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Nonflow Subtraction Beyond Two-Particle Correlations

Jiangyong Jia

2606.10258

Zaining Wang, Jiangyong Jia, Jinhui Chen, Shengli Huang, Chunjian Zhang, Zhengxi Yan

5th International Workshop on Emergent QCD Collectivity Across Scales

Outline

- Flow vs nonflow.
- Nonflow subtraction method in two-particle correlation
- Motivation for nonflow subtraction in multi-particle correlations
- Subtraction framework
  - Superposition principle and practical considerations
  - Guiding principles and classification of methods
  - Constructing the scaling factors using pp as baseline

- Performance in O+O and d+Au collisions with $\langle v_2^2\rangle$, $\langle v_2^2\delta p_T\rangle$, $c_2\{4\}$
- Summary and outlook

Flow vs nonflow

- Notoriously subtle issue in heavy ion collisions.
- Flow: ideally correlations arise from hydrodynamic response to the collision geometry.
  - Not a one-body distribution: factorization breaking in p_T and η require subleading flow modes.
  - Can be hydro but not geometry driven: jets thermalized in the medium creating jet wake.
  - Non-thermalized modes/incomplete geometry response: path-length dep. of v_n for high-p_T jets.
- Nonflow: ideally correlations of particles unrelated with the collision geometry.
  - Rely on practical definition of sources: jets, resonance decays, HBT….
  - All respect Global Momentum Conservation (GMC), appearing as away-side long-range dipolar shape.

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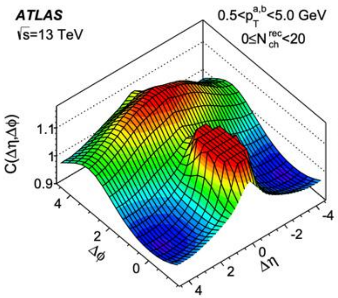
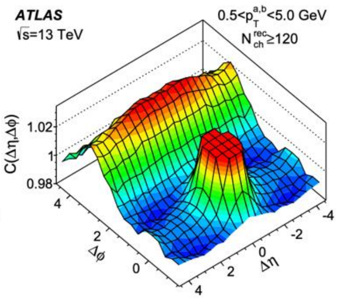
- Nonflow: ideally correlations of particles unrelated with the collision geometry.
  - Rely on practical definition of sources: jets, resonance decays, HBT….
  - All respect Global Momentum Conservation (GMC), appearing as away-side long-range dipolar shape.
- Separation of flow and nonflow.
  - Particles usually participate both flow and nonflow: a flowing ρ^0 decays to two-pions.
  - Must be defined on m-particle correlation statistically: some multiplets attributed to flow and others to nonflow.
- Nonflow subtraction assume is unmodified
  - Usually implies that nonflow subtraction works at leading-order, usually breaks down if nonflow signal dominates

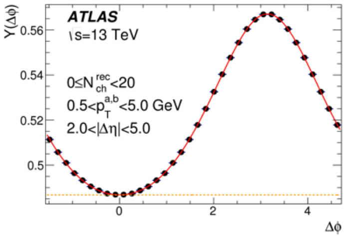
How to handle nonflow

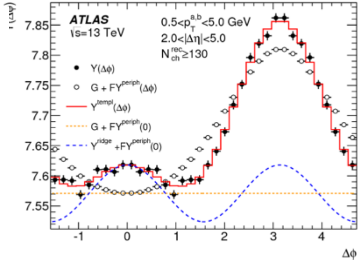
- First, separate correlation into short- and long-range correlations: SRC and LRC
  - SRC contains complex thermalization physics, flow and nonflow separation not easy.
  - LRC cleaner but still affected by away-side jets.
- Second, subtract nonflow on LRC: get rid of jets, assuming unmodified.
  - Subtraction can't be trusted when nonflow signal is very large: might be ok to subtract 50% nonflow to get 50% flow signal. But should be extremely careful when you subtract 90% and interpret the 10% as flow (method systematics are important).
  - Cause interpretation issues: Multi-particle correlation from CMS, ATLAS high p_T, e+e- ridge.
- Applicability of subtraction in small systems: low p_T region, high-multiplicity events.

