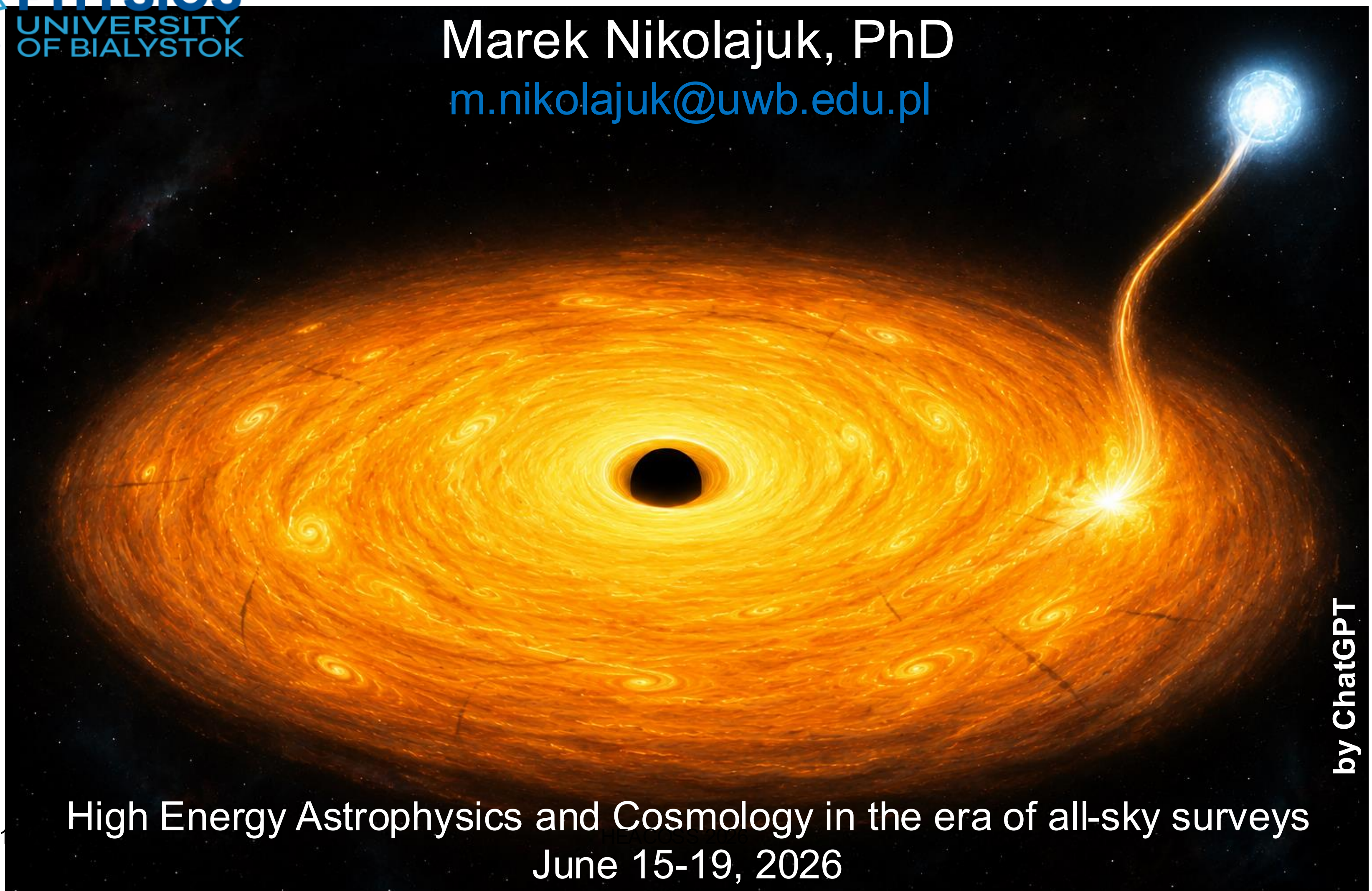
Quantum Droplets in Space:

Electromagnetic- and Gravitational-Wave Signatures of WD-BH Tidal Stripping

FACULTY OF PHYSICS
UNIVERSITY OF BIALYSTOK

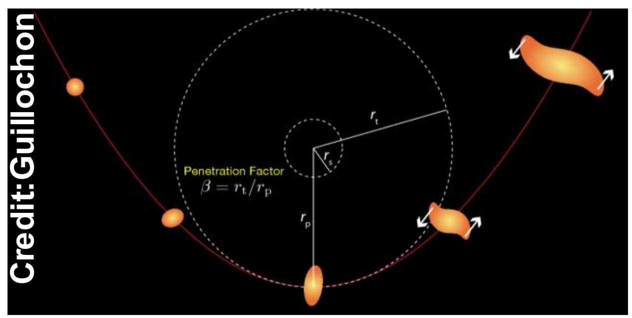
Marek Nikolajuk, PhD
m.nikolajuk@uwb.edu.pl

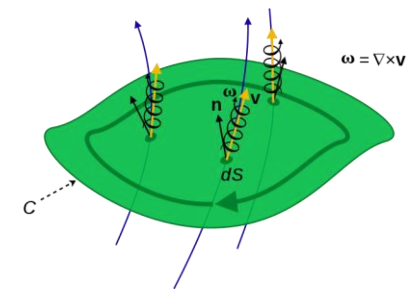


High Energy Astrophysics and Cosmology in the era of all-sky surveys
June 15-19, 2026

Talk Outline

- Intro (White Dwarfs & Tidal Disruption Events)
- Motivation
- Our method (assumptions & input parameters)
- Quantum vortices in an accretion disk inflow

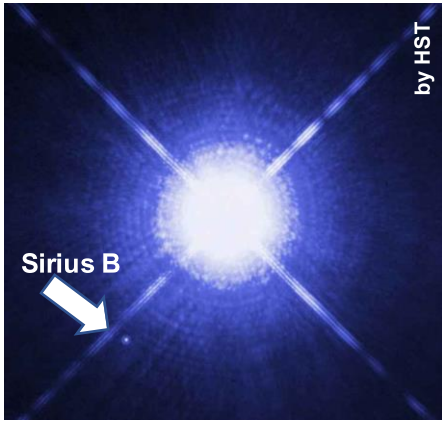




In collaboration with:
i) dr hab. Tomasz Karpiuk & prof. Mirosław Brewczyk, University of Bialystok
ii) prof. Mariusz Gajda, Institute of Physics, Polish Academy of Sciences
iii) dr Lorenzo Ducci, University of Tübingen, Germany

White Dwarf

In the interior of the WD, the charged e.g. ^{4}He nuclei are in the state of Bose-Einstein condensate, whereas the relativistic degenerate electrons form a neutralizing liquid (e.g. Gabadadze & Rosen 2008, Mosquera et al. 2010).



Reason:

- The oldest observed white dwarfs should have temperatures of the order of $10^{5\text{-}6}$ K (e.g. $\boldsymbol{T_{WD}^{core}} \approx 2.3 \times 10^{6}$ K, Das & Mukhopadhyay 2020; $T_{WD}^{surface} < 3500$ K, companion to PSRJ2222-0137, Kaplan et al. 2014)

- The critical temperature for the BE condensation for ^{4}He component becomes a few 10^{6} K (for 10^{7} g/cm^{3}) making an assumption that most of bosonic component in old & cold WDs is condensed very plausible $T_{WD} \leq T_{crit}$);

- The electron gas can be considered as an ideal Fermi gas ($T_{Fermi} \approx 10^{9} - 10^{11}$ K for 10^{4}-10^{7} g/cm^{3} (Kittel's book) => $T_{WD} < T_{Fermi}$)

TDE of WD by BH

Up to now we know ~140 TDEs (Langis+ 2026, Goldtooth+2023, Gezari 2021)
(see e.g. Open TDE Catalogue, GW-TDE Catalogue)

Only a dozen candidates of WD-BH TDEs have been reported.

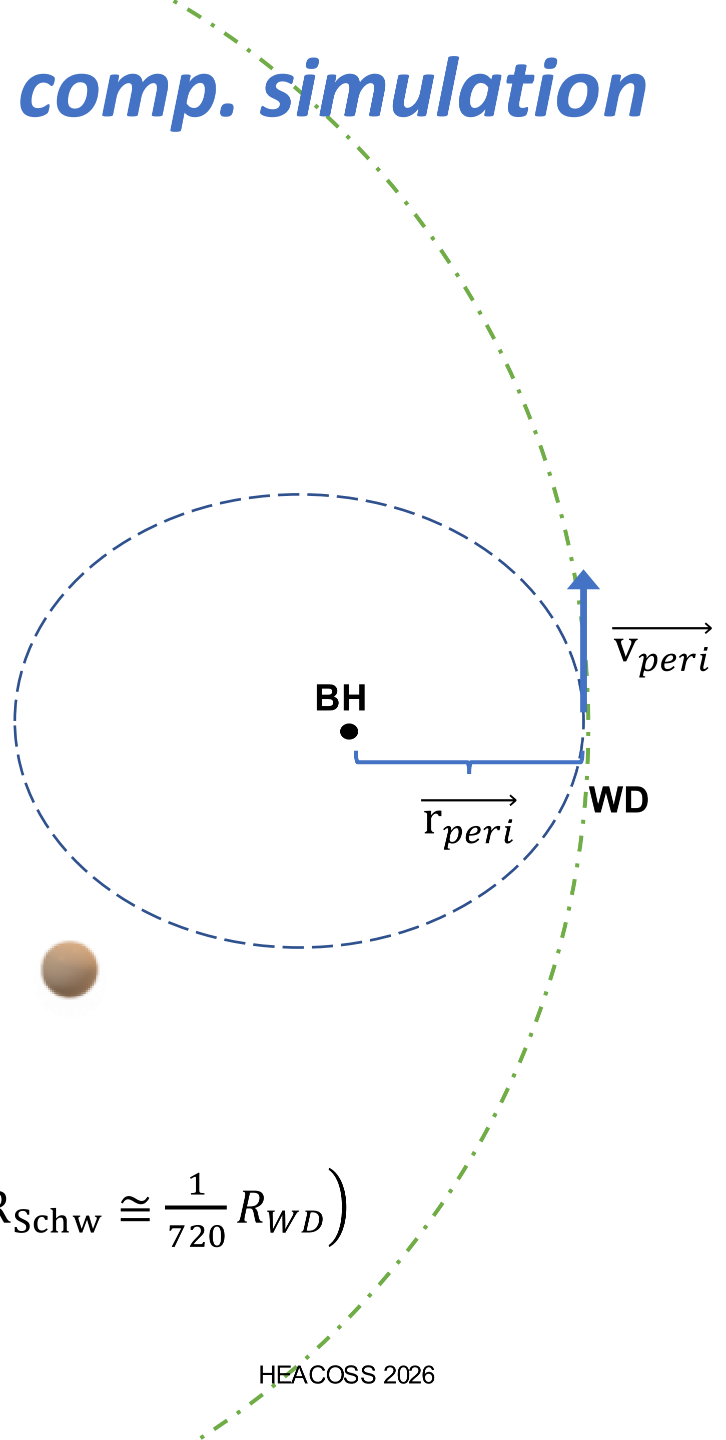
	Name	Outburst	WD+BH	Reference
Fast X-ray transient	source in a Globular Cluster of NGC 5128	• rise time: 22-51s, • 4 flares, steady state ~ 200s, • decay ~1 hour	WD + $80M_{\odot}$	Irwin et al. (2016)
Fast X-ray transient	src in a GC of NGC 4636	• 1 flare, rise time: 22s, • decay ~25 min	WD + $800M_{\odot}$	Irwin et al. (2016)
Fast X-ray transient	XRT 000519	50 – 100 sec	WD + $10^{4-5}M_{\odot}$	Jonker+ (2013)
Ultralong GRB	EP 250702a (GRB 250702B,D,E)	• 4 flares in 4 hours • rise time ~ 35-100s • tot. decay ~ 10 day • He line, no CNO lines	**He-WD** $0.17-0.6M_{\odot}$ + $< 7.5 \times 10^4 M_{\odot}$	Levan+ 2025, Li+ 2026
Ultralong GRB	Swift J1644+57 (GRB 110328A)	• rise time ~ 100s • a few flares in ~ 12 day	**He-WD** $0.2M_{\odot}$ + $10^{4-5}M_{\odot}$	Krolik & Piran (2011)
UltraCompact XRB, QPE	XMMU J1229+0753 – src in a GB RZ 2109 of NGC 4472	• 3 flares (E<1keV) in 4.3 d, rise time < 1 h, • periodicity (of flares 0.5-2keV) ~ 73.4 ks	WD 0,2 – $0.3M_{\odot}$ + BH 10 – 15 $M_{\odot}$ + star 0.1 – $1M_{\odot}$?	Clausen & Eracleous (2011), Tiengo+ 2022, Dage+ 2024
Ca-rich transients	SN 2016hnk	11 – 12 d (but longer for SN1a), $L_{peak} < 40\times\downarrow$ than typical SN1a ⇒ **shell detonation during TDE**	0.85 $M_{\odot}$ + $10^{3-5}M_{\odot}$	Sell+ (2018), Jacobson-Galán+ (2020)
Fast-blue opt.trans.	AT2018cow	• ~ hours, $L_{peak} > 5\times\uparrow$ than normal SN • **He line, no CNO lines**	**He-WD** $0.1-0.4M_{\odot}$ + $10^{3-5}M_{\odot}$	Kuin+ (2019)

Reason

Let’s focus on the old and ‘cold’ white dwarf.

