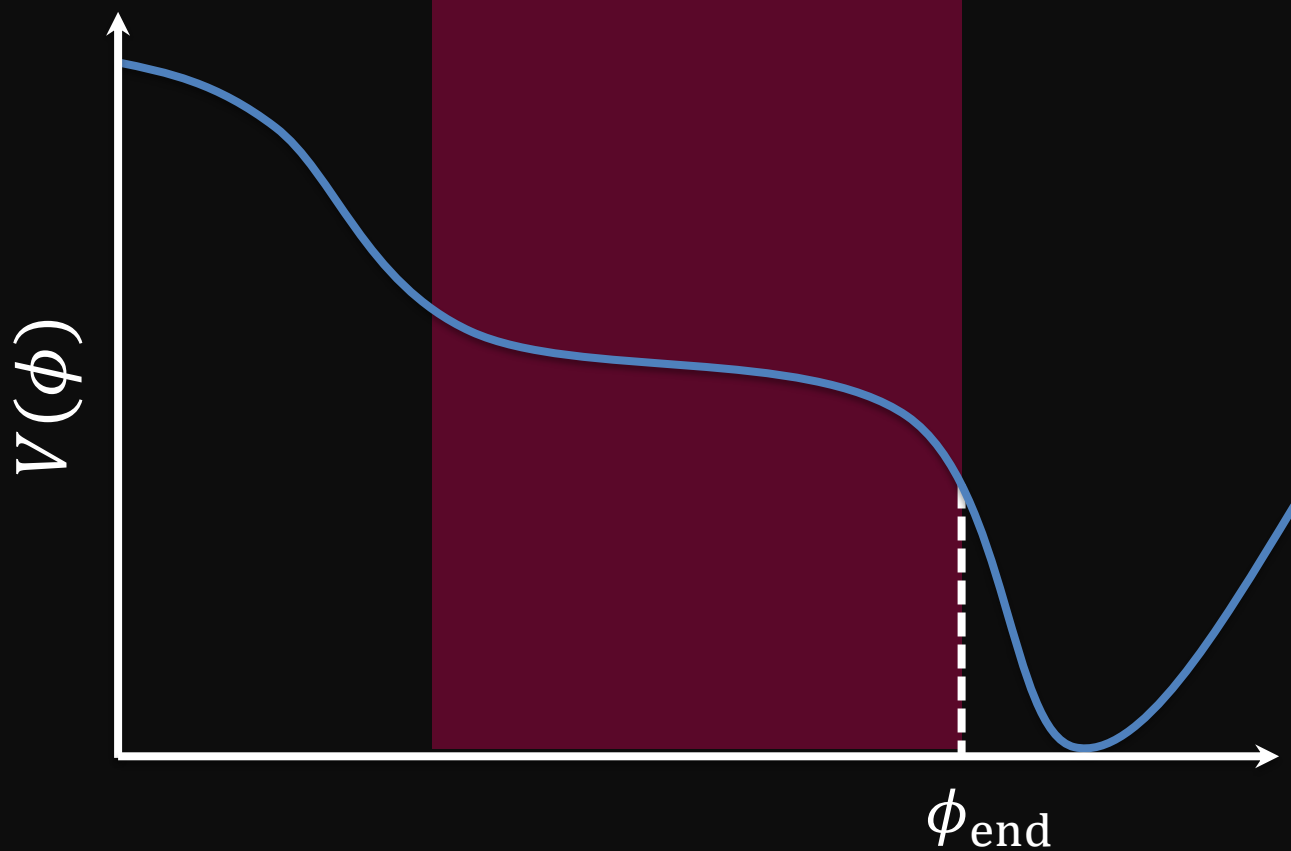


→ Homogeneous, flat universe

Inflation Potential

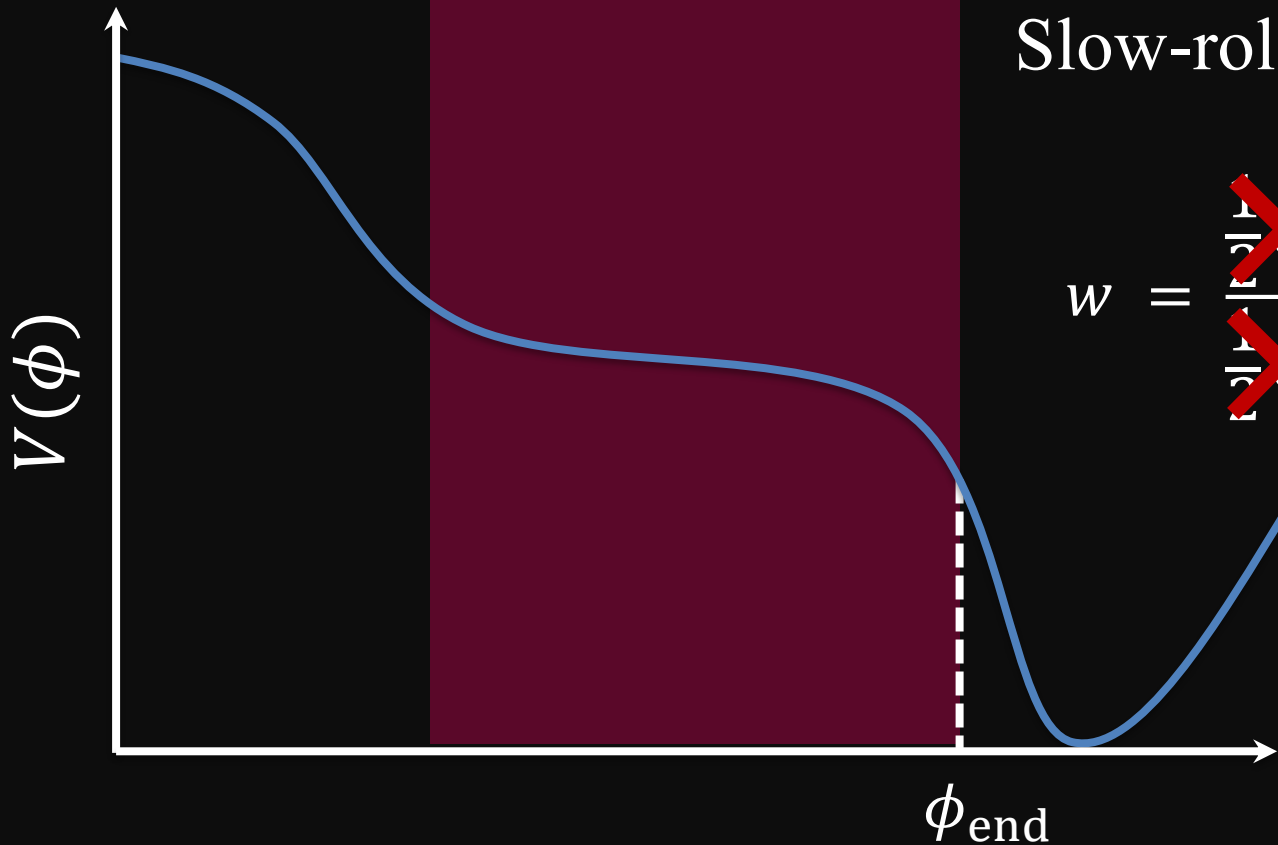
Inflation

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0$$



Inflation Potential

Inflation



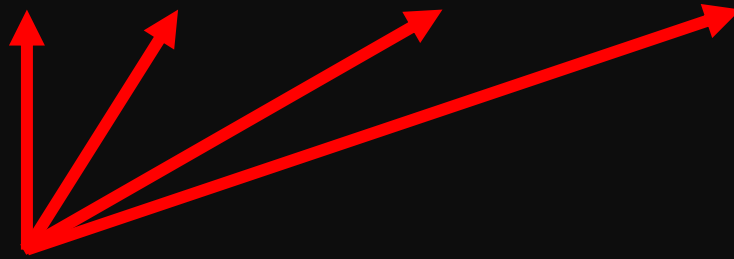
$$\cancel{\ddot{\phi}} + 3H \dot{\phi} + V'(\phi) = 0$$

$$\text{Slow-roll: } 3H \dot{\phi} \sim V'(\phi)$$

$$w = \frac{\cancel{\frac{1}{2}\dot{\phi}^2} - V(\phi)}{\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)} \approx -1$$

Inflation Potential

$$V(\phi) = V_0 + g_1 \phi + g_2 \phi^2 + g_3 \phi^3 + \dots$$



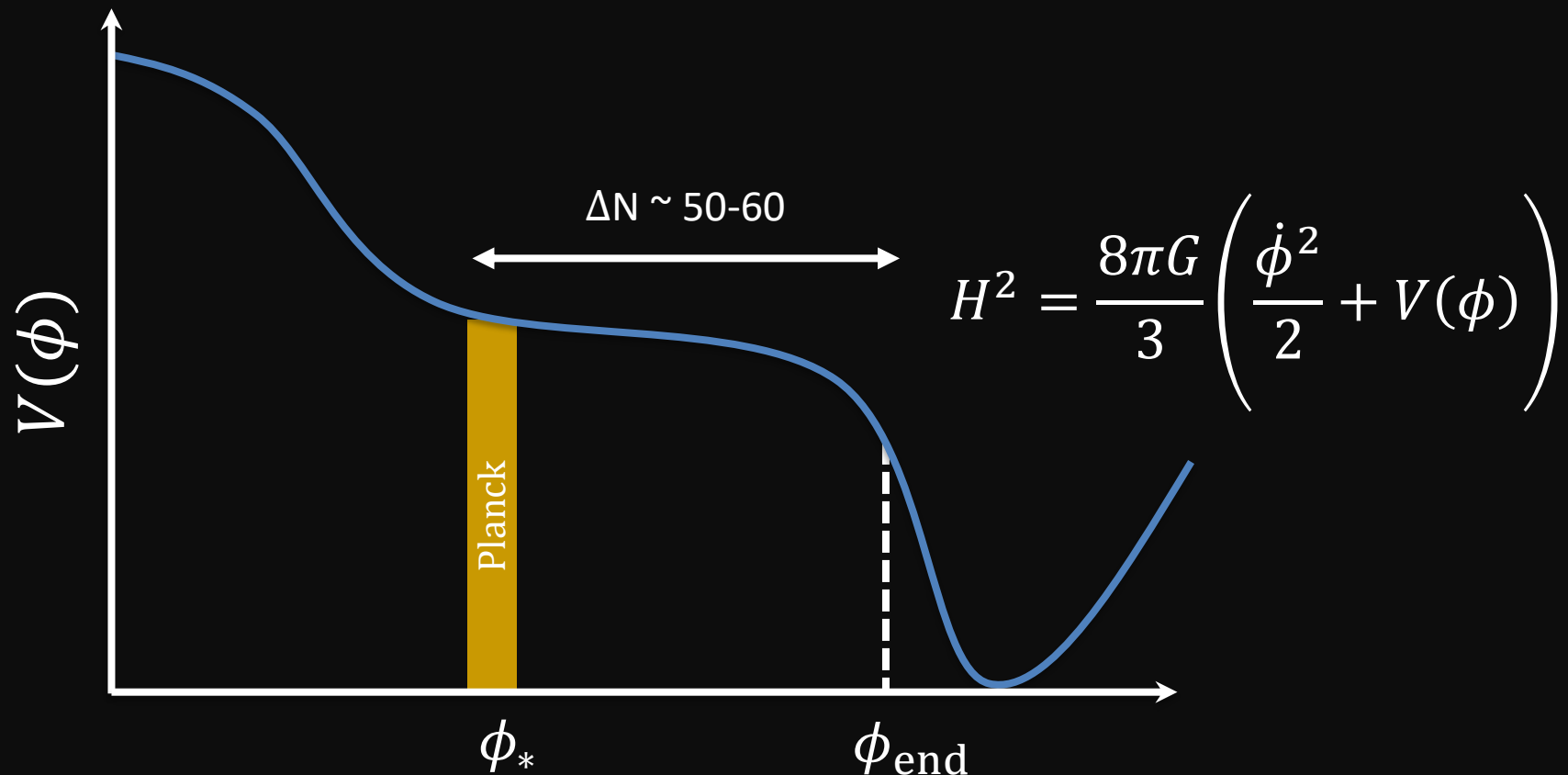
All couplings measured in the CMB

(id est @ $H = H_{\text{CMB}}$)

CMB measurements

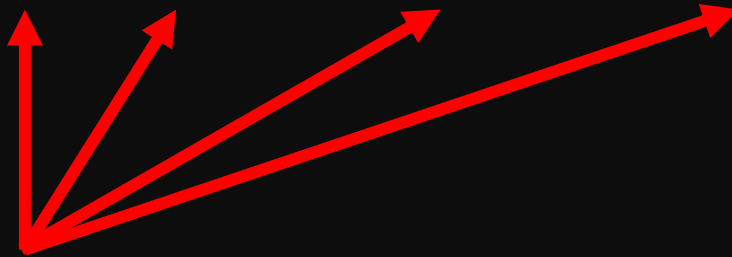
Planck:

$$k \sim 0.05 \text{ Mpc}^{-1}$$



Inflation Potential

$$V(\phi) = V_0 + g_1 \phi + g_2 \phi^2 + g_3 \phi^3 + \dots$$



All couplings measured in the CMB

(id est @ $H = H_{\text{CMB}}$)

