

Breakdown of LPM Suppression in Extremely High-Energy Bremsstrahlung

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**5th International Workshop on Emergent QCD Collectivity Across Scales:
From Small System Collisions to Jets**

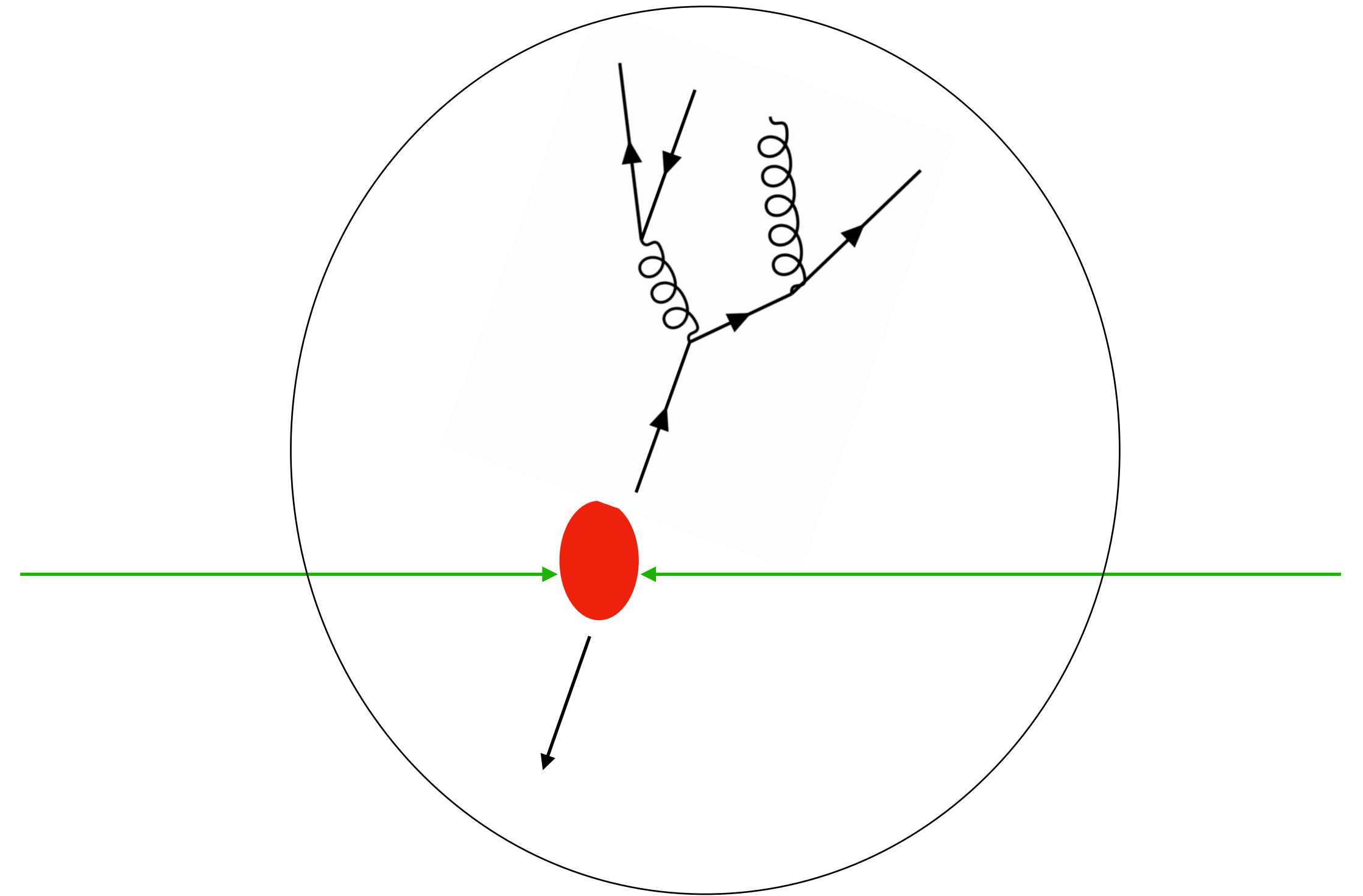
Based on work with:

Peter Arnold, Shahin Iqbal and Joshua Bautista



Particle showers

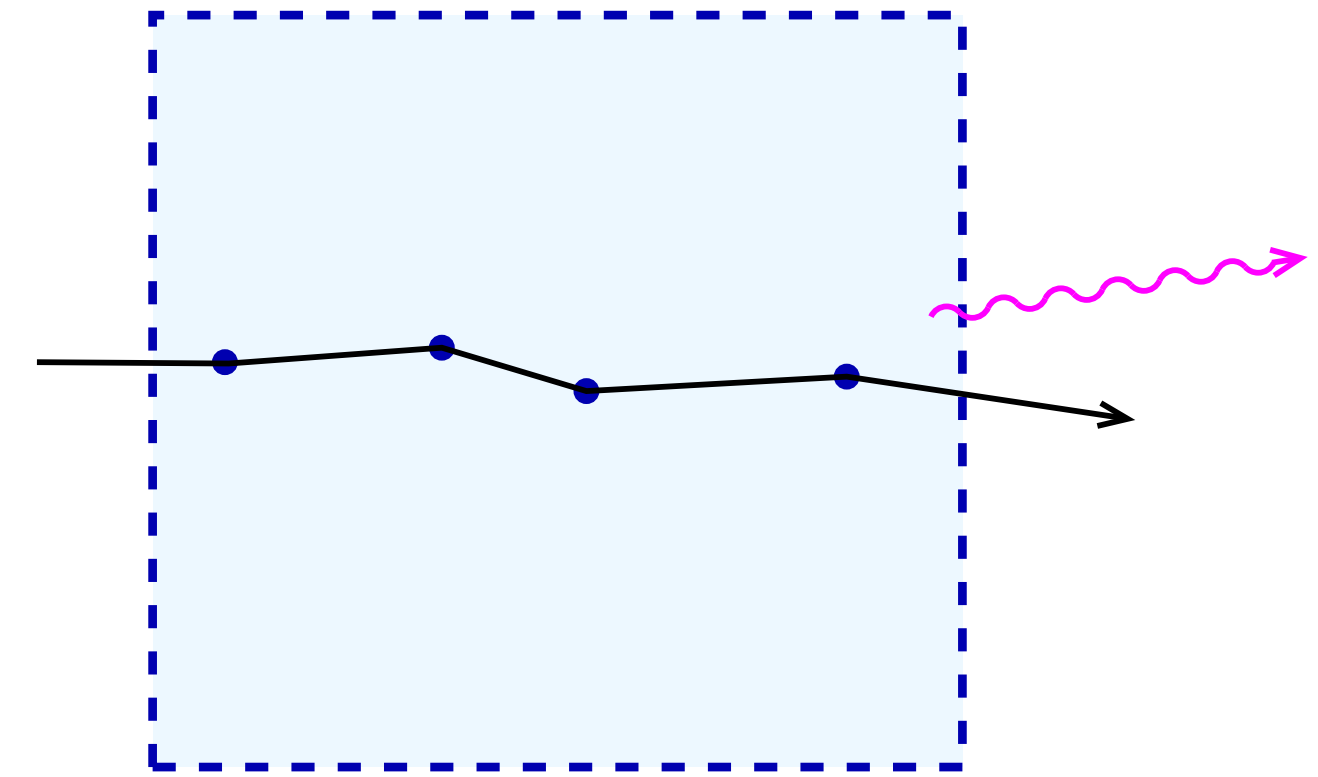
- Cosmic ray EM shower
- Shower in EM calorimeter
- A QCD shower in QGP



A qualitative picture

The rest frame of the medium

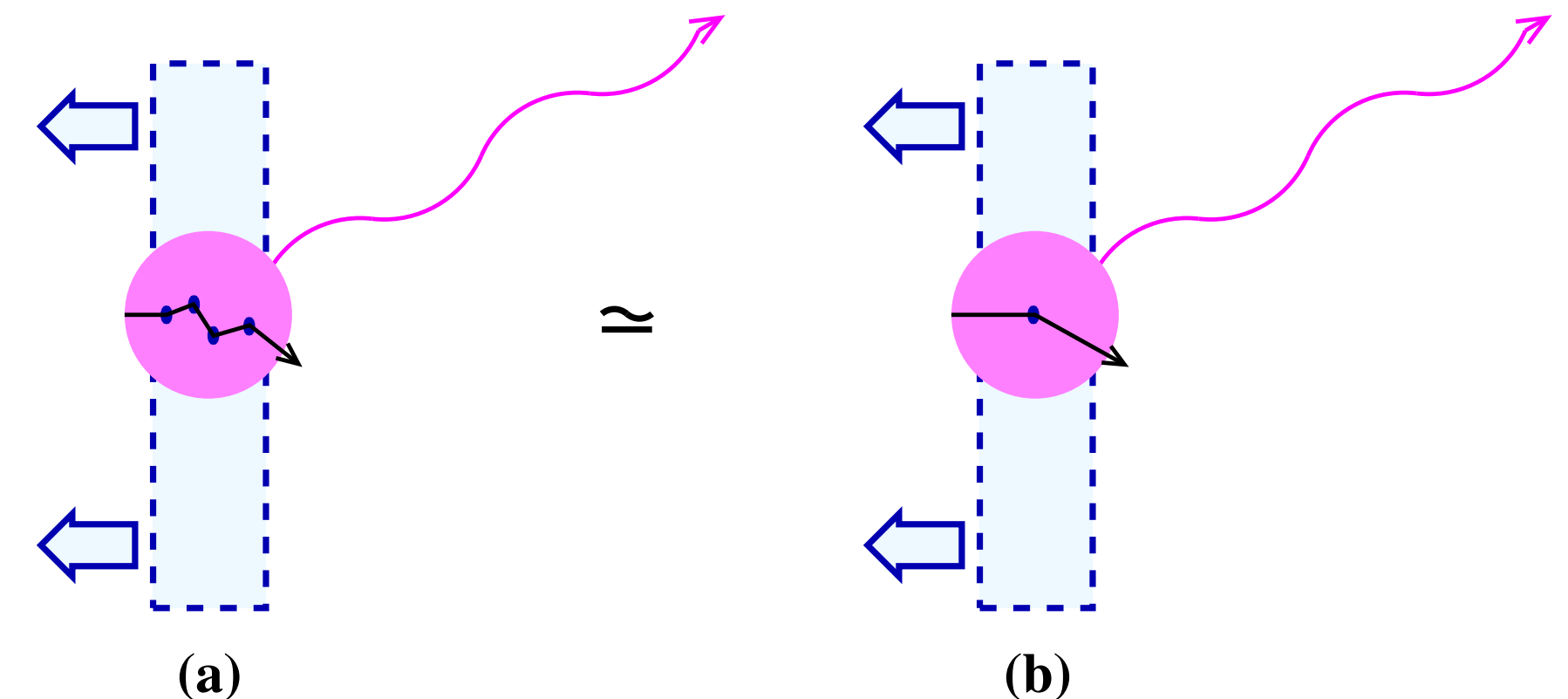
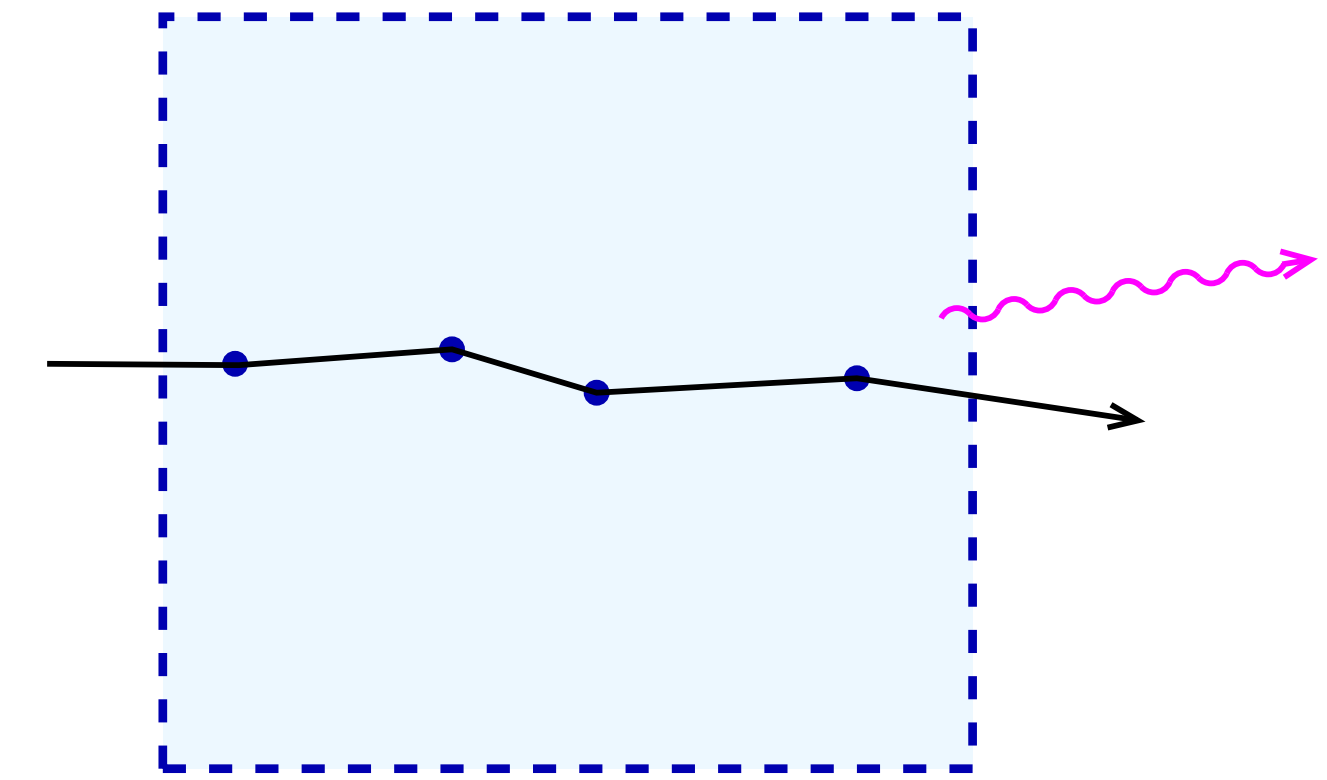
- Consider several elastic collisions in a row in the rest frame of the medium.
- Deflection angles are almost always very small at high energy, and so the bremsstrahlung radiation is nearly collinear



A qualitative picture

A boost to the right

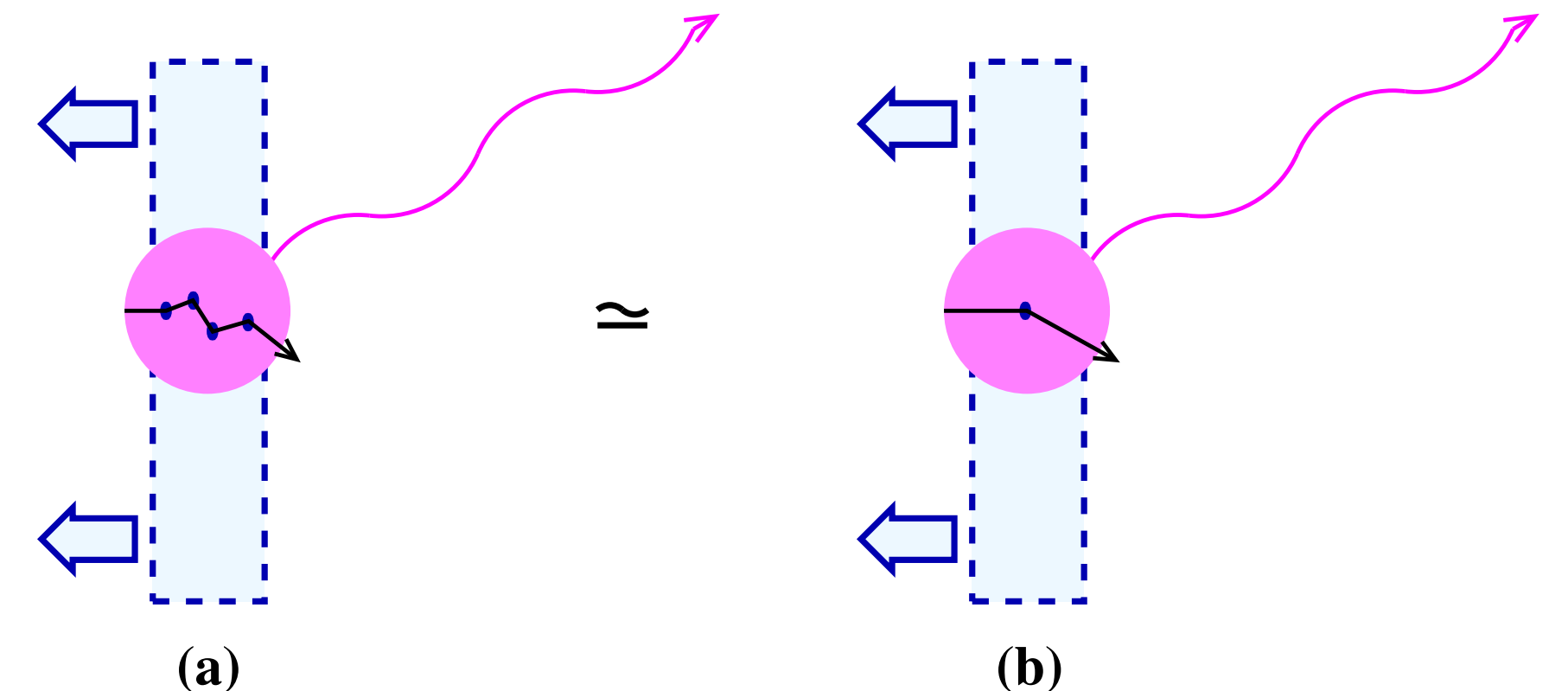
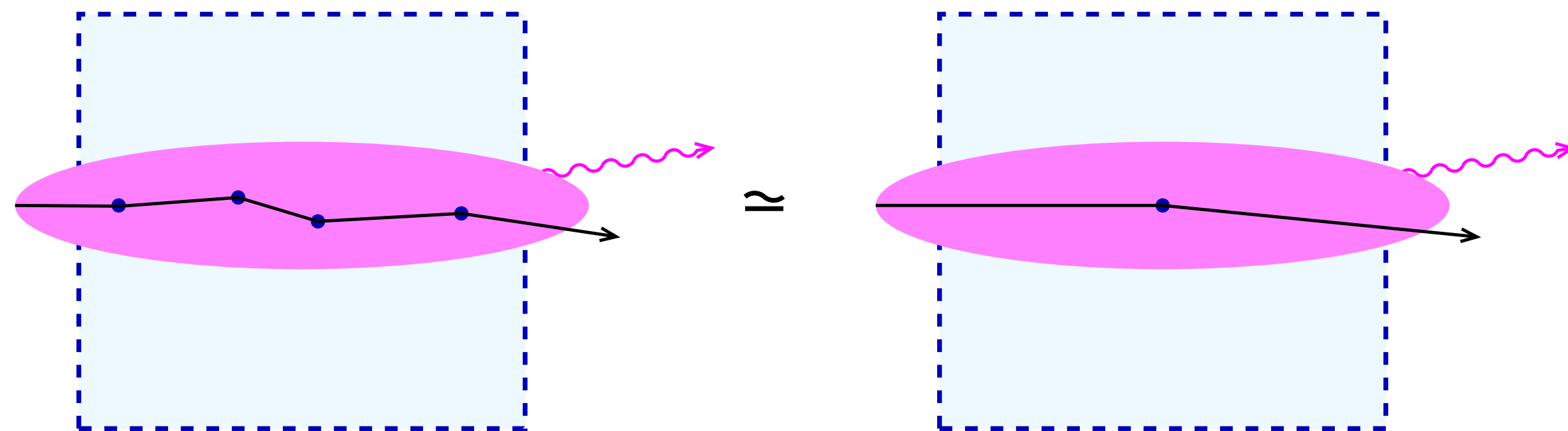
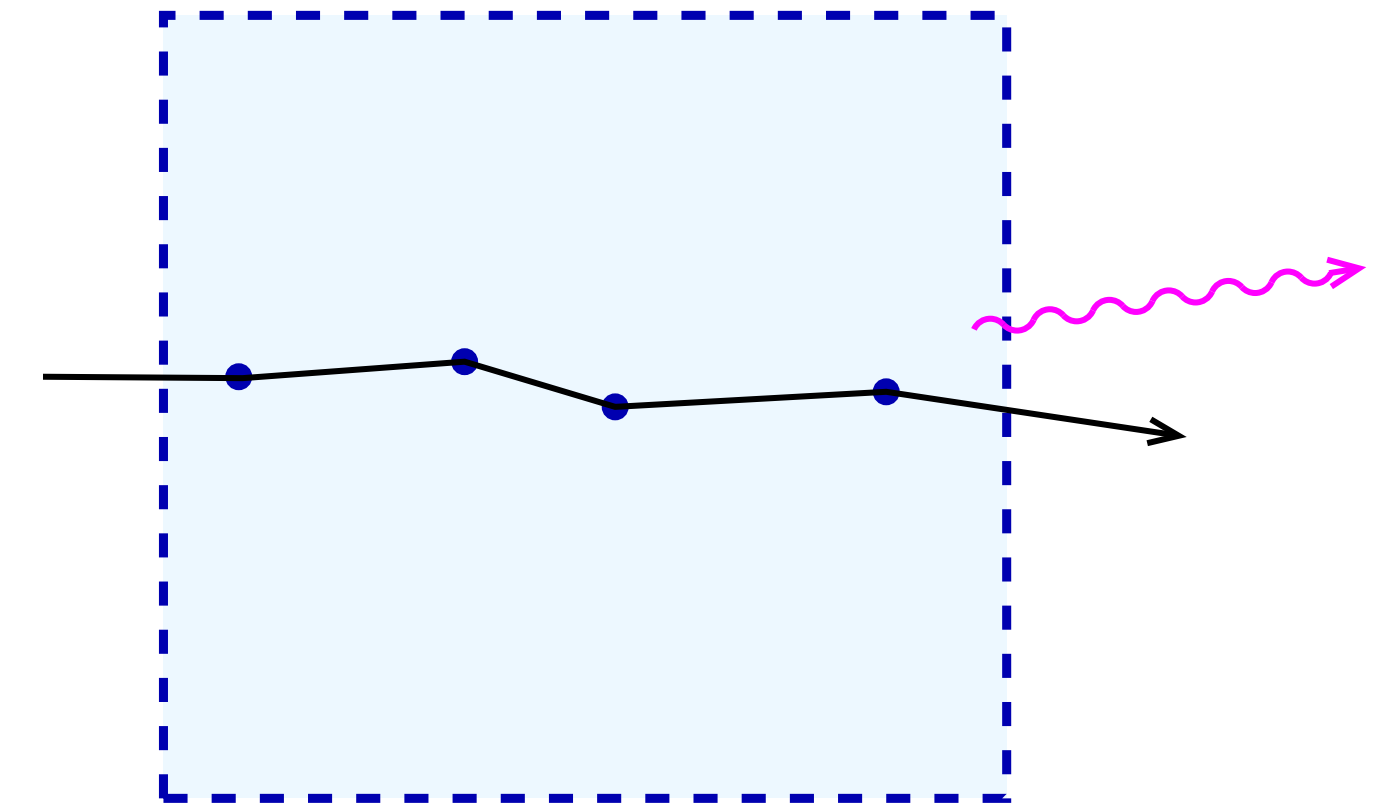
- Boosting to a frame that is moving to the right at close to the speed of light, say the center of momentum frame of the two daughters $e\gamma$ of the bremsstrahlung $e \rightarrow e\gamma$.
- The medium is extremely Lorentz contracted, the electron and bremsstrahlung photon have extremely less energy, and so the photon has extremely longer wavelength.
- The Lorentz-expanded photon wavelength may be large compared to the Lorentz-contracted elastic mean free path.
- A single photon cannot effectively resolve details smaller than its wavelength.