Let's see how the phenomenon of accretion of white dwarf matter to a black hole looks like if we take into account **quantum interactions**.

3D system in comp. simulation

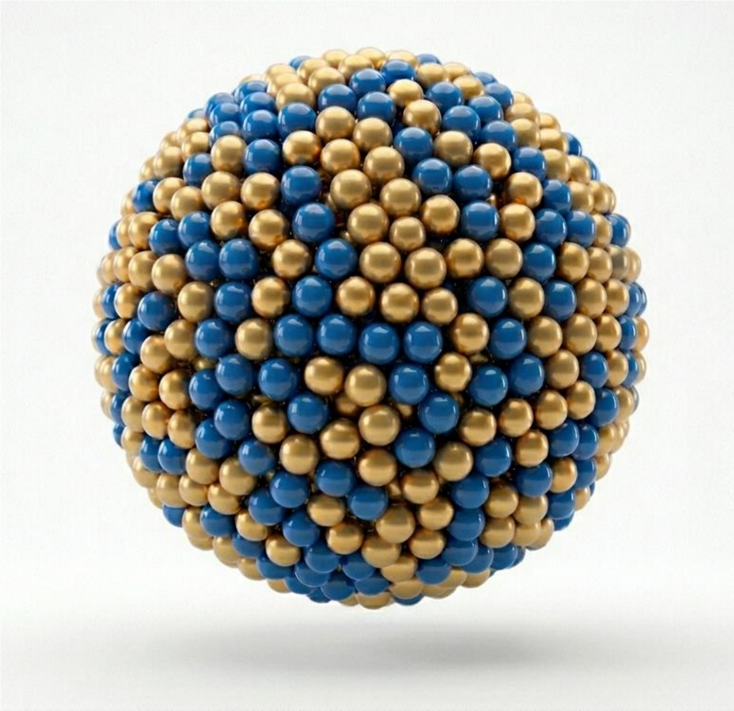


- $\boldsymbol{M_{\mathrm{WD}}(\mathrm{He}) = 0.4\ M_\odot}$
- $\boldsymbol{R_{WD}(He)} = 0.0155 R_\odot$
- $\boldsymbol{M_{\mathrm{BH}} \cong 5-6\ M_\odot}\left(i.e.\, \mathrm{R_{Schw}} \cong \frac{1}{720} R_{WD}\right)$
- $\boldsymbol{r_{peri} \cong 4\ R_{WD}}$

3D system in simulation: #1 White dwarf

Bose-Fermi droplet (i.e. Bose-Einstein condensate + fermions)

- In our case the droplet consists of 1460 bosonic (^{133}Cs) + 100 fermionic (^{6}Li) atoms (=> $R_{droplet} \approx 1\ \mu m$)



- **Note:** The droplet **does not have** self-gravity! Self-gravity is mimicked by van der Waals interactions ($F_{Waals} \propto \frac{1}{r^6}$).

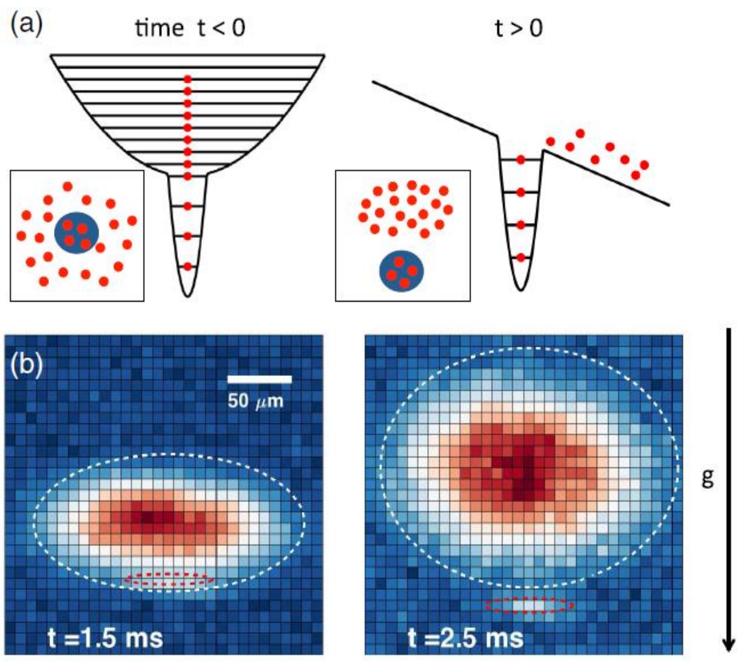
WD is only in the gravitational field of a black hole (BH).

Ceasium-Lithium mixture in a lab

PRL **119,** 233401 (2017) PHYSICAL REVIEW LETTERS week ending 8 DECEMBER 2017

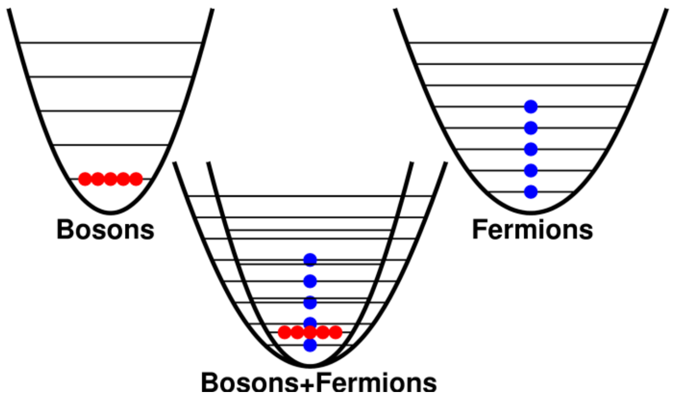
Observation of a Degenerate Fermi Gas Trapped by a Bose-Einstein Condensate

B. J. DeSalvo, Krutik Patel, Jacob Johansen, and Cheng Chin



3D system in simulation: #1 White dwarf

- We consider a Bose-Fermi mixture initially at zero temperature ($T_{ini} = 0$ K);
- **Fermions** are **spin polarized** (i.e. spin ↑), and **only 1** fermion in a quantum state;



- Particles are slow and in the first approximation we assume the simplest delta interactions (**S-wave scattering**).

Note: Bosons & fermions attract and **form a stable droplet** (attractive interaction) like in a experim. laboratory.

3D system in simulation: #2 Black hole

Black Hole:

- We assume a non-rotating black hole.
- Paczyński & Wiita pseudo-Newtonian potential.

$$V_{PN} = -\frac{GM_{BH}}{r - R_{Schw}}$$

where $R_{Schw} = \frac{2\,GM_{BH}}{c^2}$

Simul.: #3 Quantum hydrodynamic equations

$$\frac{\partial n_F(\mathbf{r},t)}{\partial t} = -\nabla \cdot (n_F \mathbf{v}_F)$$

density of fermionic fluid

velocity fields for fermions

$$m_F \frac{\partial \mathbf{v}_F(\mathbf{r},t)}{\partial t} = -\nabla \left(\frac{\partial E_F}{\partial n_F} + \frac{m_F}{2} \mathbf{v}_F^2 + \frac{\delta E_{BF}}{\delta n_F} \right)$$

Local kinetic energy of a Fermi gas

Macroscopic flow

Boson-fermion interaction energy

$$\frac{\partial n_B(\mathbf{r},t)}{\partial t} = -\nabla \cdot (n_B \mathbf{v}_B)$$

$$m_B \frac{\partial \mathbf{v}_B(\mathbf{r},t)}{\partial t} = -\nabla \left(\frac{\delta E_B}{\delta n_B} + \frac{m_B}{2} \mathbf{v}_B^2 + V_q + \frac{\delta E_{BF}}{\delta n_B} \right)$$

Simul.: #3 Quantum hydrodynamic equations

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velocity fields for bosons

Boson-boson interaction energy

$$\frac{\partial n_B(\mathbf{r},t)}{\partial t} = -\nabla \cdot (n_B \mathbf{v}_B)$$

Bosonic quantum pressure

$$m_B \frac{\partial \mathbf{v}_B(\mathbf{r},t)}{\partial t} = -\nabla \left(\frac{\delta E_B}{\delta n_B} + \frac{m_B}{2} \mathbf{v}_B^2 + V_q + \frac{\delta E_{BF}}{\delta n_B} \right)$$

Quantum hydr. equations can be turned into Schrödinger-like equations using *inverse Madelung transformation*.

$$ih\frac{\partial\psi_B}{\partial t} = H_B^{eff}\psi_B$$

$$ih\frac{\partial\psi_F}{\partial t} = H_F^{eff}\psi_F$$

\+ Paczyński-Wiita potential

$$ih\frac{\partial\psi_B}{\partial t} = \left(H_B^{eff} + V_{PN}\, m_B\right)\psi_B$$

$$ih\frac{\partial\psi_F}{\partial t} = \left(H_F^{eff} + V_{PN}\, m_F\right)\psi_F$$

where the effective nonlinear single-particle Hamiltonians:

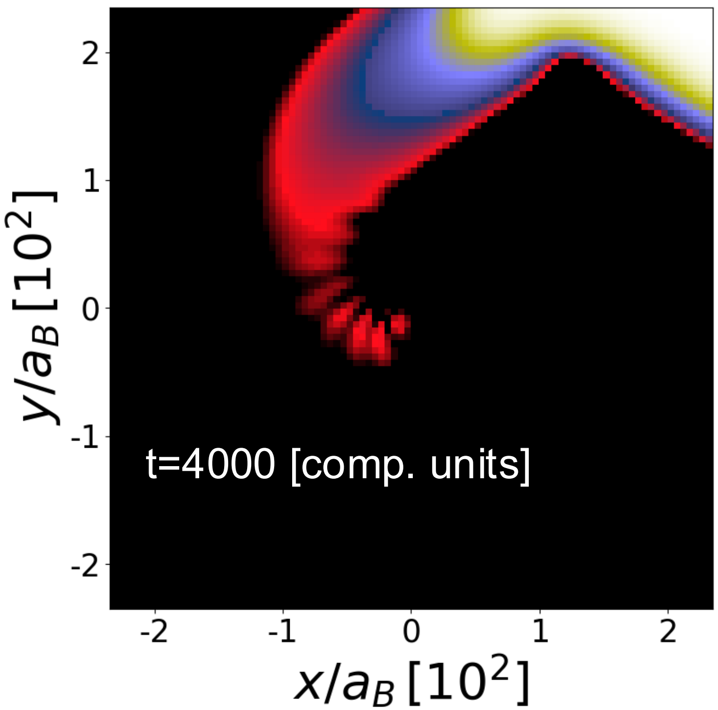
$$H_B^{eff} = -\frac{\hbar^2}{2m_B}\nabla^2 + g_B|\psi_B|^2 + \frac{5}{2}C_{LHY}\,|\psi_B|^3 + g_{BF}|\psi_F|^2 + C_{BF}|\psi_B|^{8/3}\,\mathcal{A}(w,\gamma) + C_{BF}\,|\psi_B|^2\,|\psi_F|^2\frac{\partial\mathcal{A}}{\partial\gamma}\frac{\partial\gamma}{\partial n_B}$$

$$H_F^{eff} = -\frac{\hbar^2}{2m_F}\nabla^2 + \xi'\frac{\hbar^2}{2m_F}\frac{\nabla^2|\psi_F|}{|\psi_F|} + \frac{5}{3}\kappa_k|\psi_F|^{\frac{4}{3}} + g_{BF}|\psi_B|^2 + \frac{4}{3}C_{BF}|\psi_B|^2|\psi_F|^{2/3}\,\mathcal{A}(w,\gamma) + C_{BF}\,|\psi_B|^2\,|\psi_F|^{8/3}\frac{\partial\mathcal{A}}{\partial\gamma}\frac{\partial\gamma}{\partial n_F}$$

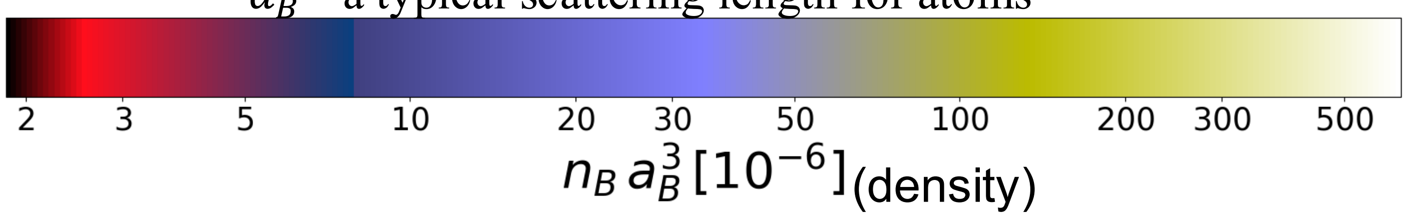
for more details see:
Debraj Rakshit et al.
New J. of Phys. 21
(2019) 073027

‘New’ phenomena in the accretion disk

Outcome: **Fragmentation** of the Bose-Fermi accreted matter due to nonlinear quantum effects.