When Hubble (and the field) evolves, the theory must be described by

$$U(\phi, H) =$$

$$U_0 + g_1(H) \phi + g_2(H) \phi^2 + g_3(H) \phi^3 + \dots$$

Exact Renormalisation Group

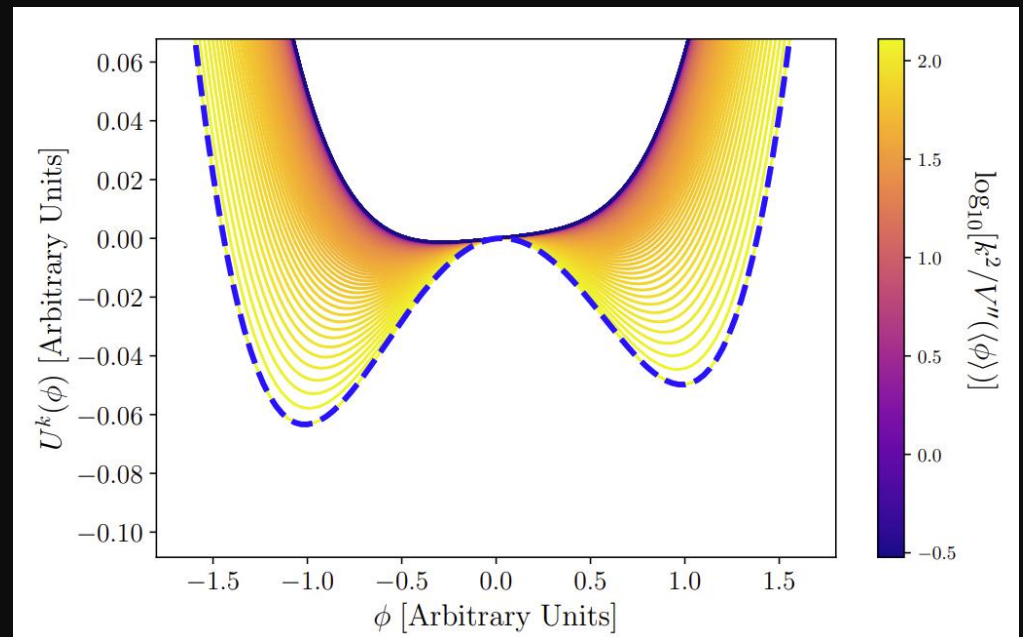
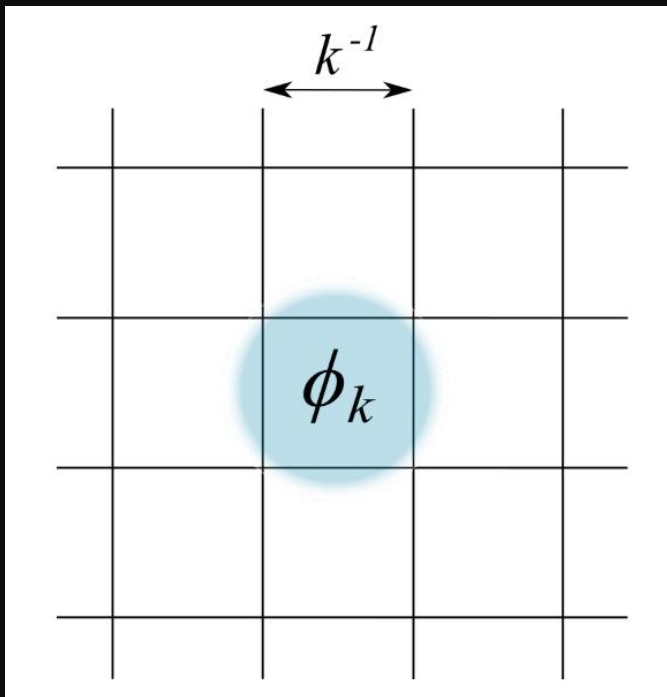
At every scale: coarse grain the theory.

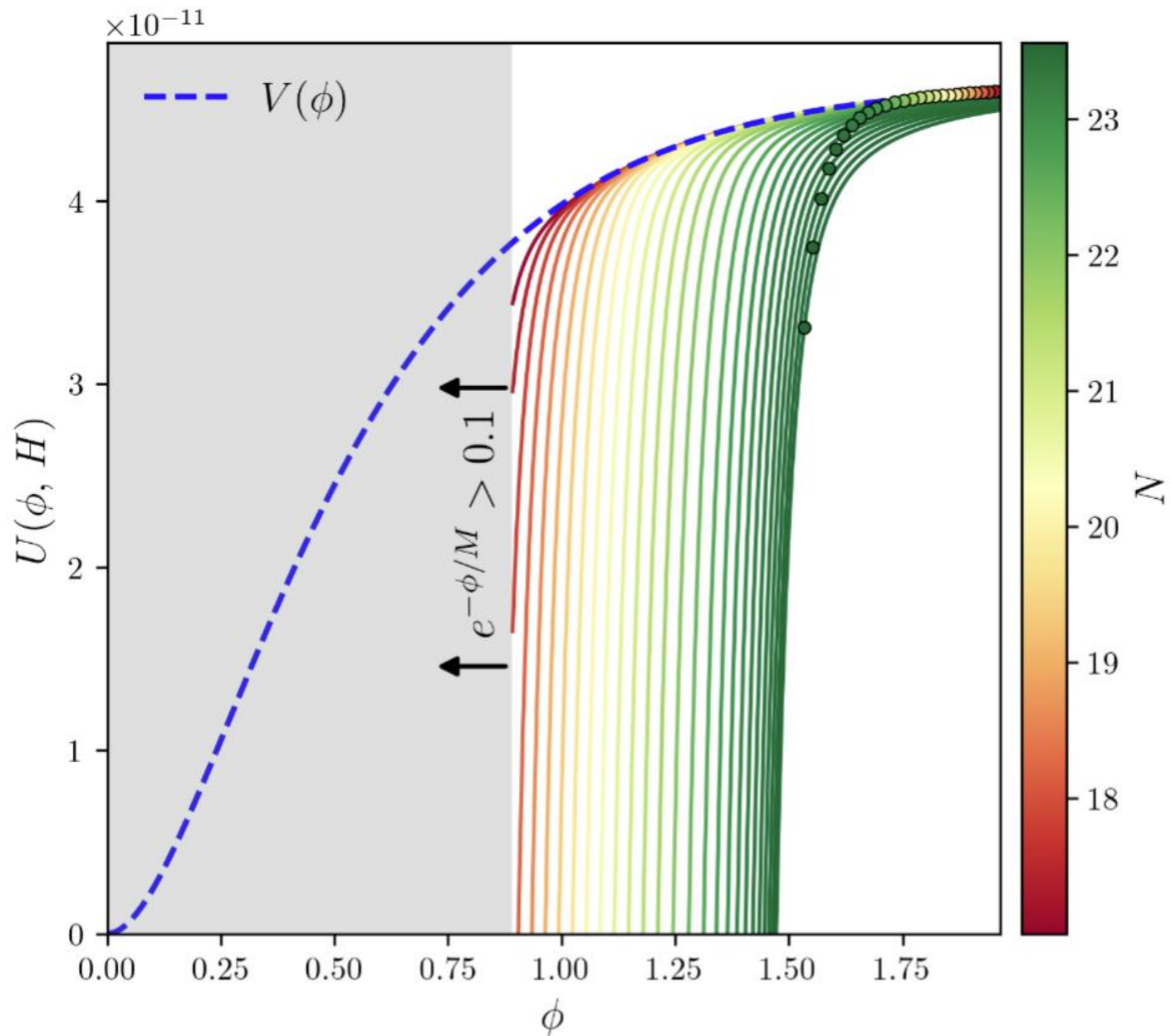
Litim, Wetterich, and many



Flow equation for the effective potential

$$\partial_k V_k(\varphi) \propto \frac{k^{d+1}}{k^2 + V_k''(\varphi)}.$$

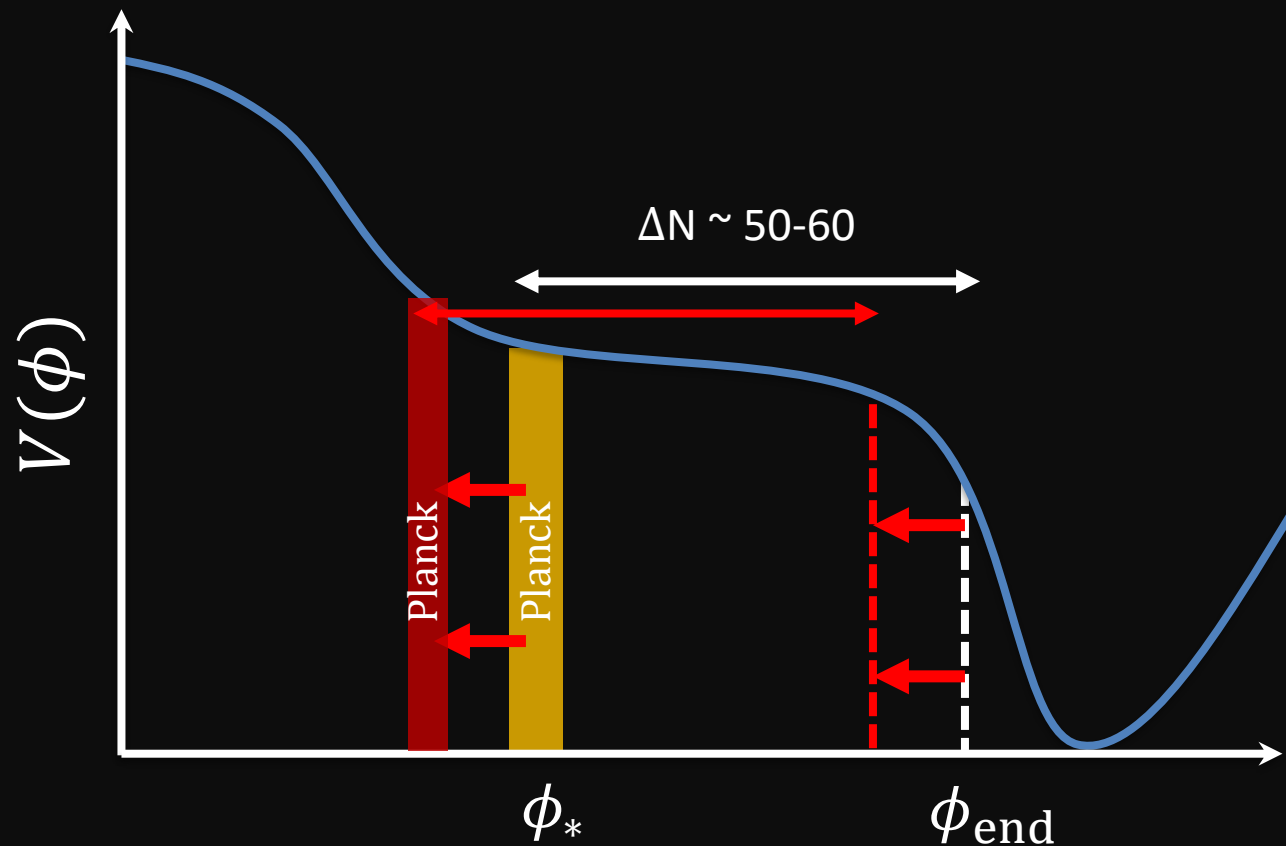




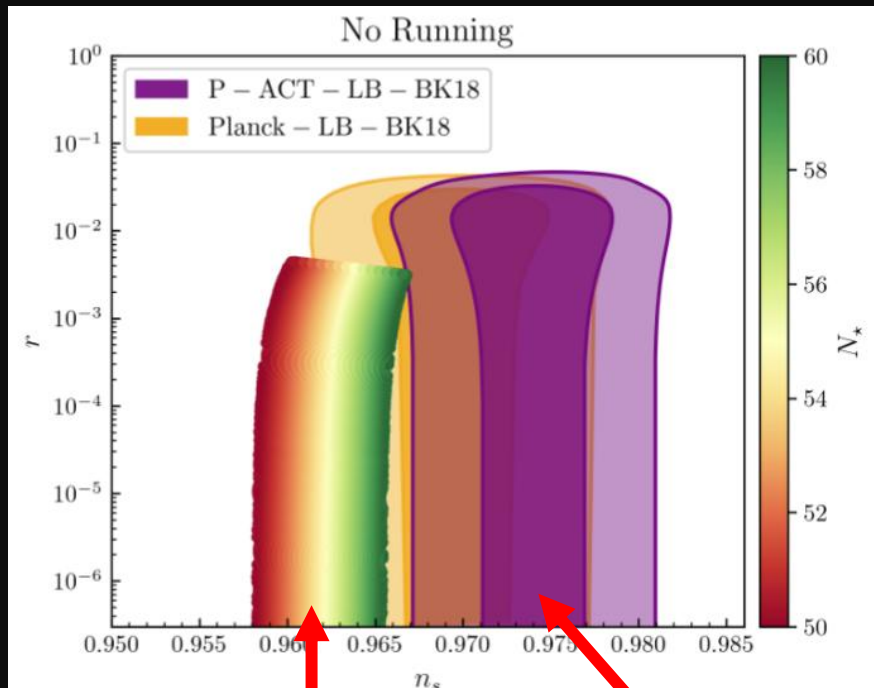
CMB measurements

Planck:

$k \sim 0.05 \text{ Mpc}^{-1}$



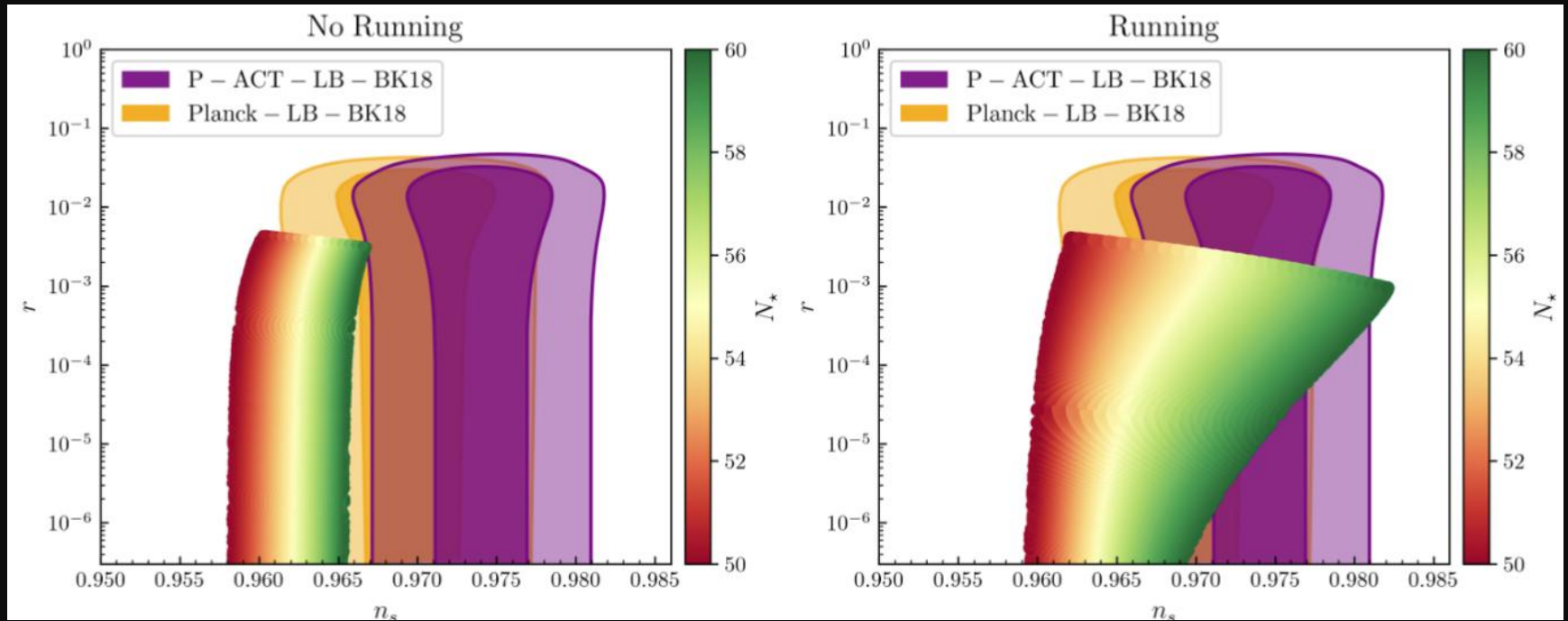
Effect on CMB observables



A popular
model prediction

The Data

Effect on CMB observables

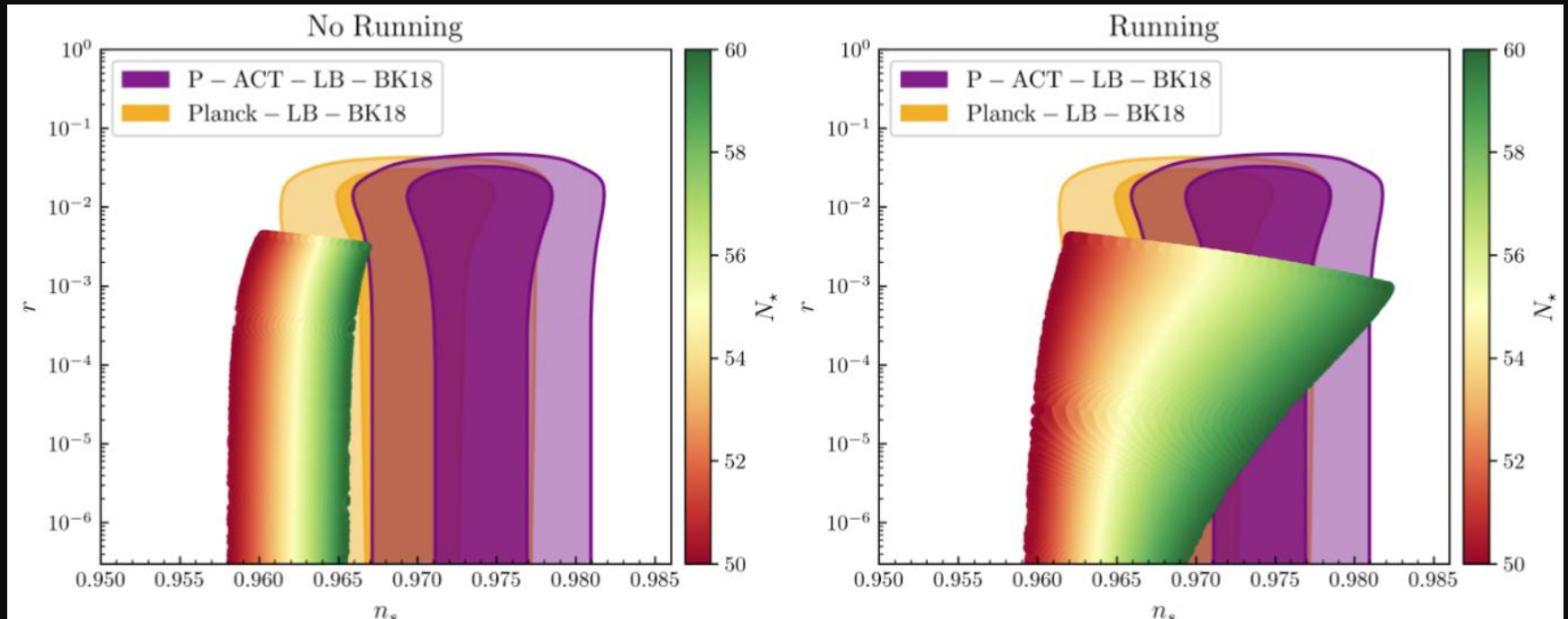


Reduced number of e-folds



Shift to larger n_s

Effect on CMB observables



Reduced number of e-folds



Shift to larger n_s

Many more avenues to be explored: Axion DM, dark energy dynamics, cosmological constant problem, bouncing cosmologies, etc.

Thank You!

Our Approach: Effective Action

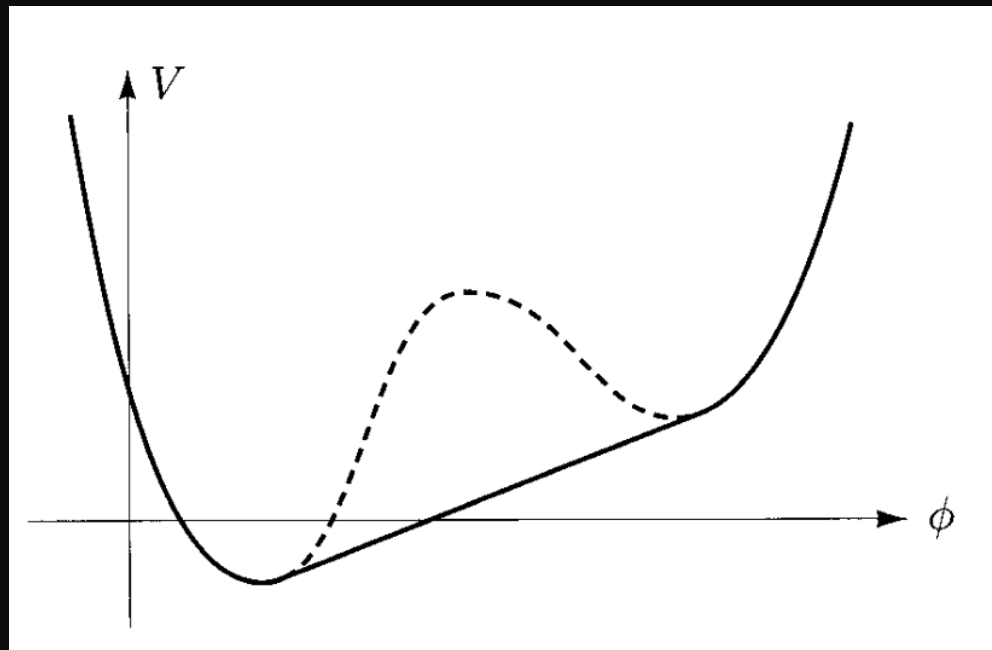
$$Z[J] = e^{-iE[J]} = \int \mathcal{D}\phi \exp \left[i \int d^4x (\mathcal{L}[\phi] + J\phi) \right].$$

$$\Gamma[\phi_{\text{cl}}] \equiv -E[J] - \int d^4y J(y)\phi_{\text{cl}}(y).$$

$$\Gamma[\phi_{\text{cl}}] = -(VT) \cdot V_{\text{eff}}(\phi_{\text{cl}}).$$


Effective Potential

(Always Convex)



Exact Renormalisation Group

4D-Euclidean space \longrightarrow 3+1 FLRW problem

$$\partial_\kappa U_\kappa(\phi) = \frac{aT^{-1}}{6\pi^2} \mathcal{D}_E^{-1} (a^{-1} \kappa^4)$$


$$\ddot{F} + H\dot{F} + (\partial_\phi^2 U - H^2 - \dot{H})F = \frac{T^{-1} H^4}{6\pi^2}$$

where $F \equiv \partial_H U$

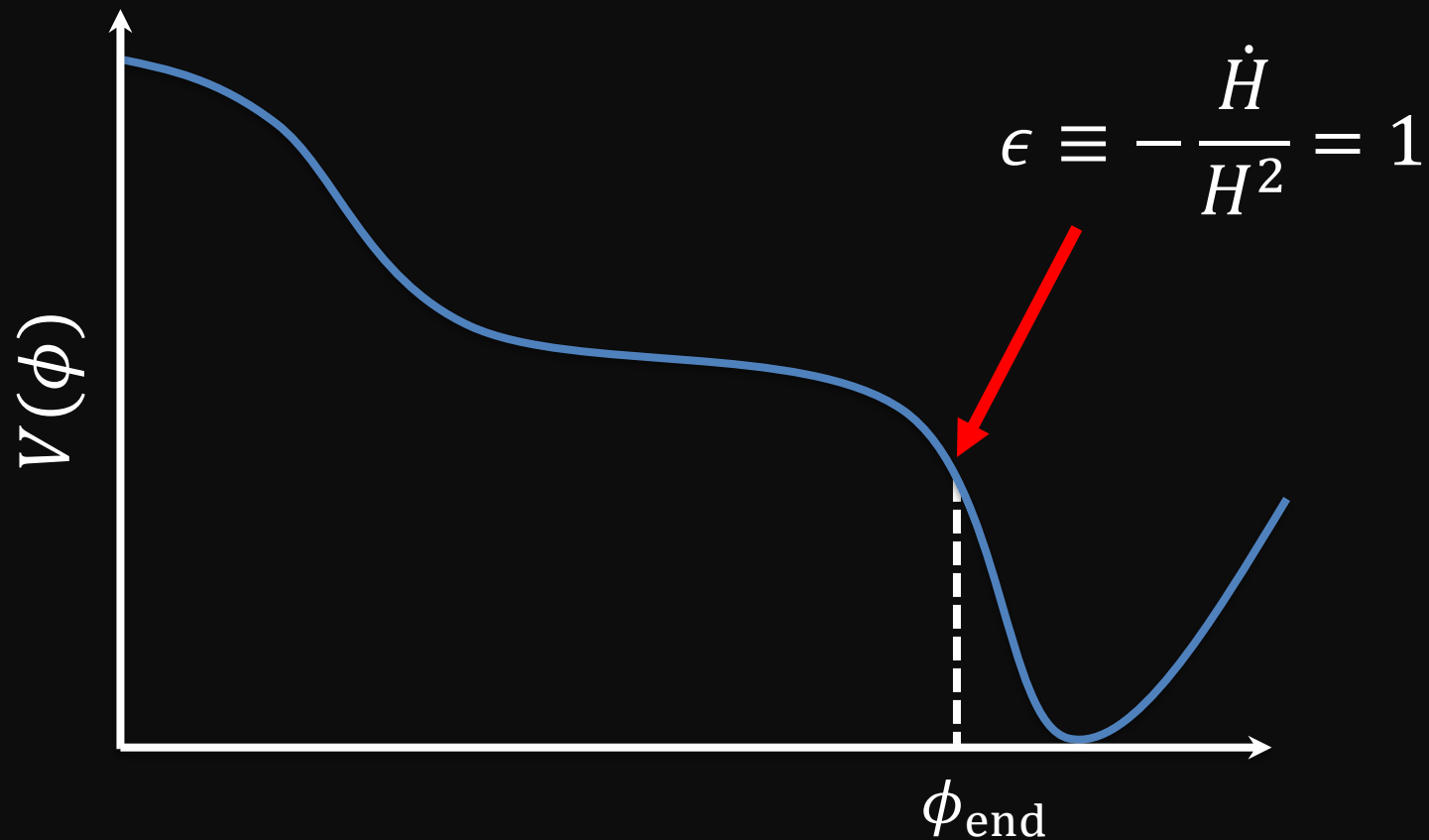
UV theory:

$$U(\phi, H_0) = V(\phi)$$

UV cutoff:

$$T^{-1} = H_0$$

Inflation Observables



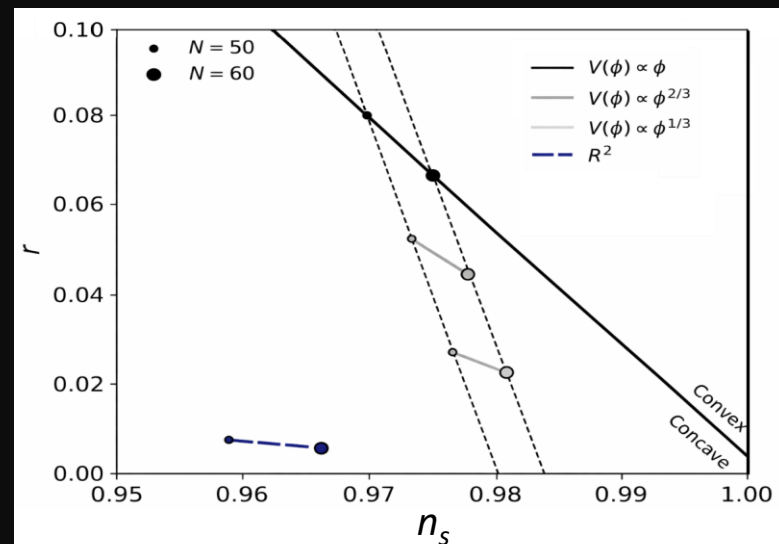
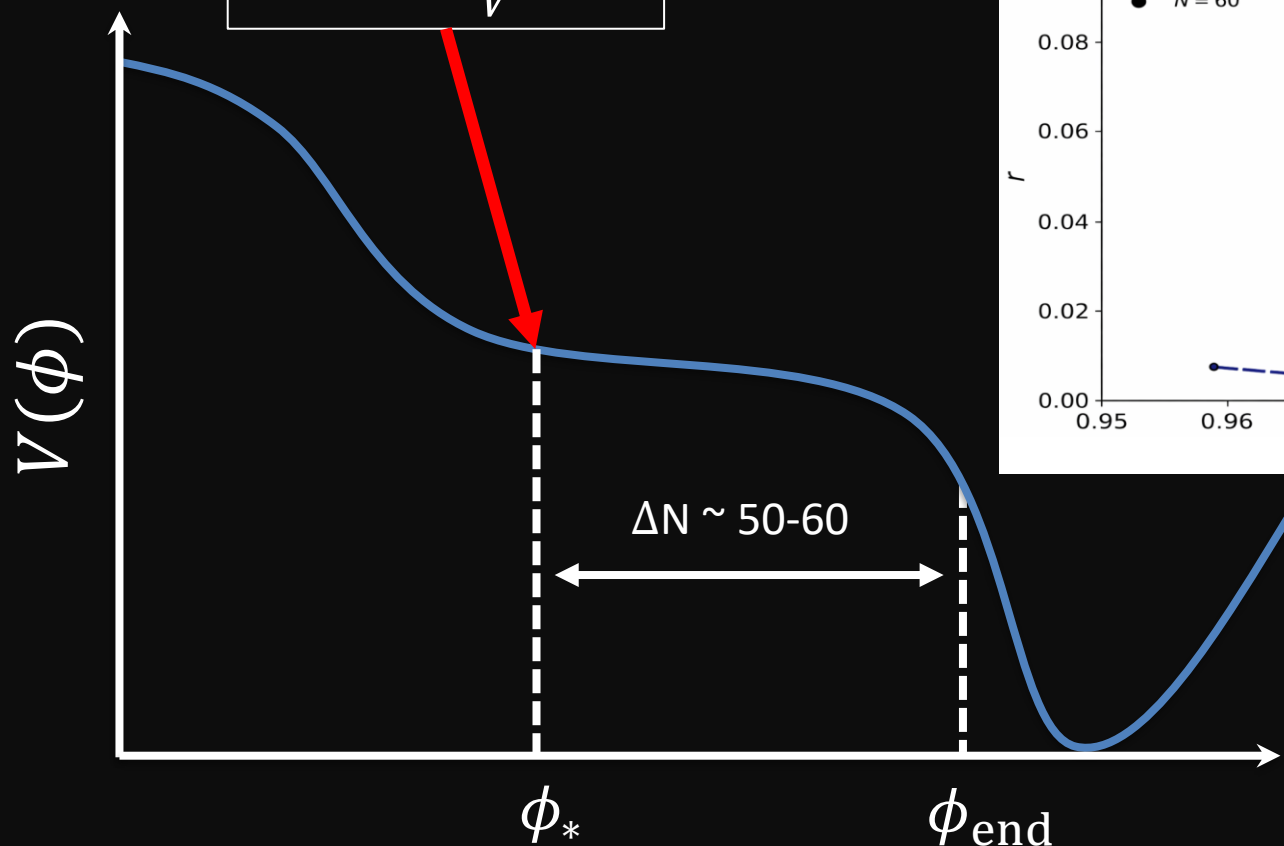
Inflation Observables

$$\epsilon_V \equiv \frac{M_p^2 V'}{2 V},$$

$$\eta_V \equiv \frac{M_p^2 V''}{V}$$

$$r = 16 \epsilon_V,$$

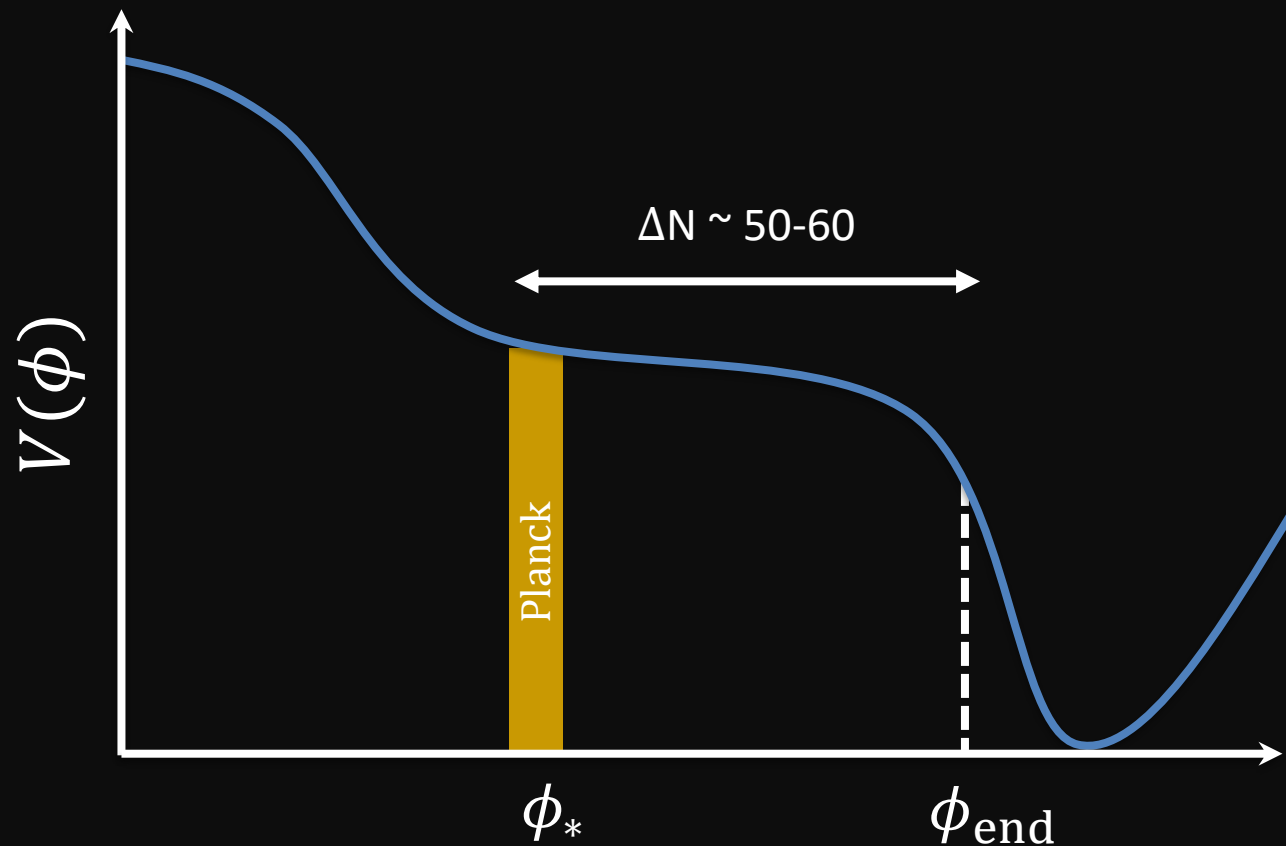
$$n_s = 1 - 6\epsilon_V + 2\eta_V$$



Inflation Model Building

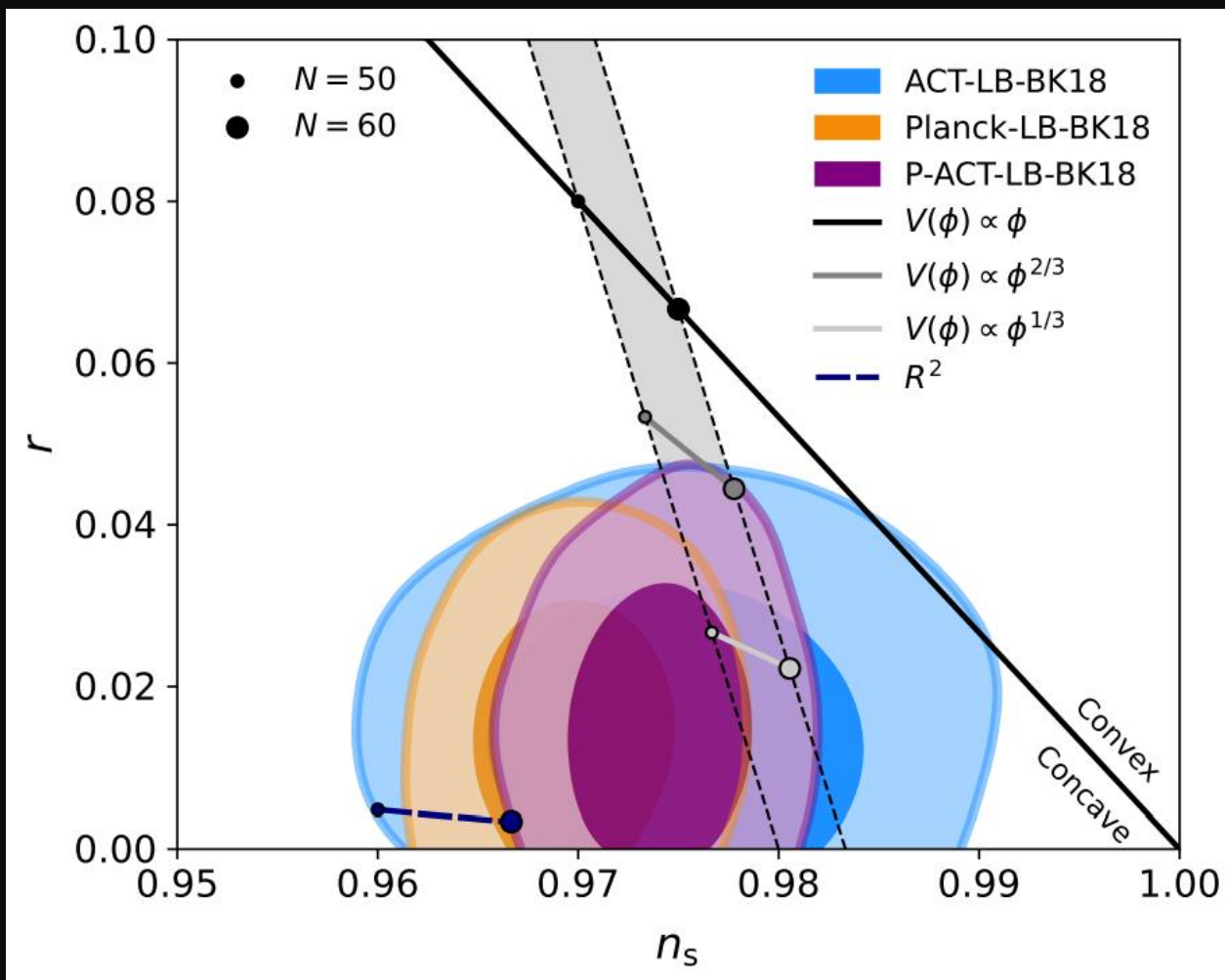
Planck:

$$k \sim 0.05 \text{ Mpc}^{-1}$$



Spectral Index Running

ACT DR6, March 2025



What about
quantum corrections?

1PI EFFECTIVE ACTION

Classical
Theory

RG Improved

$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$

Markannen *et al*, '18

Ellis *et al*, '25

$$\Delta V = \frac{1}{64\pi^2} \left[\mathcal{M}_\phi^4 \left(\log \frac{|\mathcal{M}_\phi^2|}{\mu^2} - \frac{3}{2} \right) + 2b_s \log \frac{|\mathcal{M}_\phi^2|}{\mu^2} \right]$$

$$R = 6(2H^2 + \dot{H})$$

$$b_s = -\frac{1}{180} R_{\mu\nu} R^{\mu\nu} + \frac{1}{180} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \simeq -\frac{H^4}{15}$$

$$\mathcal{M}_\phi^2 = V''(\phi) + \frac{1}{6} R$$

1PI EFFECTIVE ACTION

Classical
Theory

RG Improved

$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$

Markannen *et al*, '18

Ellis *et al*, '25

Slow-roll

$$R = 6(2H^2 + \dot{\chi})$$

$$H \approx H(\phi)$$

$$\Delta V = \frac{1}{64\pi^2} \left[\mathcal{M}_\phi^4 \left(\log \frac{|\mathcal{M}_\phi^2|}{\mu^2} - \frac{3}{2} \right) + 2b_s \log \frac{|\mathcal{M}_\phi^2|}{\mu^2} \right]$$

$$b_s = -\frac{1}{180} R_{\mu\nu} R^{\mu\nu} + \frac{1}{180} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \simeq -\frac{H^4}{15}$$

$$\mathcal{M}_\phi^2 = V''(\phi) + \frac{1}{6}R \simeq V''(\phi) - 2H^2$$

Negligible Self-Int.

$$\Delta V \sim H^4 \sim 10^{-10}V$$

1PI EFFECTIVE ACTION

Classical
Theory

RG Improved

$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$

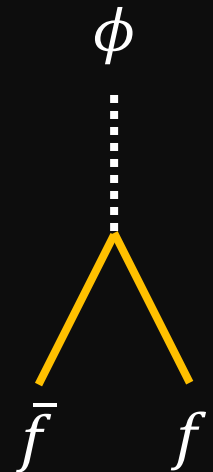
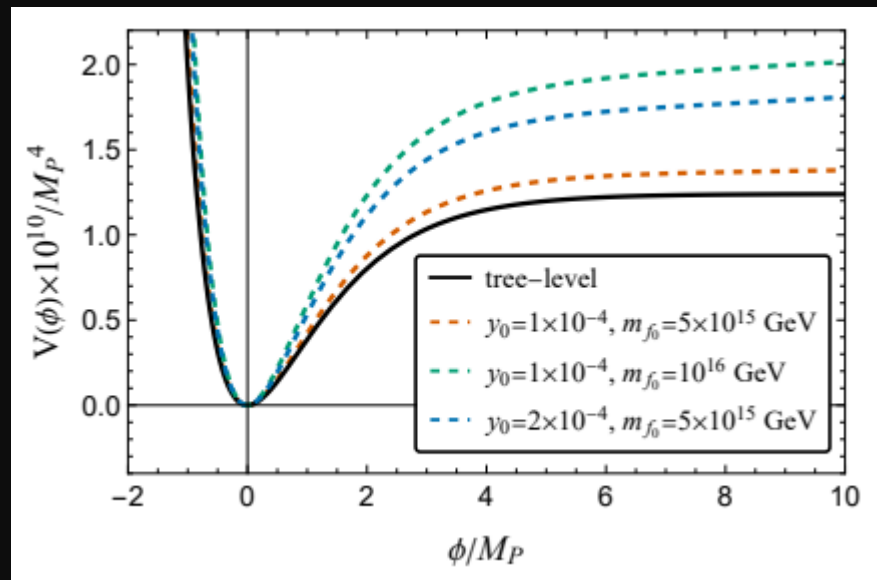
Markannen *et al*, '18