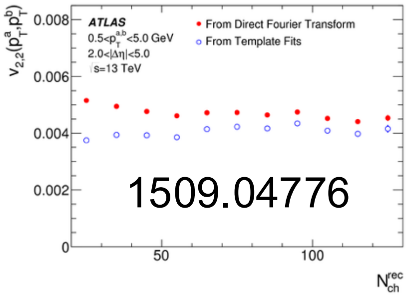
Caution on subtraction











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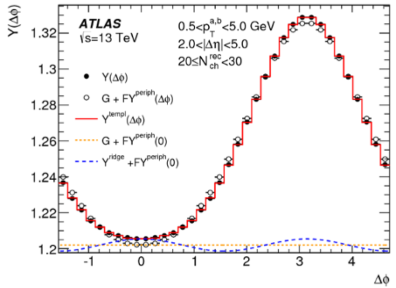
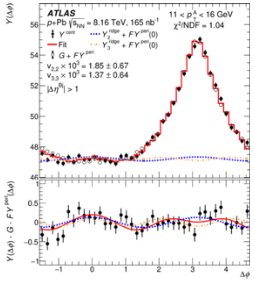


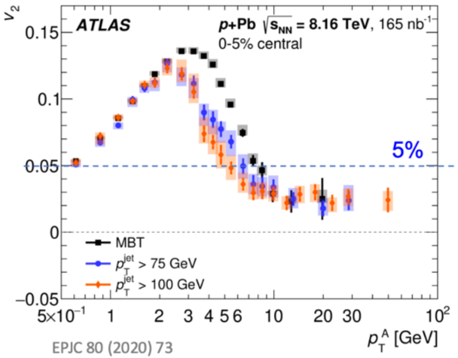
Dominated by nonflow from c_1, nonflow in c_2 is small

Caution on subtraction

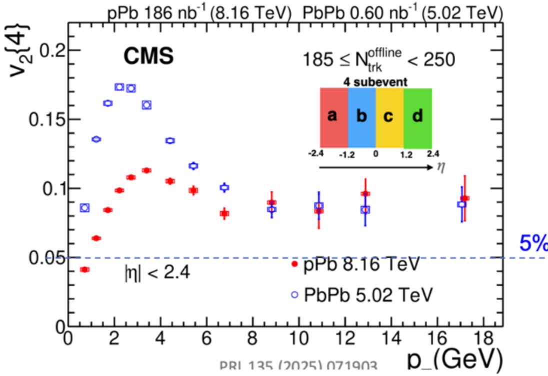
Potential method biases everywhere, need to be carefully studied!

2-particle corr. w/ template nonflow sub.





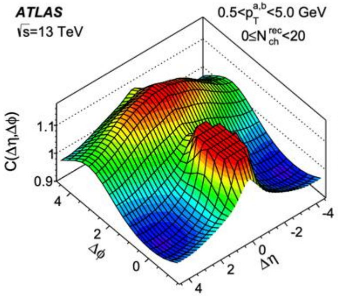
4-particle corr. w/ 4-subevent

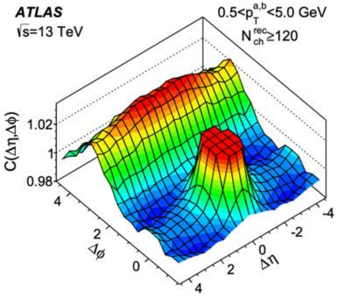


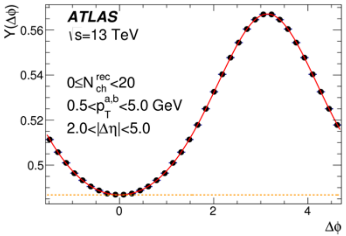
Dominated by nonflow from c1, but nonflow in c_2 is also large

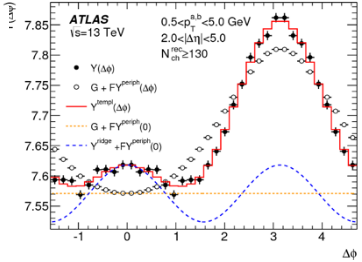
$$v_n\{4\} = -d_n\{4\}/(-c_n\{4\})^{3/4}$$