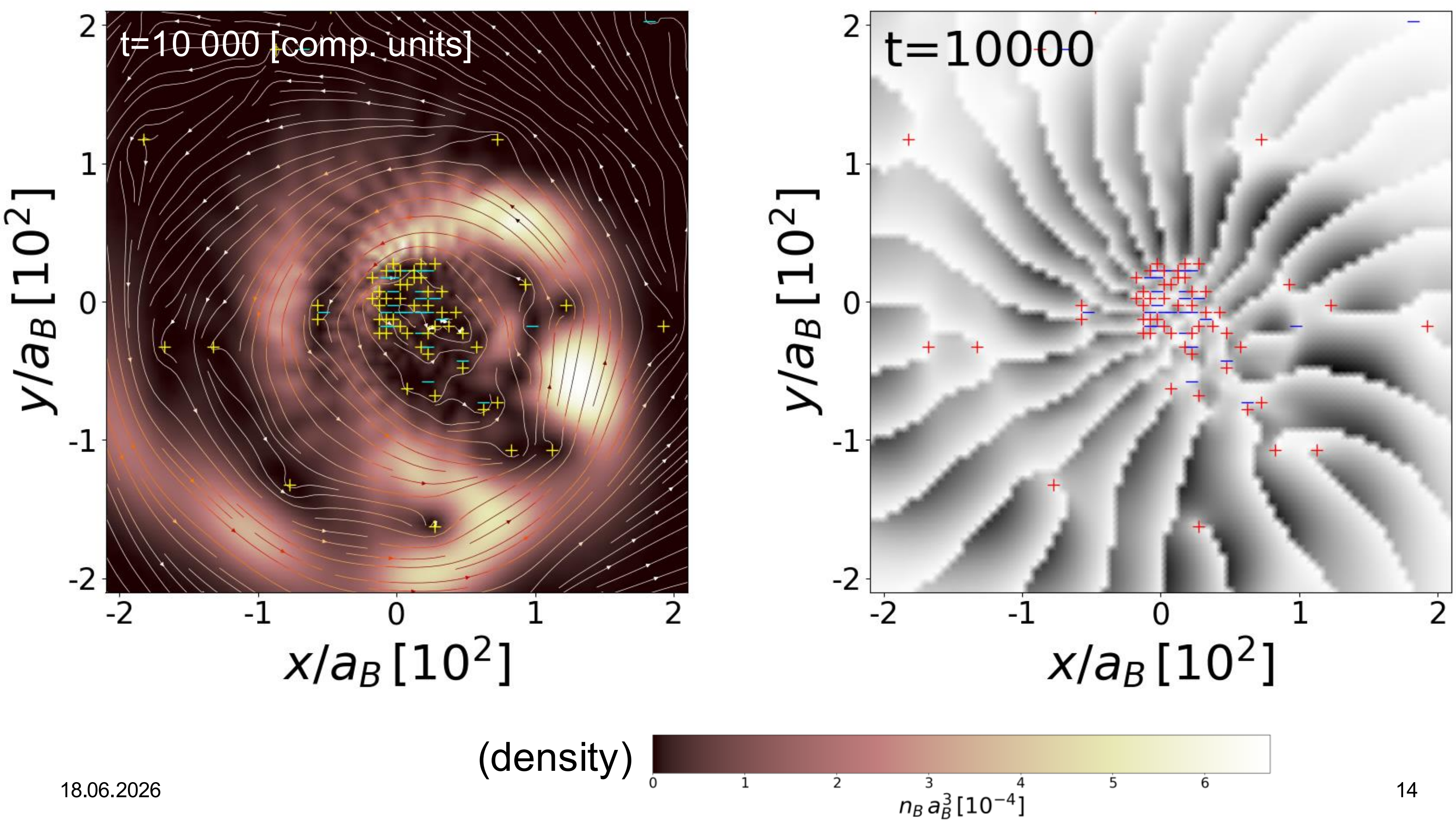


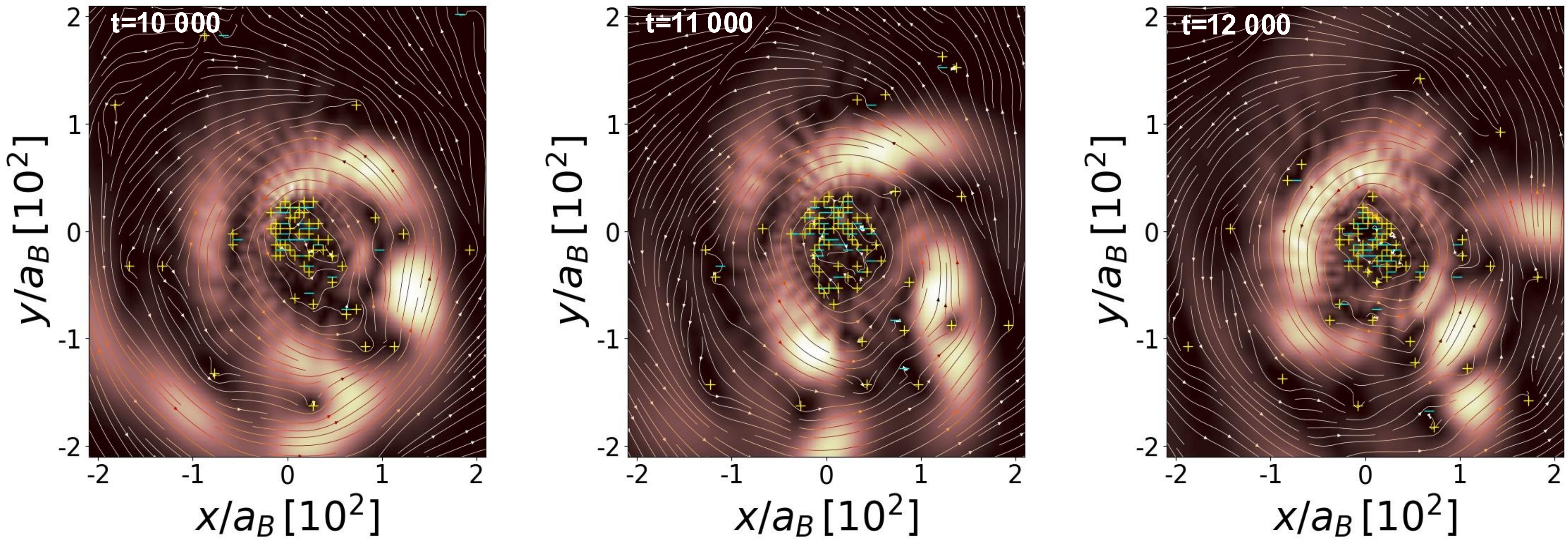
a_B - a typical scattering length for atoms



‘New’ phenomena in the accretion disk

Outcome: **Quantum vortices** are formed in the accreted matter.





- Vortices rotating in the same direction, repel each other, break up into those with less charge (n =1) and tend to congregate close to the BH.
- The vortices exist also in low density regions.
- The **central region** of the vortex is filled with **non-boson matter or is empty**.

The vortex diameter is about a few % of the droplet diameter (up to 15% in the high density region).

Electromag. radiation of falling Bose-Fermi matter

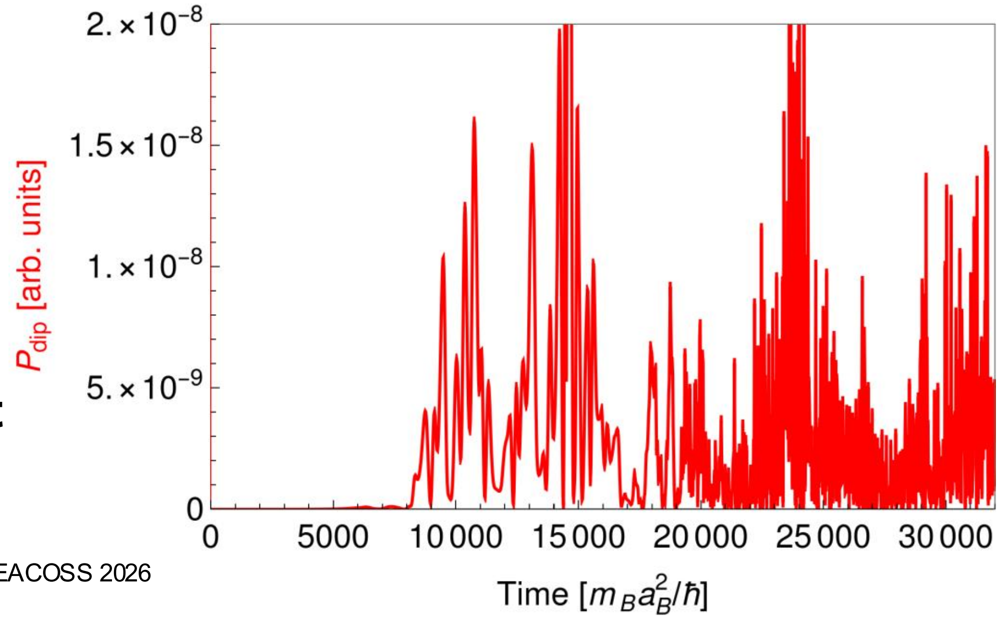
The power radiated by

- an electric dipole moment $\vec{\boldsymbol{p}}$ in the far-field approximation (Jackson 1999, Griffiths 1999):

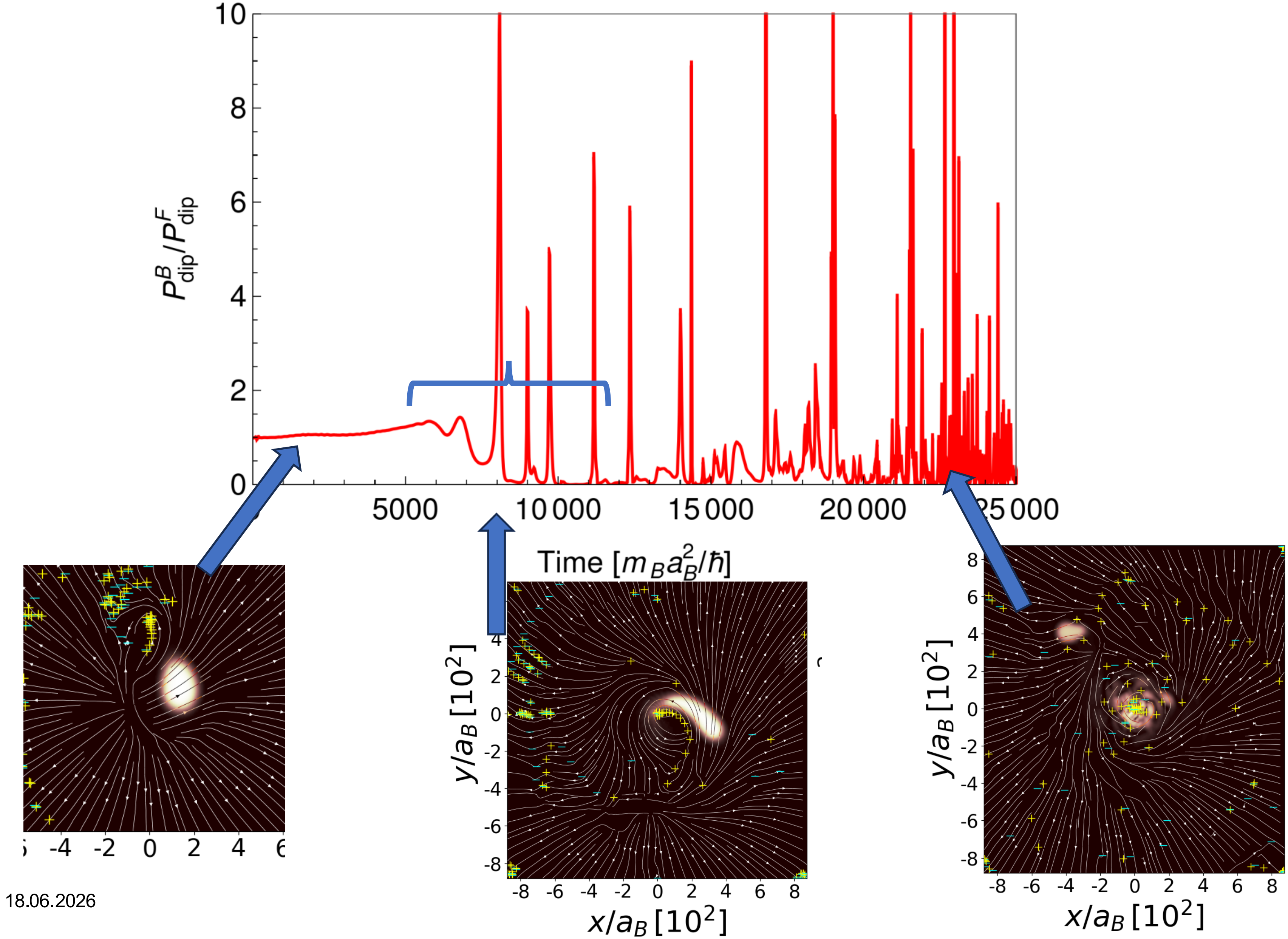
$$P_{dip} = \frac{2}{3c^2}\left|\ddot{\vec{\boldsymbol{p}}}\right| \qquad \text{where } \vec{\boldsymbol{p}} = \int \vec{r}\,\rho_{el}(\vec{r})\,d^3r$$

$\rho_{el}(\vec{r}) = q_B n_B(\vec{r}) - q_F n_F(\vec{r})$ - is the charge density; $n_B(\vec{r}), n_F(\vec{r})$ - number density for bosons and fermions;
q_B, q_F - electric charges.