A qualitative picture

Boost back

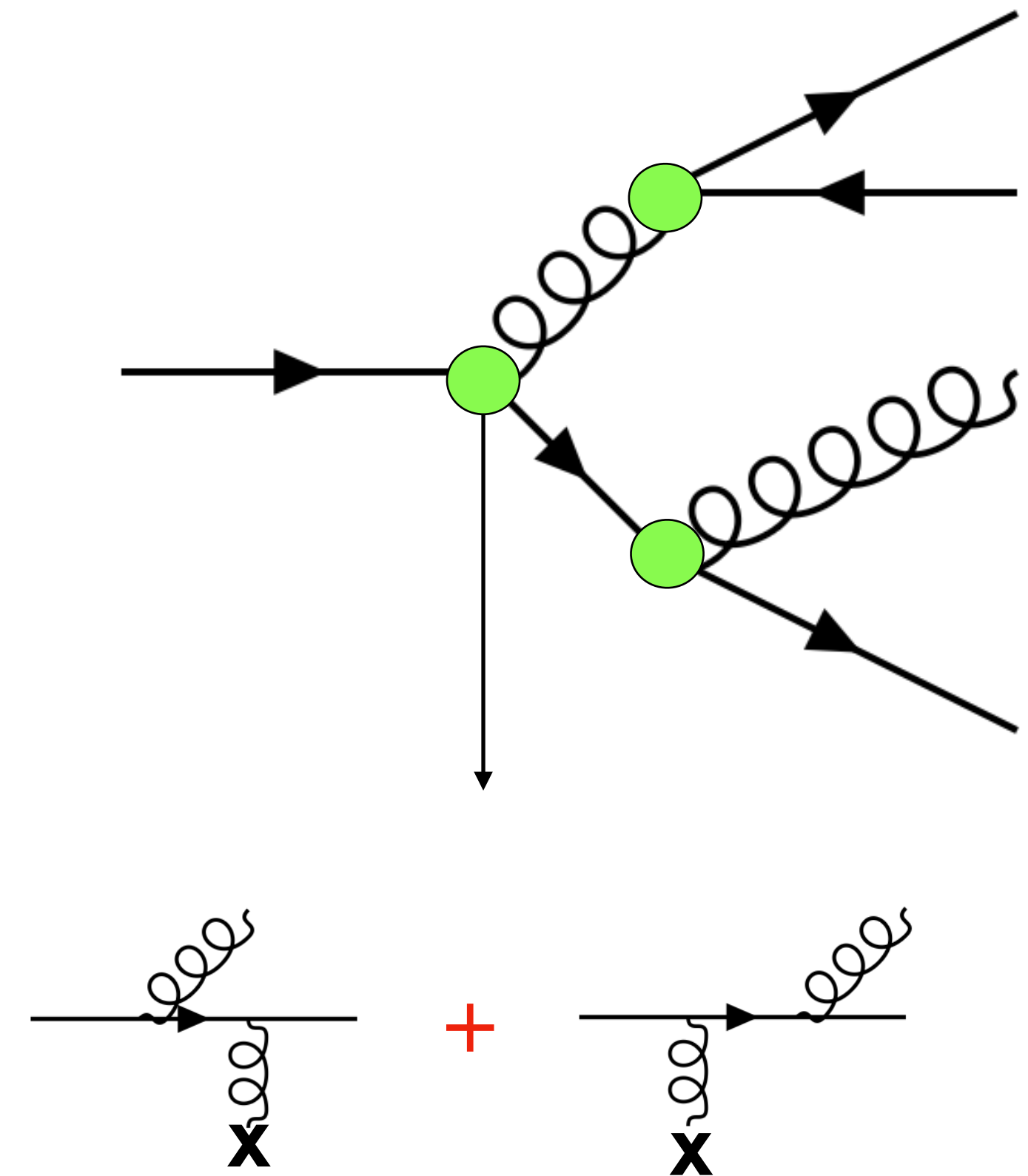
- Boosting back to the rest frame of the medium, the unresolved region (magenta circle) becomes long and stretched. This length corresponds to the “formation time/length” t_{form} for the bremsstrahlung photon.
- Multiple collisions with the medium do not give independent chances for bremsstrahlung but instead give only one chance.



In-medium showers

Beithe-Hitler rate

- Each collision with the medium offers a chance for emission.
- Prob of splitting $\sim \alpha$ per collision
- The green region is the quantum mechanical duration of the splitting known as the formation time.

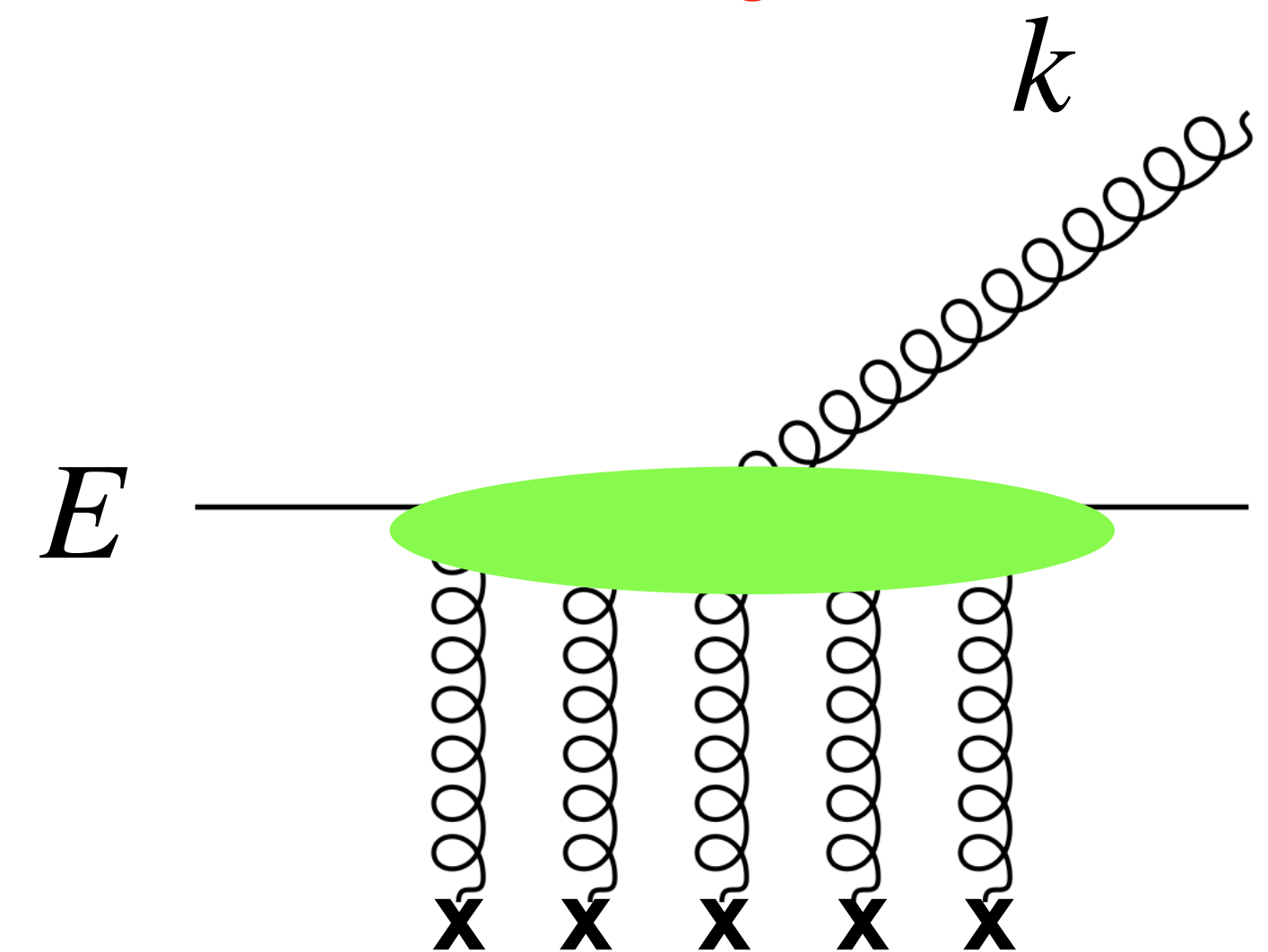


In-medium showers

LPM effect

- When $t_{form} \gg \tau_{scatt}$, many scatterings take place within one formation time. This is known as the Landau-Pomeranchuk & Migdal (LPM) effect.
- Prob of splitting $\sim \alpha$ per formation time.
- This reduction in the rate was figured out in the 1950's for QED (LPM) and for QCD (1990s) by BDMPS-Z.

Formation time grows with energy.

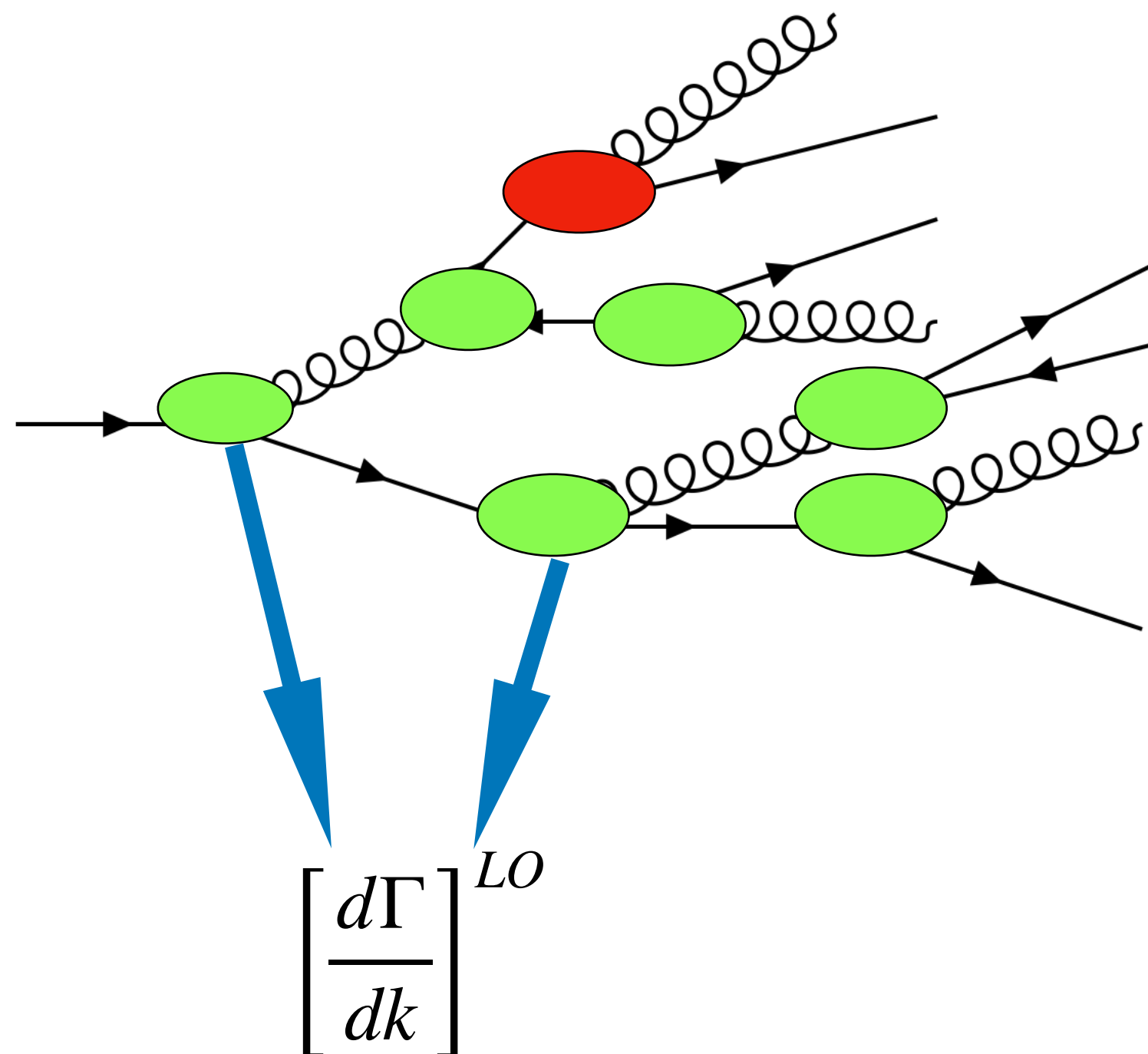


I will call the rate for splitting accounting for this effect $\left[\frac{d\Gamma}{dk} \right]^{LO}$.

Shower type

Weakly or strongly coupled ?

- Can we use the LPM splitting rate to model shower development ?



LPM splitting rate

One may statistically model shower development by treating high-energy particles classically between splittings, and rolling dice based on the splitting rates to decide when and how each particle splits.

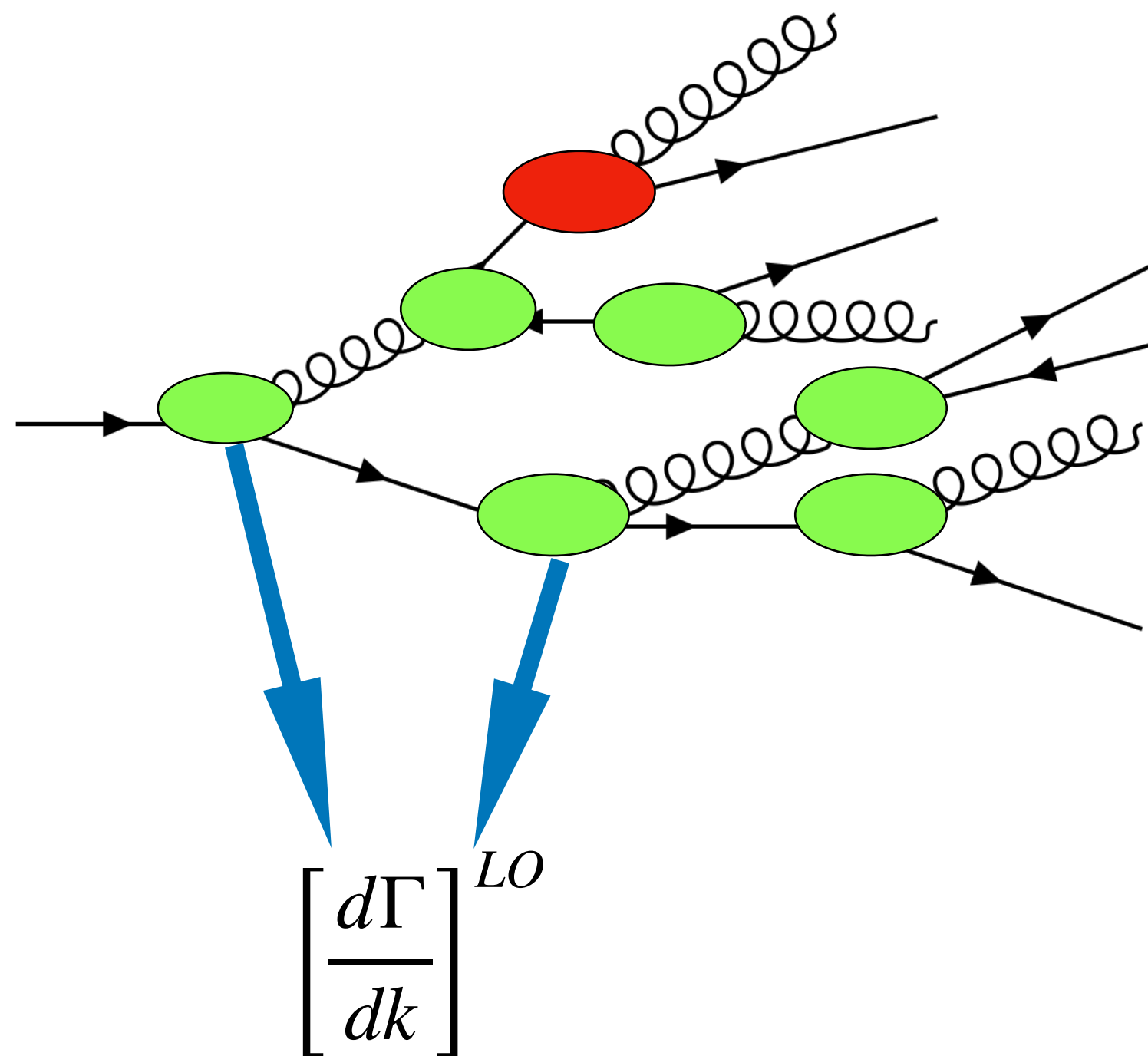
We call this a weakly coupled shower .

Shower type

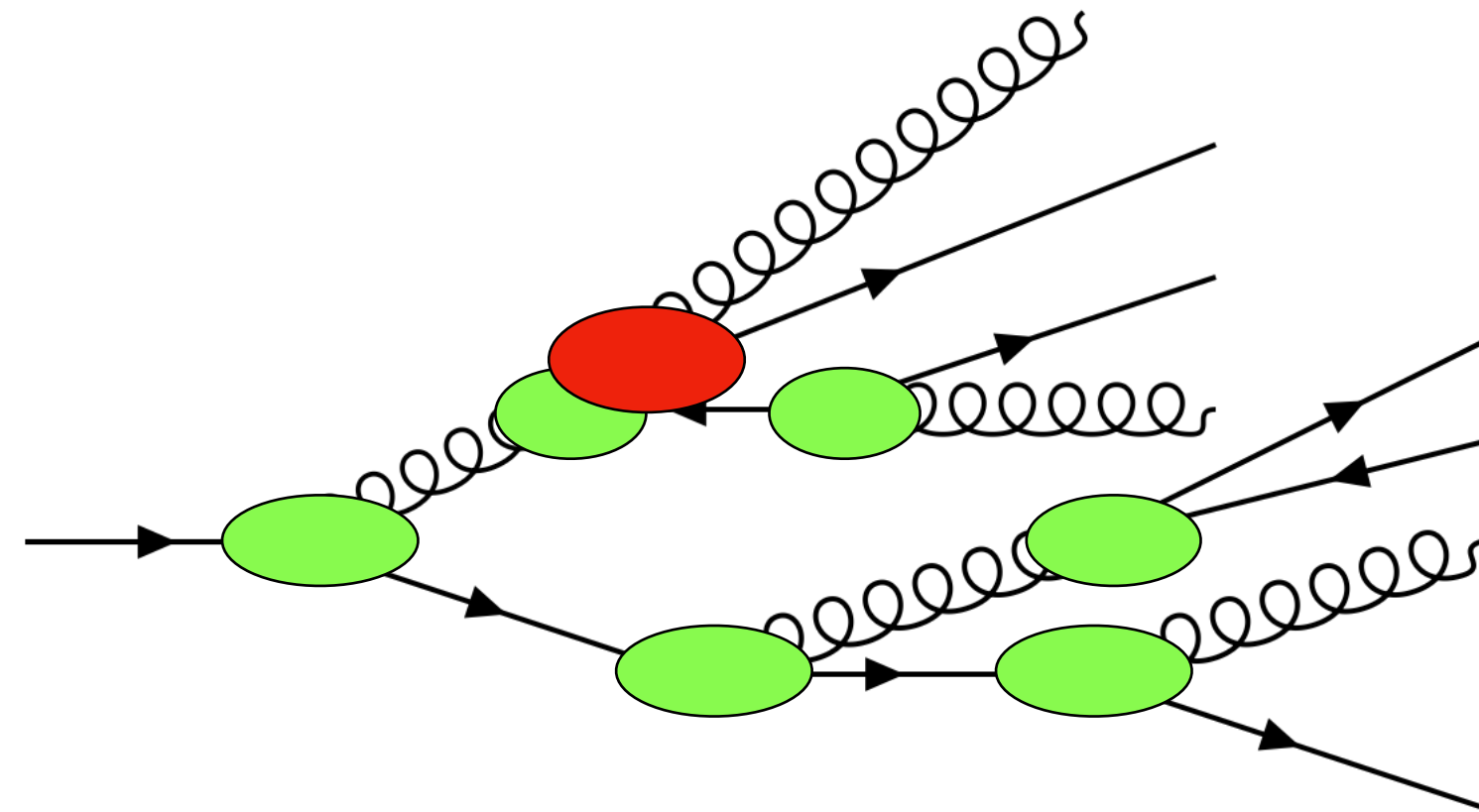
Weakly or strongly coupled ?

- Or can splittings overlap?

Formation time grows with energy.