Ellis *et al*, '25

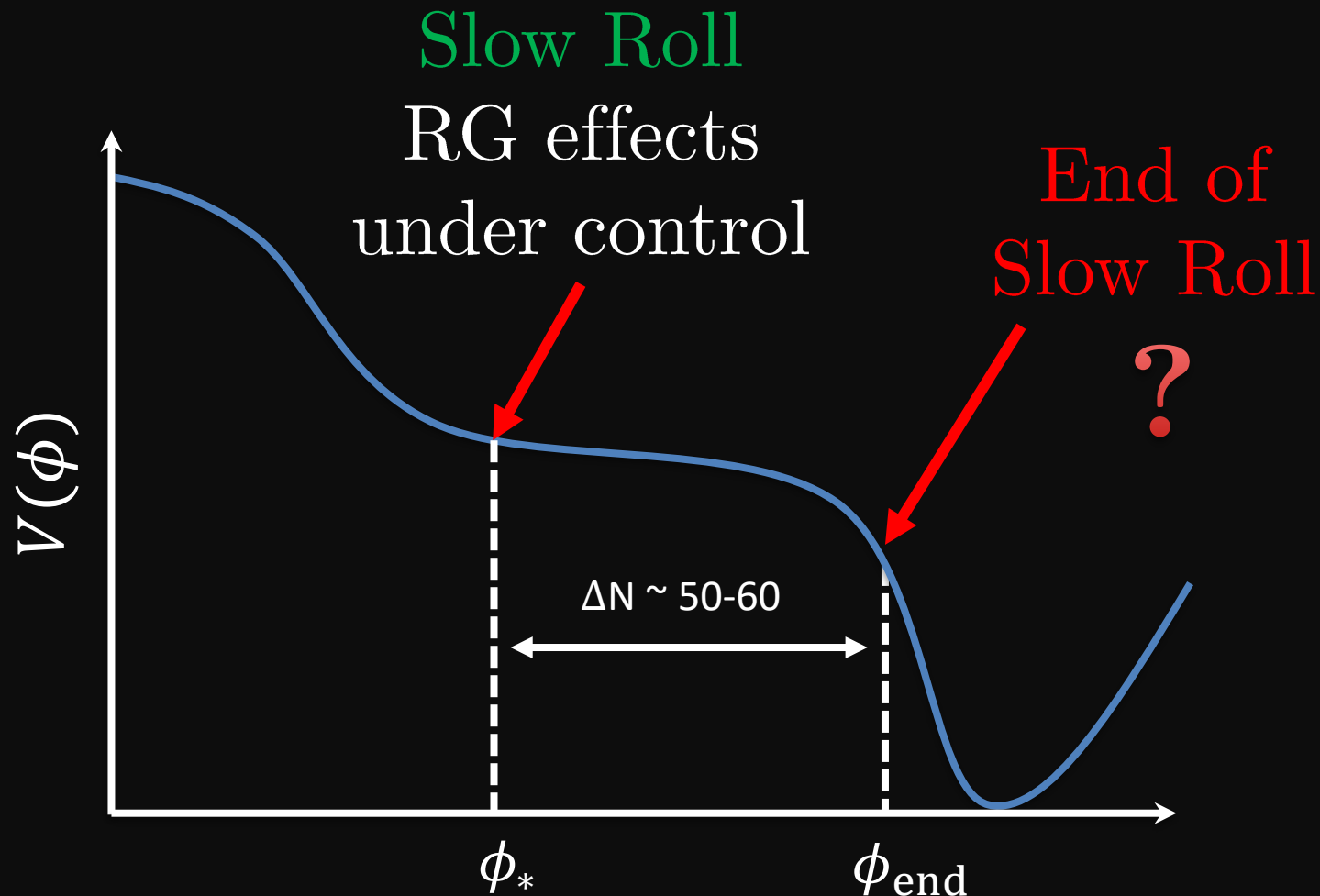
Slow-roll

$$R = 6(2H^2 + \dot{\phi}^2)$$

$$H \approx H(\phi)$$



Inflation Observables



1PI EFFECTIVE ACTION

Classical
Theory

RG Improved

$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$

Markannen *et al*, '18

Ellis *et al*, '25

$$\Delta V = \frac{1}{64\pi^2} \left[\mathcal{M}_\phi^4 \left(\log \frac{|\mathcal{M}_\phi^2|}{\mu^2} - \frac{3}{2} \right) + 2b_s \log \frac{|\mathcal{M}_\phi^2|}{\mu^2} \right]$$

$$R = 6(2H^2 + \dot{H})$$

$$b_s = -\frac{1}{180} R_{\mu\nu} R^{\mu\nu} + \frac{1}{180} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

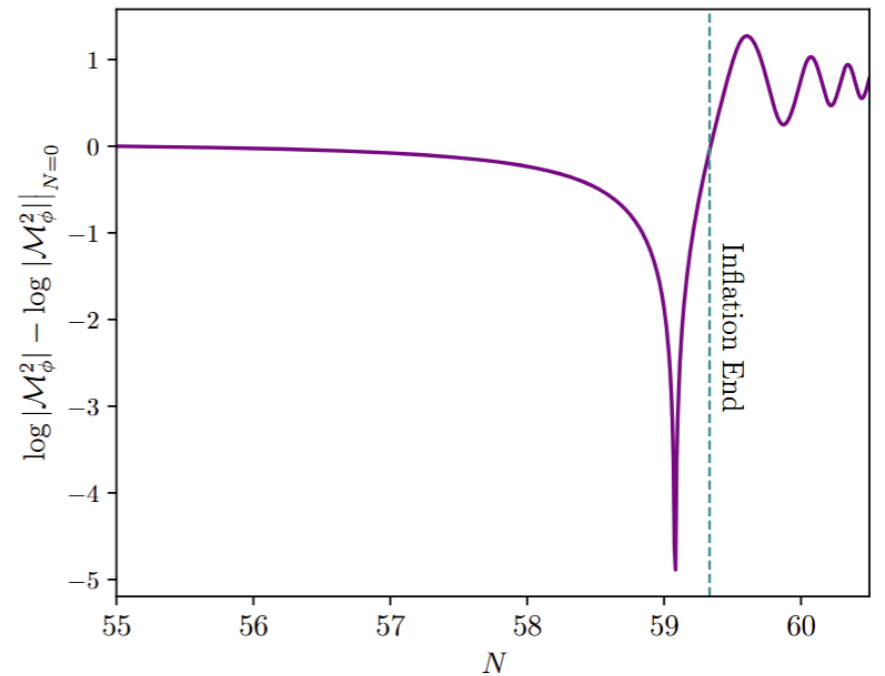
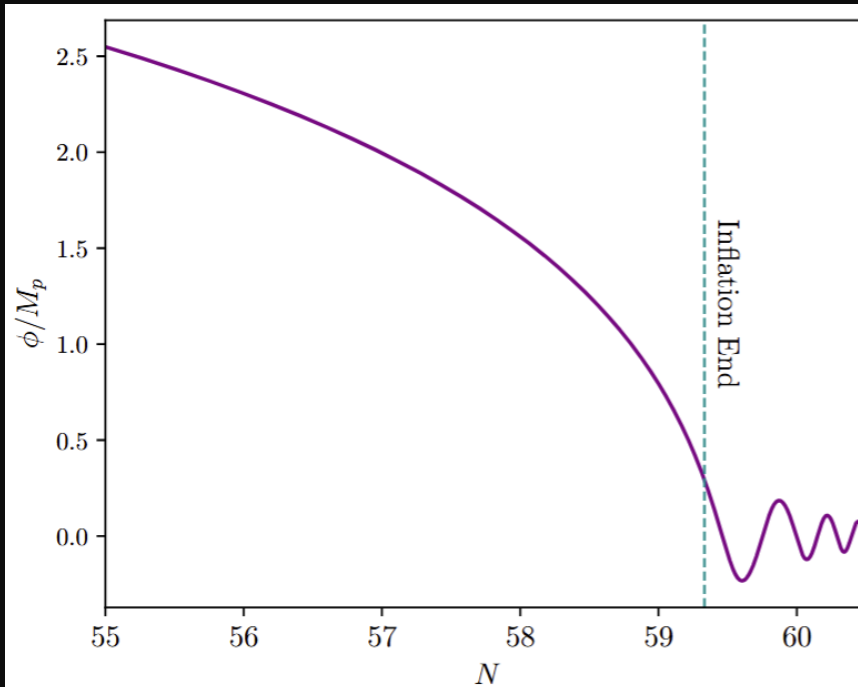
$$\mathcal{M}_\phi^2 = V''(\phi) + \frac{1}{6} R$$

1PI EFFECTIVE ACTION

Classical
Theory

RG Improved

$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$



HOW CAN WE DO BETTER?

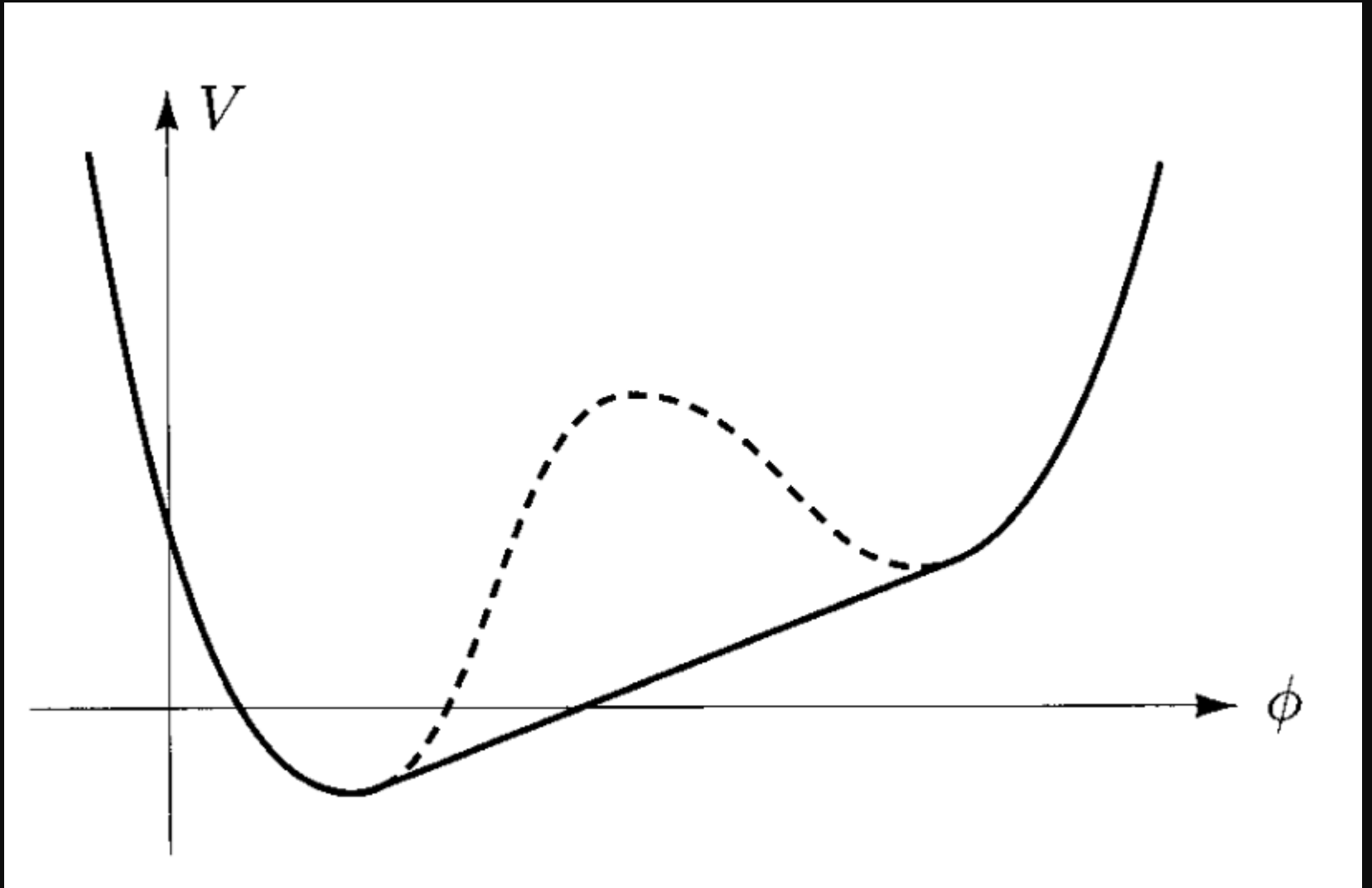
BACK TO BASICS: EFFECTIVE ACTION

$$Z[J] = e^{-iE[J]} = \int \mathcal{D}\phi \exp \left[i \int d^4x (\mathcal{L}[\phi] + J\phi) \right].$$