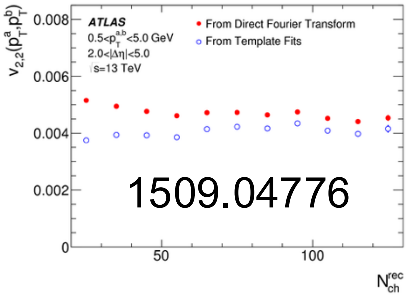
Factorization assumption broken

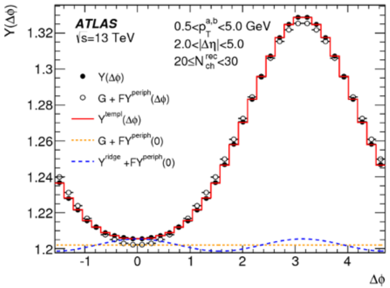












Dominated by nonflow from c_1, nonflow in c_2 is small

Nonflow subtraction method in 2PC

- Assume nonflow shape is unmodified and can be estimated from LM or pp events

$$\langle v_n^2\rangle(N) = \langle v_n^2\rangle^{\rm obs}(N) - f(N)\times\langle v_n^2\rangle^{\rm obs}_{\rm LM}$$

- f(N) ~ $1/N_{ch}$, a.la. independent source scenario for nonflow.

- Selection basis modify this scaling, especially if $p(N_{ch})$ has a sharp fall off:
  - Nonflow enhanced at large N_{ch} and suppressed at small N_{ch}. Bias is stronger for near-side jet

- Three nonflow subtraction methods assuming zero-flow in LM.

$$f_1 = N_{\rm LM}/N$$

c_0-scaling

$$f_2 = \langle v_1^2\rangle^{\rm obs} / \langle v_1^2\rangle^{\rm obs}_{\rm LM} \approx f_1 \times \frac{\rm Y_{away}}{\rm Y_{away,LM}}$$

c_1-scaling

$$f_3 = f_1 \times \frac{\rm Y_{near}}{\rm Y_{near,LM}}$$

$\mathrm{Near_{jet}}$-scaling

Nonflow subtraction method in 2PC

- LM may have flow → Template method
  - Fourier expansion of correlation function:
    $$Y(\Delta\phi) = N^a(1+2\sum_{n=1}^{\infty}\left\langle v_n^2\right\rangle^{\rm obs}\cos n\Delta\phi)$$
  - Peripheral subtraction removes the dipole component
    $$Y(\Delta\phi) - F\,Y(\Delta\phi)_{\rm LM} = G\left(1+2\sum_{n=2}^{\infty}\left\langle v_n^2\right\rangle\cos n\Delta\phi\right)$$

$$N^a\left\langle v_1^2\right\rangle^{\rm obs} = F\,N^a_{\rm LM}\left\langle v_1^2\right\rangle^{\rm obs}_{\rm LM} \Longrightarrow F = f_2/f_1 \qquad G = N^a - F\,N^a_{\rm LM} = N^a\,(1-f_2)$$

  - Template subtracted result
    $$\left\langle v_n^2\right\rangle^{tmp} = \frac{\left\langle v_n^2\right\rangle^{\rm obs} - f_2\left\langle v_n^2\right\rangle^{\rm obs}_{\rm LM}}{1-f_2} = \frac{\left\langle v_n^2\right\rangle - f_2\left\langle v_n^2\right\rangle_{\rm LM}}{1-f_2}$$ If nonflow all subtracted