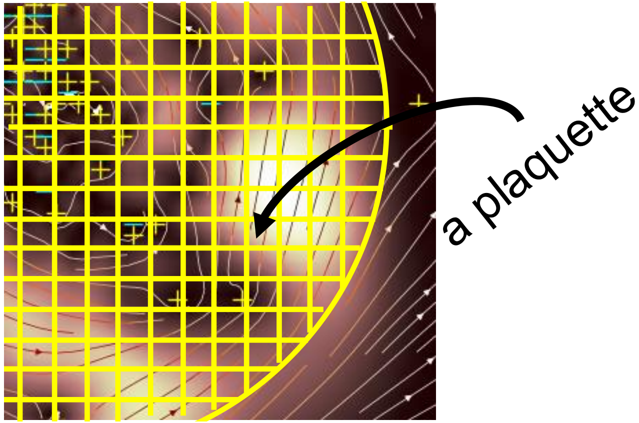
Case: Hyperbolic orbit

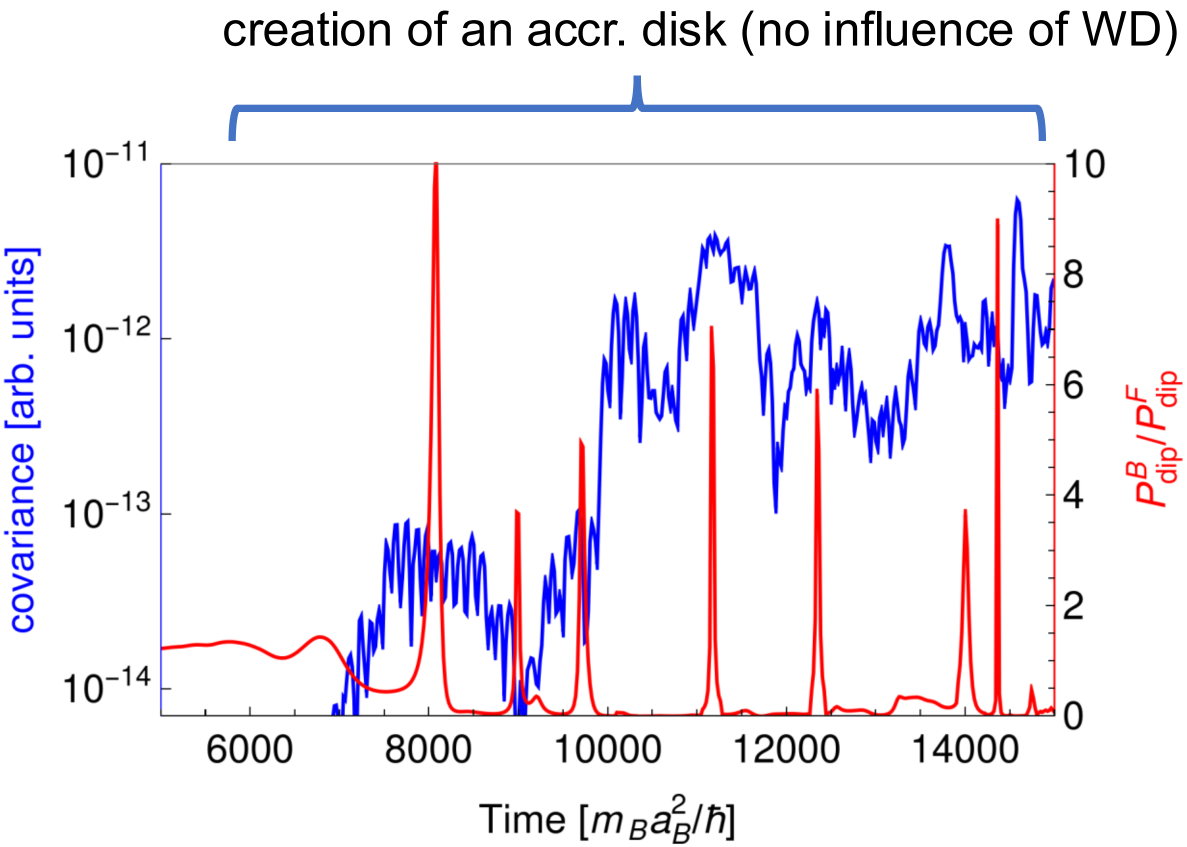


Evolution of $P_{dip}^{Bos} / P_{dip}^{Ferm}$ in time



$P_{dip}^{Bos}/P_{dip}^{Ferm}$ & a presence of vortices





$$\mathbf{cov(X, Y)} = \langle \mathbf{XY} \rangle - \langle \mathbf{X} \rangle \langle \mathbf{Y} \rangle$$

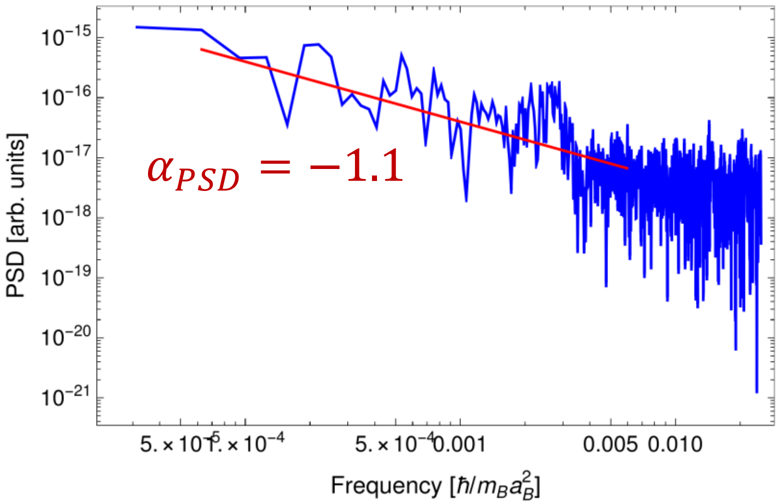
$X(t) = P_{dip}$ or P_{quad}
$Y(t) =$ there is (1) or no (0) vortex in the plaquette.

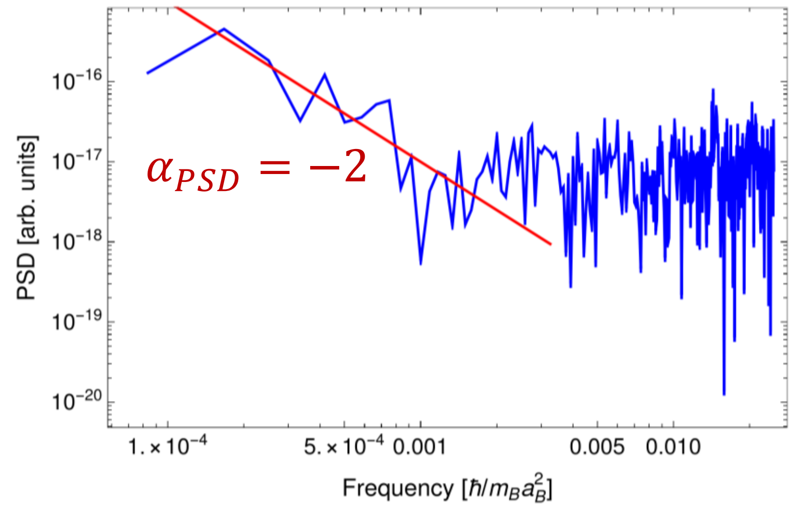
$$\langle A \rangle = \frac{1}{T}\int_0^T A(t)dt$$

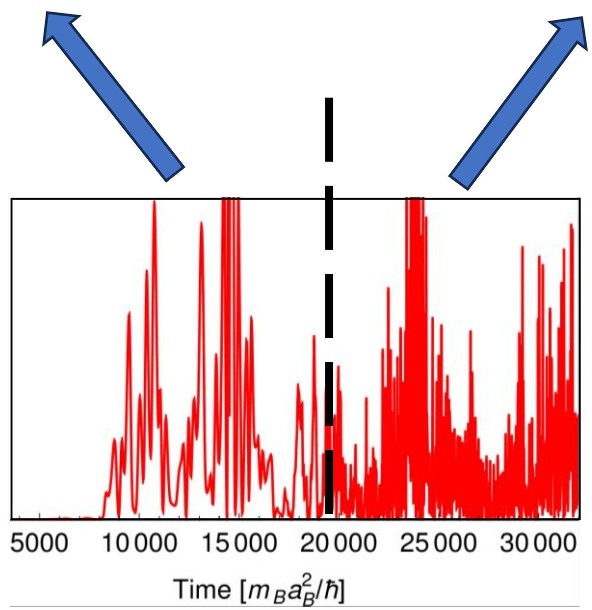
The red spikes clearly correlate with time intervals of high covariance.

Variability of the boson-fermion accretion disk

$$\text{PSD} = f^{\alpha_{PSD}}$$







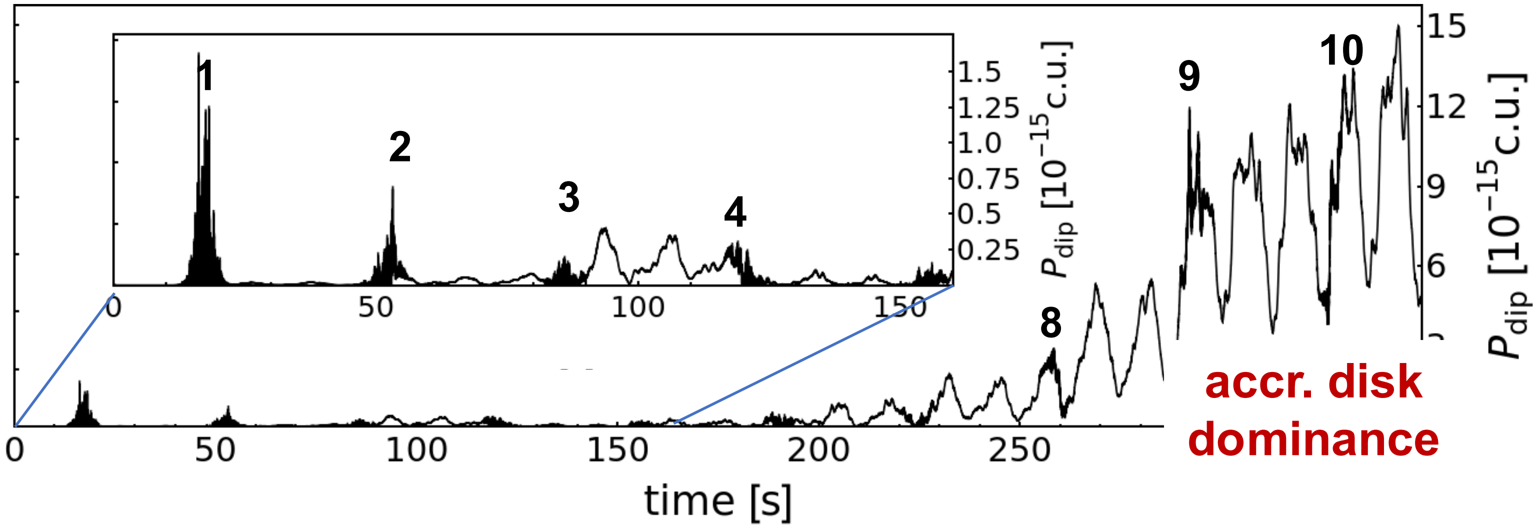
Electromag. radiation of falling Bose-Fermi matter

$\mathbf{5}M_{\odot}(BH) + \mathbf{0.4}M_{\odot}(WD) \quad \& \quad \beta = 0.73$