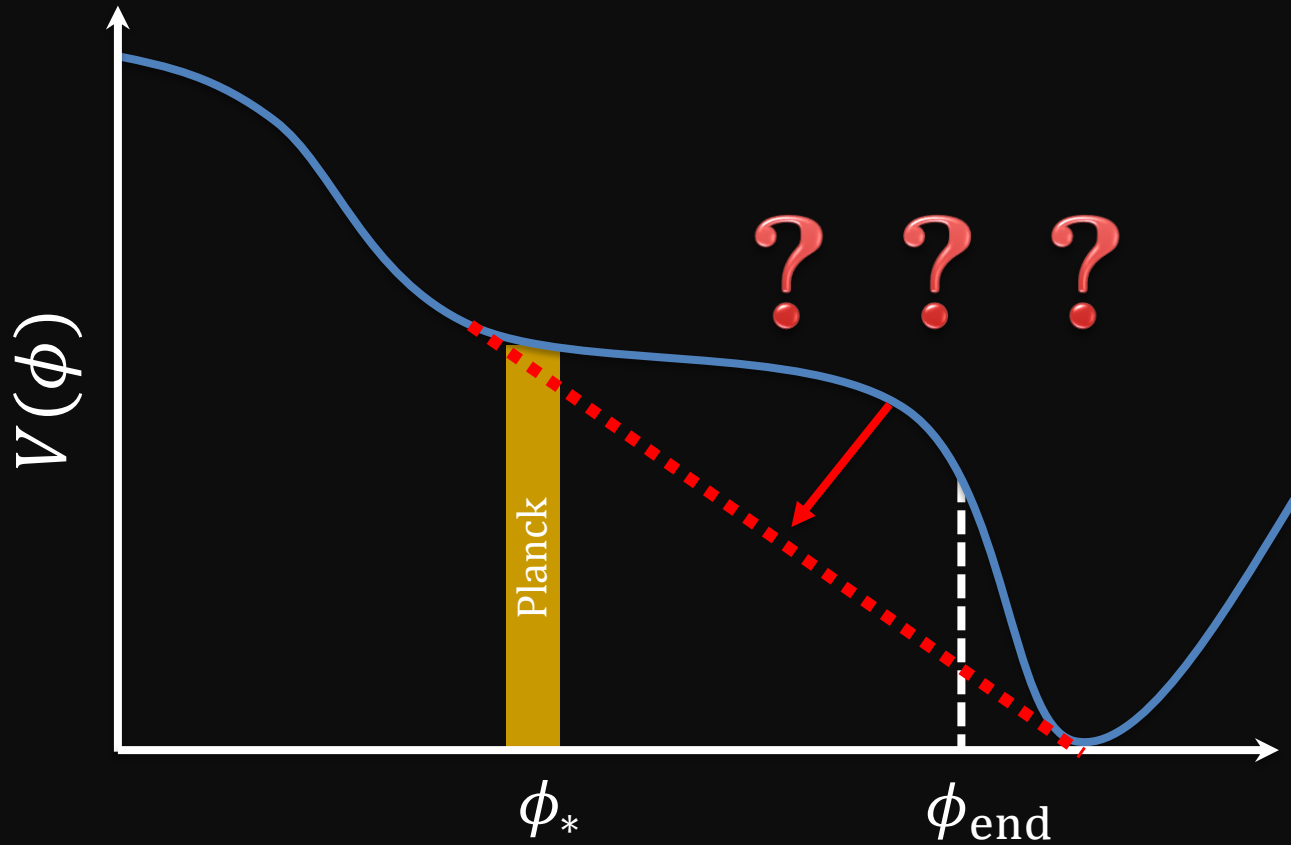
$$\Gamma[\phi_{\text{cl}}] \equiv -E[J] - \int d^4y J(y)\phi_{\text{cl}}(y).$$

$$\Gamma[\phi_{\text{cl}}] = -(VT) \cdot V_{\text{eff}}(\phi_{\text{cl}}).$$

EFFECTIVE ACTION



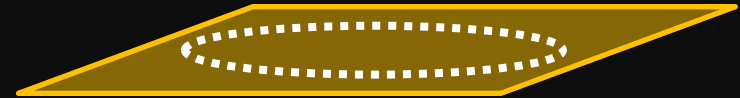
EFFECTIVE ACTION



EFT of inflation

\longleftrightarrow
 H^{-1}

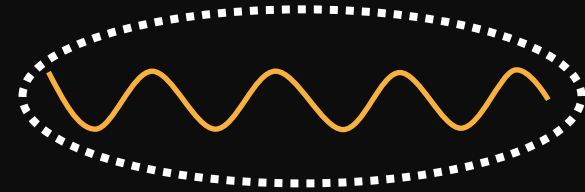
Homogeneous Background:



=

Quantum Perturbations

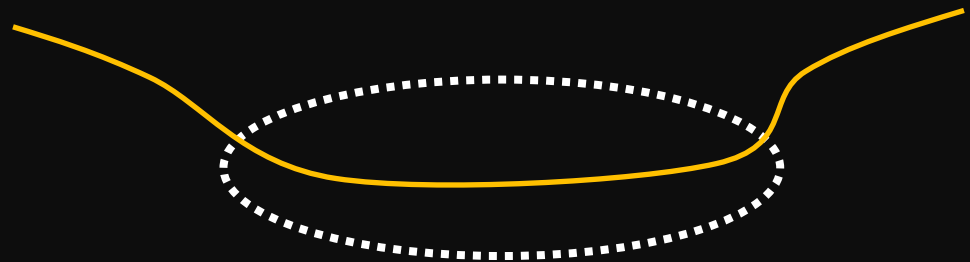
(integrated out)



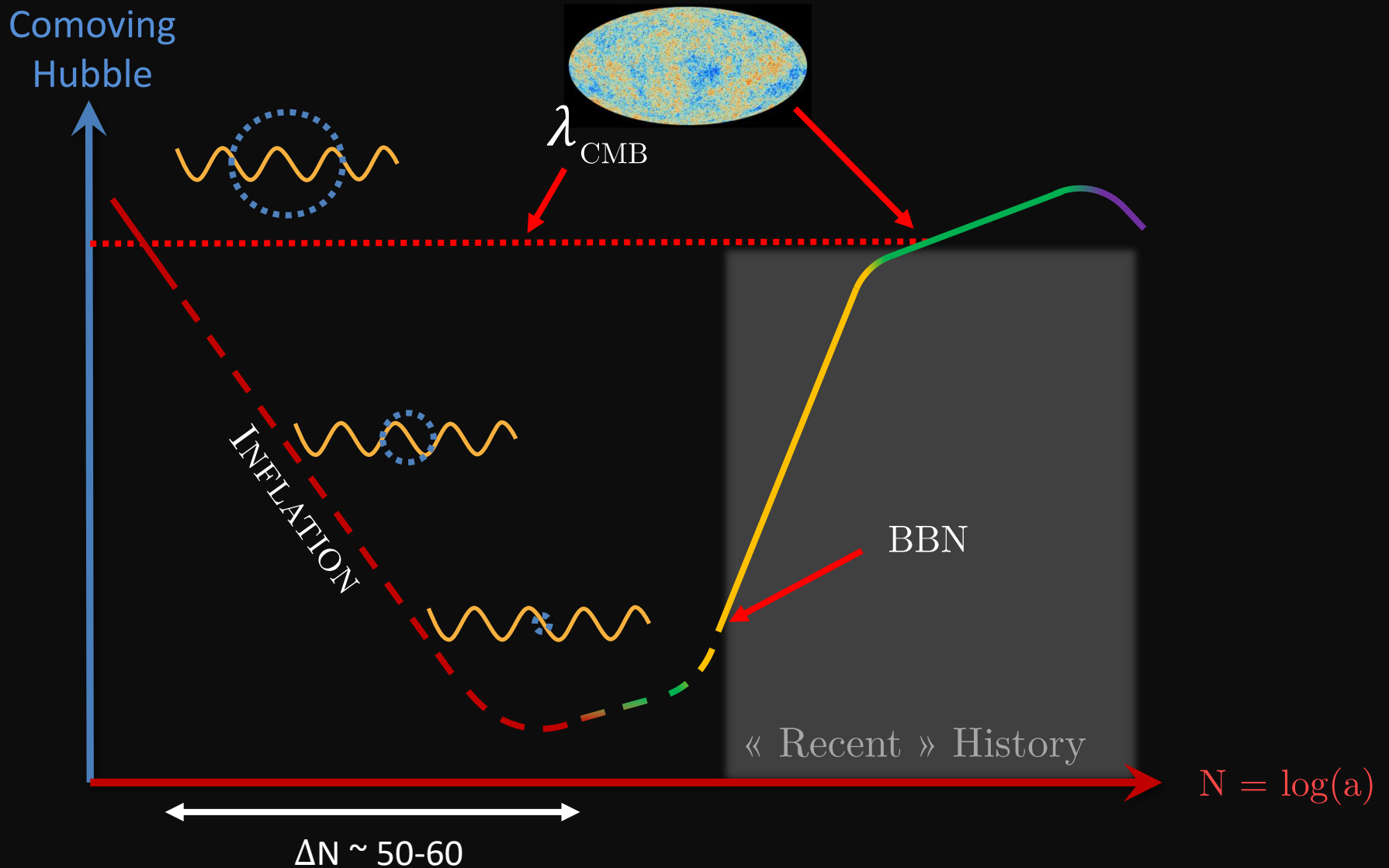
+

Classical Perturbations

(super-horizon)



EFT of inflation

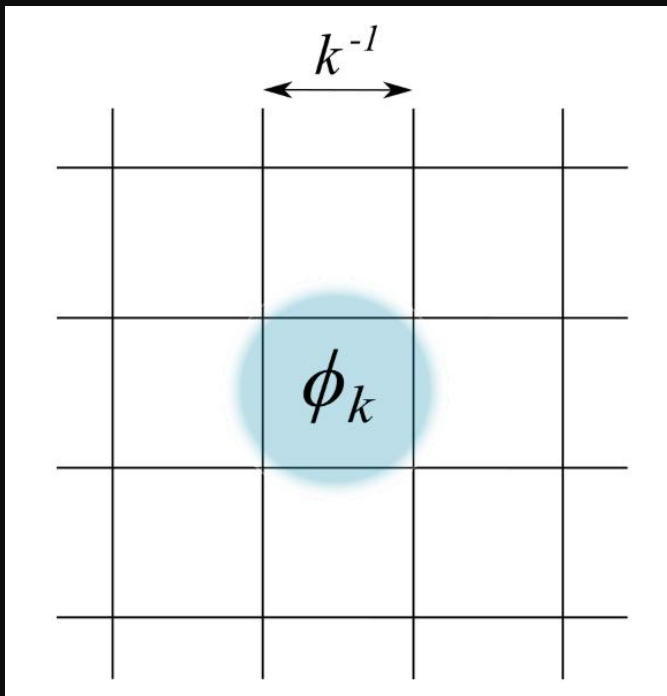


EXACT RENORMALISATION GROUP

At every scale: coarse grain the theory.

- Short wavelengths are integrated out.
- Large wavelengths live in the effective field theory.

Litim, Wetterich, and many



$$Z_k[J] = \int \mathcal{D}\phi \exp\left(-S[\phi] - \Delta S_k[\phi] + \int J\phi\right)$$

$$\Delta S_k[\phi] = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \phi(-q) R_k(q) \phi(q).$$

$$R_k(q) \xrightarrow{q^2 \ll k^2} k^2, \quad R_k(q) \xrightarrow{q^2 \gg k^2} 0.$$

$$W_k[J] = \ln Z_k[J].$$

$$\Gamma_k[\varphi] = \sup_J \left(\int J\varphi - W_k[J] \right) - \Delta S_k[\varphi].$$

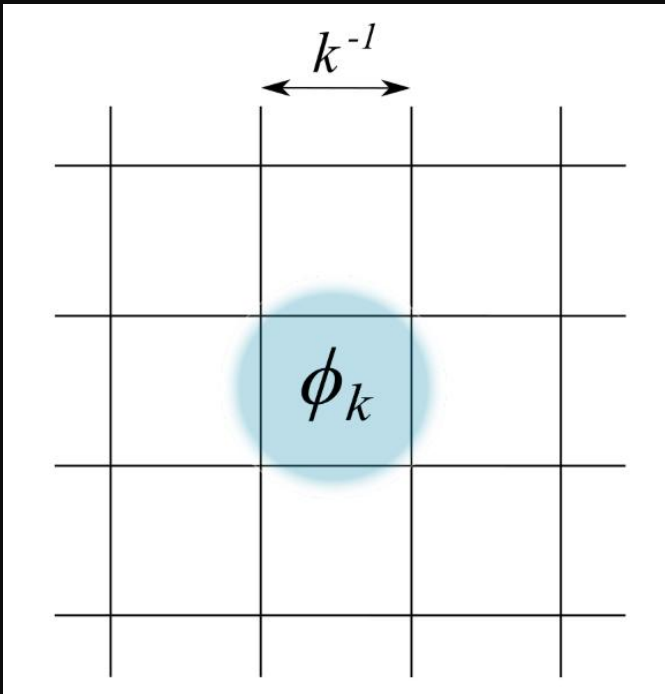
$$\Gamma_{k \rightarrow \Lambda} \rightarrow S, \quad \Gamma_{k \rightarrow 0} \rightarrow \Gamma.$$

EXACT RENORMALISATION GROUP

At every scale: coarse grain the theory.

- Short wavelengths are integrated out.
- Large wavelengths live in the effective field theory.

Litim, Wetterich, and many



$$\frac{\partial}{\partial t} \Gamma_k[\varphi] = \frac{1}{2} \text{Tr} \left[(\Gamma_k^{(2)}[\varphi] + R_k)^{-1} \partial_t R_k \right].$$

$$\frac{\partial}{\partial t} \Gamma_k = \frac{1}{2} \times \text{circle with shaded blob}$$

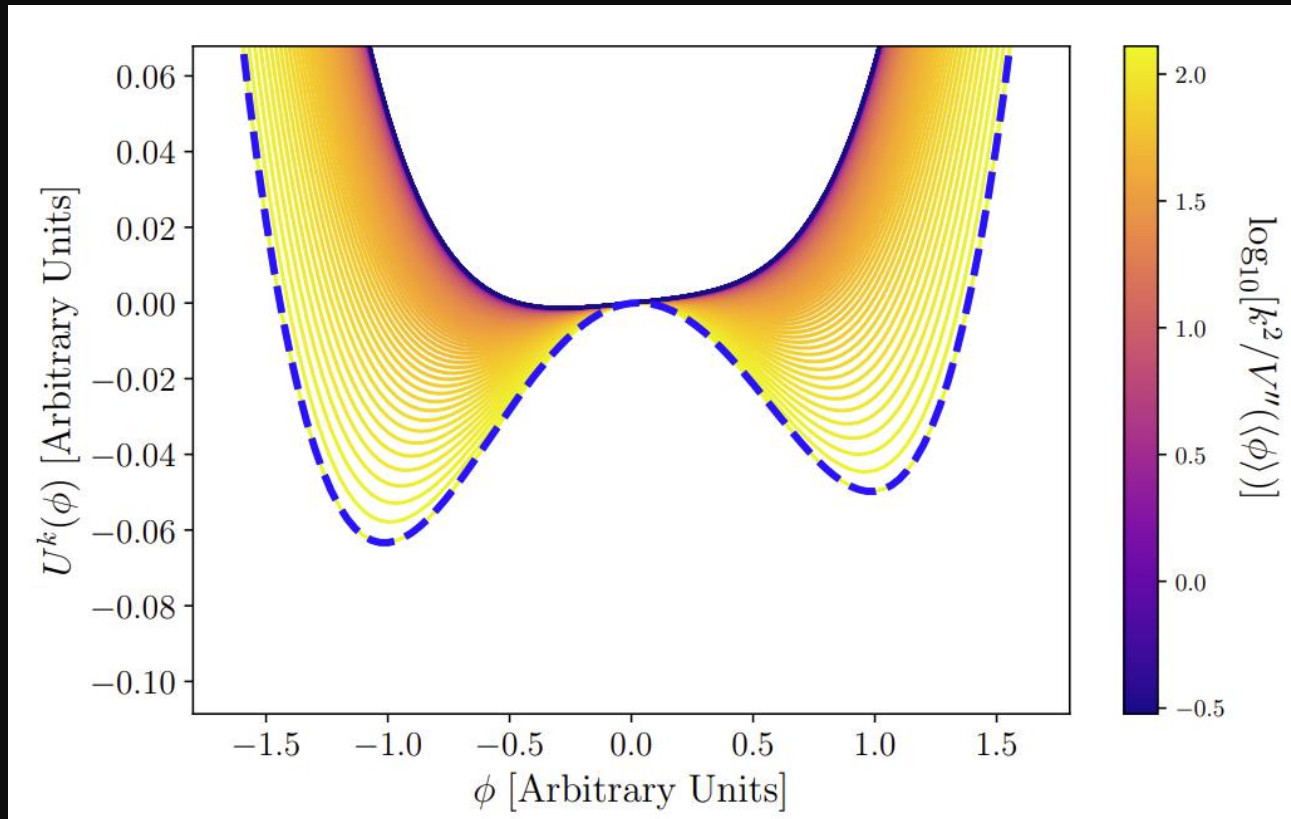
$$\times = \frac{\partial}{\partial t} R_k$$