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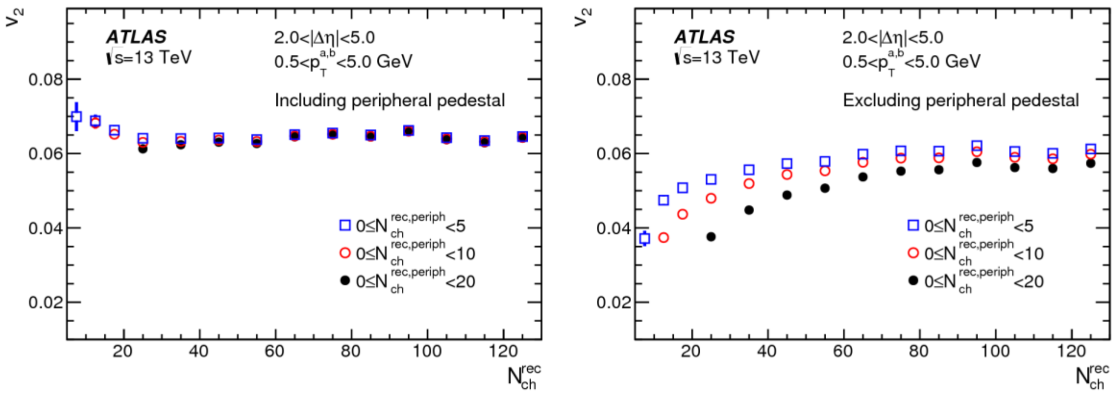
- Possible bias
  - $v_n > v_{n\mathrm{LM}}$, underestimate
  - $v_n < v_{n\mathrm{LM}}$, overestimate

- In pp: systematic trends of lower LM, shows that flow is ~ N_{ch} independent in LM→ no bias.
- Not the case in pPb and PbPb



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- Rewrite the equation $$\left\langle v_n^2\right\rangle = \left\langle v_n^2\right\rangle^{\mathrm{tmp}} - f_2\left(\left\langle v_n^2\right\rangle^{\mathrm{tmp}} - \left\langle v_n^2\right\rangle_{\mathrm{LM}}\right)$$

- Improved template: correct N_{ch}-dependence of flow starting with the 2nd bin

$$\left\langle v_n^2\right\rangle = \left\langle v_n^2\right\rangle^{\mathrm{tmp}} - f_{2,\mathrm{nextLM}}\left(\left\langle v_n^2\right\rangle^{\mathrm{tmp}} - \left\langle v_n^2\right\rangle^{\mathrm{tmp}}_{\mathrm{nextLM}}\right)$$

Nonflow subtraction method in 2PC

$$\langle v_n^2\rangle = \langle v_n^2\rangle^{\mathrm{tmp}} - f_{2,\mathrm{nextLM}} \underbrace{\left(\langle v_n^2\rangle^{\mathrm{tmp}} - \langle v_n^2\rangle^{\mathrm{tmp}}_{\mathrm{nextLM}}\right)}$$

Account for N_{ch} dependence of v_n.

Correction is more important for v_3

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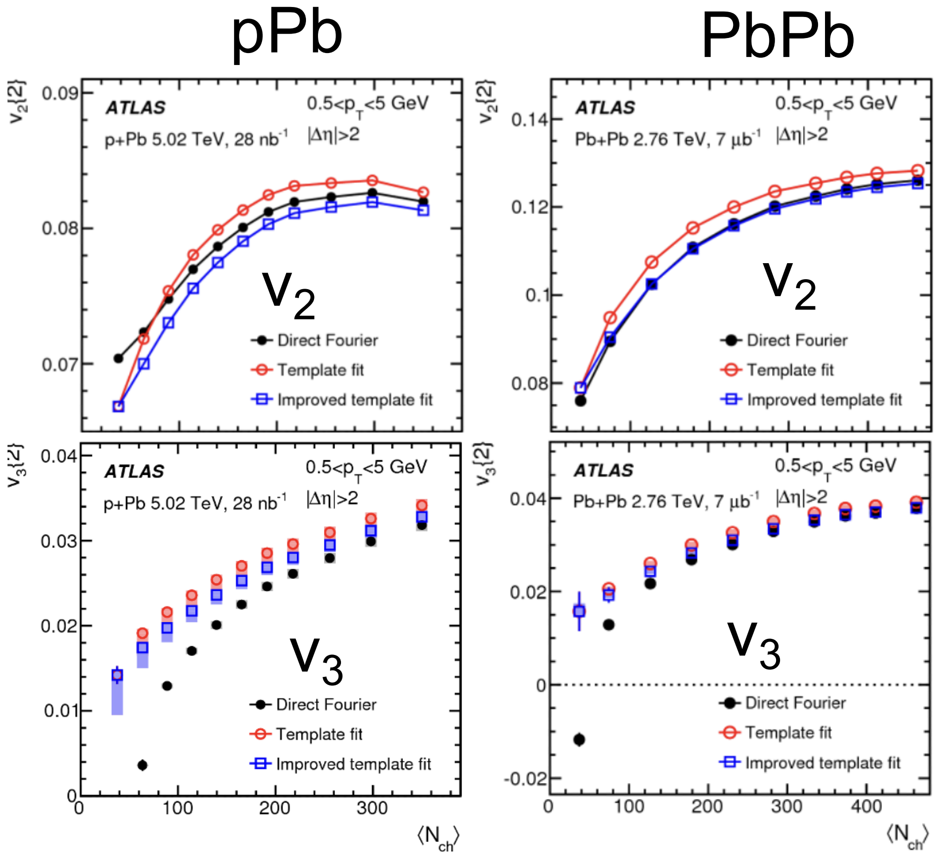
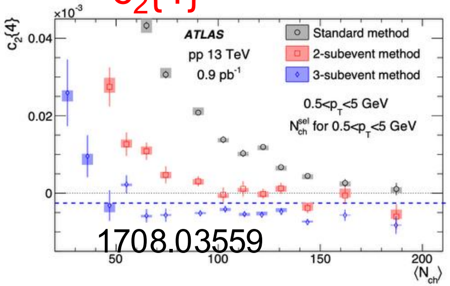