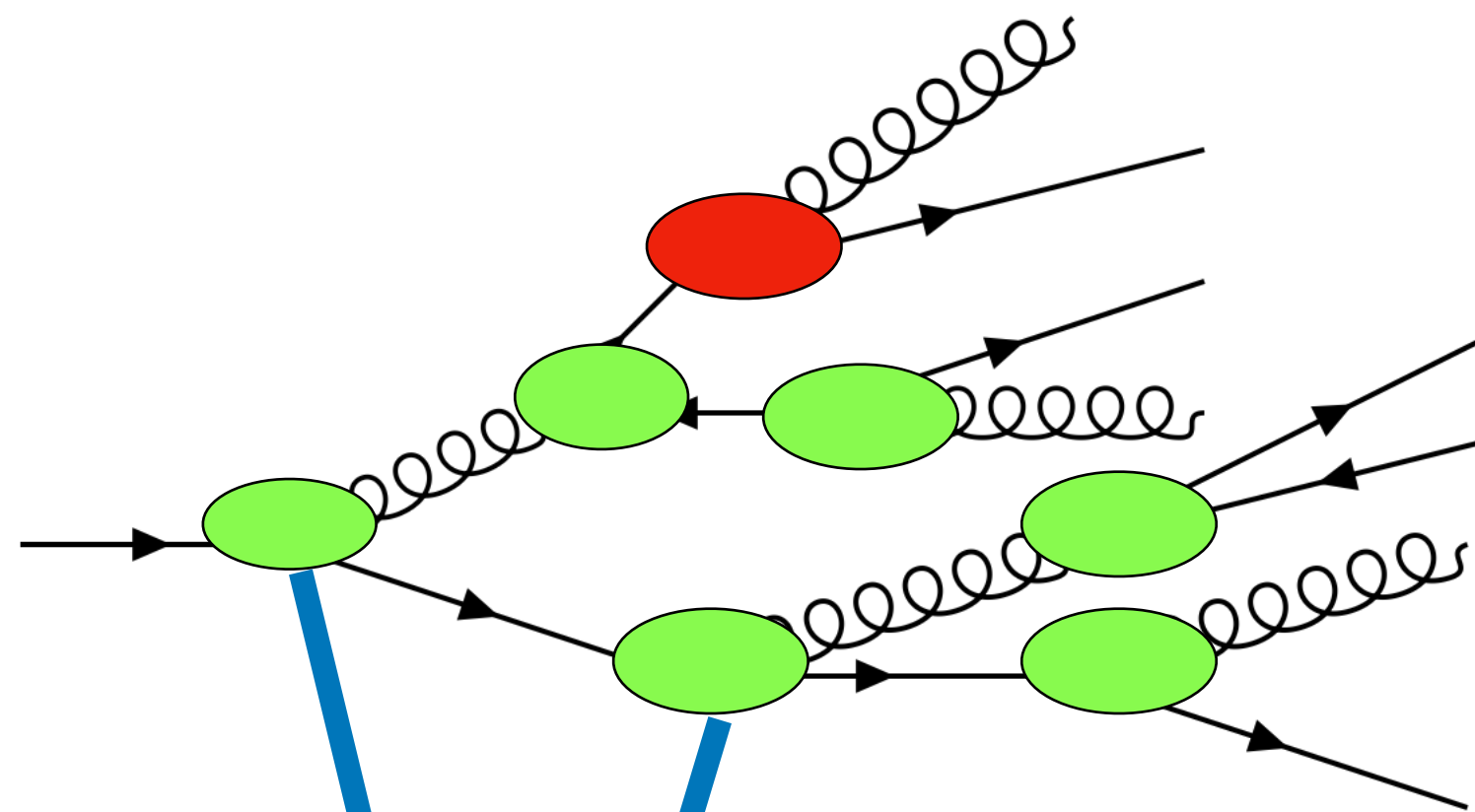


LPM splitting rate

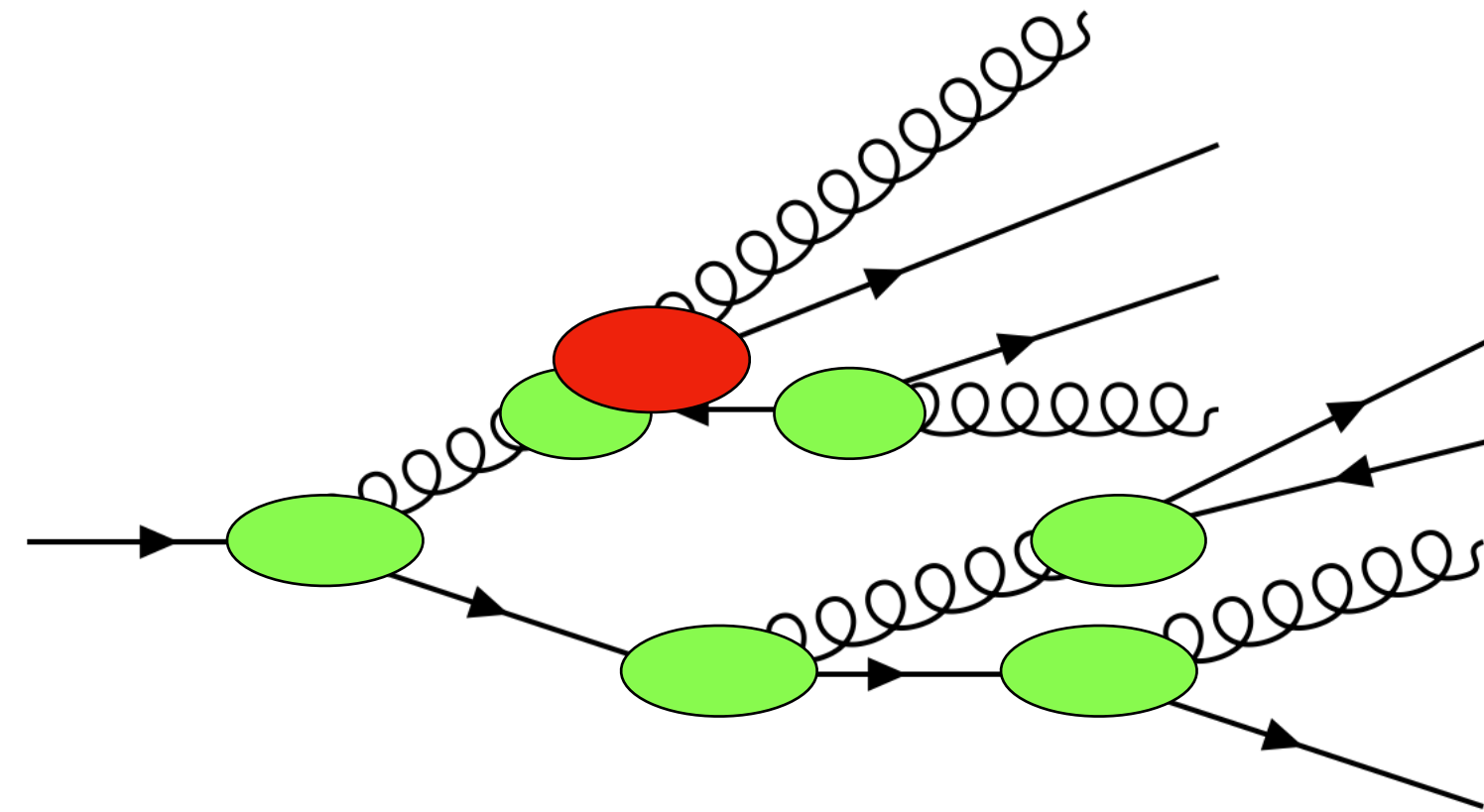


Shower type

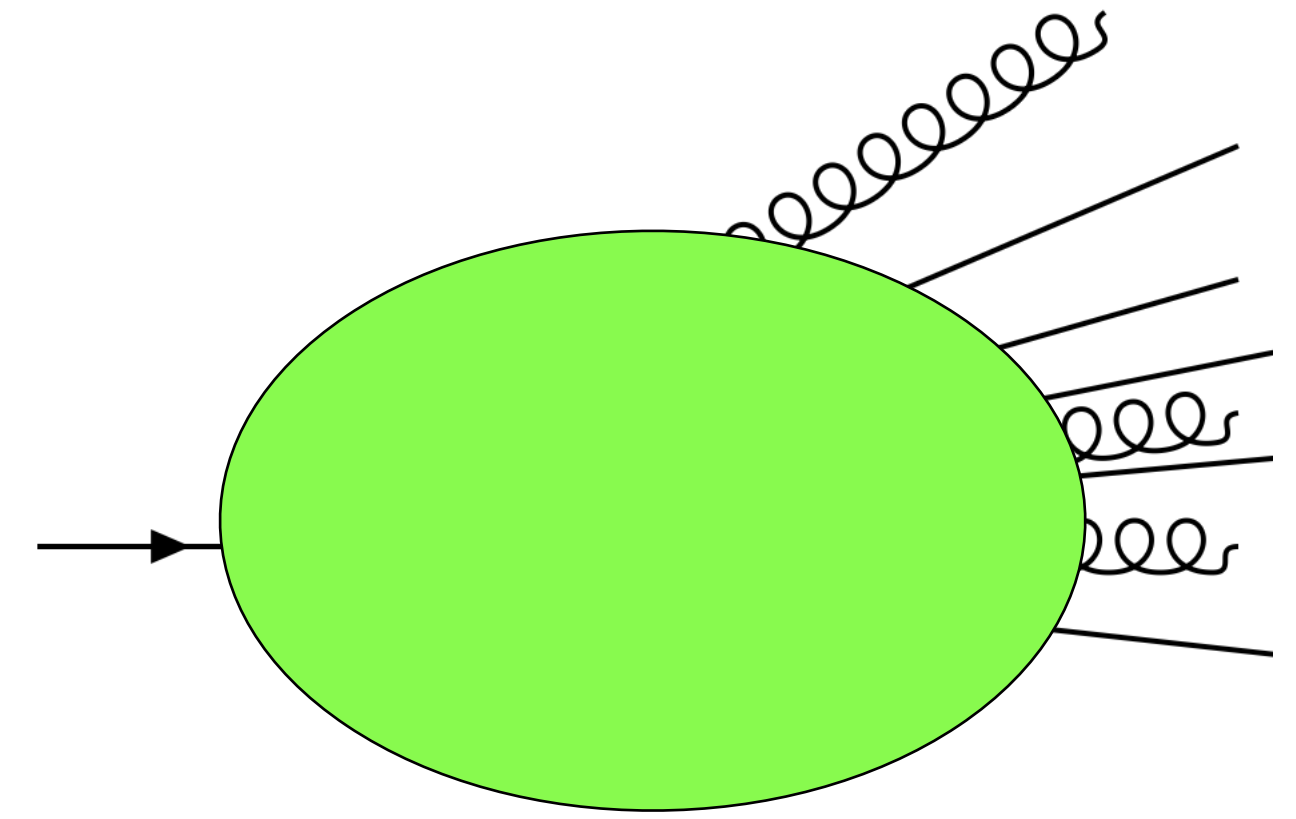
Weakly or strongly coupled ?



LPM splitting rate



If formation times are large compared to times between splittings, one may not treat different splittings as quantum mechanically independent.

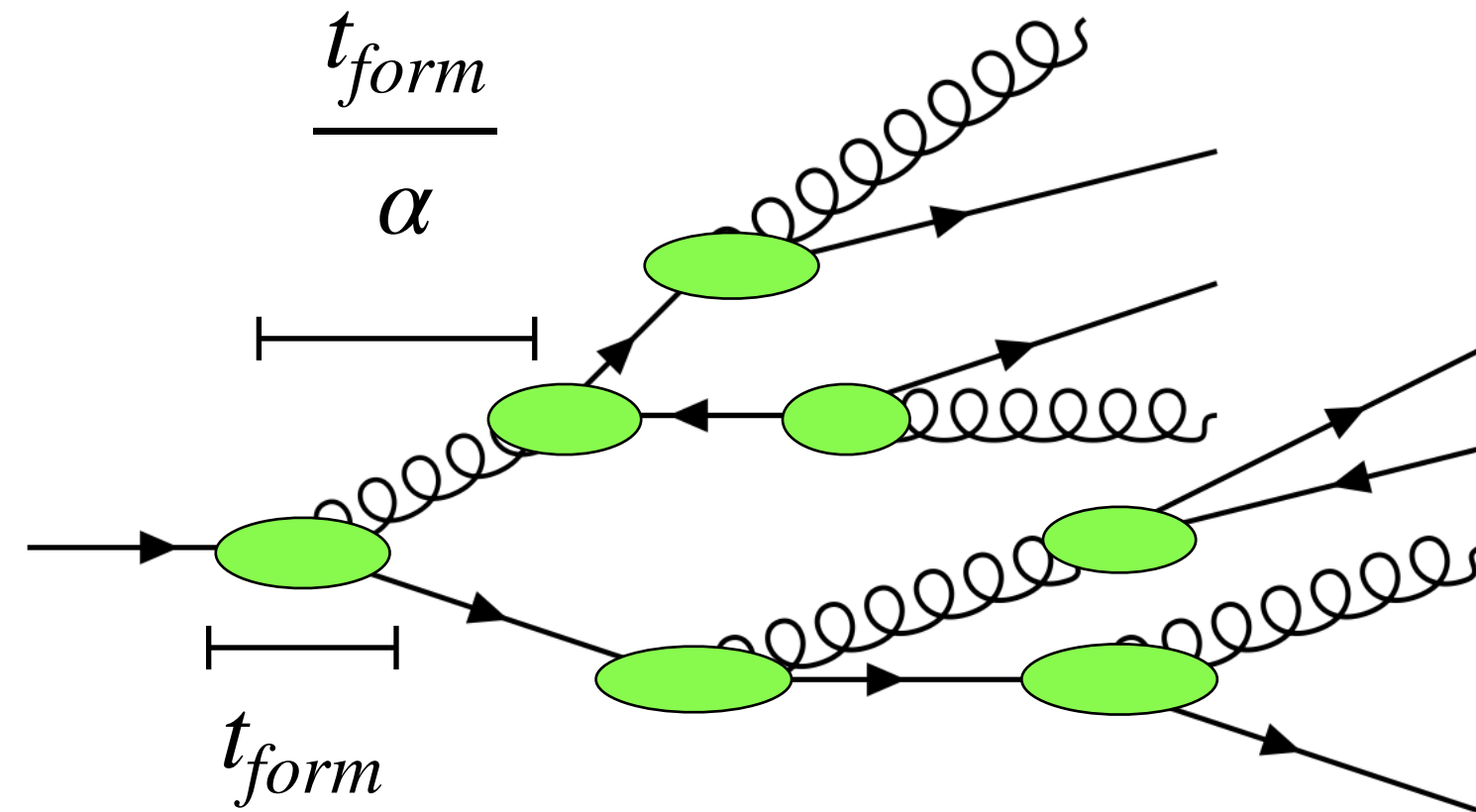


Resort to ADS/CFT

Hierarchy of scale

Weakly or strongly coupled ?

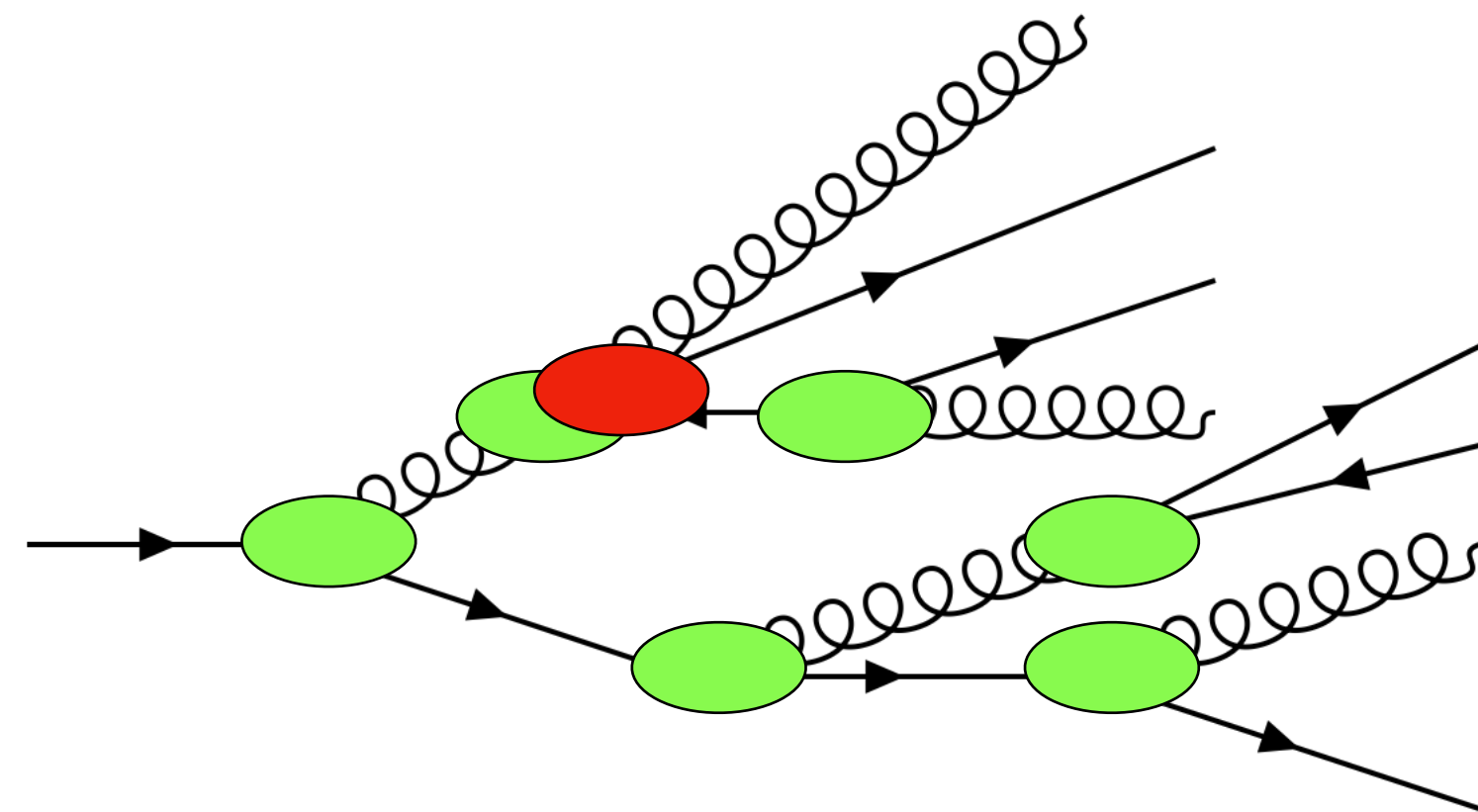
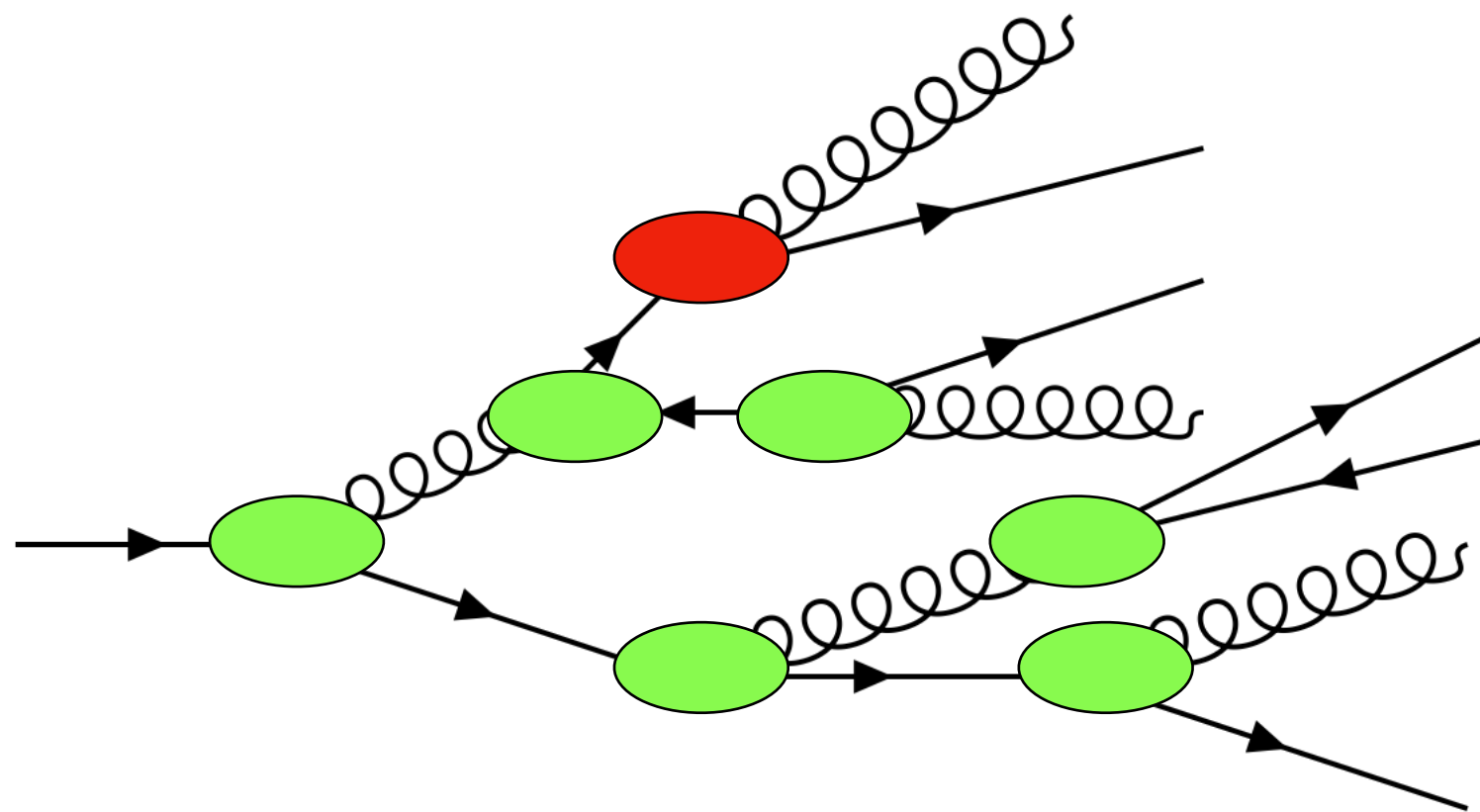
- Prob of splitting $\sim \alpha$ per formation time. The prob of overlapping splitting is $\alpha(\mu)$, where μ is the scale at which splittings happen. Here it depends on E of the parent not T of the QGP. However, it turns out the prop of splitting is accompanied by a large double log .



Our approach

a theoretical test

- Treat $\alpha(\mu)$ as small, but calculate the correction to the weakly coupled picture. For reasonable values of $\alpha(\mu)$, how large are these corrections ?



Our approach

Assumptions

- The medium is static, homogeneous and large enough to stop the shower.
- Start with a single high energy parton that is very close to on-shell.
- Use the $\hat{q}$ approximation for elastic scattering with the medium $\langle \Delta p_{\perp}^2 \rangle = \hat{q}L$.

In that case, $\mu \sim (\hat{q}E)^{1/4}$ and $t_{form} \sim \sqrt{E/\hat{q}}$.

- The large N_c limit [with $N_f = 0$ (pure gluons) or $N_f \gg N_c$ (many quark flavors)].
- Integrate over $p_{\perp}$.

Rate Diagrams

Leading Order

- Brem or pair production

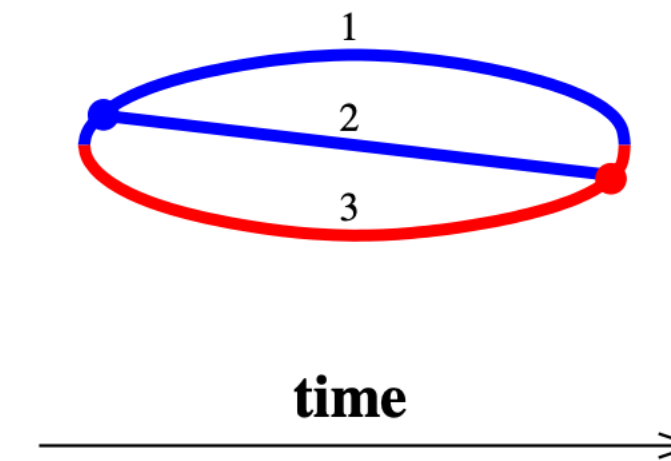
$$\text{LPM/BDMPS-Z rate} = \left| \begin{array}{c} \text{diagram with blue lines} \end{array} \right|^2 = \begin{array}{c} \text{diagram with blue lines} \\ \times \left(\begin{array}{c} \text{diagram with red lines} \end{array} \right)^* \end{array}$$

time →

- The solid lines represents (a quark or a gluon)

The LO calculation

Step 1: High energy approximation



- Let's ignore medium interactions for now, and look at the particles 1,2 and 3.
- Re-interpret the diagram as *three* particles evolving forward in time. Translate calculation into a 3-particle “QM” problem.
- For particles (1,2) in the **amplitude** $e^{-iE_n t}$ and for particle 3 in the **conjugate amplitude** $e^{+iE_n t}$.
- Define a hamiltonian H for the total evolution e^{-iHt} , with $H = E_1 + E_2 - E_3$.
- Use $E = \sqrt{p_z^2 + p_\perp^2} \simeq |p_z| + \frac{p_\perp^2}{2p_z}$, to get $H \simeq \frac{p_{\perp 1}^2}{2|p_{z1}|} + \frac{p_{\perp 2}^2}{2|p_{z2}|} - \frac{p_{\perp 3}^2}{2|p_{z3}|}$.
- This looks like a 2-d QM kinetic energy term $p_{zi} \longrightarrow M_i$, except $M_3 < 0$.

The LO calculation

Step 2: the “potential energy”

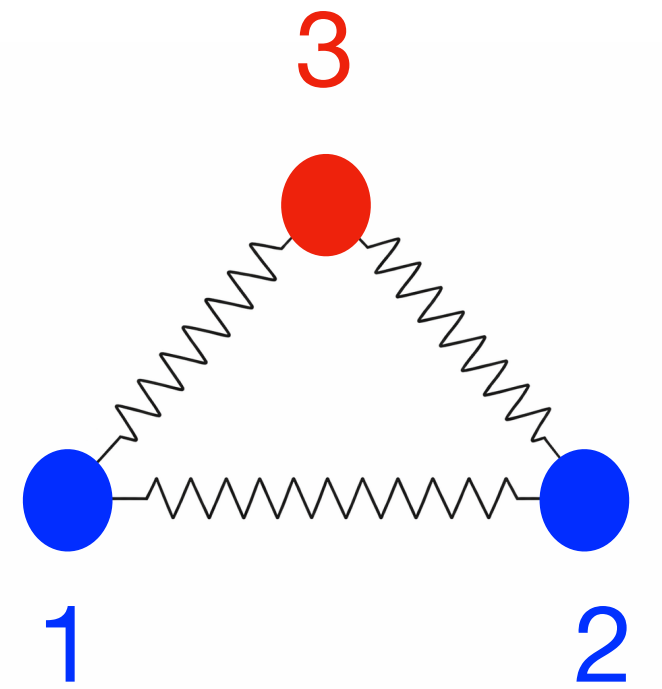
- As discussed, in the $\hat{q}$ approximation, the potential takes the form

$$V(\Delta b) = -\frac{i}{4}\hat{q}(\Delta b)^2.$$

- For $g \rightarrow gg$,

$$V(b_1, b_2, b_3) = V(b_{12}) + V(b_{23}) + V(b_{31}) = -\frac{i\hat{q}_A}{8} [(b_2 - b_1)^2 + (b_3 - b_2)^2 + (b_1 - b_3)^2]$$

- This decomposition into 2-body potentials is valid for (i) weak coupling, (ii) for any coupling in the $\hat{q}$ approximation (iii) and any coupling in the large N_c limit.



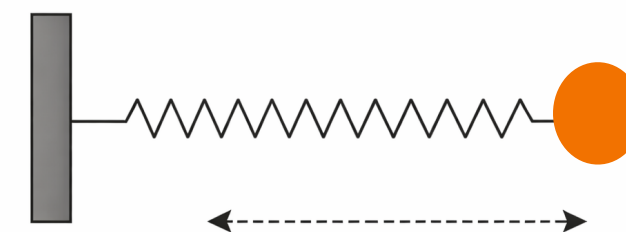
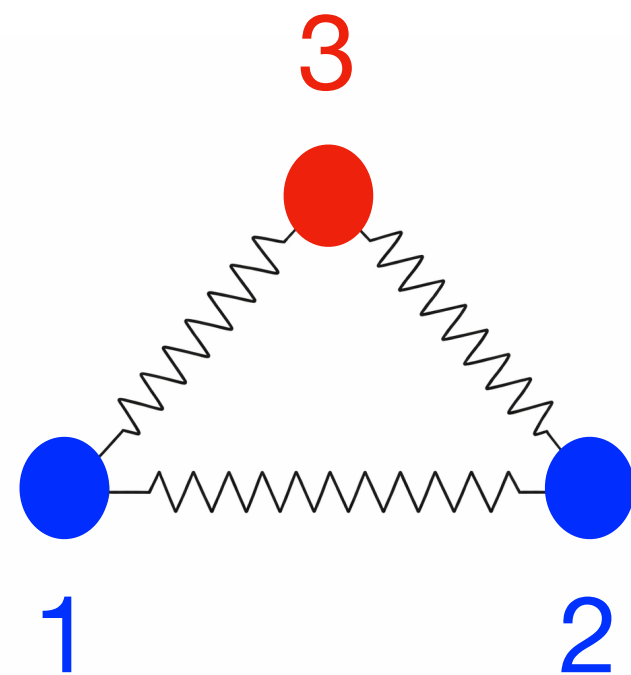
The LO calculation

Step 3: Reduce N to N-2

- We can use (i) 2d translation invariance and (ii) 3d rotational invariance, to reduce the 3-particle harmonic oscillator problem to an effective 1-particle problem.

- $H = \frac{P_{\perp}^2}{2M} + \frac{M\Omega_0^2 B^2}{2}$, with a complex frequency Ω_0 .

- $P_{\perp} = x_j p_{\perp i} - x_i p_{\perp j}$ and $B = \frac{b_i - b_j}{x_i + x_j}$, which satisfies $[P_{\perp}, B] = -i$.