—(shaded blob)— = full k -dependent propagator

$$R_k(q) = (k^2 - q^2)\theta(k^2 - q^2)$$

$$\partial_k V_k(\varphi) \propto \frac{k^{d+1}}{k^2 + V_k''(\varphi)}.$$

EXACT RENORMALISATION GROUP



S. Abel, LH, JHEP 08 (2025) 198

EXACT RENORMALISATION GROUP

4D-Euclidean space \longrightarrow 3+1 FLRW problem

$$\begin{aligned}
 Z_k[j] &\equiv e^{(i/\hbar)W_k[j]} & (25) \\
 &= \int \mathcal{D}[\varphi] \exp \left((i/\hbar)S[\Phi] + (i/\hbar) \int dt \sqrt{-g} \int d^3x j\varphi \right. \\
 &\quad \left. - (i/2\hbar) \int dt \sqrt{-g} \int_p \varphi(t, \vec{p}) \varphi(t, -\vec{p}) (C_k - i\varepsilon) \right) ,
 \end{aligned}$$

$$\begin{aligned}
 \frac{\delta \Gamma_k}{\delta k(t)} &= \frac{\sqrt{-g(t)}}{2} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \delta(\vec{p} + \vec{q}) \\
 &\quad \int dt' \delta(t - t') \mathcal{O}(t, t', \vec{p}, \vec{q}) \partial_k C_k(p) , & (5)
 \end{aligned}$$

where

$$\begin{aligned}
 \mathcal{O}(t, t', \vec{p}, \vec{q}) &= \\
 &\left[\sqrt{|g|} (C_k(p) - i\varepsilon) \delta(\vec{p} + \vec{q}) \delta(t - t') - \frac{\delta^2 \Gamma_k}{\delta \varphi_b(t, \vec{p}) \delta \varphi_b(t', \vec{q})} \right]^{-1} , & (6)
 \end{aligned}$$

EXACT RENORMALISATION GROUP

4D-Euclidean space \longrightarrow 3+1 FLRW problem

+ Local Potential Approximation

$$\Gamma_k[\phi] = \int dt \sqrt{-g} \int d^3x \left(-\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - U_k(\phi) \right)$$

+ Litim Top-Hat Regulator

$$\partial_\kappa U_\kappa(\phi) = \frac{aT^{-1}}{6\pi^2} \mathcal{D}_E^{-1} (a^{-1} \kappa^4) , \quad (9)$$

where

$$\mathcal{D}_E = -\frac{d^2}{dt^2} - 3H \frac{d}{dt} + \kappa^2 + \partial_\phi^2 U_\kappa(\phi) , \quad (10)$$

$T^{-1} \equiv \lim_{t' \rightarrow t} \delta(t - t')$ is a cut-off frequency

EXACT RENORMALISATION GROUP

4D-Euclidean space \longrightarrow 3+1 FLRW problem

+ Specify the coarse-graining scale:

$$\kappa = \sigma H$$

(natural choice: $\sigma = 1$)

+ Multiply by inverse propagator

$$\partial_\kappa U_\kappa(\phi) = \frac{aT^{-1}}{6\pi^2} \mathcal{D}_E^{-1} (a^{-1} \kappa^4)$$

$$\ddot{F} + H\dot{F} + (\partial_\phi^2 U - H^2 - \dot{H})F = \frac{T^{-1} H^4}{6\pi^2}$$

where $F \equiv \partial_H U$


RG running of inflation

Bare potential $V(\phi)$ \longrightarrow RG-Improved potential $U(\phi)$

Static field: $\phi \sim \text{cst.}$ \longrightarrow $H = H(\phi)$ \longrightarrow $U = V_{RGI}(\phi)$

Valid in the **deep slow-roll era only**

Rolling field: $\phi(t), \dot{\phi}(t)$ \longrightarrow $\phi(t), H(t)$ \longrightarrow $U = U(\phi, H)$



Allows for **deviations from slow-roll**

RG running of inflation

UV theory:

$$U(\phi, H_0) = V(\phi)$$

UV cutoff:

$$T^{-1} = H_0$$

$$\begin{aligned} F'' &= \frac{H_0 H^2}{6\pi^2} - \left(1 + \frac{H'}{H}\right) F' - \left(\frac{\partial_\phi^2 U}{H^2} - \frac{H'}{H} - 1\right) F, \\ \phi'' &= -\left(3 + \frac{H'}{H}\right) \phi' - \frac{\partial_\phi U}{H^2}, \\ H' &= -\frac{H}{2M_p^2} \left((\phi')^2 + \frac{F'}{3H}\right). \end{aligned}$$

(17)

EXACT RENORMALISATION GROUP

- de Sitter Case

$$H_0 F_{\text{dS}} = -\frac{H_0^4}{6\pi^2(1 - \partial_\phi^2 U_{\text{dS}}/H_0^2)}$$

$$U_{\text{dS}} = 3M_p^2 H_0^2 - \frac{H_0^4}{6\pi^2(1 - \partial_\phi^2 U_{\text{dS}}/H_0^2)}$$

Compatible with

$$\phi' = H' = F' = 0$$

- Inflation Case

E-Model:

$$U(\phi, H_0) = V(\phi) = V_0 \left(1 - e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_p}} \right)^2 .$$

EXACT RENORMALISATION GROUP

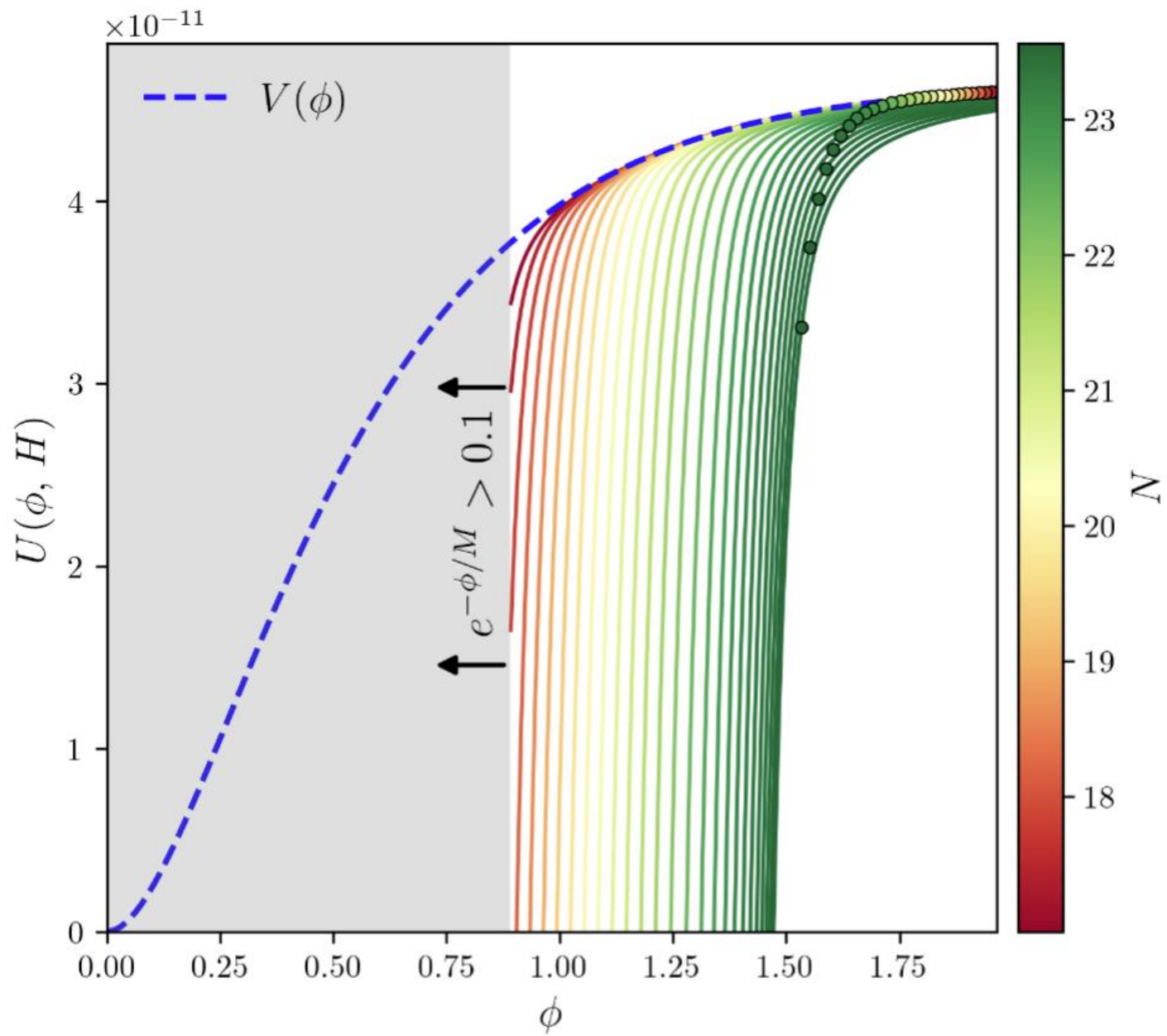
E-models

$$U(\phi, H_0) = V(\phi) = V_0 \left(1 - e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_p}} \right)^2 .$$

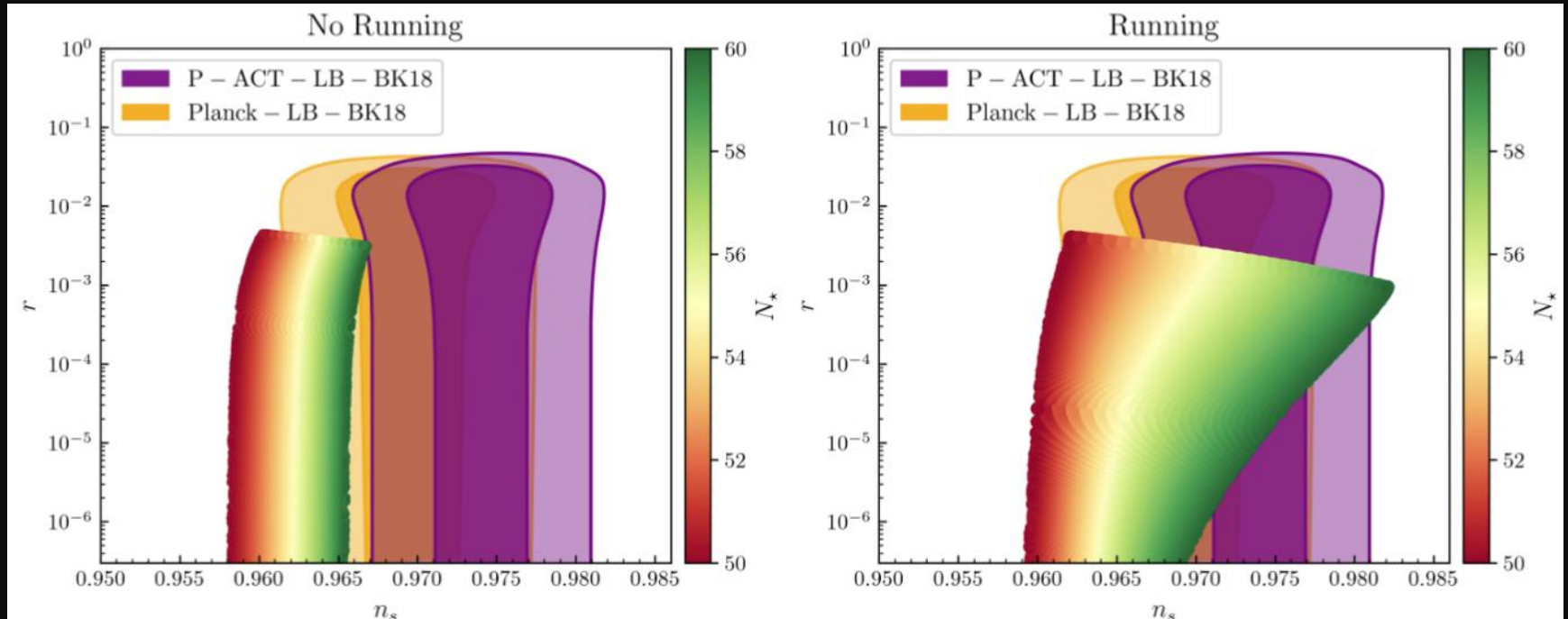
$$U_H(\phi) = \sum_{n=0}^{\infty} U_n(H) e^{-n\phi/M}$$

$$F = \sum_{n=0}^{\infty} F_n e^{-n\phi/M} , \text{ with } F_n = \frac{dU_n}{dH}$$

$$\begin{aligned} F_n'' &= \delta_{n,0} \frac{H_0 H^2}{6\pi^2} - F_n' \left[-2n \frac{\phi'}{M} + \left(1 + \frac{H'}{H} \right) \right] \\ &+ F_n \left[n \frac{\phi''}{M} - \left(n \frac{\phi'}{M} \right)^2 + \left(1 + \frac{H'}{H} \right) \left(1 + n \frac{\phi'}{M} \right) \right] \\ &- \frac{1}{H^2 M^2} \left(\sum_{l=0}^n l^2 U_l F_{n-l} \right) , \quad n \geq 0. \end{aligned} \quad (26)$$