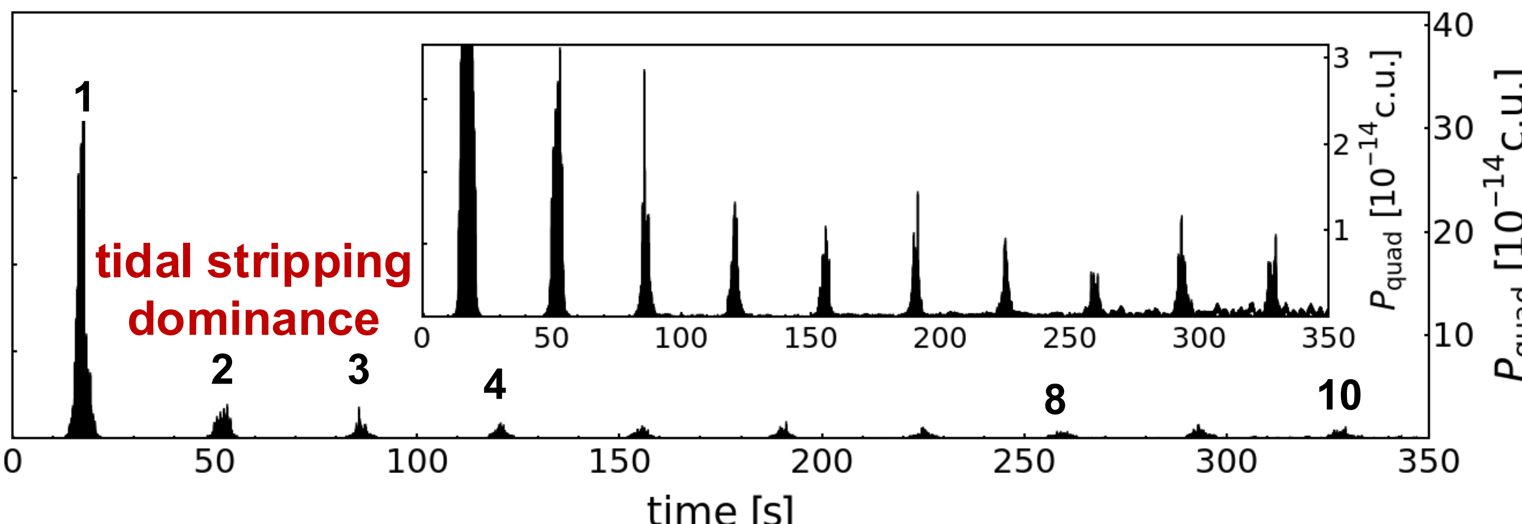
$(time[s] \cong 10^4\,(m_B a_B^2/\hbar) \cdot time[c.u.])$

Case: closed orbit 10 passages of WD through periastron with tidal stripping

P_{dipole}



$P_{quadrupole}$



Gravit. radiation of falling Bose-Fermi matter

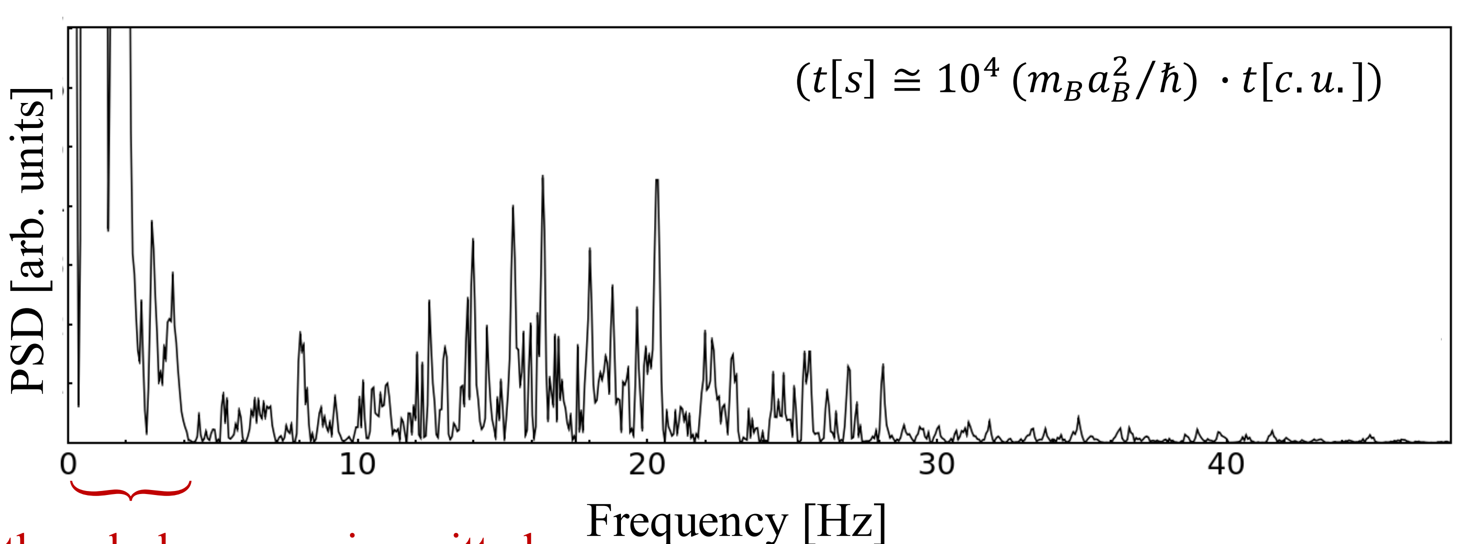
In the lowest order, the gravitational radiation depends on a third time derivative of a mass quadrupole moment $Q^m_{\alpha\beta}$, and is given by a famous Einstein's quadrupole formula:

$$P_{grav} = \frac{G}{45c^5}\sum_{\alpha\beta}\left(\dddot{Q}^m_{\alpha\beta}\right)^2$$

(Maggiore 2008)

where $Q^m_{\alpha\beta} = \int \left(3r_\alpha r_\beta - r^2\delta_{\alpha\beta}\right)\left(m_B n_B(\vec{r}) + m_F n_F(\vec{r})\right)d^3r$

m_B and m_F — masses of bosonic and fermionic component particles



More than 95 % of the whole energy is emitted within the range of frequencies < 4 Hz

Conclusion

• **quantised vortices** (→ a possible explanation for the flicker noise and hot spots on the accretion disk surface),

• a characteristic **flickering pattern changes over time** as the system transforms from its initial phase to maturity (the same like from observations of 18 TDEs, Chakraboty+ 2026)

• the accret. disk radiates mainly through the electric dipole moment,

• our simulations also show that the gravitational energy is emitted mainly at very low frequencies (< 4Hz).

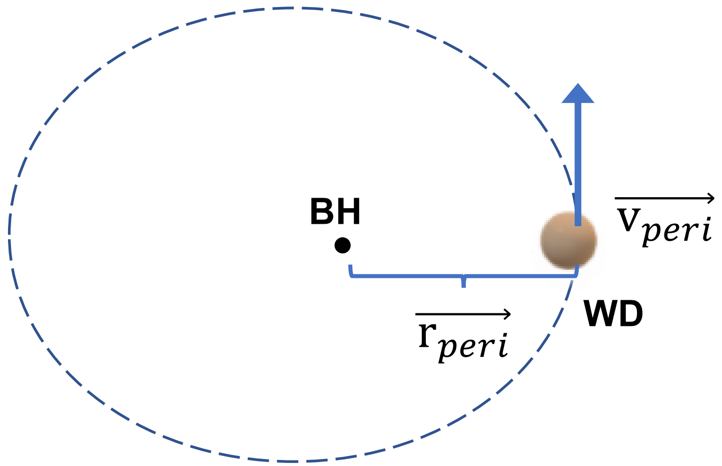
Extra materials

- Nikołajuk, Karpiuk & Brewczyk, arXiv:2606.12049 (2026)
  "Searching for cosmic vortices"
- Nikołajuk, Karpiuk, Ducci & Brewczyk, Astrophysical Journal, vol. 980, id. 256 (2025)
  "Studying radiation of a white dwarf star falling on a black hole"
- Karpiuk, Nikolajuk, Gajda & Brewczyk, *Scientific Reports*, vol. 11, id. 2286 (2021)
  "Modelling quantum aspects of disruption of a white dwarf star by a black hole"

3D system in simulation: #4 run

Input parameters:

- $M_{drplt} = 3.23 \times 10^{-22}$ kg
- $R_{drplt} \simeq 1\ \mu m$
- $\mathrm{r}_{periastron} \simeq 4\ R_{drlpt}$
- $\mathrm{r}_{apastron} \simeq 7\ R_{drlpt}$ or bigger
- $(GM_{BH})_{num} \ \simeq -1$ [comp.units]



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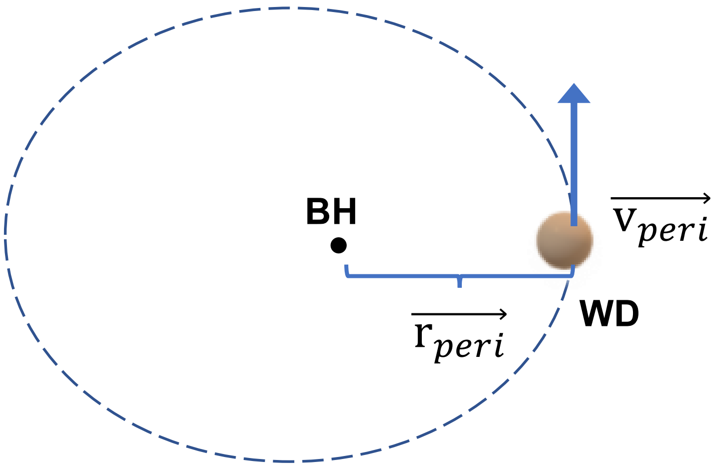
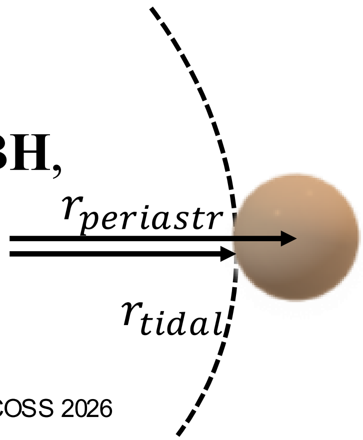


From calculations:

- $\boldsymbol{\beta} \cong \mathbf{1}$ (a periastron passage)
- $\boldsymbol{\beta} = \mathbf{0.73}$ (i.e. **stripping of WD by BH**, several dozen periastron passages)

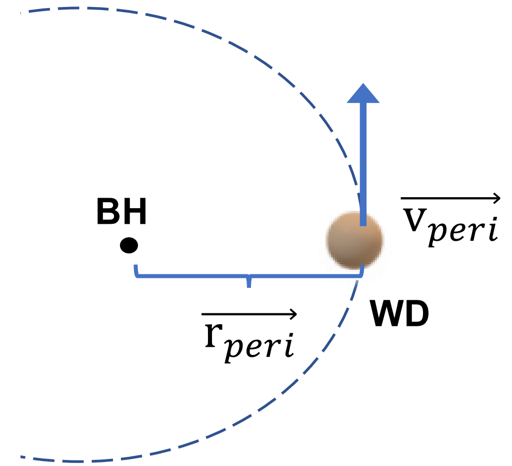
$$\beta \stackrel{\text{def}}{=} \frac{r_{tidal}}{r_{periastr}}$$



Scalling up

If we assume that droplet parameters correspond to a real He-WD parameters:

- $M_{drplt} = 3.23 \times 10^{-22}$ kg $\Rightarrow$ $\boldsymbol{M_{\mathbf{WD}}\,(\mathbf{He}) = 0.4\, M_{\odot}}$
- $R_{drplt} \simeq 1\,\mu m \quad \Rightarrow \qquad \boldsymbol{R_{WD}(He)} = 0.0155 R_{\odot}$
- $E_{drplt} \simeq -1\ [\hbar^2/(m_B a_B^2)] \Rightarrow \boldsymbol{E_{astro}^{WD(0.4M_{\odot})}} \cong -6 \times 10^{42}[\mathrm{J}]$



Then

- $(M_{BH})_{num} \simeq -1/G$ [comp.units] $\Rightarrow$ $\boldsymbol{M_{\mathbf{BH}} \cong 5 - 6\, M_{\odot}}$
  $\left(i.e.\, \mathrm{R_{Schw}} \cong \frac{1}{720} R_{WD}\right)$

Scaling up

$$\begin{cases} r_{astro}[\mathrm{m}] & = & \mathcal{A}\, r_{num}\, [\text{code unit}_1 \equiv a_B] \\ m_{astro}[\mathrm{kg}] & = & \mathcal{B}\, m_{num}\, [\mathrm{c.u.}_2 \equiv m_B] \\ t_{real}[\mathrm{s}] & = & \mathcal{C}\, t_{num}\, [\mathrm{c.u.}_3 \equiv m_B a_B^2/\hbar] \end{cases}$$

- $r_{astro} \equiv \mathrm{R_{WD}} = 0.0155\, \mathrm{R}_\odot$ & $r_{droplet=num} = 1\, \mu\mathrm{m} \Rightarrow$ $\mathcal{A} = 1.1 \times 10^{13}$
- $\mathrm{m_{astro}} \equiv \mathrm{M_{WD}} = 0.4\, \mathrm{M}_\odot$ & $m_{droplet=num} \equiv 3.23 \times 10^{-22}\, \mathrm{kg}$ $\Rightarrow$ $\mathcal{B} = 2.4 \times 10^{51}$

$$\mathcal{C} = ?$$

Scaling up

The unit of energy is given by $\frac{(unit\ of\ mass)(unit\ of\ length)^2}{(unit\ of\ time)^2}$.