The LO calculation

Putting it all together

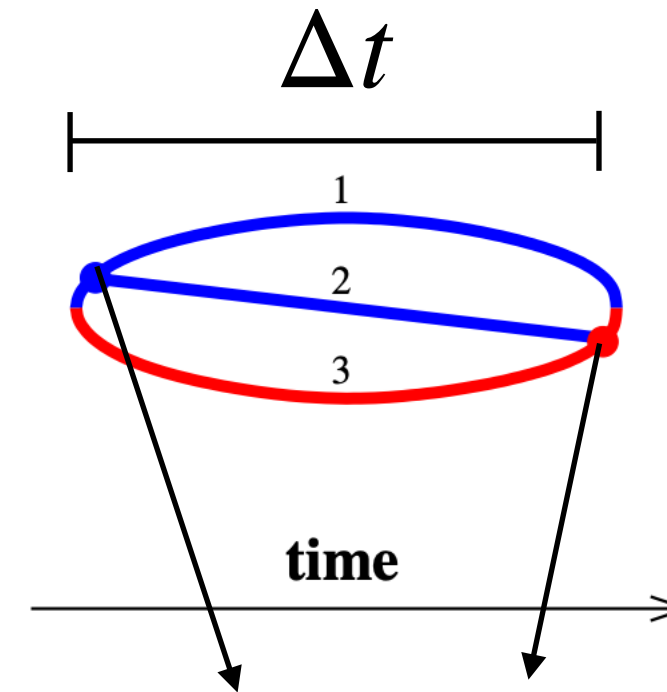
- Zakharov's form of the calculation

$$\frac{d\Gamma}{dx} = \alpha_s P(x) 2 \operatorname{Re} \int_0^\infty d(\Delta t) \nabla_B \cdot \nabla_{B'} \langle B, \Delta t | B', 0 \rangle \Big|_{B=0, B'=0}$$

DGLAP splitting function

the $\hat{q}$ approximation $\rightarrow$ HO propagator

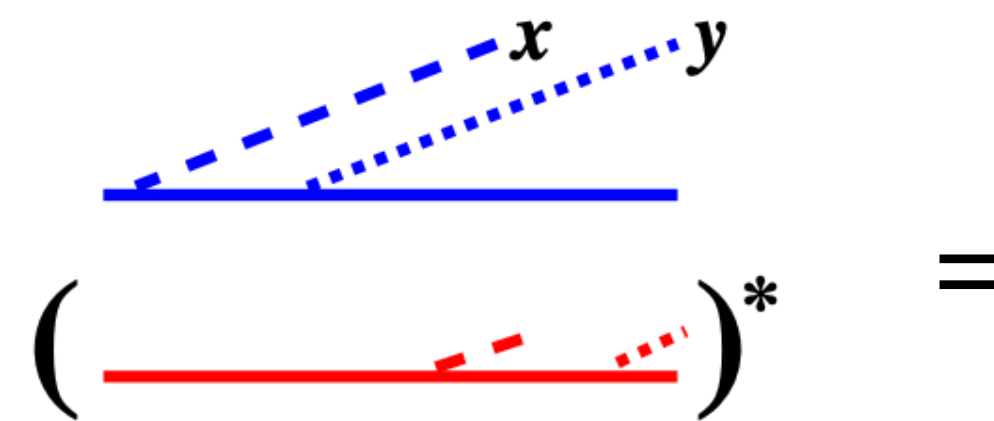
- The result is simple $\frac{d\Gamma}{dx} = \frac{\alpha_s}{\pi} P(x) \operatorname{Re} \{ i\Omega_0 \}$, with complex $\Omega_0 = \sqrt{\frac{-i\hat{q}_A}{2x(1-x)E}}$



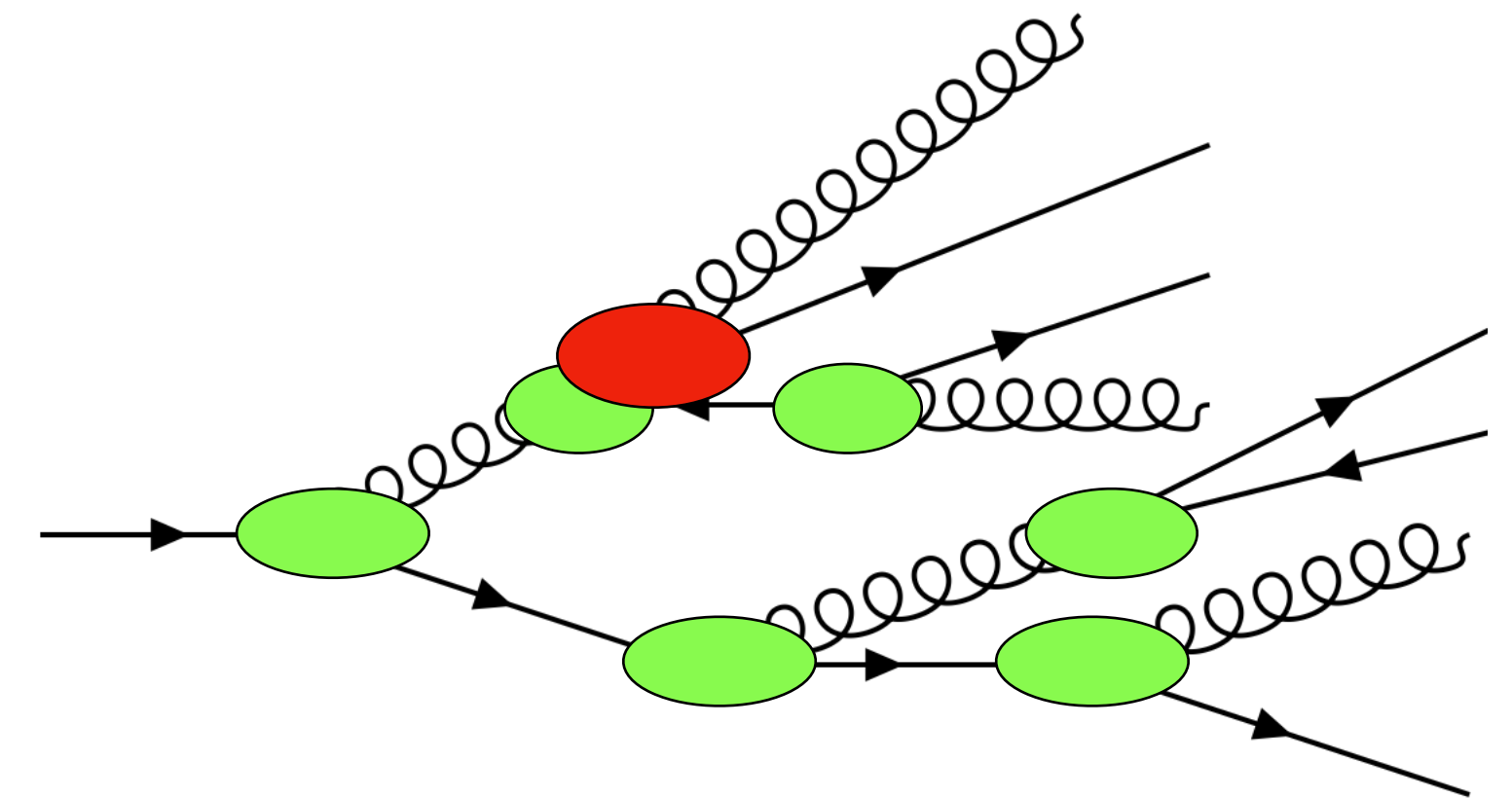
The NLO calculation

Overlapping formation times

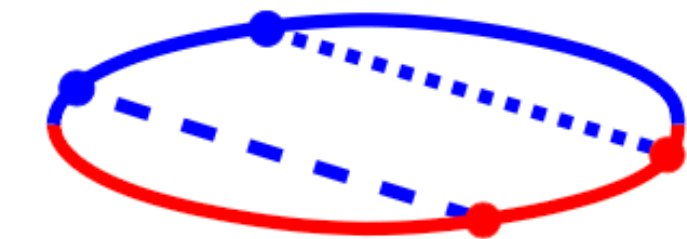
- Let's consider an example of such a diagram in Zakharov's description.



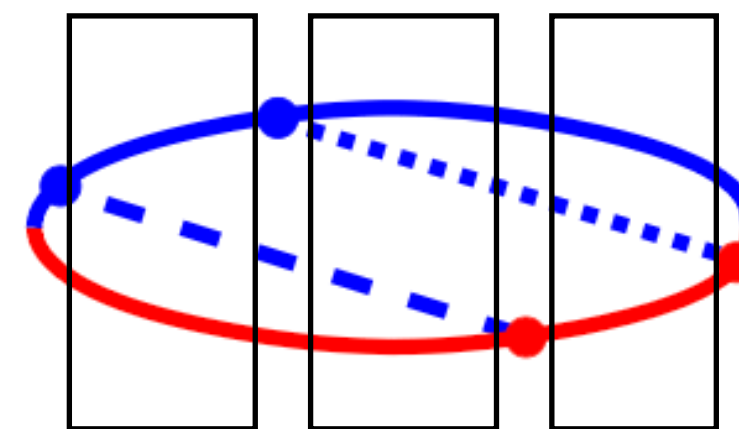
- As we saw before, we put together QFT matrix elements for each vertex, then evolve in between with QM propagators.



time →



time →



3-particle 4-particle 3-particle

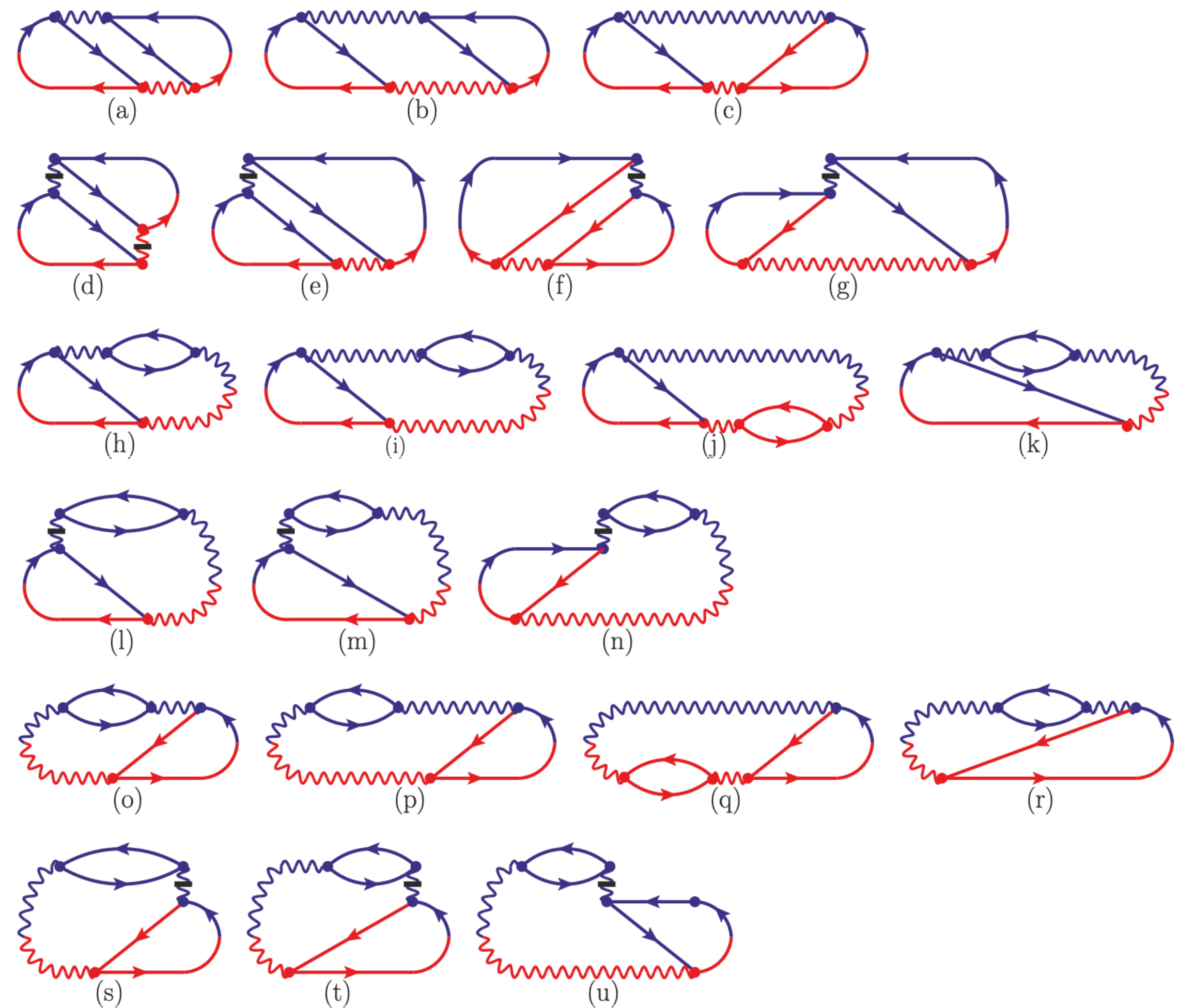
1-particle 2-particle 1-particle

Using symmetries of the problem →

The NLO calculation

Time-ordered diagrams (Real & virtual)

- For large- N_f QCD

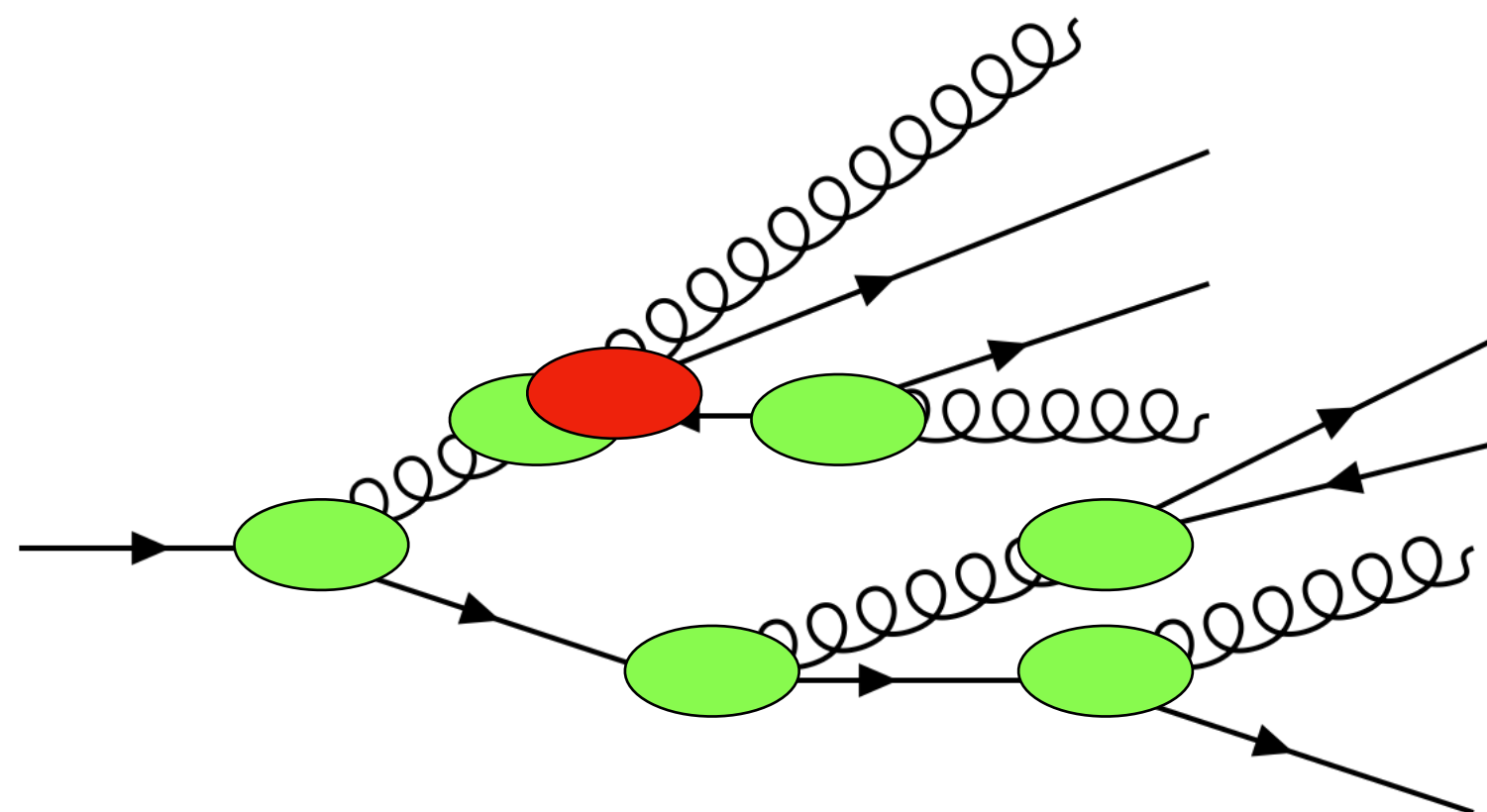


QED result

Overlap corrections = $-99\% N_f \alpha(\mu)$
($N_f \gg 1$, many flavors)

[P.Arnold & S.Iqbal 2019]

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QCD result

Overlap corrections = $-1\% N_c \alpha_s(\mu)$
(pure gluons)

[P.Arnold & **OE** & S.Iqbal]

Phys.Rev.Lett. 131 (2023) 16, 162302

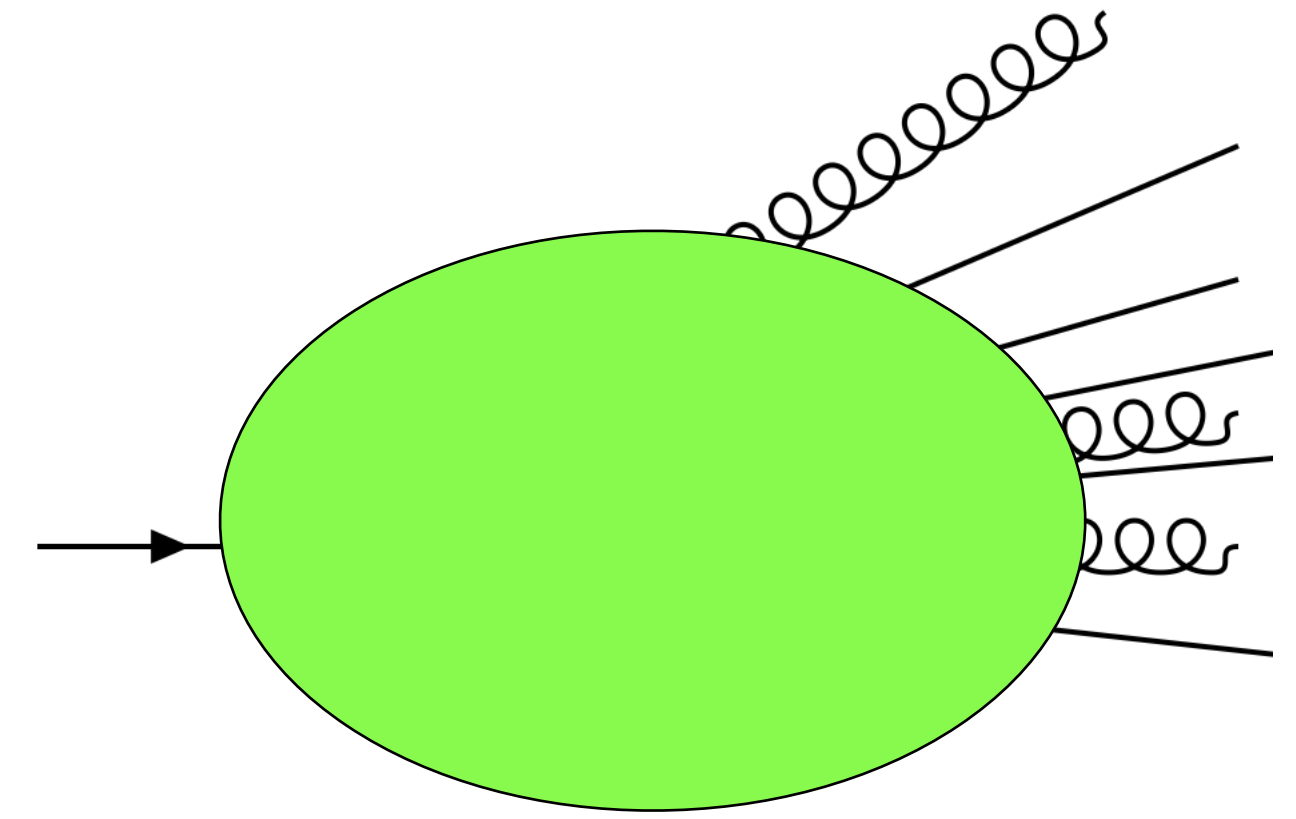
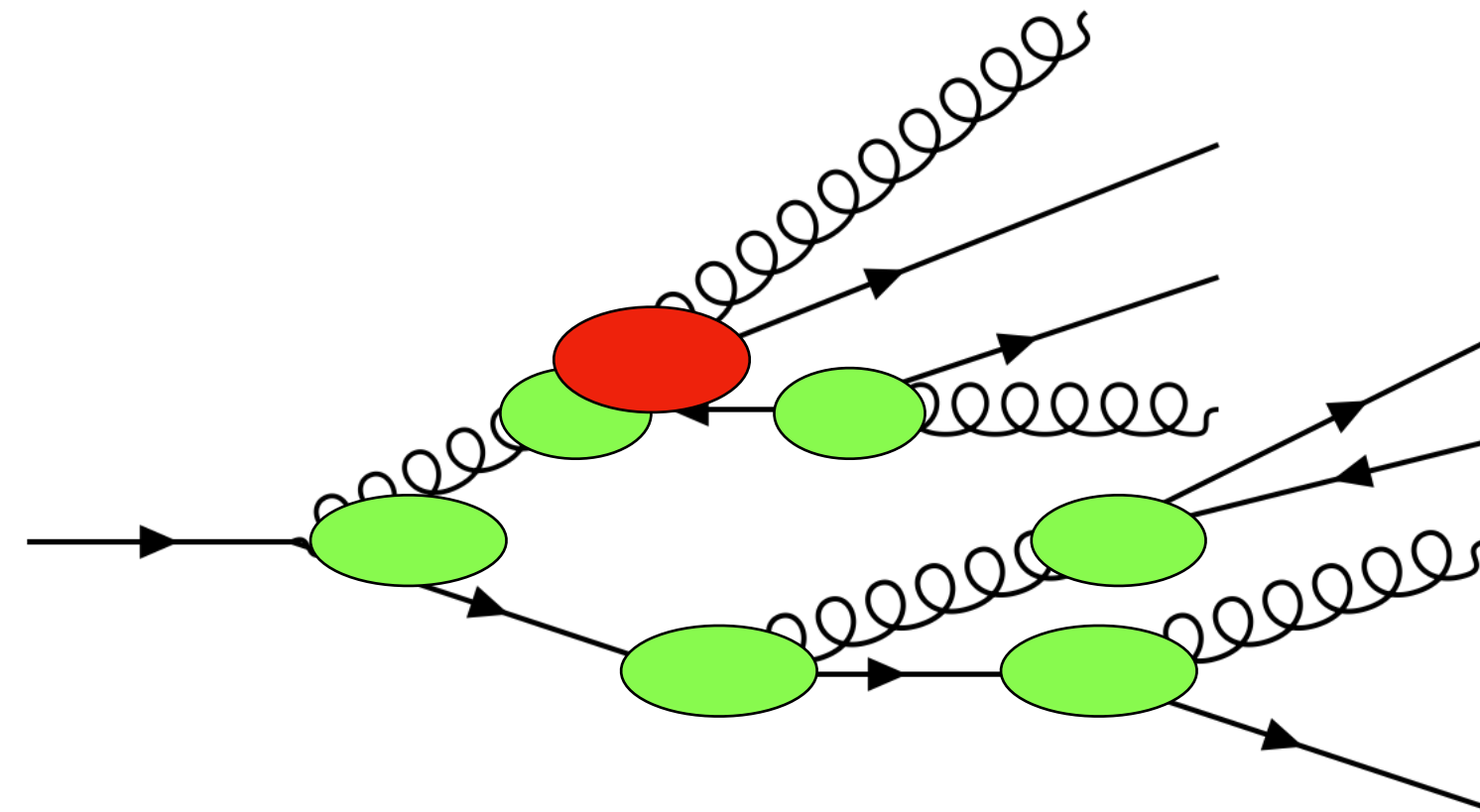
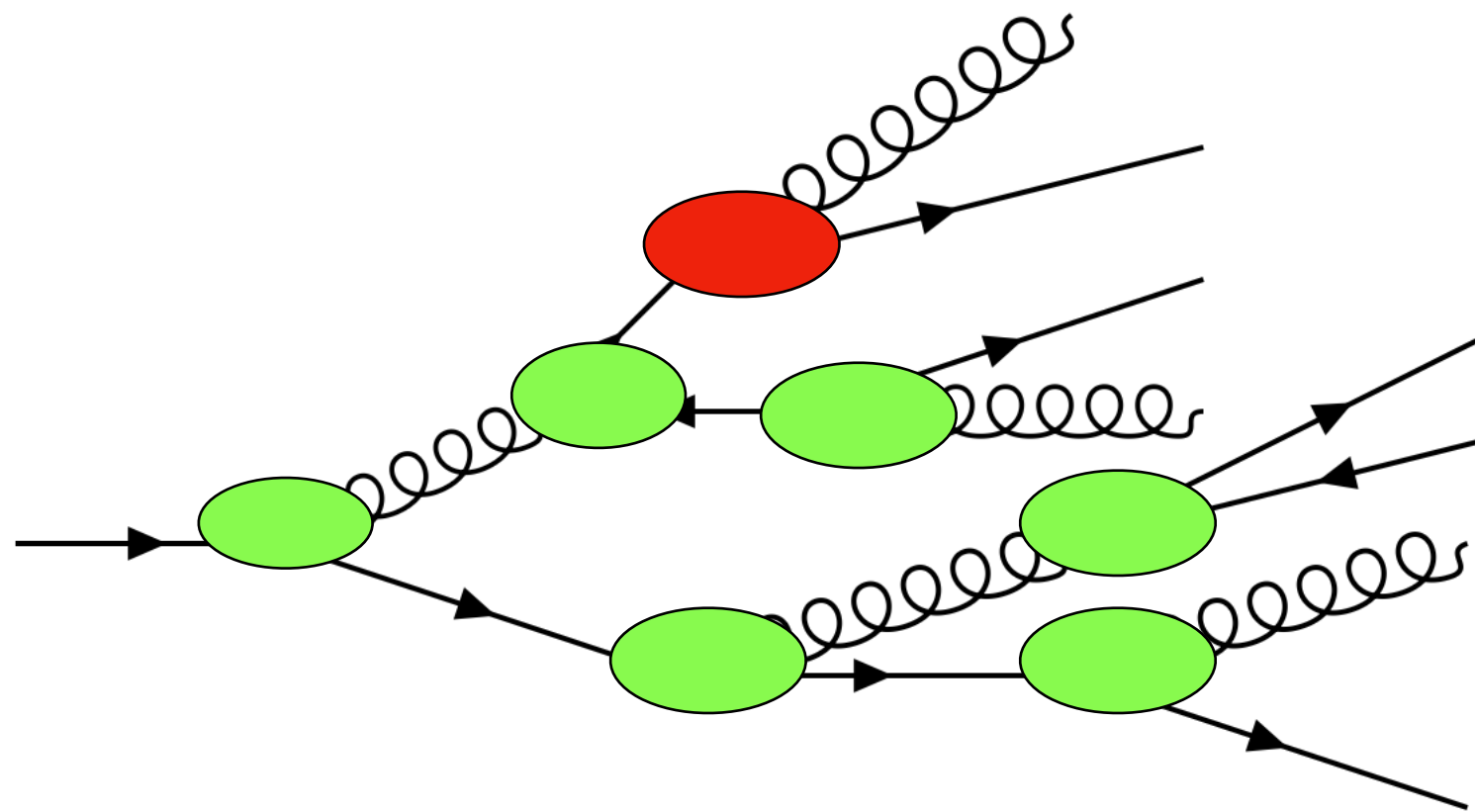
Overlap corrections = $-0.4\% N_f \alpha_s(\mu)$
($N_f \gg N_c \gg 1$, many quarks)