EXACT RENORMALISATION GROUP



Effect on Observables:

Reduced number of e-folds \longrightarrow Shift to larger n_s

FURTHER REFINEMENTS

1PI Matching

Adiabatic Limit + Minkowsky \longrightarrow Vanilla 1PI result? **YES!**

$$H = 0 \text{ and } \dot{\kappa} = \dot{\phi} = \dot{T} = 0$$



$$\partial_{\kappa} U_{\kappa}(\phi) = \frac{T^{-1}}{6\pi^2} \frac{\hbar \kappa^4}{\kappa^2 + \partial_{\phi}^2 U_{\kappa}(\phi)} .$$

if one identifies T^{-1} with $3\kappa/16$

Adiabatic Limit + FLRW

$$U_{\kappa}(\phi) = U_i(\phi) + \mathcal{O}(\hbar)$$

$$\mathcal{M}_i^2 = \partial_{\phi}^2 U_i(\phi) + (\xi - 1/6)R$$

$$\partial_{\kappa} U_{\kappa}(\phi) = \frac{\hbar}{32\pi^2} \frac{\kappa^5}{\kappa^2 + \mathcal{M}_i^2} + \mathcal{O}(\hbar^2)$$

$$U_{ren}^{(1)} = U_i + \frac{\hbar}{64\pi^2} \left(\mathcal{M}_i^2\right)^2 \left(\ln\left(\frac{\mathcal{M}_i^2}{\mu^2}\right) + C\right)$$

1PI Matching

if one identifies T^{-1} with $3\kappa/16$



$$T = T(H)$$

New Formulation:

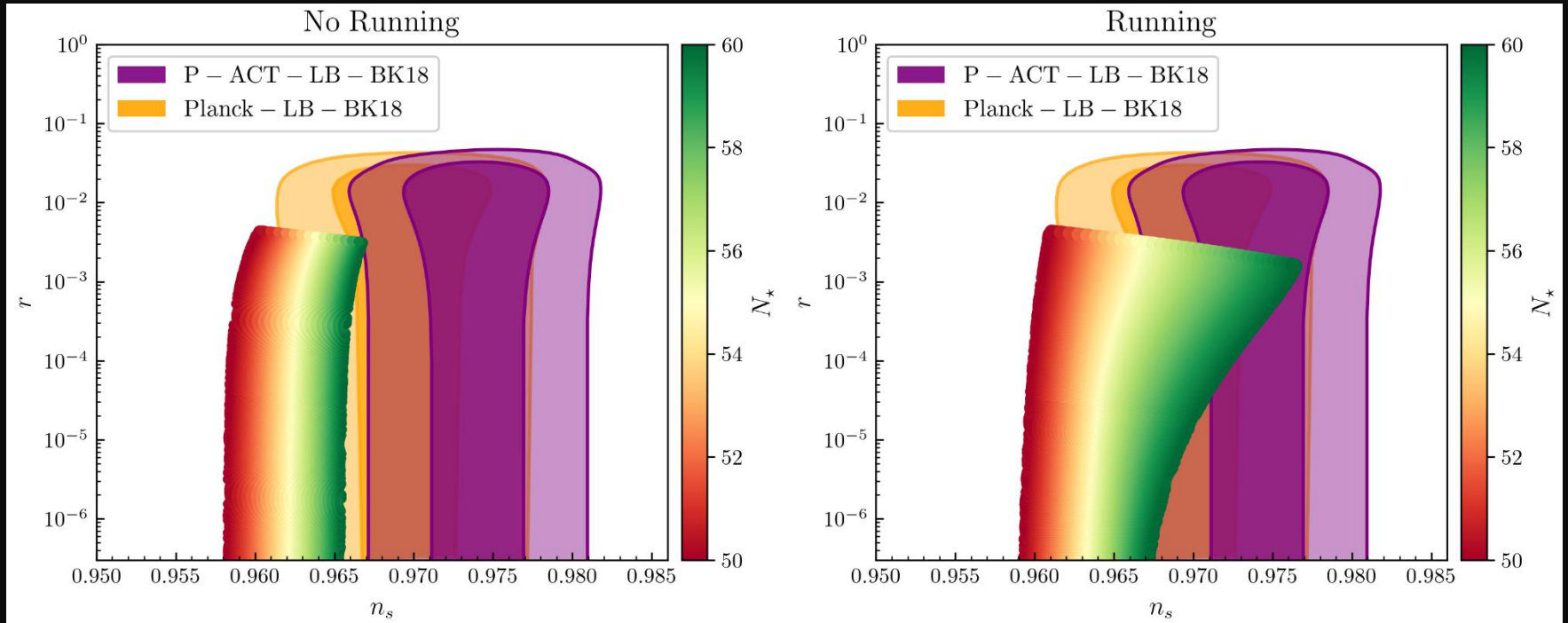
$$F \equiv \partial_H U$$



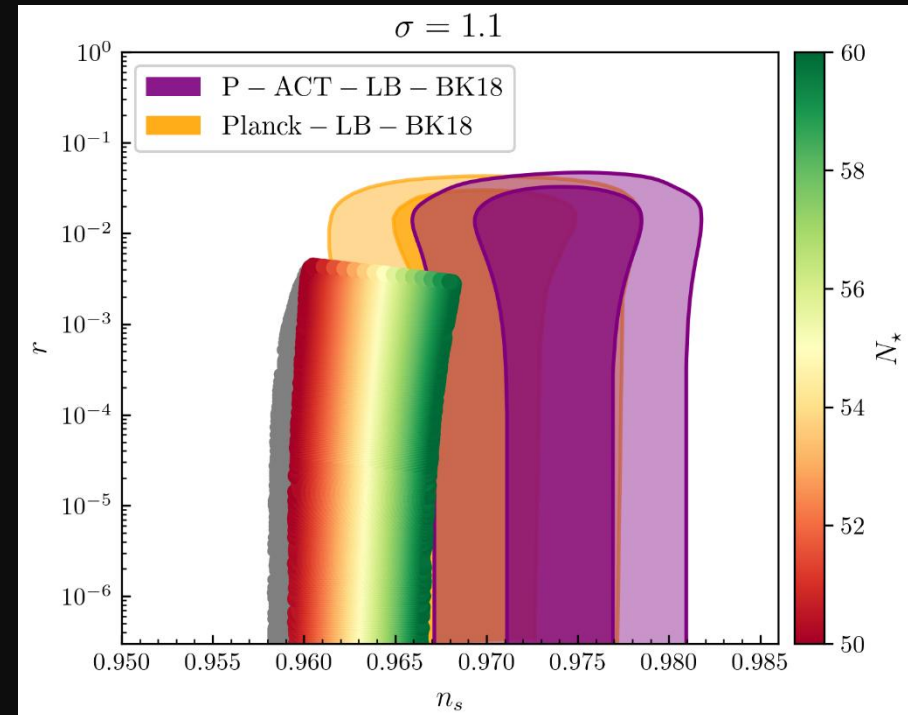
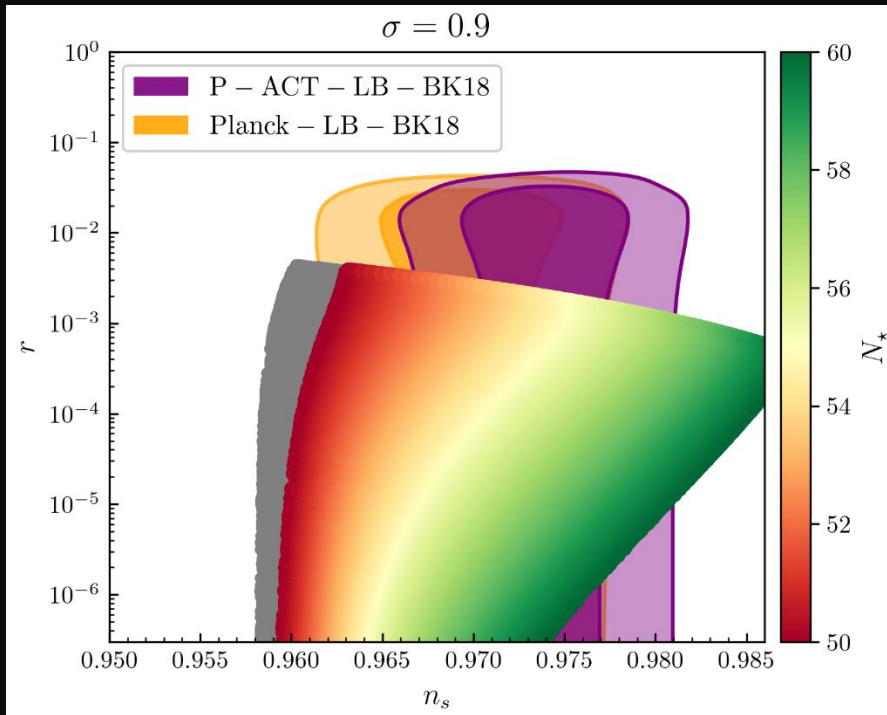
$$X \equiv H^{-1} \partial_H U$$

$$\begin{aligned} X'' &= \frac{\sigma^6 H^2}{32\pi^2} - \left(1 + \frac{H'}{H}\right) X' \\ &\quad - \left(\frac{\partial_\phi^2 U}{H^2} - \frac{H'}{H} - 2 + \sigma^2\right) X, \\ \phi'' &= - \left(3 + \frac{H'}{H}\right) \phi' - \frac{\partial_\phi U}{H^2}, \\ H' &= -\frac{H}{2M_p^2} \left[(\phi')^2 + \frac{1}{3} \left(\frac{H'}{H} X + X' \right) \right]. \end{aligned} \tag{80}$$

1PI Matching



Coarse-Graining Size Effect



DISCUSSION

Open questions:

- What happens after inflation?
- What is the role of metric fluctuations?
- What about other potentials?
- What about RG corrections to the Kinetic term?
- Where has the spinodal instability gone?

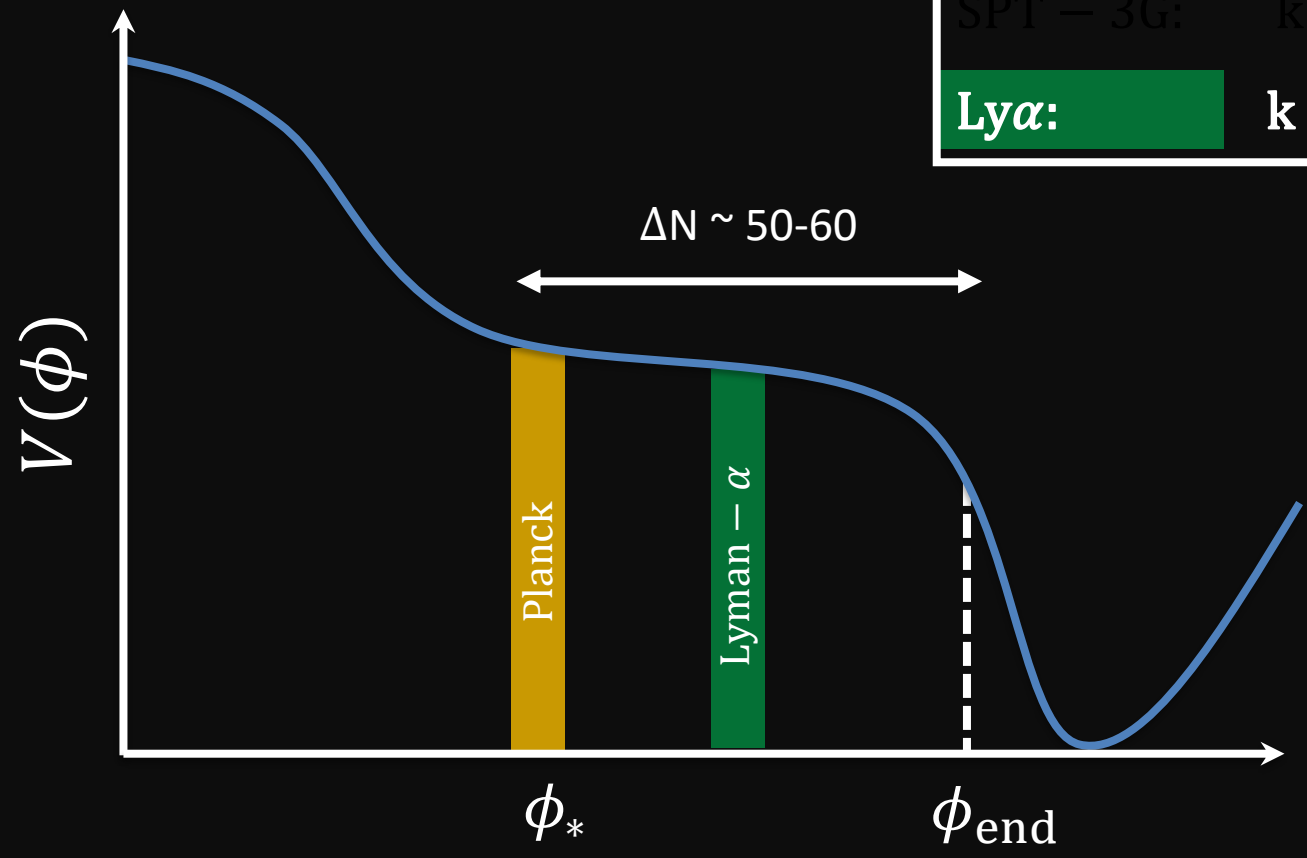
Future Directions:

- Application to oscillating scalars and oscillon formation
- Application to quintessence scenarios
- UV-IR interactions: CTP formalism and stochastic inflation

谢谢！

Constraints Across Scales

Planck:	$k \sim 0.05 \text{ Mpc}^{-1}$
ACT:	$k \lesssim 0.6 \text{ Mpc}^{-1}$
SPT – 3G:	$k \lesssim 0.7 \text{ Mpc}^{-1}$
Lyα:	$k \sim 0.2 - 2.5 \text{ Mpc}^{-1}$



Constraints Across Scales