We have to multiply E_{num} by $\mathcal{B}\mathcal{A}^2/\mathcal{C}^2$ to get astronomical one:

$$E_{real}[J] = \frac{\mathcal{B}\mathcal{A}^2}{\mathcal{C}^2} E_{num}[\mathrm{c.u.} = \hbar^2/(m_B a_B^2)]$$

It means that:

$$E_{astro}^{WD}[J] = \frac{\mathcal{B}\mathcal{A}^2}{\mathcal{C}^2} E_{num}[\mathrm{c.u.}]$$

Based on data from the simulation & calculations:

- $E_{num} \cong -1\ [\hbar^2/(m_B a_B^2)]$
- $E_{astro}^{WD} \cong -6 \times 10^{42}[\mathrm{J}]$

$\mathcal{C} = 9500$

Scaling up

The duration of the stripping is roughly given by the dynamical timescale:

$$t_{dyn} \simeq \sqrt{\frac{R_{WD}^3}{GM_{WD}}}$$

From our simulations the duration of the stripping $\simeq 10^4$ [c.u.$_3$]

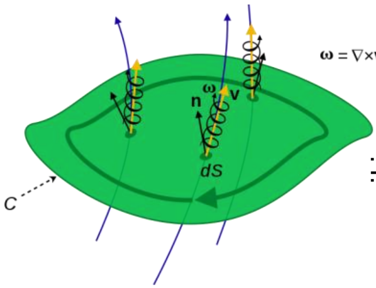
$$\sqrt{\frac{R_{WD}^3}{GM_{WD}}} = \mathcal{C}\ 10^4[\mathrm{c.\,u.}]$$

$\mathcal{C} \simeq\ 9300$

Circulation of the velocity field, $\vec{v}_B$, of particles orbiting the vortex is quantised and is equal to:

$$\Gamma = \oint_C \vec{v}_B \cdot \overrightarrow{dl} = \pm n 2\pi \frac{\hbar}{m_B}$$

The "phase charge" in the vortex $= \pm n\, 2\pi\,, \quad n = 1,2,3, \ldots$.



source: wiki

The **central region** of the vortex is filled with **non-boson matter or is empty**.

REPORT

Science, 292, 476 (2001)

Observation of Vortex Lattices in Bose-Einstein Condensates

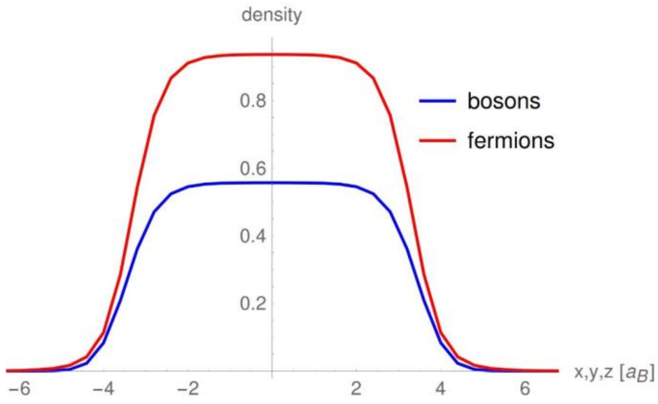
J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle

Abstract

Quantized vortices play a key role in superfluidity and superconductivity. We have observed the formation of highly ordered vortex lattices in a rotating Bose-condensed gas. These triangular lattices contained over 100 vortices with lifetimes of several seconds. Individual vortices persisted up to 40 seconds. The lattices could be generated over a wide range of rotation

3D system in simulation: #1 White dwarf

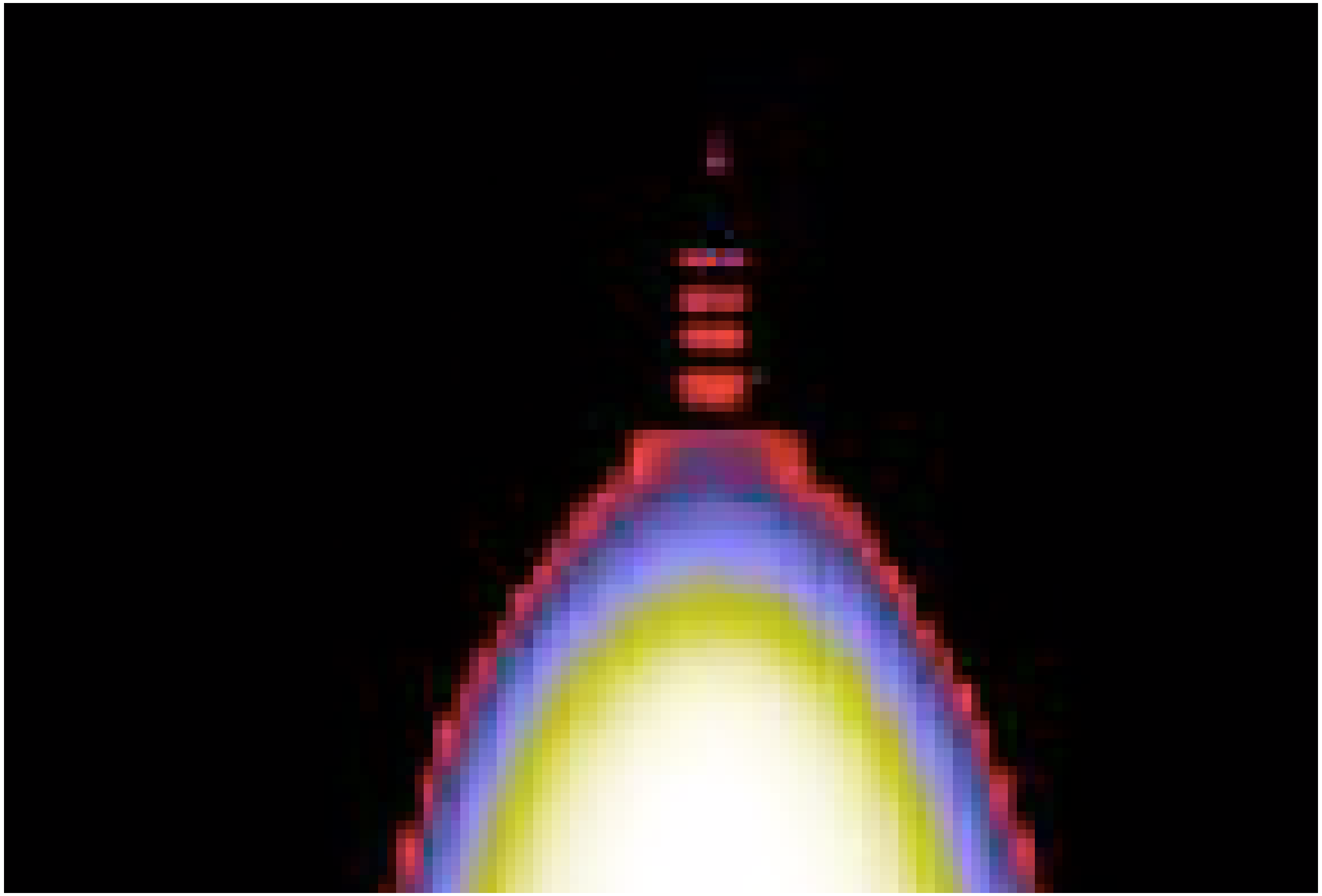
Remark:
We consider the $^{133}Cs + {}^{6}Li$ droplet, however, it can be **any** Bose-Fermi mixture.



Karpiuk et al. (2004),
Phys. Rev. Lett. 93, 100401

New phenomena in the accretion disk

This is called “train of solitons”



WD falls in a straight line into a BH

Fast, Ultraluminous X-ray bursts

LETTER

doi:10.1038/nature19822

Irwin et al. Nature **538**, 356 (2016)

Ultraluminous X-ray bursts in two ultracompact companions to nearby elliptical galaxies

Jimmy A. Irwin[1], W. Peter Maksym[2], Gregory R. Sivakoff[3], Aaron J. Romanowsky[4,5], Dacheng Lin[6], Tyler Speegle[1], Ian Prado[1], David Mildebrath[1], Jay Strader[7], Jifeng Liu[8,9] & Jon M. Miller[10]

Figure 1 | The environment of Centaurus A. … e a source of ultraluminous X-ray flares (indicated by the white circle) near the elliptical galaxy Centaurus A (NGC 5128; centre).

Fast, Ultraluminous X-ray bursts

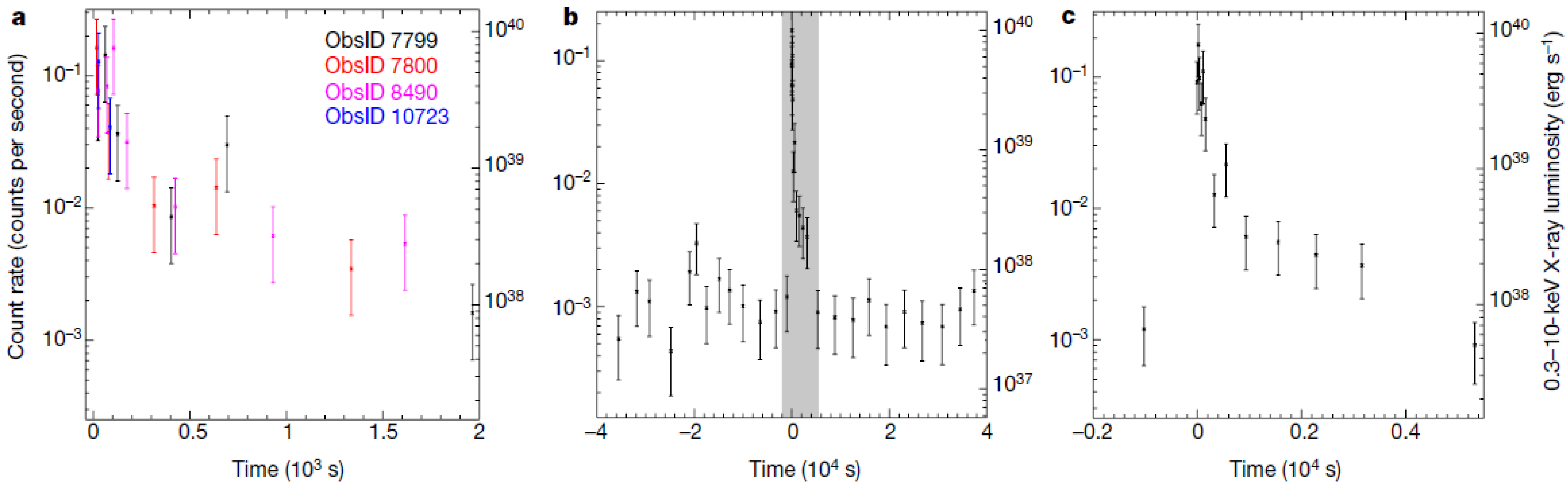


Figure 2 | Individual and combined background-subtracted X-ray light curves for Source 2 in the NGC 5128 globular cluster or ultracompact dwarf. a, The X-ray light curves for the four Chandra flares show similar behaviour. Each time bin contains five photons. **b,** The combined light curve of the four flares illustrates the fast rise and slow decay of the flares. Each time bin contains ten photons. The time is given relative to the beginning of the flare. **c,** Zooming in on the grey shaded region in **b** reveals that the luminosity during the flare rose quickly and remained steady in an ultraluminous state for approximately 200 s before decaying back to its persistent level after about 1 h. All error bars represent 1σ uncertainties.

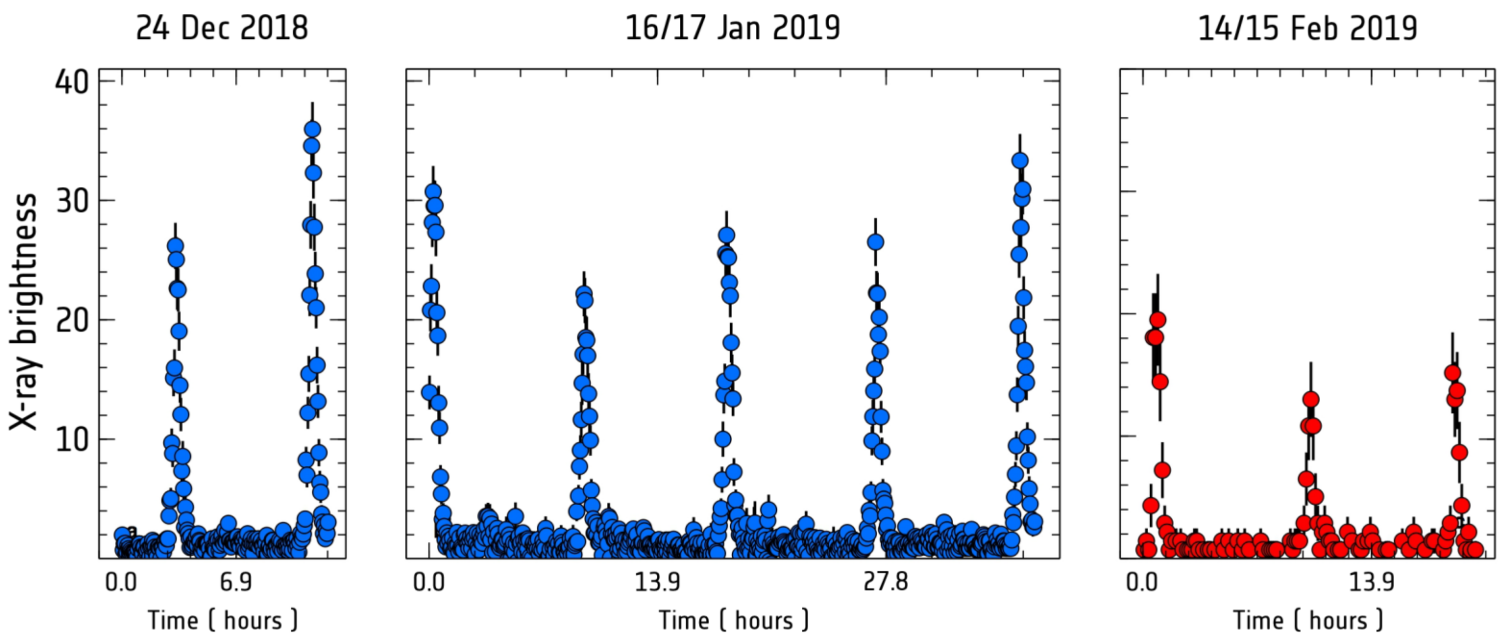
- The rise time < 1 min (30.03.2007),
- 4 flares, time(ave. steady state) ~ 200s, $L_{peak}(0.3 - 10\ \mathrm{keV}) \sim 9 \times 10^{39}$ erg/s
- Decay ~1 hour.