[P.Arnold & **OE** & S.Iqbal]

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Comments

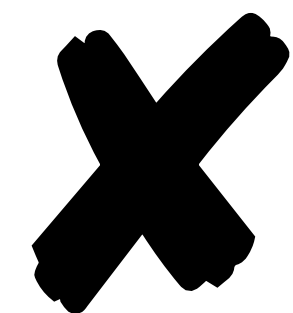
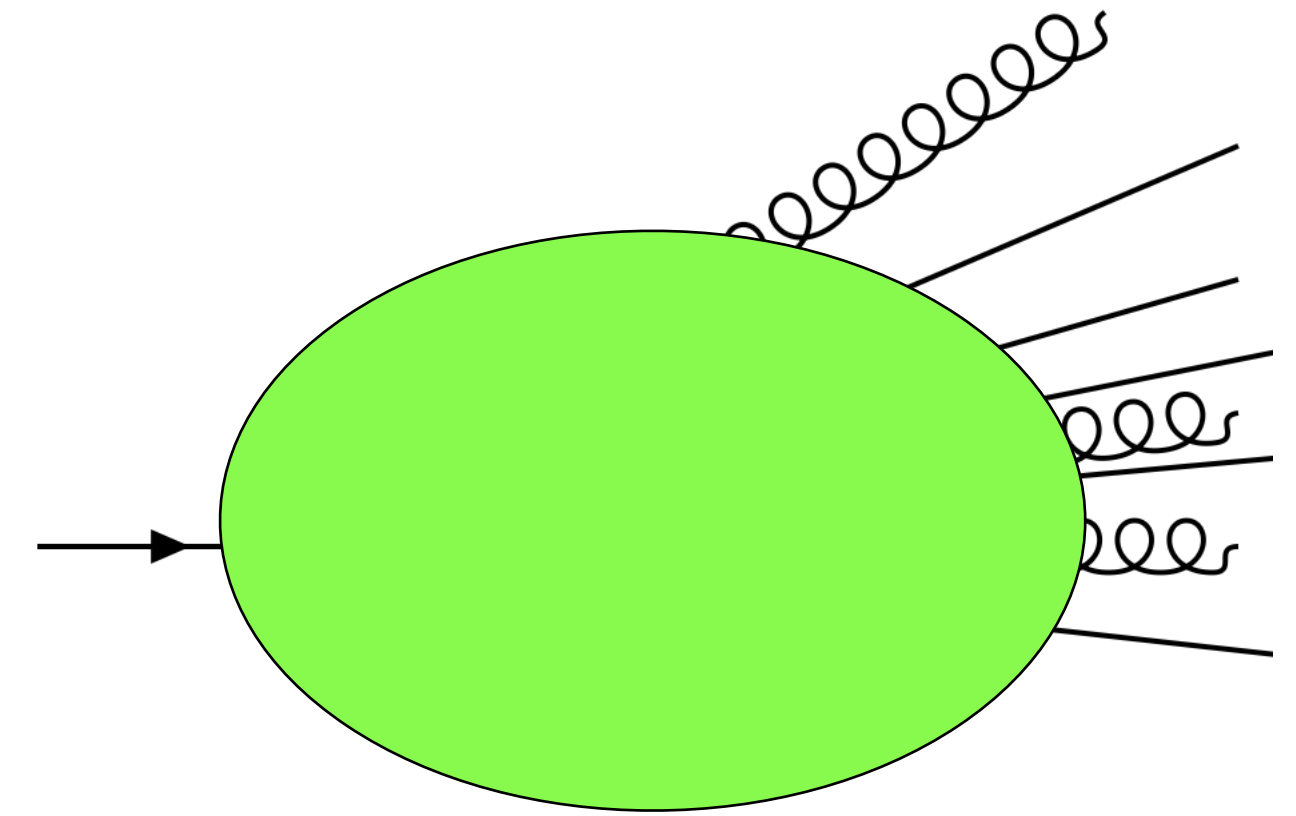
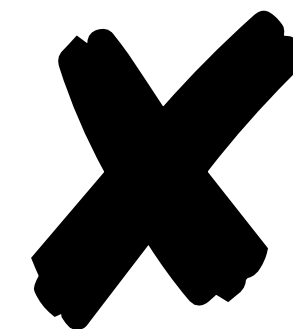
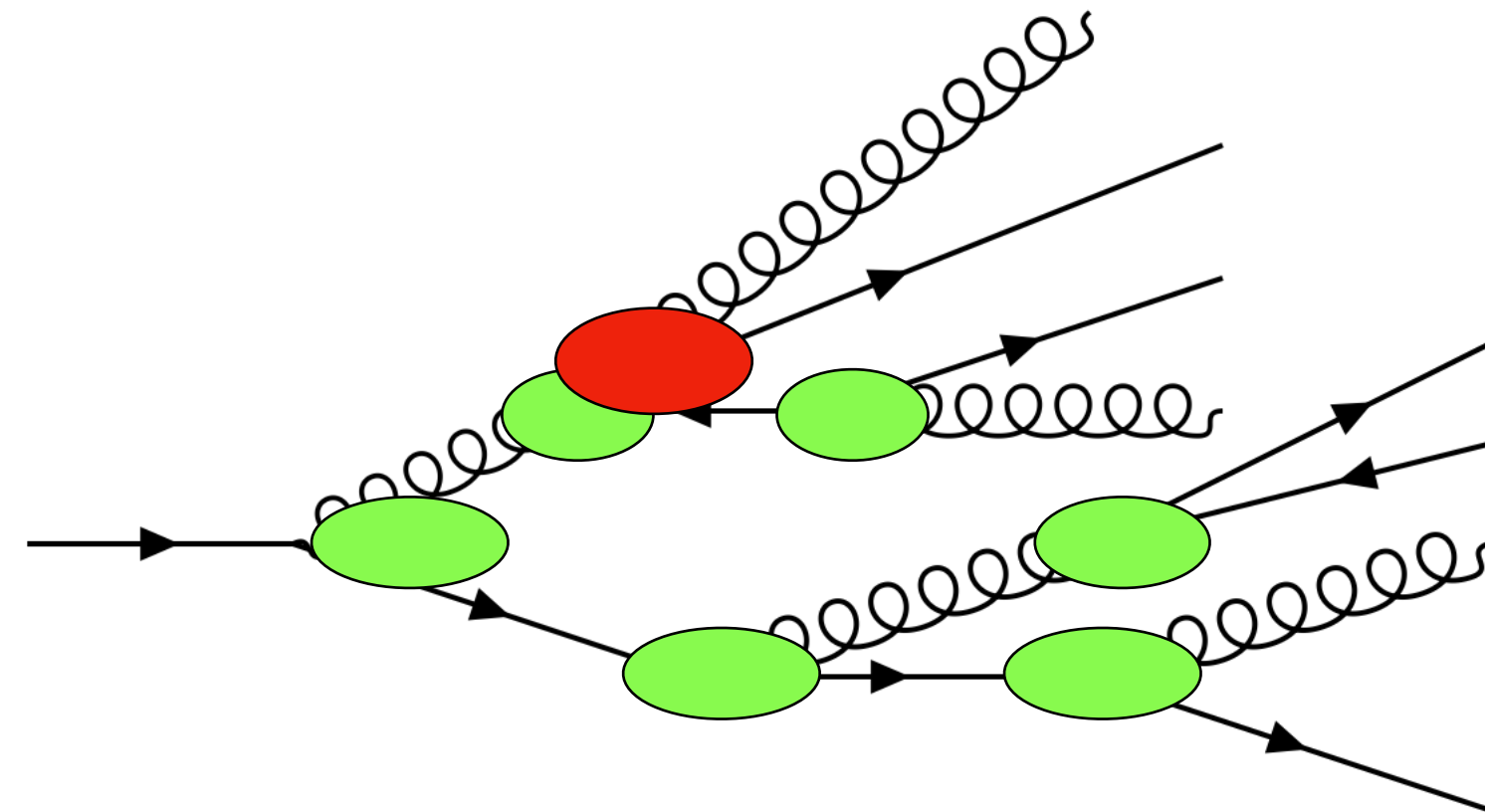
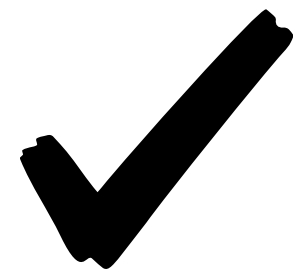
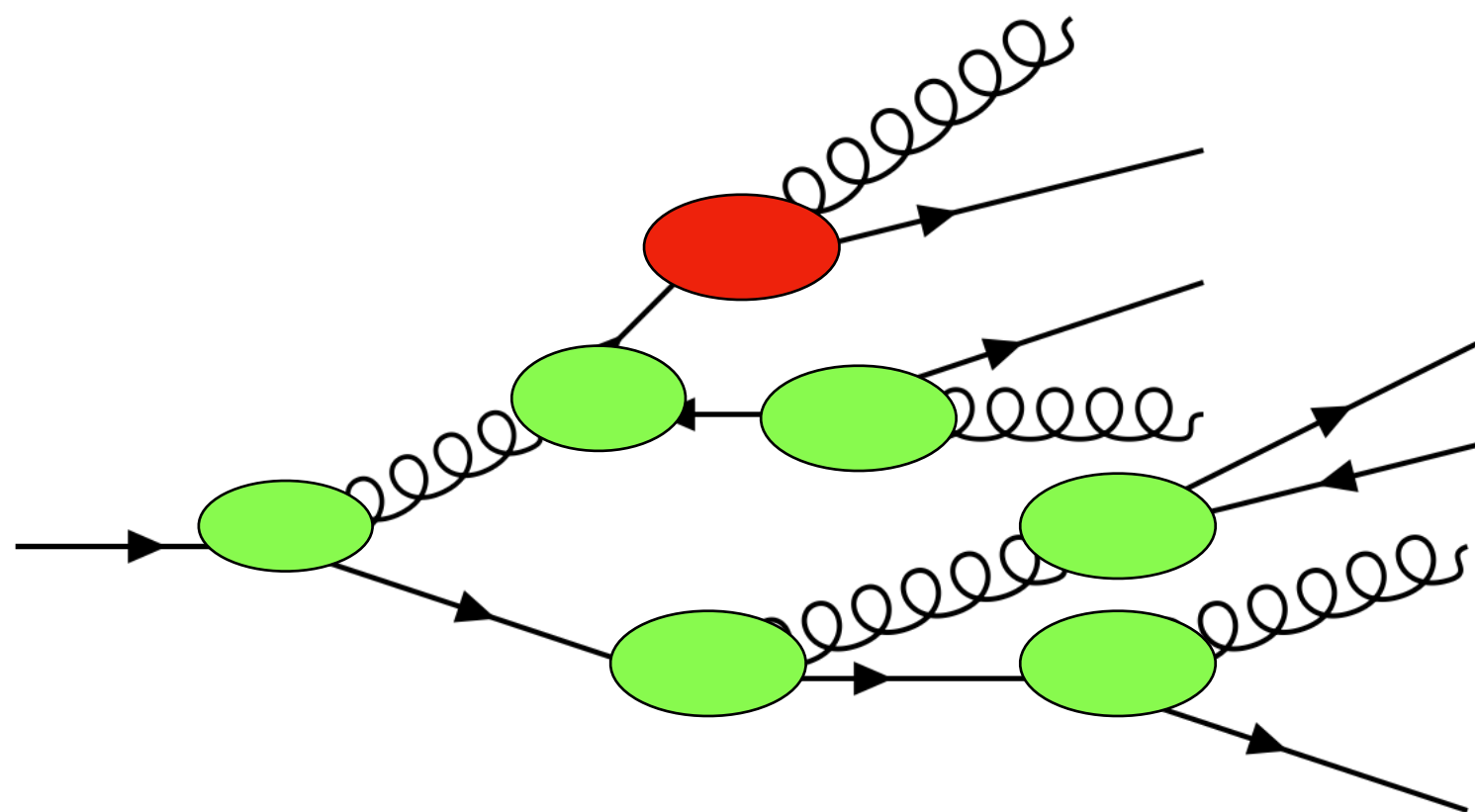
QCD



Comments

QCD: Weakly coupled shower

- Using our simplifying assumptions and theoretical observables :



- Provided one treats $\hat{q}$ as a parameter that can be measured at one scale and then evolved to other scales (known for LL).

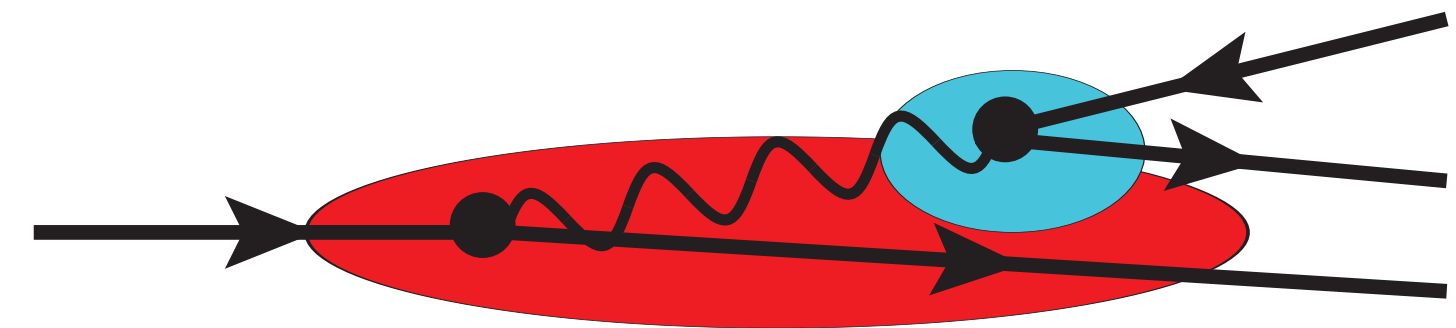
Comments

QED

- Large corrections coming from overlapping soft bremsstrahlung with a subsequent pair production.

- $t_{form}^{e \rightarrow e\gamma} \sim \sqrt{\frac{E}{x_\gamma \hat{q}}}$ in the limit $x_\gamma \rightarrow 0$.

- The soft e^+e^- can be easily scattered by the QED medium, which destroys the coherence of the brem.
- Soft gluons already interact easily with the QCD medium and so overlap does not make the same qualitative change.



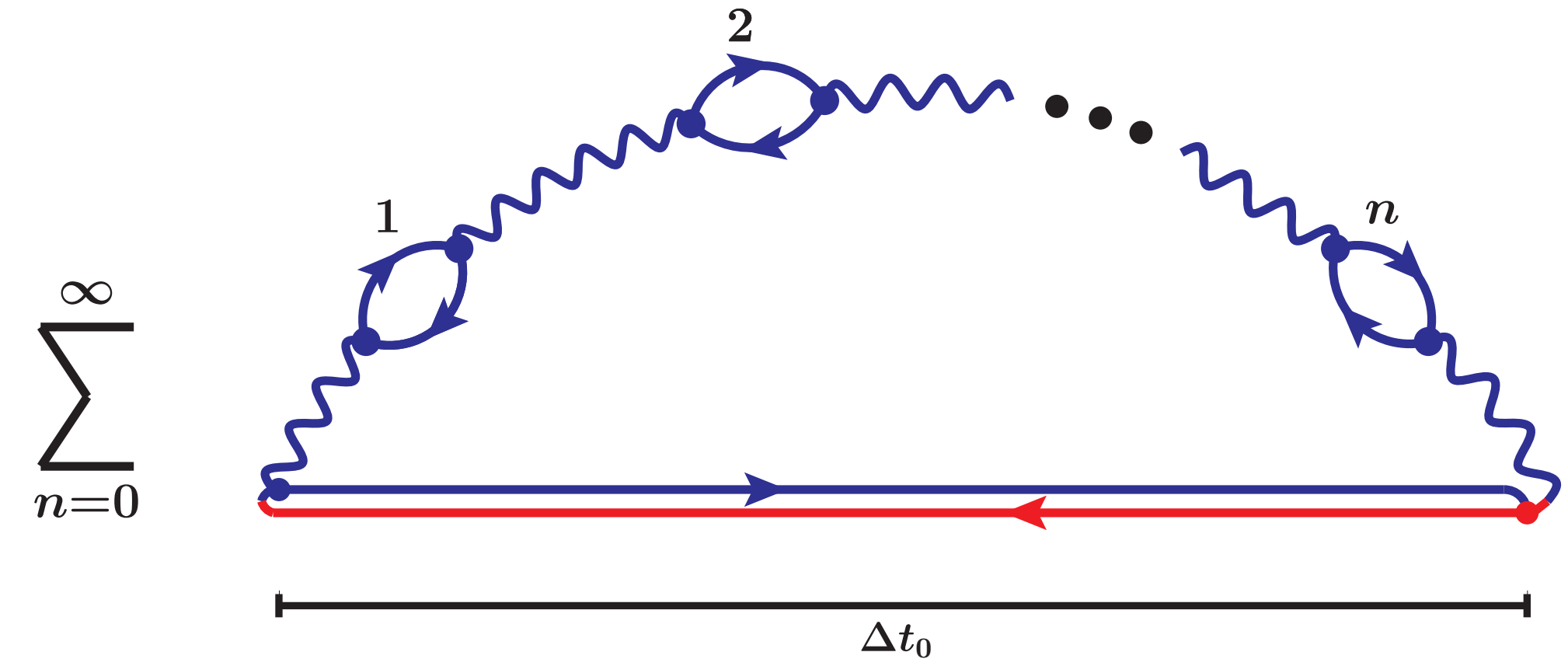
Comments

QED

- Our theoretical observable depends on the NLO/LO ratio

$$\frac{\delta \left[\frac{d\Gamma}{dx_\gamma} \right]_{e \rightarrow e}}{\left[\frac{d\Gamma}{dx_\gamma} \right]_{e \rightarrow e}} \sim \frac{N_f \alpha}{x_\gamma} \text{ in the limit } x_\gamma \rightarrow 0.$$

- A precise calculation of the rate for $x_\gamma \lesssim N_f \alpha$ in QED would require resumming higher-order effects.



The cost of adding a loop is $\frac{N_f \alpha}{x_\gamma}$.

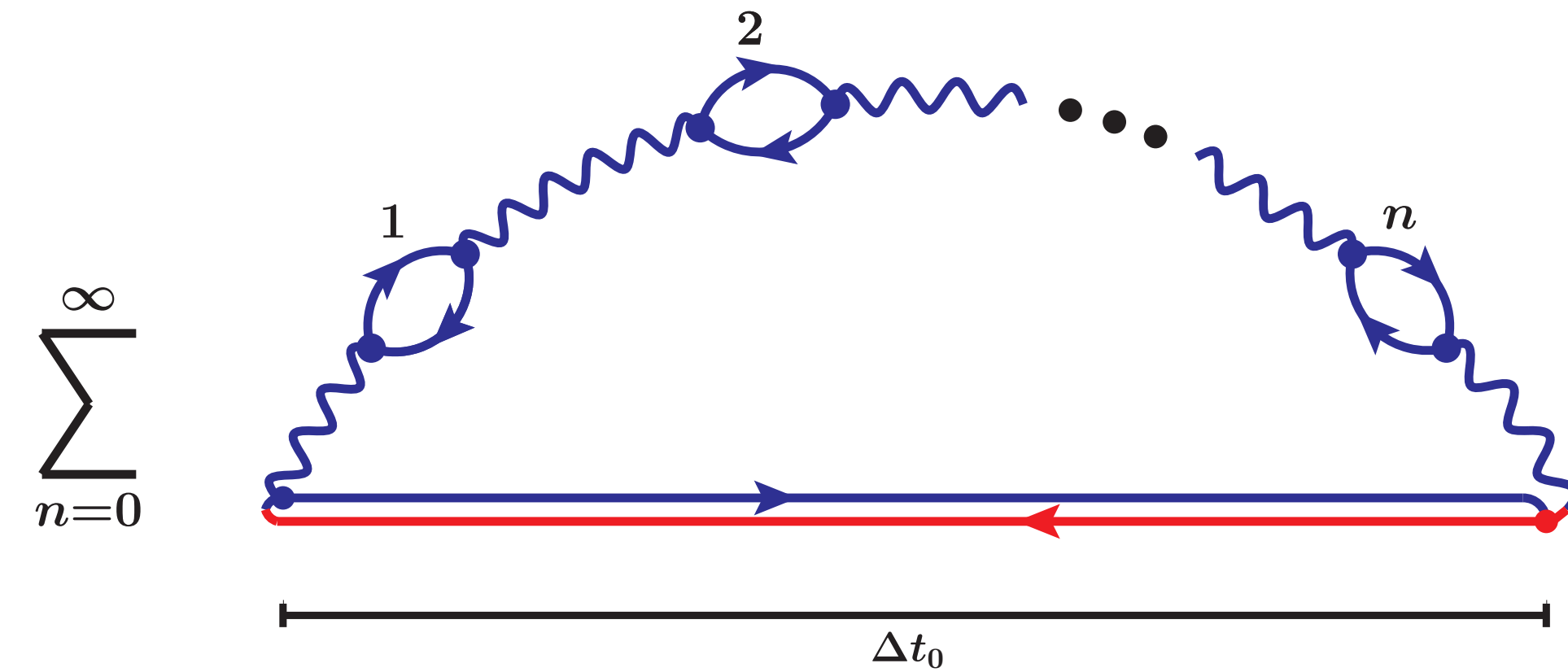
Bubble Resummation

Resumed rate

- In the soft photon limit, the rate factorizes into:

$$\left[\frac{d\Gamma}{dx_\gamma} \right]_{n \geq 0} \simeq 2Re \int_0^\infty d(\Delta t_0) \left[\frac{dg}{dx_\gamma d\Delta t_0} \right]_{brem} e^{-g_{pair} \Delta t_0}$$

Note $\Gamma_{pair} = 2Re g_{pair}$

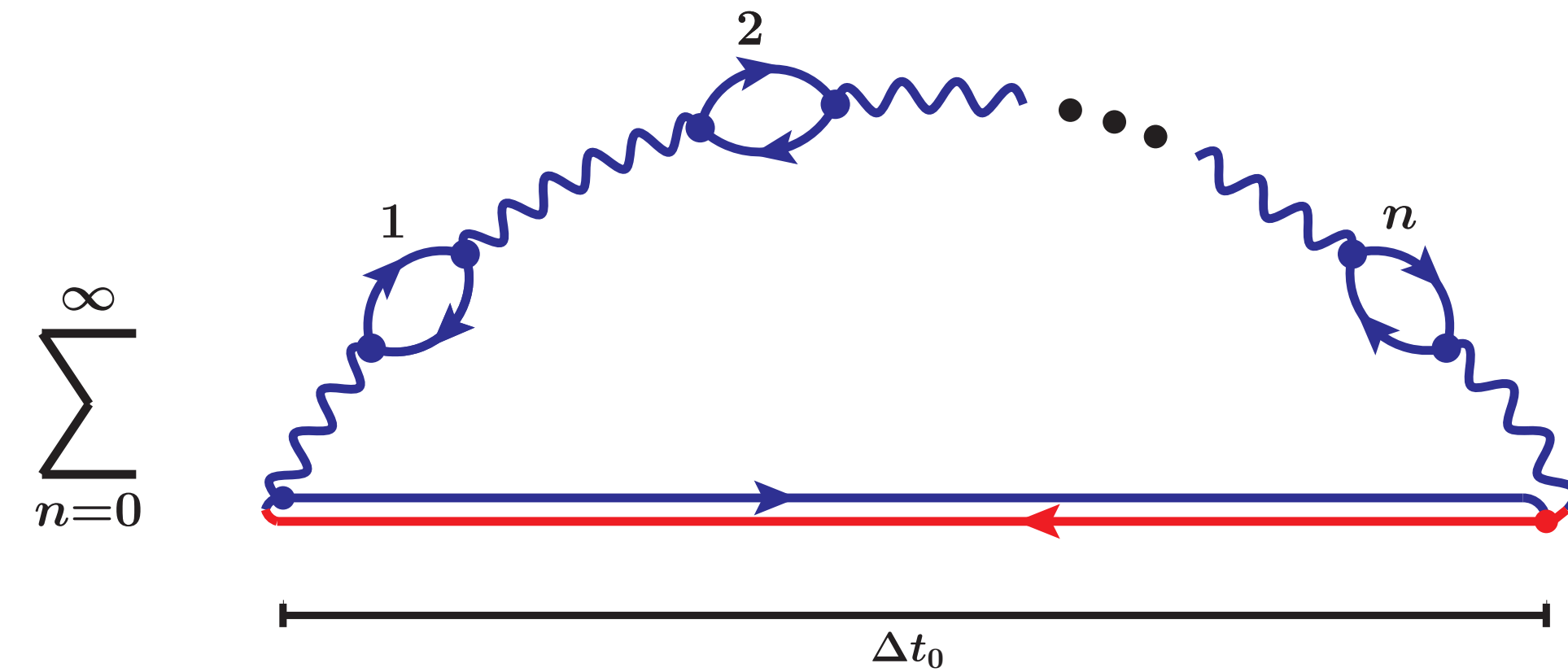


Bubble Resummation

Resumed rate

- In the soft photon limit, the resumed LPM rate becomes

$$\left[\frac{d\Gamma}{dx_\gamma} \right]_{\text{LPM}+} = \left[\frac{d\Gamma}{dx_\gamma} \right]_{\text{LPM}} + \delta \left[\frac{d\Gamma}{dx_\gamma} \right]$$



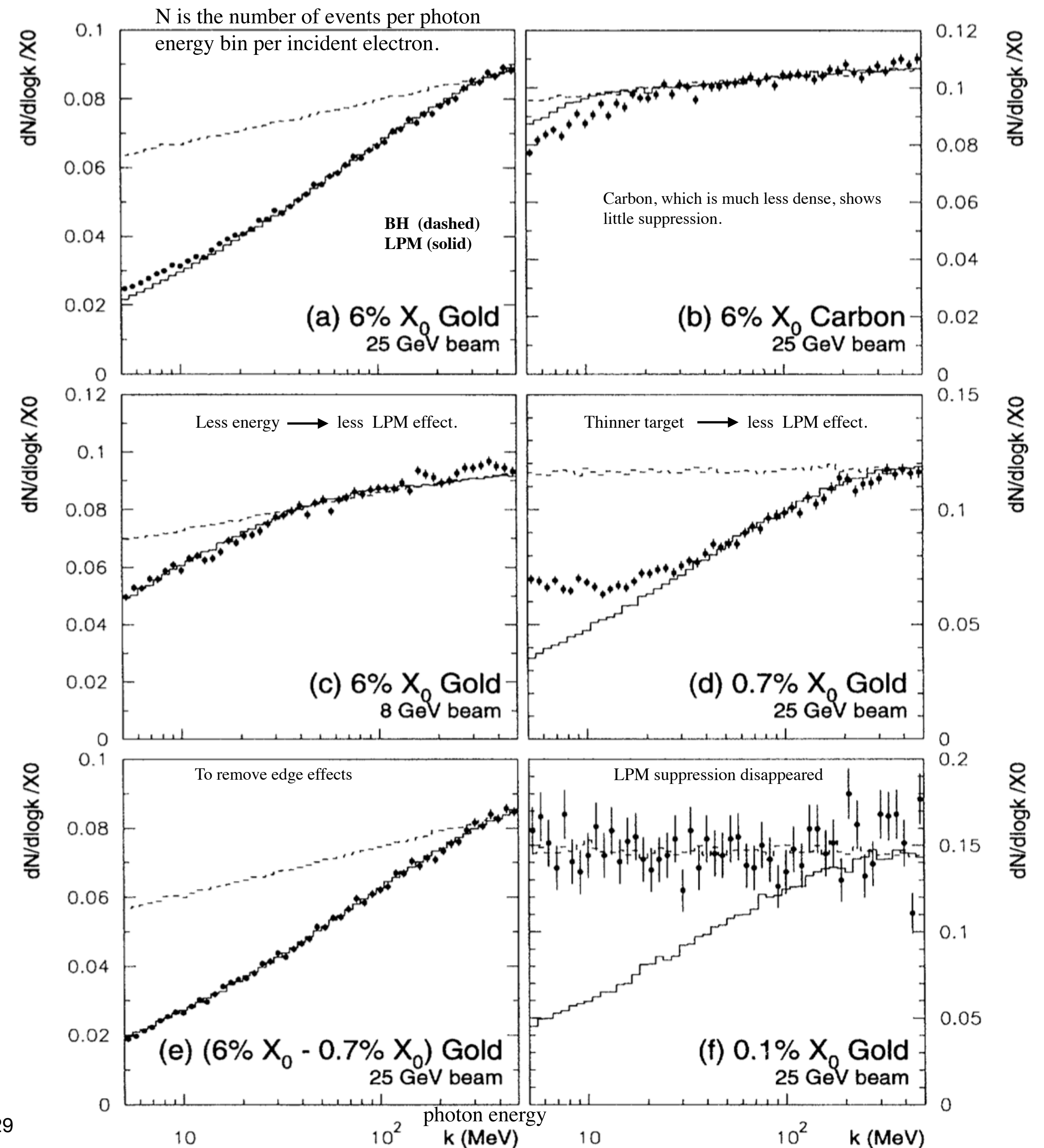
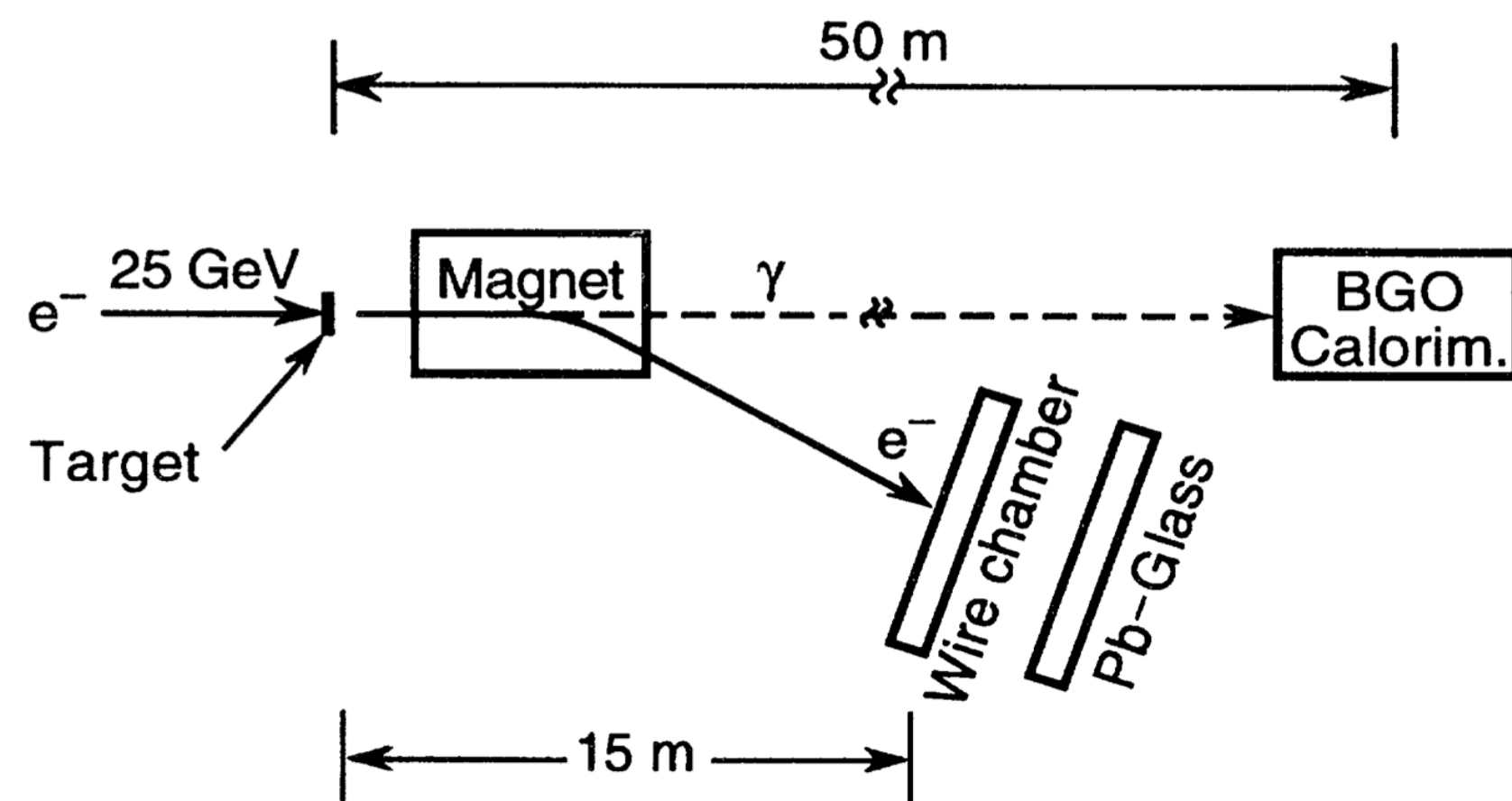
The first LPM experiment

SLAC [Anthony *et al.*, PRL.75.1949]

They sent 8 and 25 GeV electrons through thin gold targets and measured the bremsstrahlung spectrum.

At low photon energies, the radiation rate was suppressed compared to Bethe-Heitler.

The suppression matches Migdal's LPM prediction to within 5%.



The second LPM experiment

CERN [Hansen *et al.*, PRL 91, 014801]

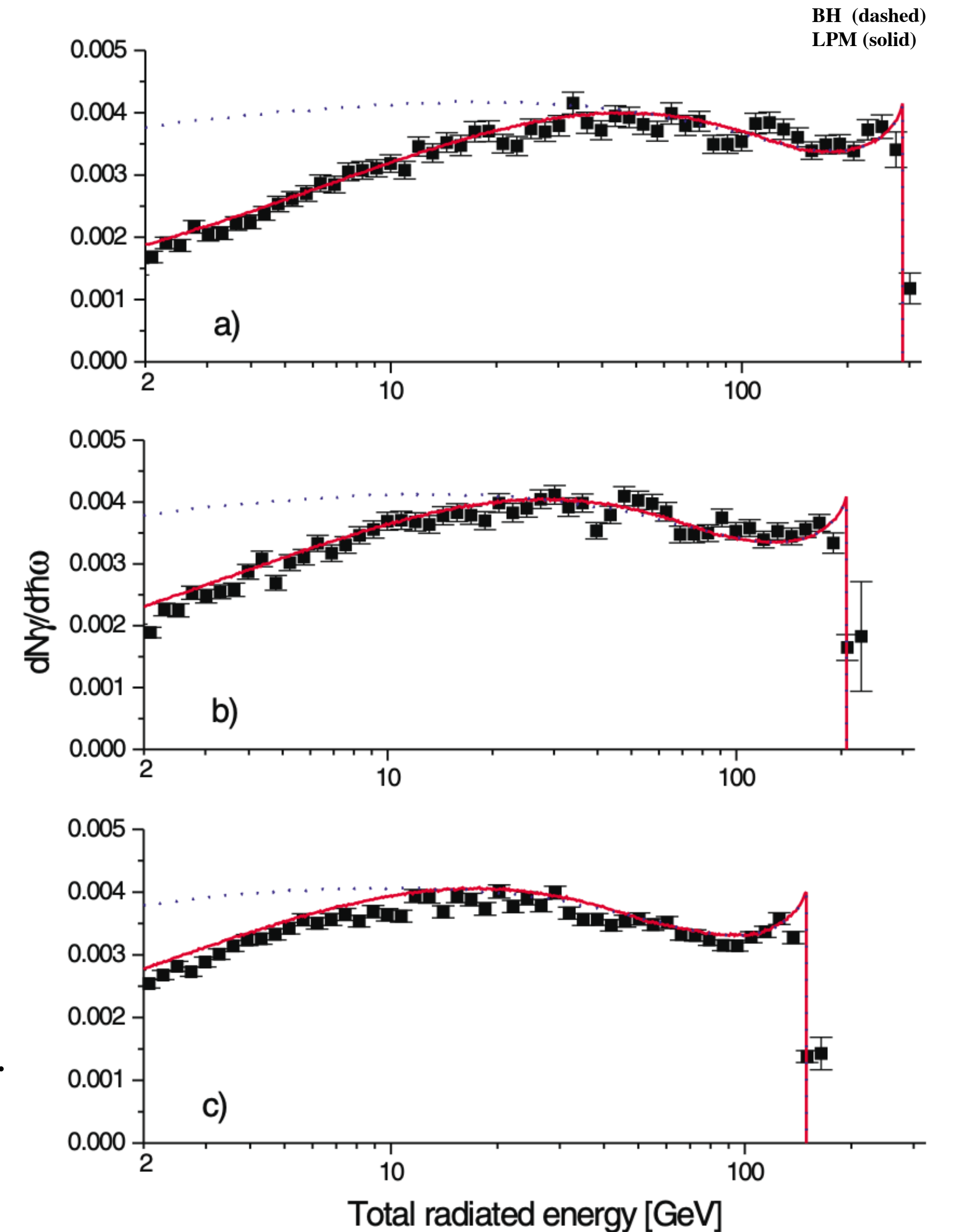
Bremsstrahlung spectrum (a) 287, (b) 207, and (c) 149 GeV electrons on 0.128 mm Ir (4.36 % X_0).

- $t_{form}^{e \rightarrow e\gamma} \sim \sqrt{\frac{E}{x_\gamma \hat{q}}}$ in the limit $x_\gamma \rightarrow 0$.

$$t_{form} \gg \tau_{el}$$

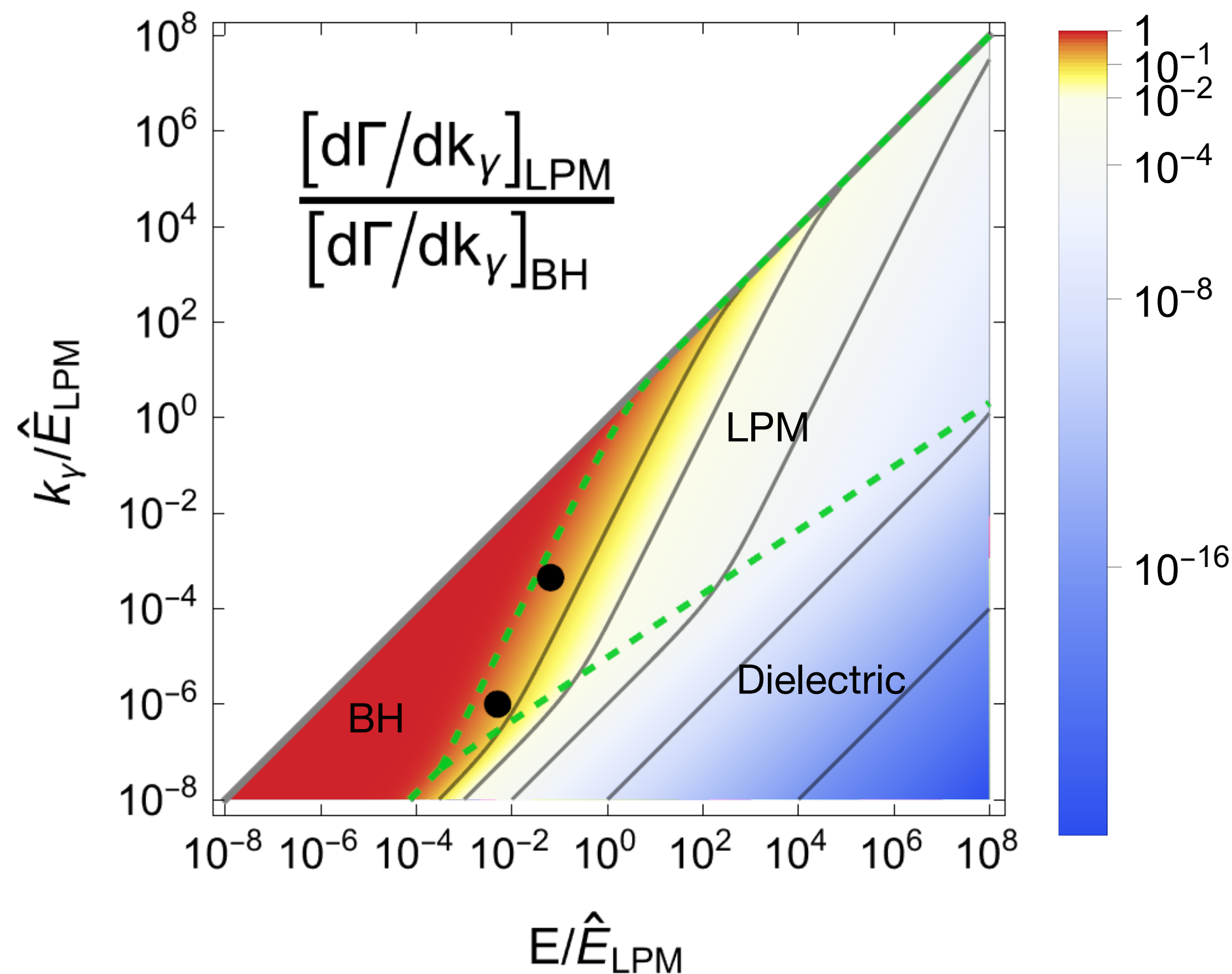
$$k_\gamma \ll \frac{E^2}{E_{LPM}}$$

The LPM effect is found at photon energies approximately proportional to E^2 .



Results

LPM rate



Here $\alpha = 1/137$ and $m_\gamma/m = 10^{-4}$ (roughly the value for Lead or Gold).

Given the scaling of the axes, the plots are independent of the value of E_{LPM} .

The value of m_γ/m only affects the location of the dielectric-effect regions.

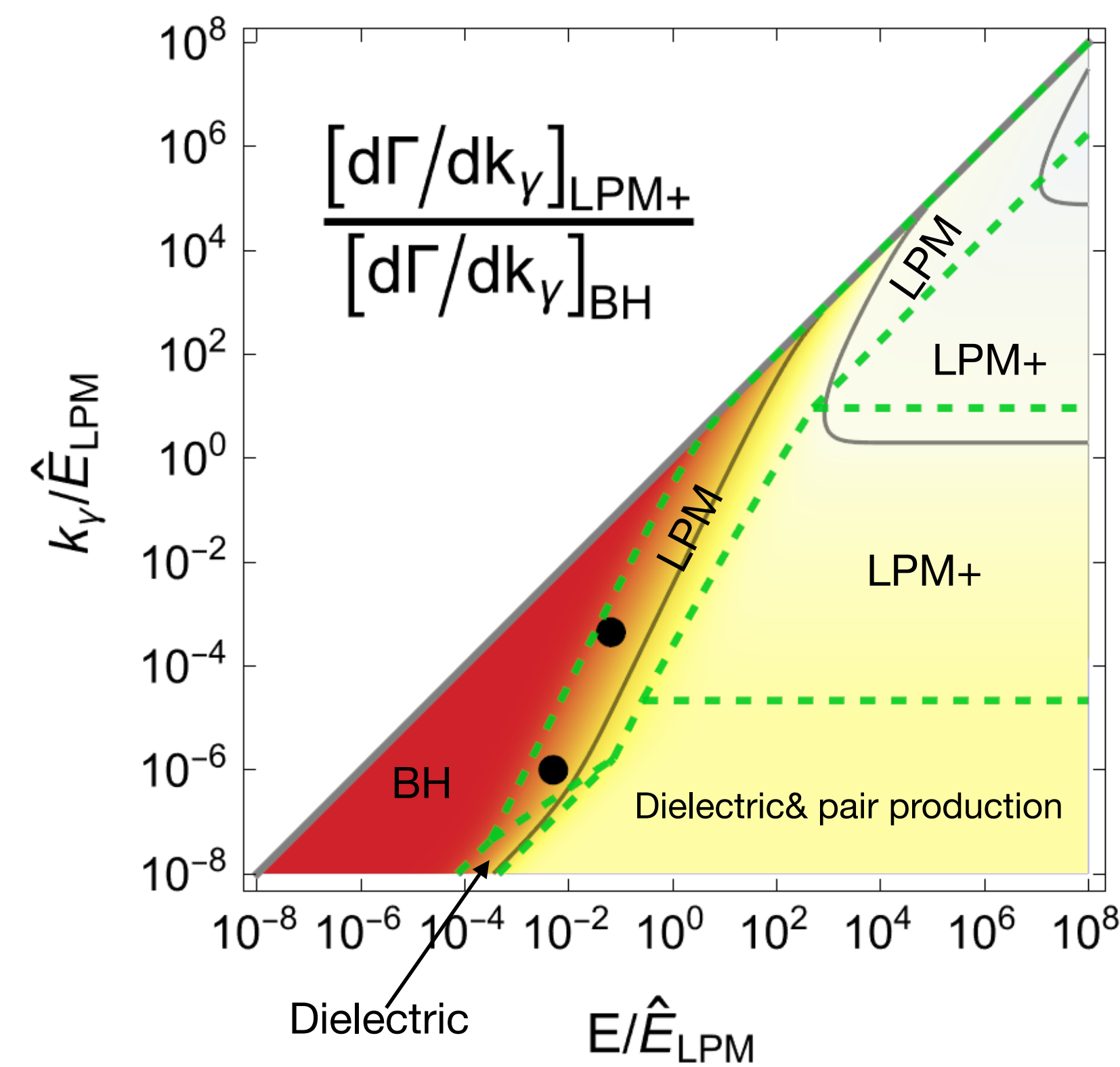
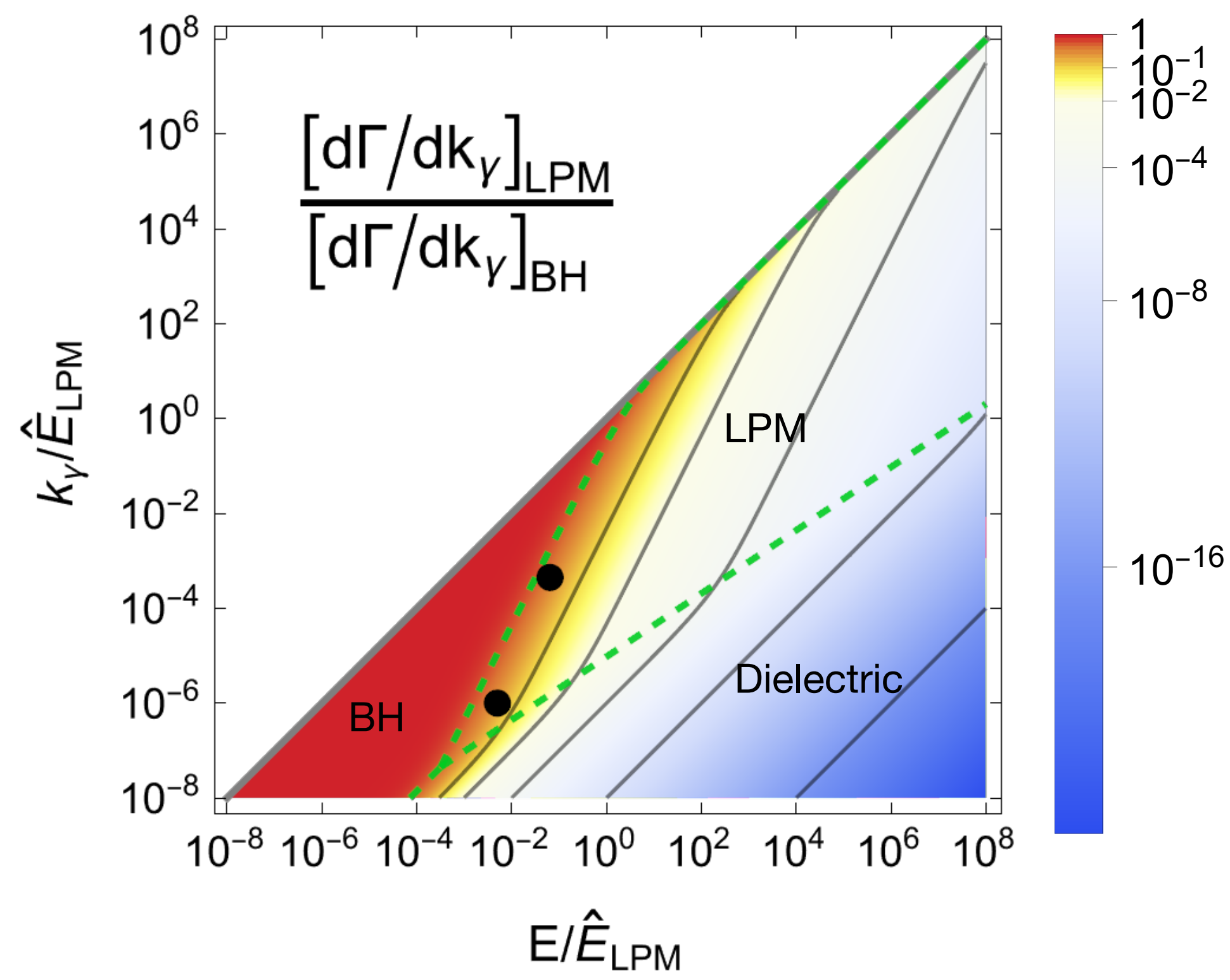
The two black dots indicate 2 examples each of (k_γ, E) from the range of values tested by two dedicated LPM experiments: $(5 \text{ MeV}, 25 \text{ GeV})$ in Gold and $(2 \text{ GeV}, 287 \text{ GeV})$ in Iridium, for which $\hat{E}_{LPM} \simeq 2E_{LPM} \simeq 5.0$ and 4.5 TeV respectively.