Main peak of the flares varied in duration: ~ 50-700 sec.

X-ray Quasi-Periodic Eruptions

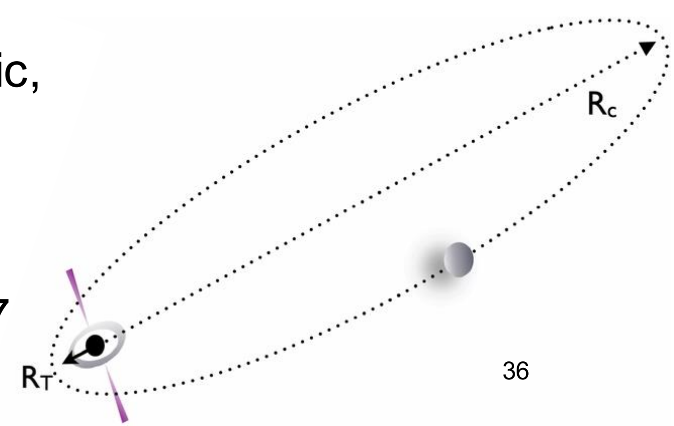
Miniutti et al. Nature **573**, 381 (2019)

GSN 069 (2MASX J01190896-3411305) – Seyfert 2



- Separation between bursts ~ 8h17min,
- rise & decay time ~1.8 ks, profile is close to symmetric, $L_{peak}(0.4 - 2\ \mathrm{keV}) \sim 5 \times 10^{42}$ erg/s

Swift J1644+57
Krolik & Piran (2011)



X-ray Quasi-Periodic Eruptions

Bykov, Gilfanov, Sunyaev, Medvedev, MNRAS, **540**,30 (2025)

TDE AT2019vcb (aka Tormund)

" The XMM–Newton and eROSITA flares provide strong evidence that the TDE AT2019vcb is a bona fide quasi-periodic eruption (QPE) source.
Our work further strengthens **the direct connection between TDEs and QPE**…"

Hernández-García et al.,Nature Astronomy, (2025)

The galaxy SDSS1335+0728 was stable for two decades. It exhibited an increase in optical brightness in December 2019. Since February 2024, X-ray emission has been detected, revealing extreme ~4.5-d QPEs with high fluxes and amplitudes, and a ~25-d superperiod.

This discovery suggests that QPEs **are linked not solely to tidal disruption events but more generally to newly formed accretion flows**, which we are witnessing in real time in a turn-on AGN candidate.

Electromagn. radiation of falling Bose-Fermi mater

The power radiated by

- an electric dipole moment $\vec{p}$ in the far-field approximation (Jackson 1999, Griffiths 1999):

$$P_{dip} = \frac{2}{3c^2}\left|\ddot{\vec{p}}\right| \qquad \text{where } \vec{p} = \int \vec{r}\,\rho_{el}(\vec{r})d^3r$$

$\rho_{el}(\vec{r}) = q_B n_B(\vec{r}) - q_F n_F(\vec{r})$ — is the charge density,

$n_B(\vec{r})$ and $n_F(\vec{r})$ — particle number density for bosons and fermions

q_B and q_F — effective electric charges of bosonic and fermionic components

Electromagn. radiation of falling Bose-Fermi mater

The next order contribution to electromagnetic radiation originates from a change of the electric quadrupole moment $Q_{\alpha\beta}$ (contribution is coming from a time variation of the magnetic dipole moment remains of the same order).

The power radiated by

- an electric quadrupole moment $Q_{\alpha\beta}$ in the far-field approximation:

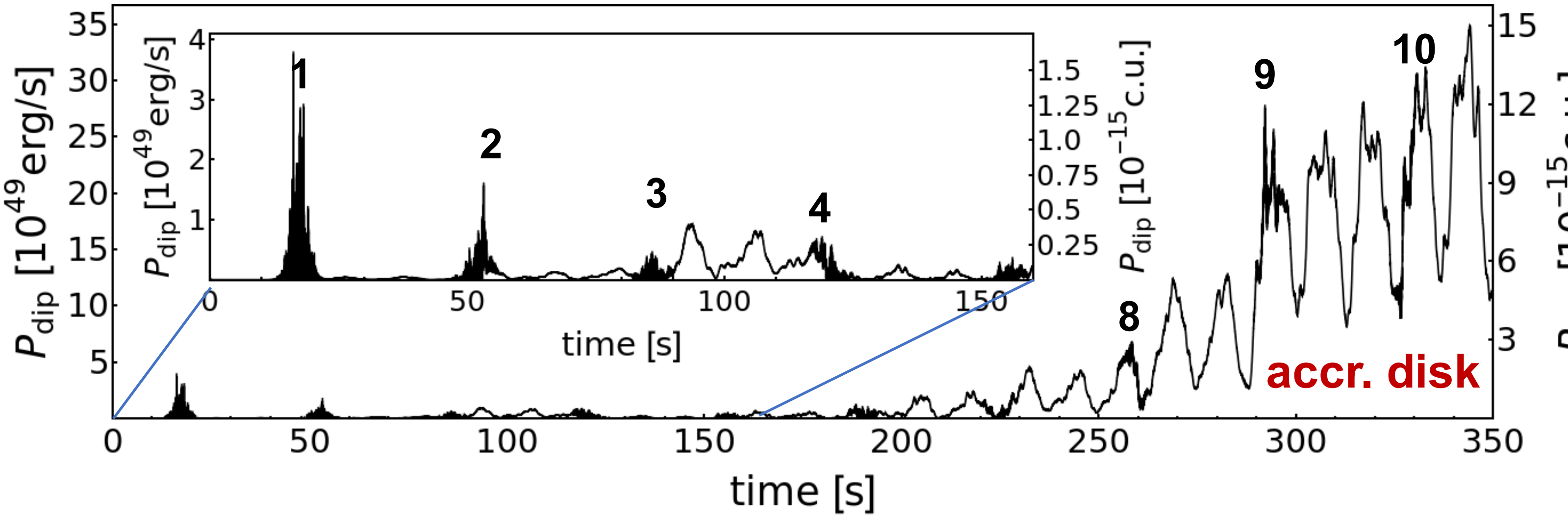
$$P_{quad} = \frac{1}{180c^5} \sum_{\alpha\beta} \left(\dddot{Q}_{\alpha\beta}\right)^2 \quad \text{where } Q_{\alpha\beta} = \int (3r_\alpha r_\beta - r^2 \delta_{\alpha\beta})\, \rho_{el}(\vec{r}) d^3 r$$

Indices α and β refer to Cartesian components

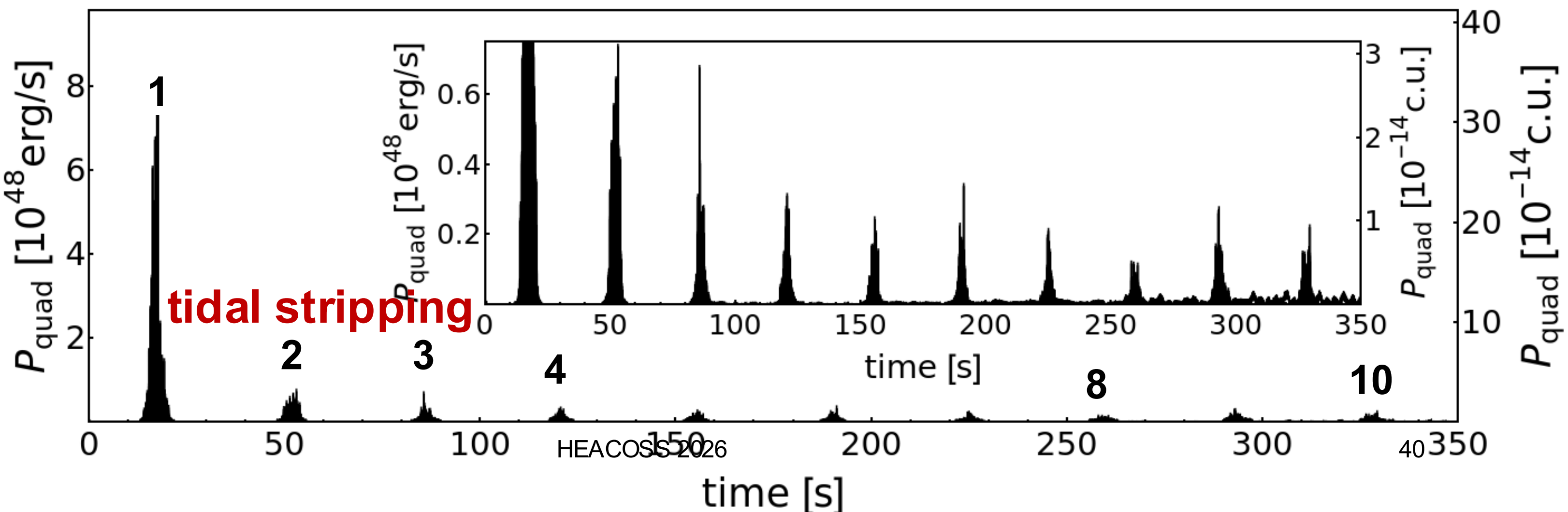
Electromagn. radiation of falling Bose-Fermi mater

$\mathbf{5}M_{\odot}(BH) + \mathbf{0.4}M_{\odot}(WD) \quad \& \quad \beta = 0.73$

P_{dipole}



$P_{quadrupole}$

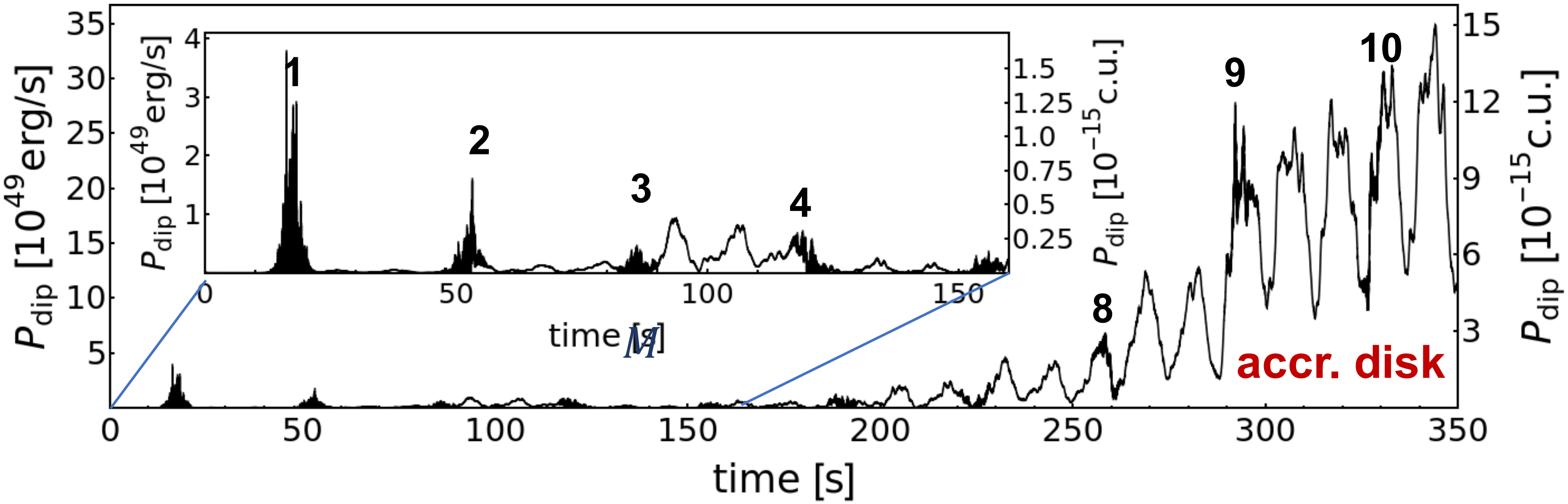


Electromagn. radiation of falling Bose-Fermi mater

$\mathbf{5}M_{\odot}(BH) + \mathbf{0.4}M_{\odot}(WD) \quad \& \quad \beta = 0.73$