[Klien, Reviews of Modern Physics, Vol. 71, No. 5, October 1999]

E_{LPM} is an energy scale determined by properties of the medium and is, for example, about $E_{LPM} = 2.5 \text{ TeV}$ for Gold.

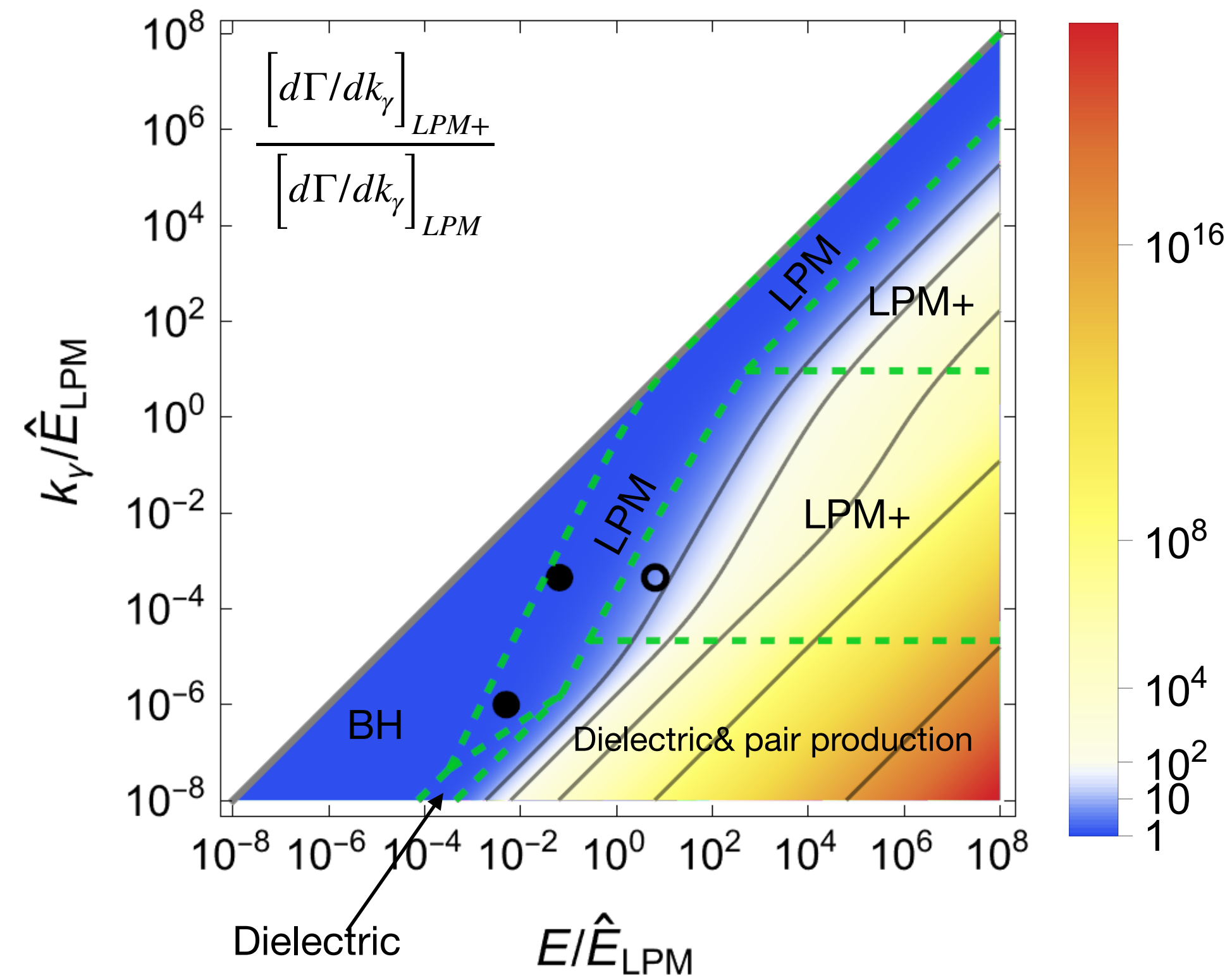
Results

LPM rate



Results

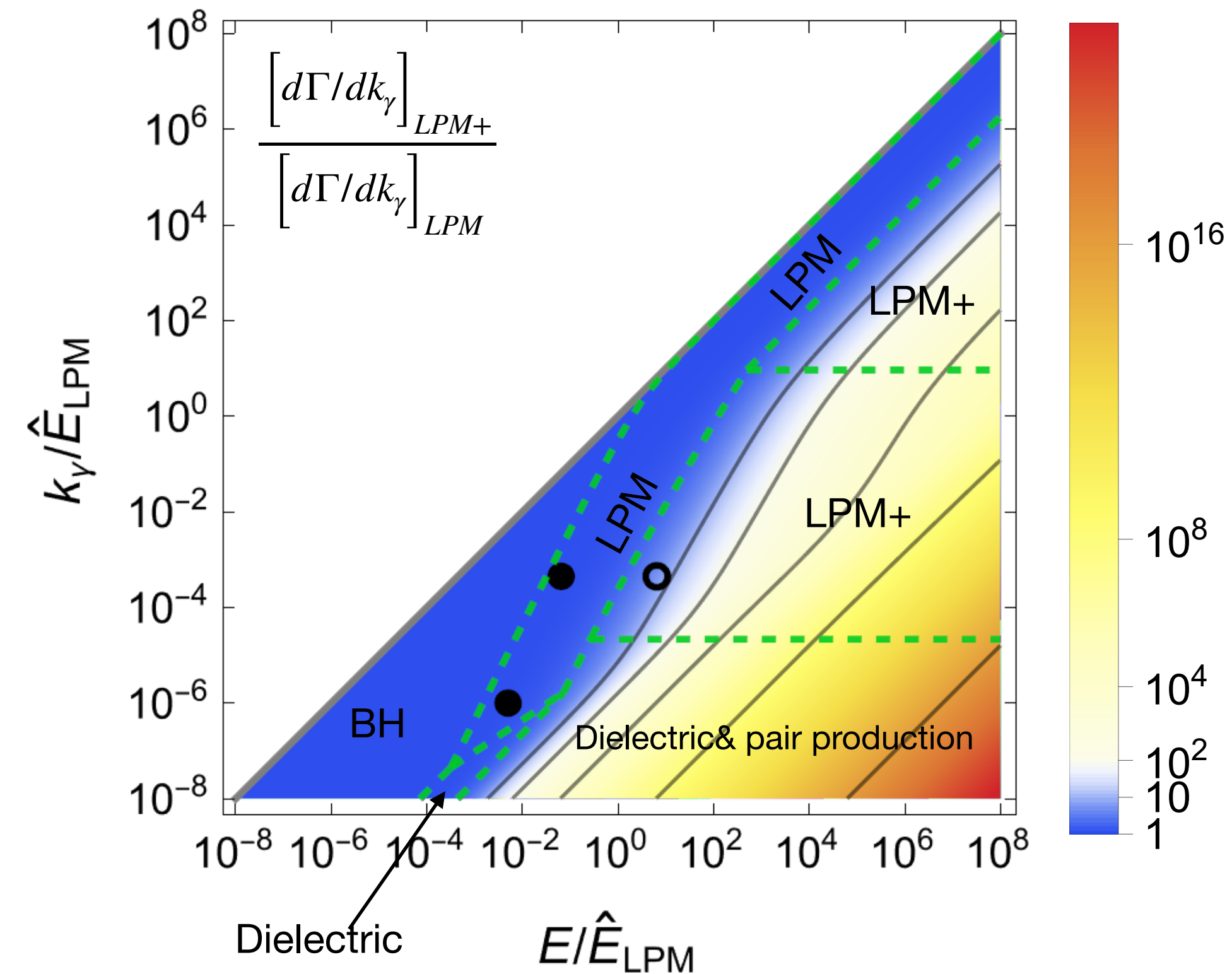
LPM vs. LPM+



Log-log contour plot of the LPM+ /LPM ratio of the LPM+rate (which includes overlapping pair production) to the LPM rate (which does not).

Results

LPM vs. LPM+



Log-log contour plot of the LPM+ /LPM ratio of the LPM+rate (which includes overlapping pair production) to the LPM rate (which does not).

If a~50 TeV proton beam became available (FCC-hh accelerator), and if the concept of the CERN LPM experiment could be scaled up by a factor of 100 in electron energy, that might open the possibility of exploring the small open circle where the LPM correction is significant.

[P.Arnold & **OE** & S.Iqbal]

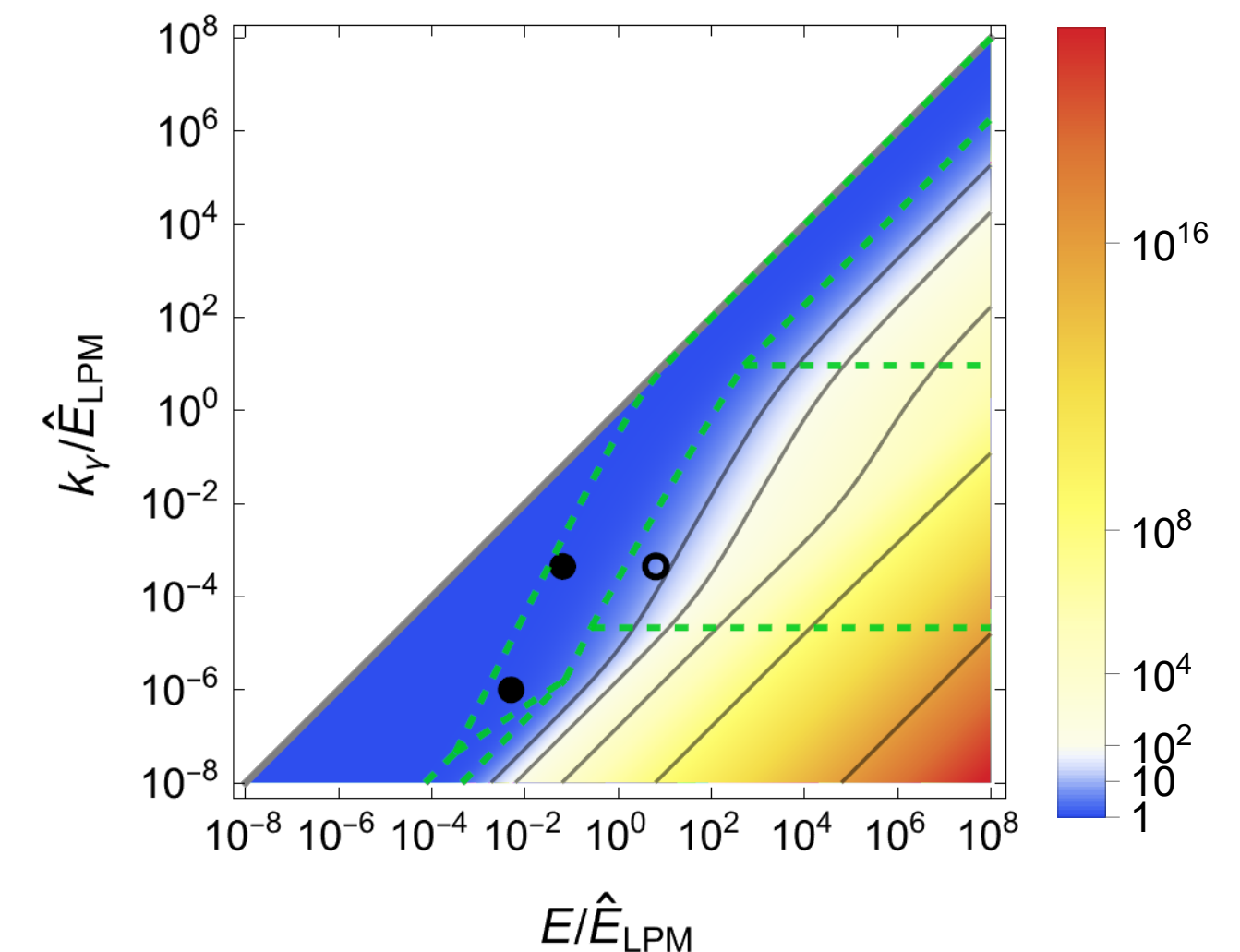
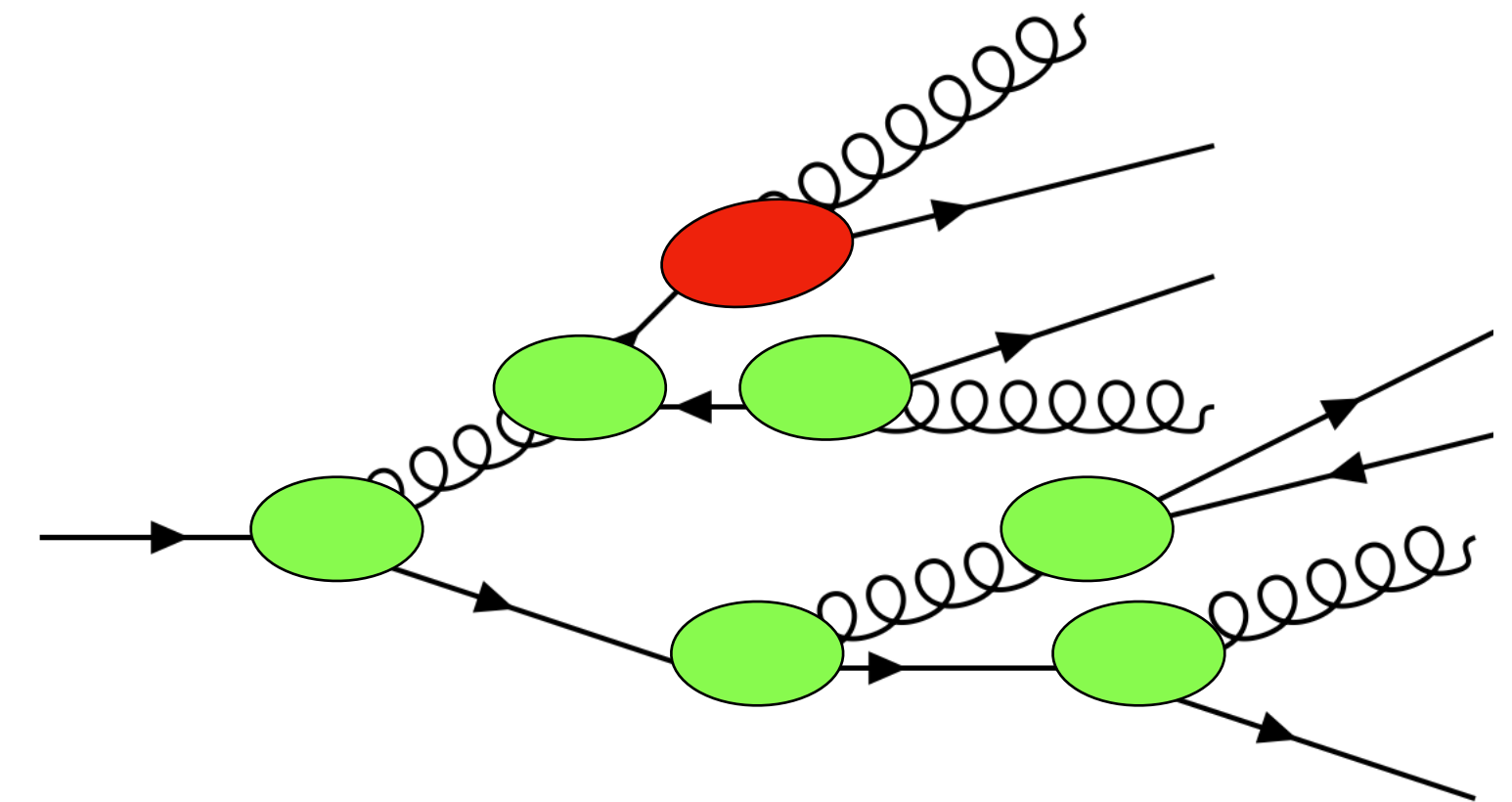
JHEP 03 (2026) 015

ArXiv:2604.18685 [hep-ph]

ArXiv:2605.03002 [hep-ph]

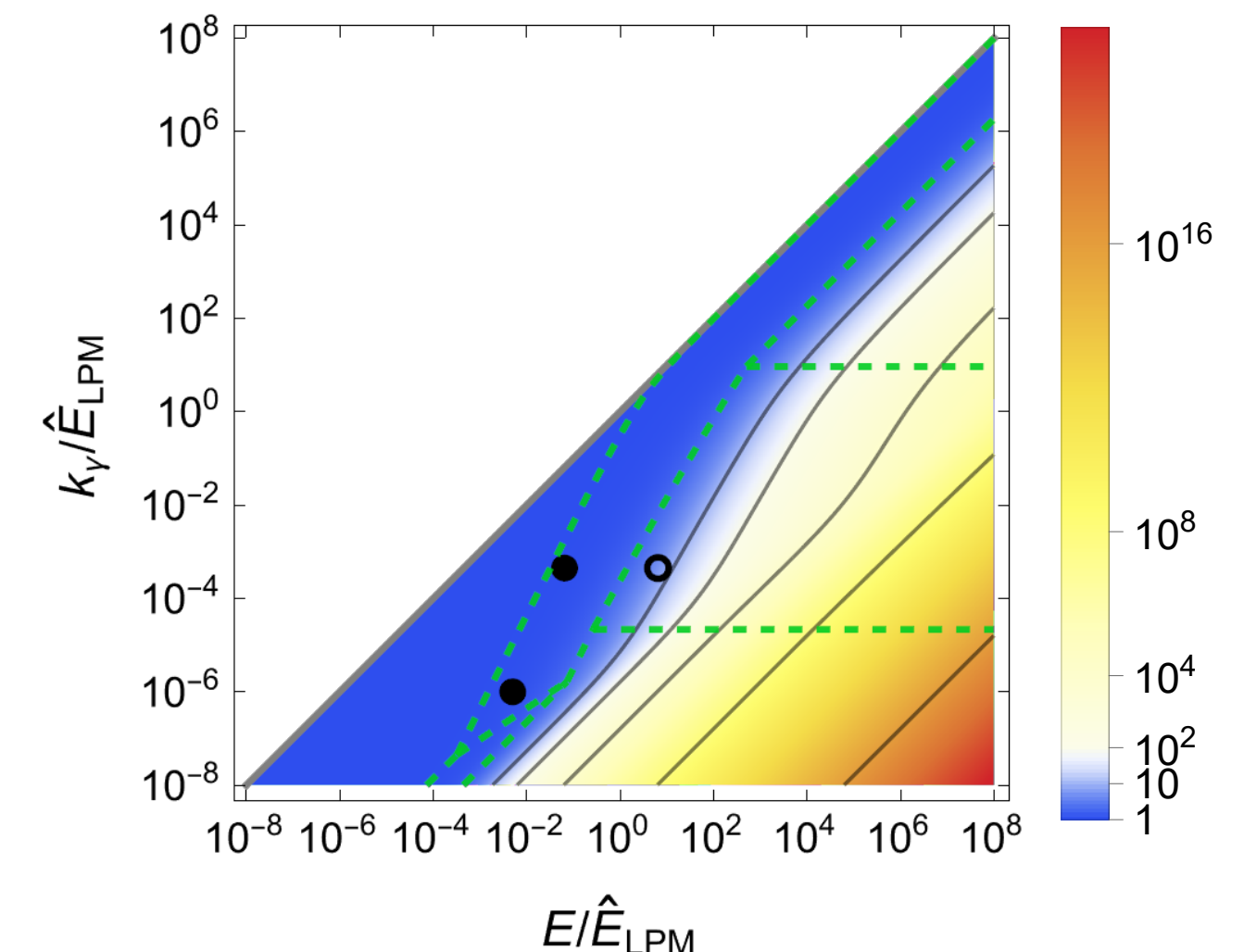
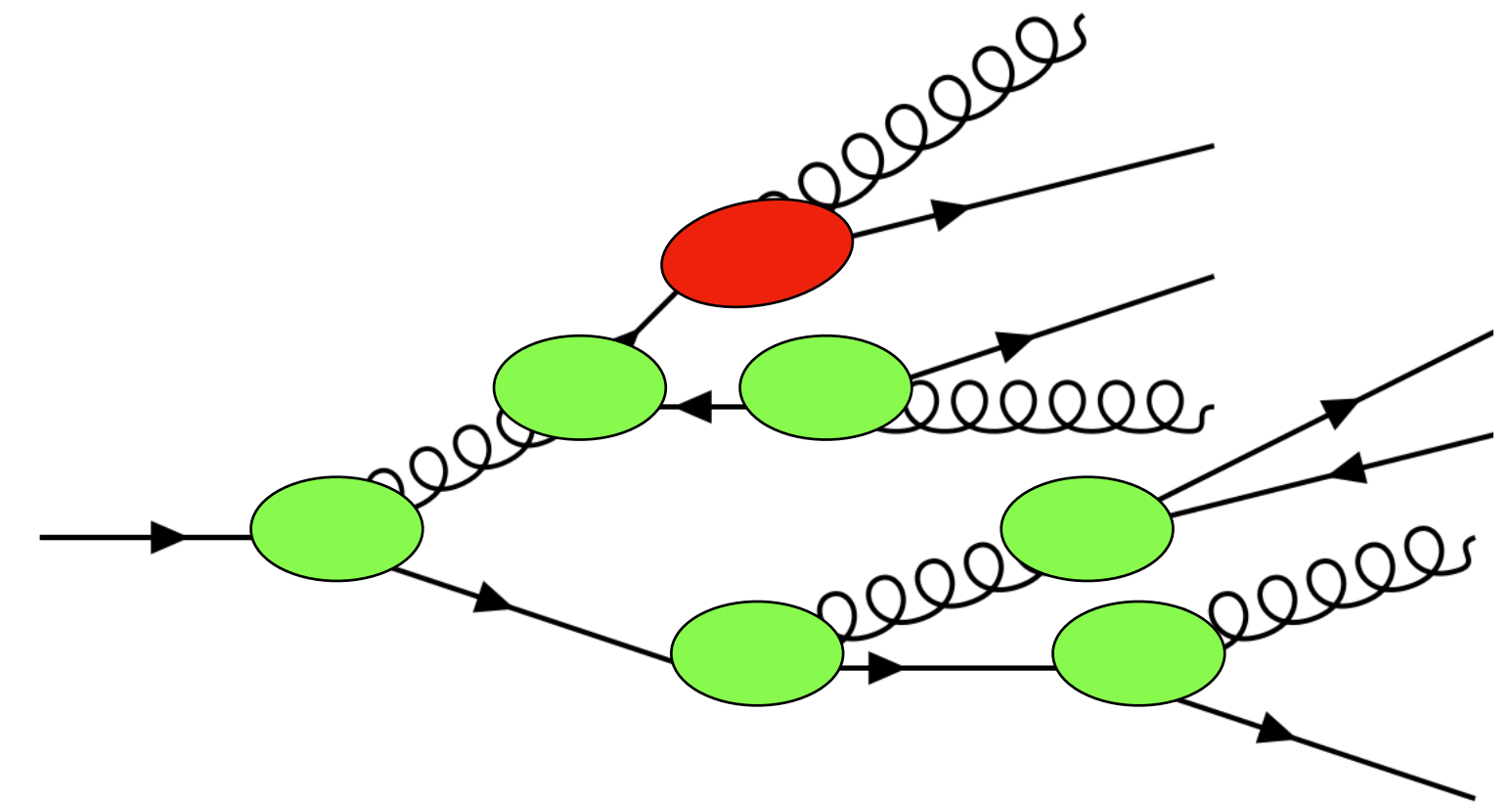
Summary and Conclusion

- QCD showers are weakly coupled. Provided one treats $\hat{q}$ as a parameter that can be measured at one scale and then evolved to other scales (known for LL).
- QED, on the other hand, gets large modifications from overlapping bremsstrahlung with pair production, significantly increasing the bremsstrahlung rate compared to the LPM calculation.