10 passages of WD through periastron with tidal stripping

P_{dipole}

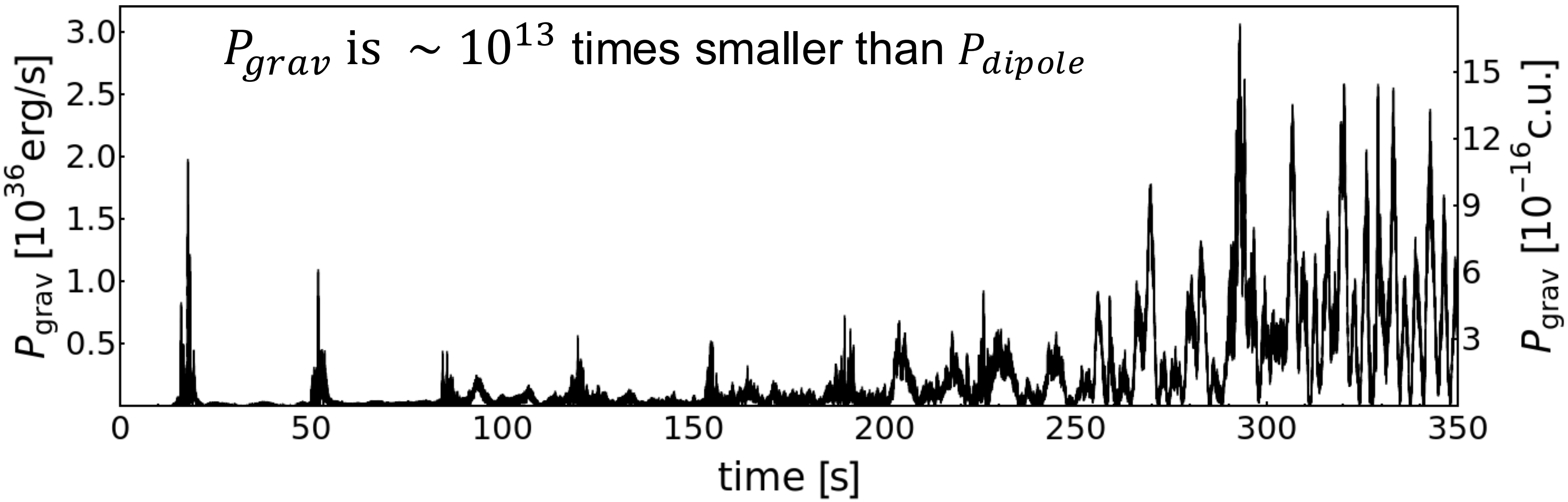


Accreted mass in each periastron passage:

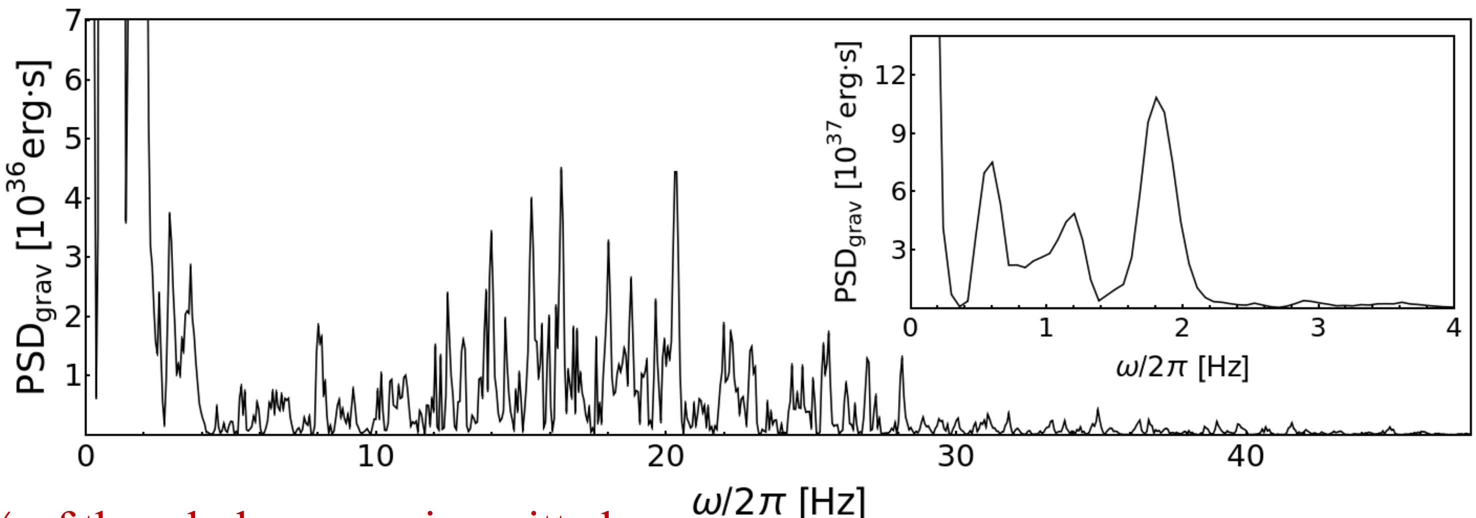
$\dot{m} = 5 \times 10^{-4} M_{WD}$ => L=mc^2 = ??? erg/s

Max. power emitted in el-mag.:: L=10% x mc^2 = ??? erg/s

Gravitational radiation of falling Bose-Fermi mater



Power Density Spectrum



More than 95 % of the whole energy is emitted within the range of frequencies $\omega/2\pi < 4$ Hz

Gravitational radiation of falling Bose-Fermi mater

- The operating terrestrial laser interferometer detectors (like **LIGO, Virgo and KAGRA**), are sensitive to GWs at frequencies around 10^2 Hz,
- the Laser Interferometer Space Antenna (**LISA**) will be most sensitive to GWs with frequencies around 10^{-2} Hz,

- The **AION** project has built a prototype of **an atom interferometer** using the ^{133}Sr clock transition and it will be sensitive to GWs with an intermediate range of frequencies ~1 Hz.

Baynham et al. (2025)
arXiv:2504.09158

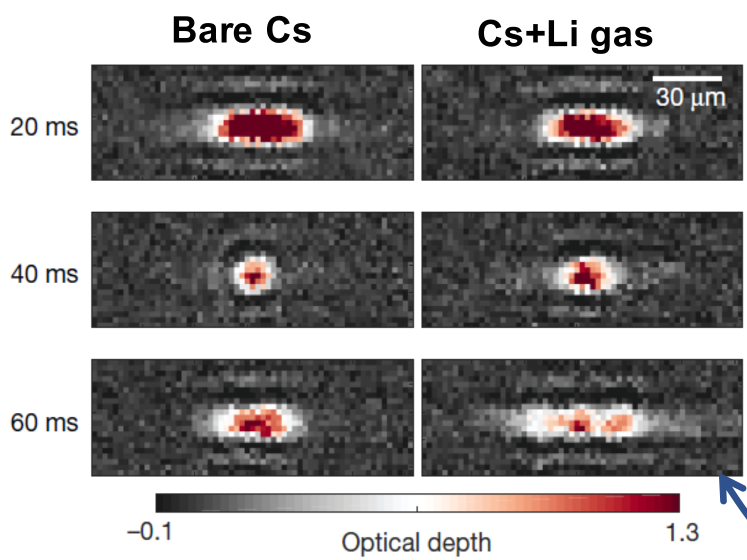
Bose-Fermi solitons (experiment)

LETTER

Nature, 568,61-64 (2019)
https://doi.org/10.1038/s41586-019-1055-0

Observation of fermion–mediated interactions between bosonic atoms

B. J. DeSalvo[1,2]*, Krutik Patel[1,2], Geyue Cai[1] & Cheng Chin[1]



The magnetic field is abruptly changed from 885.5 to 880.32 G (at time t=0).

B-F train of solitons

Fig. 4 | Formation of Bose–Fermi solitons. a, In situ images of a caesium BEC 75 ms after a quench from $B = 885.5$ G ($a_{BB} = 120a_0$) to the lower fields indicated on the left. For positive a_{eff}, there is no qualitative

Hydrodynamic equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{\mathrm{v}}) = 0$$

Continuity equation

$$\frac{\partial \vec{\mathrm{v}}}{\partial t} + \overrightarrow{(\mathrm{v} \cdot \nabla)\mathrm{v}} = -\frac{1}{\rho} \nabla p + \vec{g}$$

Euler equations (the 2nd Newton law)

$$\rho \left(\frac{\partial \vec{\mathrm{v}}}{\partial t} + (\vec{\mathrm{v}} \cdot \vec{\nabla}) \vec{\mathrm{v}} \right) = -\vec{\nabla} p + \rho \vec{g} + \eta \Delta \vec{\mathrm{v}} + \left(\xi + \frac{1}{3} \eta \right) \nabla (\vec{\nabla} \cdot \vec{\mathrm{v}})$$

Navier-Stockes equation

Quantum hydrodynamic equations

$$\frac{\partial n_F}{\partial t} = -\nabla \cdot \left(n_F \, \mathbf{v}_F \right)$$

$$m_F \frac{\partial \mathbf{v}_F}{\partial t} = -\nabla \left(\frac{\delta T}{\delta n_F} + \frac{m_F}{2} \mathbf{v}_F^2 + \frac{\delta E_{BF}}{\delta n_F} \right)$$

$$T = \int d\mathbf{r} \left(\kappa_k \, n_F^{5/3} - \xi \frac{\mathrm{h}^2}{8 m_F} \frac{\left(\nabla n_F \right)^2}{n_F} \right)$$

$$E_{BF} = \int d\mathbf{r} \, n_B(\mathbf{r}) \, n_F(\mathbf{r}) + C_{BF} \int d\mathbf{r} \, n_B(\mathbf{r}) \, n_F^{4/3}(\mathbf{r}) \, A(w, \alpha)$$

Quantum hydrodynamic equations

$$\frac{\partial n_B}{\partial t} = -\nabla \cdot \left(n_B \mathbf{v}_B \right)$$

$$m_B \frac{\partial \mathbf{v}_B}{\partial t} = -\nabla \left(\frac{\delta E_B}{\delta n_B} + \frac{m_B}{2} \mathbf{v}_B^2 + V_q + \frac{\delta E_{BF}}{\delta n_B} \right)$$

$$V_q = -\frac{\hbar^2}{2 m_B} \frac{\nabla^2 \sqrt{n_B}}{\sqrt{n_B}}$$

$$E_B = \frac{1}{2} \int d\mathbf{r}\, g_B n_B^2 + C_{LHY} \int d\mathbf{r}\, n_B^{5/2}(\mathbf{r})$$

Motion of a test particle

$$\frac{1}{2} m \dot{r}^2 + V_{eff}(r) = E$$

$$m r^2 \dot{\phi} = l$$

$$V_{eff}(r) = -\frac{GMm}{r} + \frac{l^2}{2mr^2} - \frac{GMl^2}{mc^2r^3}$$

$$E = \frac{\varepsilon^2 - m^2c^4}{2mc^2}$$

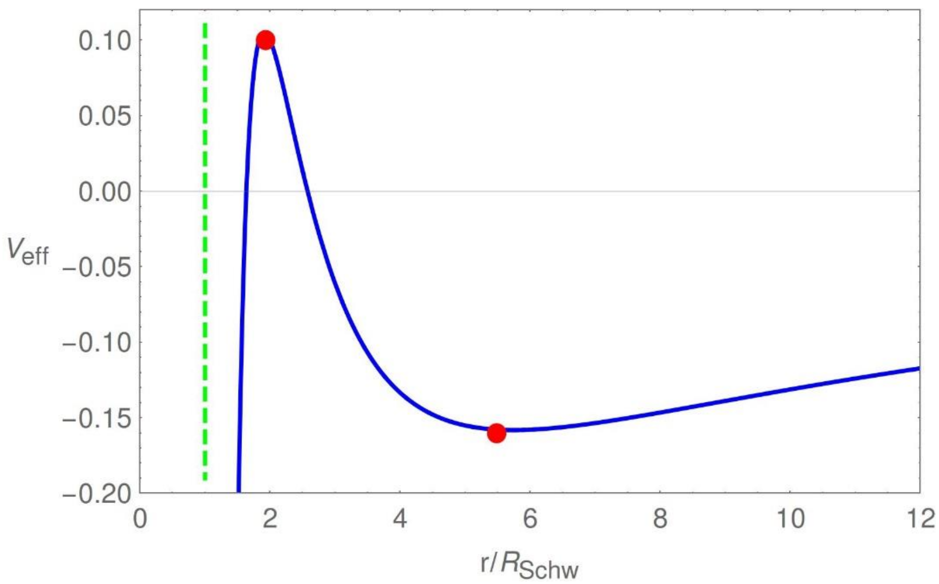
Pseudo-Newtonian potential (by Paczyński & Wiita)

$$V_{PN} = -\frac{GM}{r - R_s}$$

$$V_{eff}^{PN} = -\frac{GM}{r - R_s} + \frac{l^2}{2mr^2}$$

$$\frac{dV_{eff}^{PN}}{dr} = 0 \quad \& \quad \frac{d^2V_{eff}^{PN}}{dr^2} = 0$$

$$r_{ms} = 3R_s$$



$$V_{eff}^{Schw} = -\frac{GM}{r} + \frac{l^2}{2mr^2}\left(1 - \frac{R_s}{r}\right)$$