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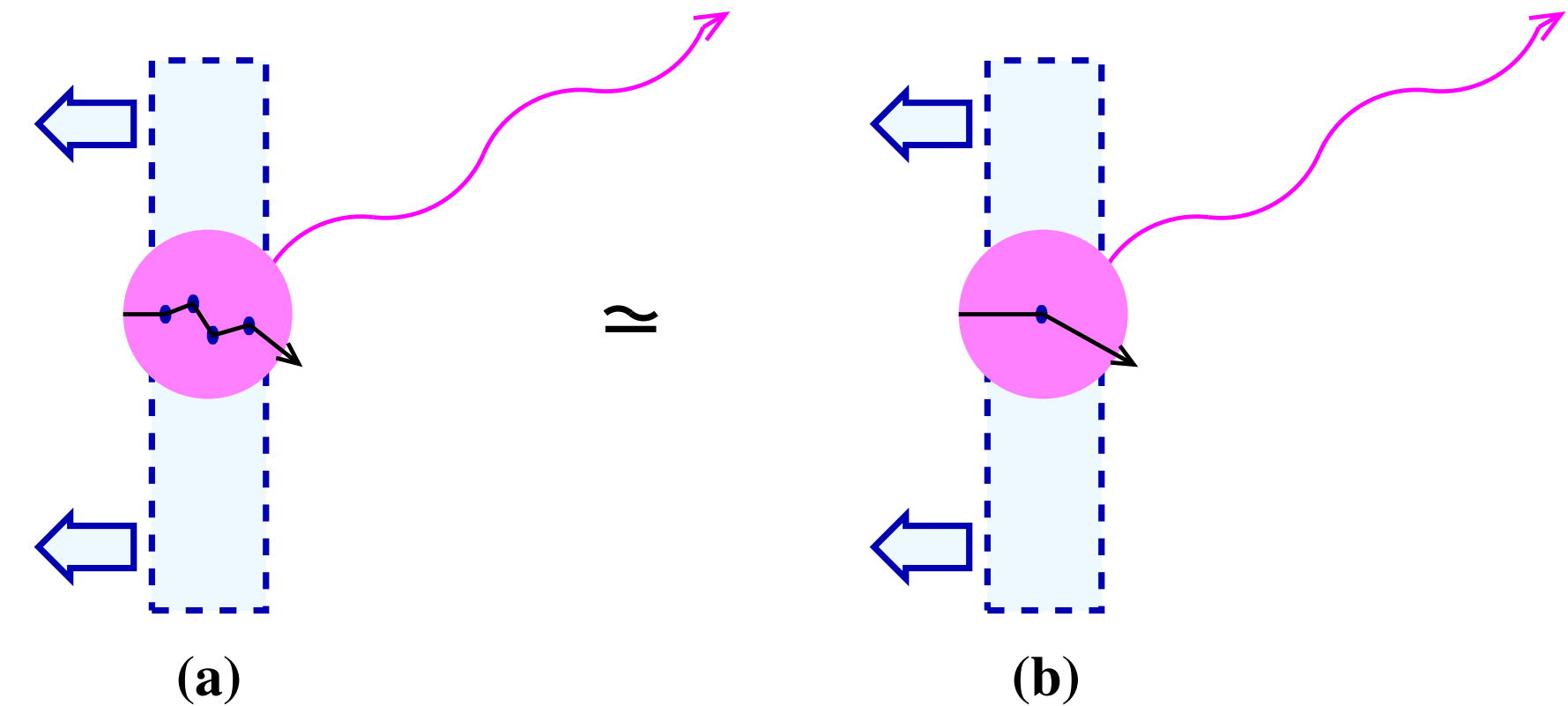
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THANKS FOR YOUR ATTENTION

The LPM formation time

- The bremsstrahlung is insensitive to details of the collisions that fit within a wavelength.
- This could be written as $|\Delta x_\mu k^\mu| \lesssim 1$ (true in any frame).



- The formation time will be given as

$$1 \sim |\Delta x_\mu k^\mu| \simeq k_\gamma t_{form} (1 - \cos \theta_{\gamma e}) \sim k_\gamma t_{form} \theta_{\gamma e}^2 \longrightarrow t_{form} \sim \frac{1}{k_\gamma \theta_{\gamma e}^2}$$
- In QED, $\theta_{\gamma e} \sim \theta_{scatt} \sim \frac{\Delta p_\perp}{p_z}$

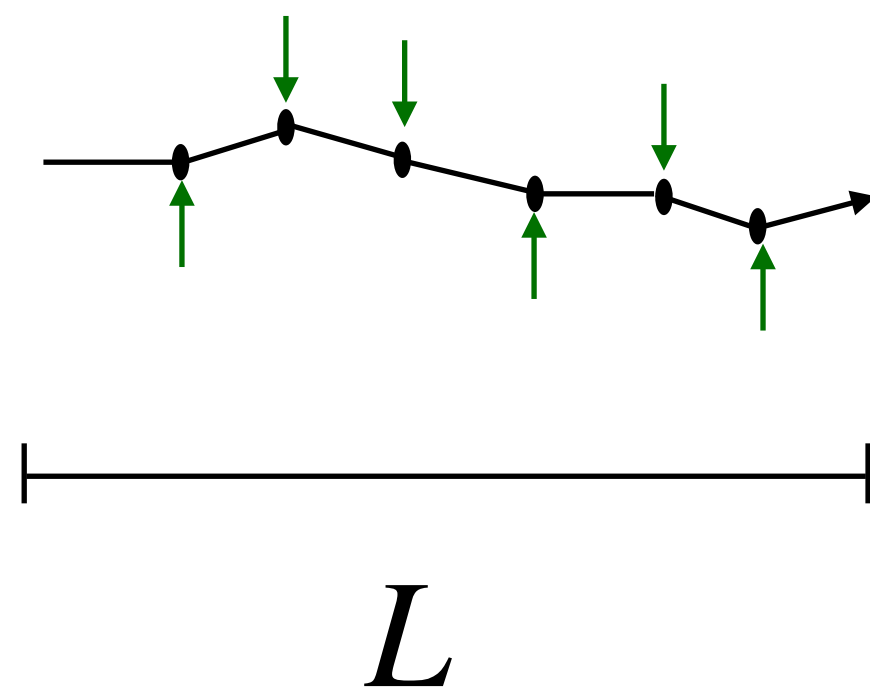
$\nearrow$
 $\frac{\sqrt{\hat{q} t_{form}}}{E}$ for LPM
 $\searrow$
 $\frac{m}{E}$ for BH

What do we mean by a “potential” ?

The $\hat{q}$ approximation

- Random walk in the $p_{\perp}$ space, which can be described by a diffusion eq for the probability distribution $\Psi(p_{\perp}, t)$.

$$\partial_t \Psi(\mathbf{p}_{\perp}, t) = -\Gamma_{\text{el}} \Psi(\mathbf{p}_{\perp}, t) + \int d^2 q_{\perp} \frac{d\Gamma_{\text{el}}}{d^2 q_{\perp}} \Psi(\mathbf{p}_{\perp} - \mathbf{q}_{\perp}, t)$$



$$\langle \Delta p_{\perp}^2 \rangle \simeq \hat{q} L$$

What do we mean by a “potential” ?

The $\hat{q}$ approximation

- Fourier transforming to position space:

$$i\partial_t \Psi(\mathbf{b}, t) = V(\mathbf{b}) \Psi(\mathbf{b}, t) = -i \int d^2 q_{\perp} \frac{d\Gamma_{\text{el}}}{d^2 q_{\perp}} (1 - e^{i\mathbf{q}_{\perp} \cdot \mathbf{b}}) \Psi(\mathbf{b}, t)$$

- High energy bremsstrahlung is nearly collinear and so corresponds to small values of transverse displacements $\mathbf{b}$.
- Taking the small- $\mathbf{b}$ limit, one finds
$$V(b) = -\frac{i}{4} \hat{q} b^2 \quad \hat{q} = \int d^2 q_{\perp} \frac{d\Gamma_{\text{el}}}{d^2 q_{\perp}} q_{\perp}^2$$
- Sometime called the harmonic oscillator approximation, but with an imaginary spring constant.

Bubble Resummation

The soft photon approximation $x_\gamma \ll 1$

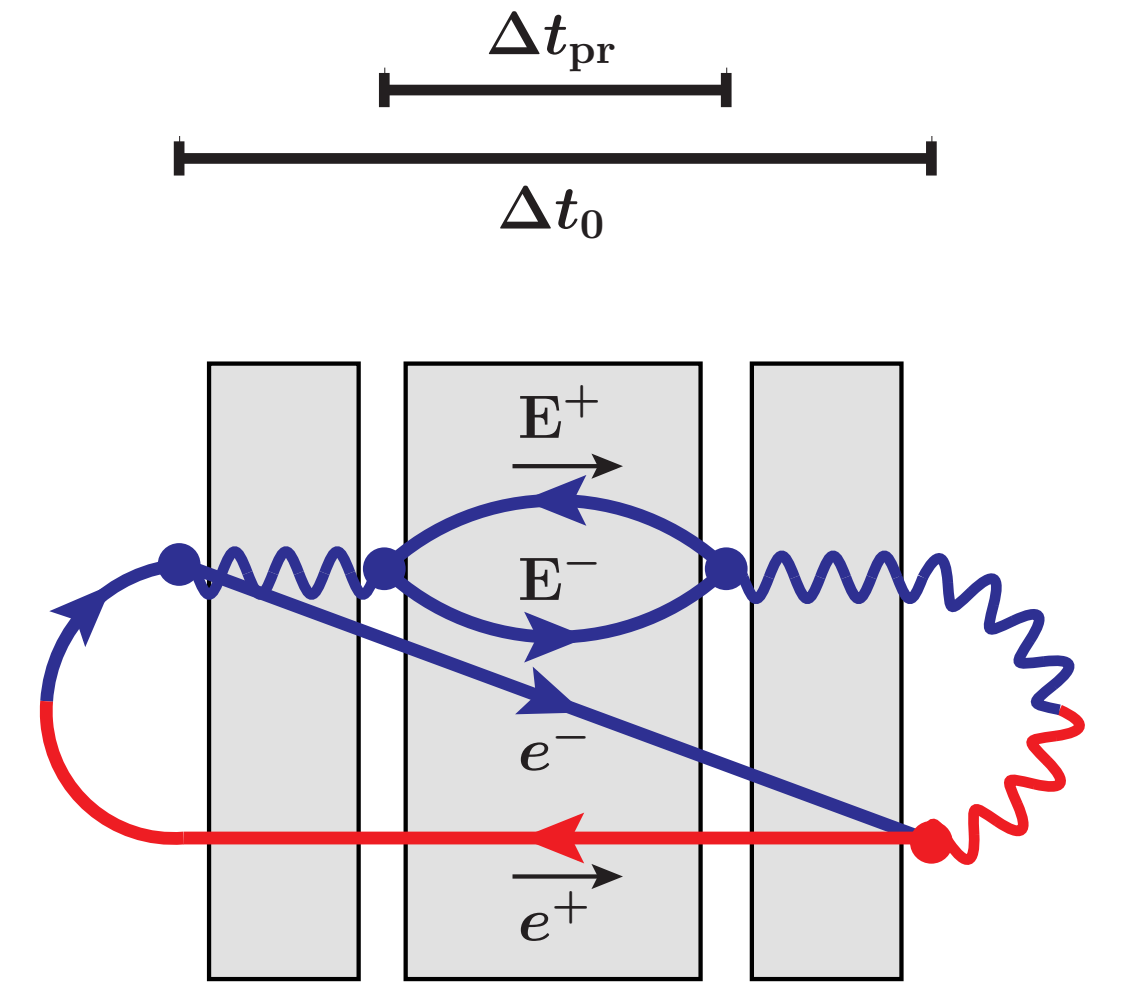
- In the soft photon limit:

$$p_\perp^{e \rightarrow e\gamma} \sim \left(\hat{q} t_{form}^{e \rightarrow e\gamma} \right)^{1/2} \sim \left(\hat{q} E / x_\gamma \right)^{1/4} \quad \longrightarrow \quad |b_{e^-} - b_{e^+}| \sim \left(\hat{q} E / x_\gamma \right)^{-1/4}$$

$$p_\perp^{\gamma \rightarrow e^- e^+} \sim \left(\hat{q} t_{form}^{\gamma \rightarrow e^- e^+} \right)^{1/2} \sim \left(\hat{q} x_\gamma E \right)^{1/4} \quad \longrightarrow \quad |b_{E^-} - b_{E^+}| \sim \left(\hat{q} x_\gamma E \right)^{-1/4}$$

- Because of this, a good approximation to the 4-body potential is

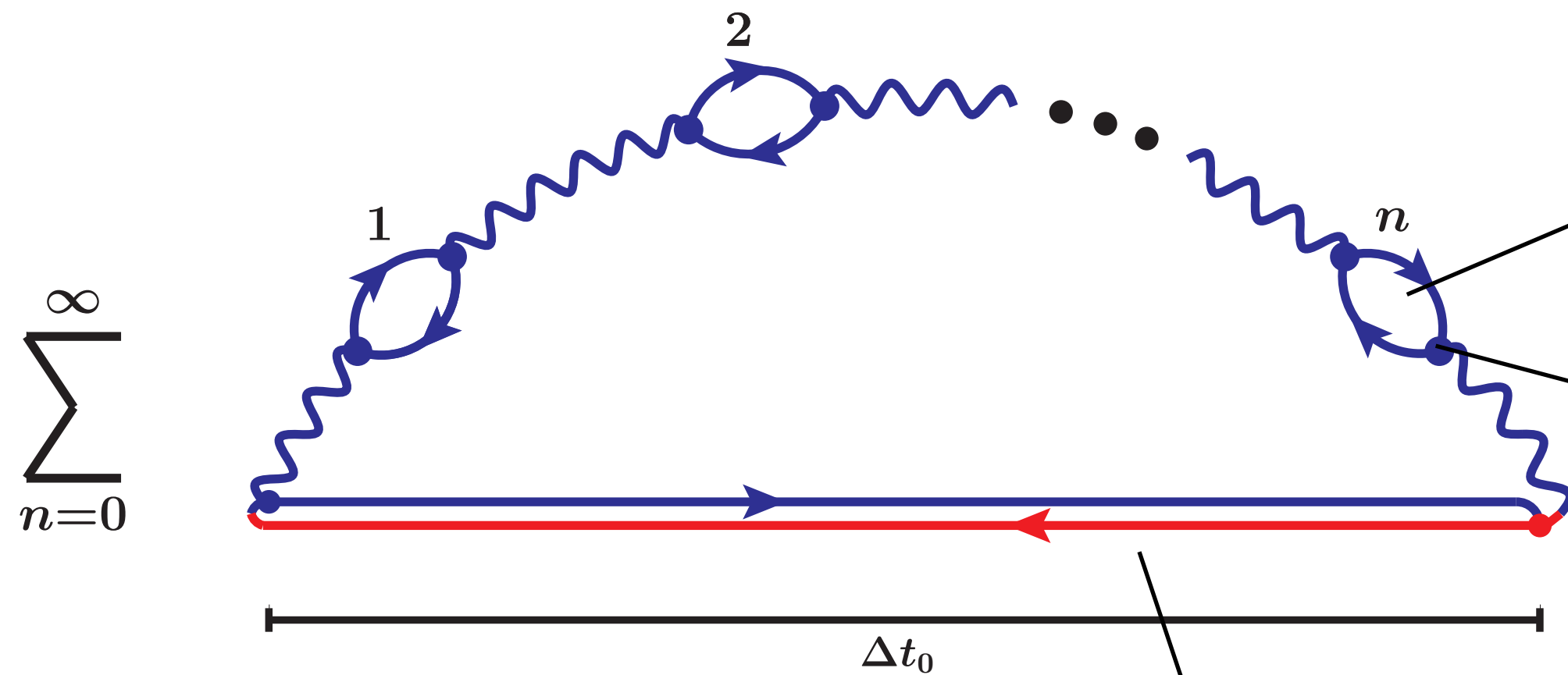
$$V_4(b_{e^-}, b_{e^+}, b_{E^-}, b_{E^+}) = -\frac{i\hat{q}}{4}(b_{E^-} - b_{E^+})^2 - \frac{i\hat{q}}{4}(b_{e^-} - b_{e^+})^2$$



Bubble Resummation

Putting it all together

- The modified LPM rate (LPM+) is $\left[\frac{d\Gamma}{dx_\gamma} \right]_{n \geq 0} \simeq 2\text{Re} \int_0^\infty d(\Delta t_0) \left[\frac{dg}{dx_\gamma d\Delta t_0} \right]_{brem} e^{-g_{pair}\Delta t_0}$



The HO propagator for the pair gets modified
 $\langle B, \Delta t | B', 0 \rangle_{pr} \rightarrow \langle B, \Delta t | B', 0 \rangle_{pr} e^{-ih_{pr}\Delta t_{pr}}$

The splitting vertex get a mass term

$$\alpha P_{\gamma \rightarrow e}(\eta) \nabla_{B'} \cdot \nabla_B \rightarrow \alpha \left[P_{\gamma \rightarrow e}(\eta) \nabla_{B'} \cdot \nabla_B + m^2 \right]$$

The HO propagator of the brem gets modified

$$\langle B, \Delta t | B', 0 \rangle_{brem} \rightarrow \langle B, \Delta t | B', 0 \rangle_{brem} e^{-ih_\gamma \Delta t_0}$$

The question

Size of the correction from overlapping splittings ?

- Previous calculation by (Blaizot, Mehtar-Tani/Iancu/Wu) showed that the probability of a hard splitting overlapping with soft bremsstrahlung k ($T \ll k \ll E$) is enhanced by a large double logarithm in QCD.
- The probability of overlap is suppressed not just by a factor of α_s , but by $\alpha_s \ln^2 \frac{E}{T}$ (a large effect for large E).
- But, these logs can be factorized and absorbed into $\hat{q}$:

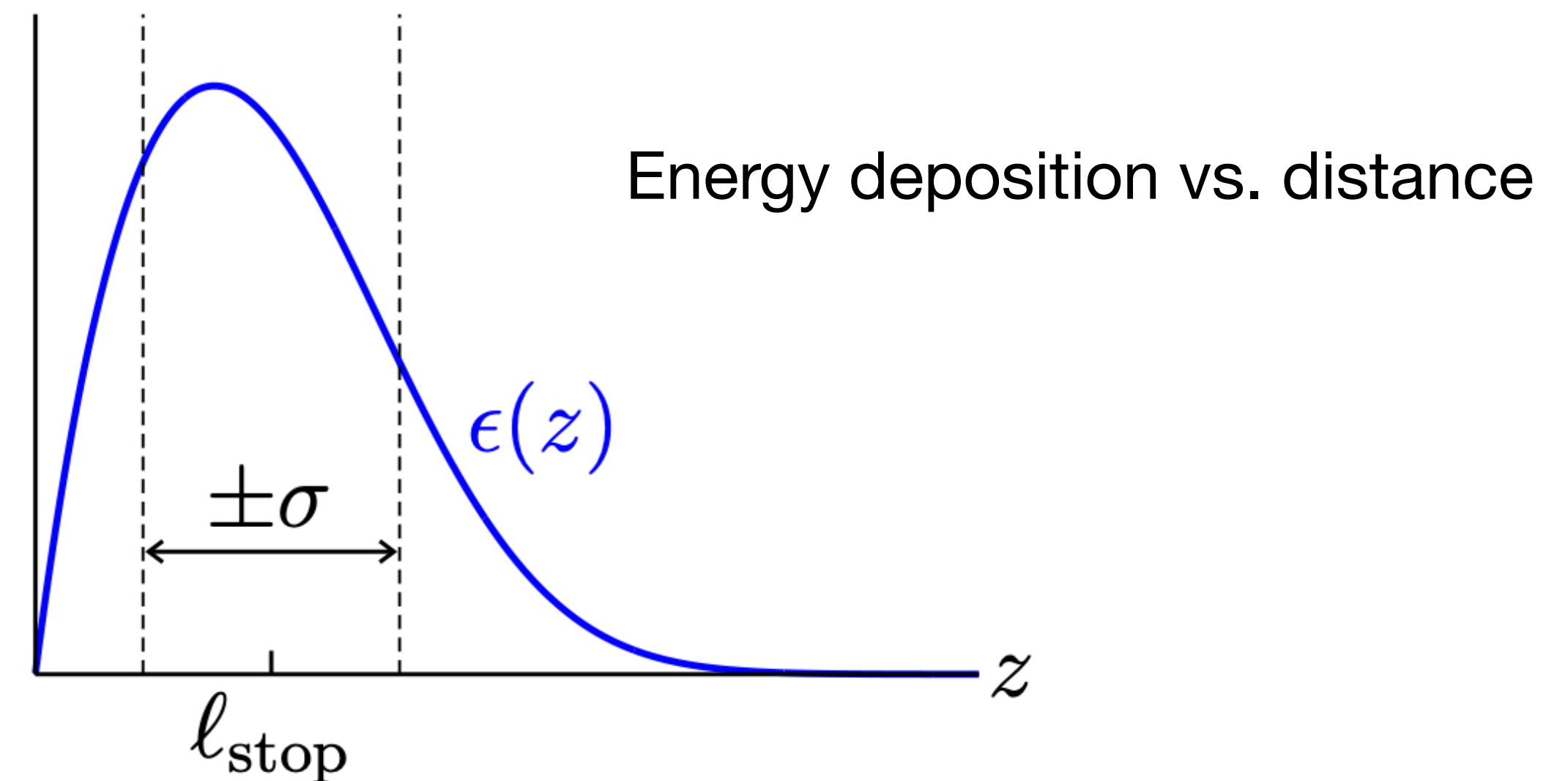
$$\hat{q} \rightarrow \hat{q}_{eff}(\mu)$$

The modified question

How big are overlapping effects that cannot be absorbed into $\hat{q}$?

- Look at ratios where $\hat{q}$ dependence cancels.
- For example:

The ratio σ/ℓ_{stop} or any characteristic of the “shape” of the distribution.



Results

- Large- N_f QED :

$$\frac{\sigma}{\ell_{stop}} = \left(\frac{\sigma}{\ell_{stop}} \right)_{LO} \left[1 - 0.99 N_f \alpha(\mu) \right]$$

[P.Arnold & S.Iqbal 2019]

- Large- N_c QCD ($N_f = 0$, gluons only):

$$\frac{\sigma}{\ell_{stop}} = \left(\frac{\sigma}{\ell_{stop}} \right)_{LO} \left[1 - 0.01 N_c \alpha_s(\mu) \right]$$

[P.Arnold & **OE** & S.Iqbal 2023]

- Large- N_c QCD ($N_f \gg N_c$, many quarks):

$$\frac{\sigma}{\ell_{stop}} = \left(\frac{\sigma}{\ell_{stop}} \right)_{LO} \left[1 - 0.004 N_f \alpha_s(\mu) \right]$$

[P.Arnold & **OE** & S.Iqbal 2025]