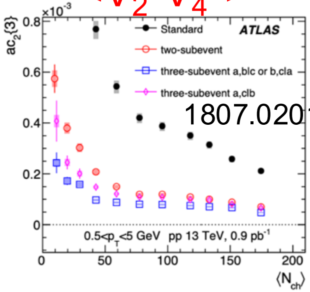
Why nonflow subtraction in multi-particle correlations

- Use multi-particle correlation to show flow is collective
  - Quantify residual nonflow after subevent requirement, which could be large with complex behavior, results uninterpretable.
  - Distinguish residual nonflow from flow decorrelations in p_T and η.
  - Subevent also throw away statistics→ a factor of $4^4/4!=11$ in 4 subevent.

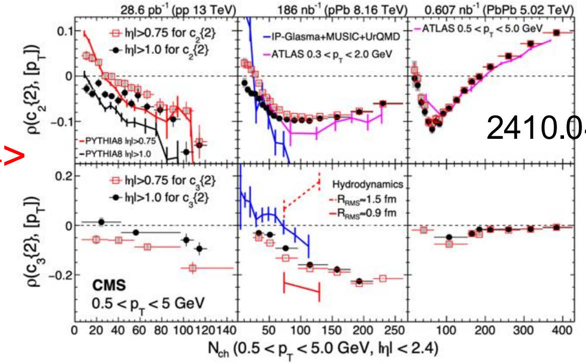
$c_2\{4\}$

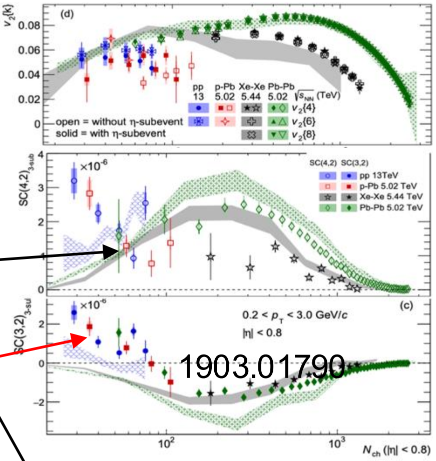


$\langle V_2^2V_4^*\rangle$



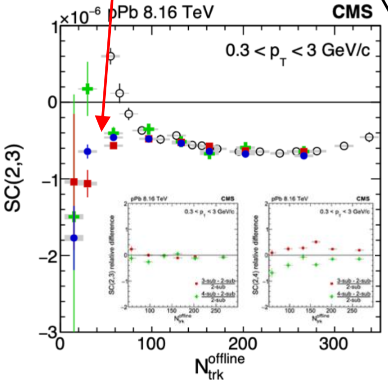
$\langle v_2^2 dp_T\rangle$

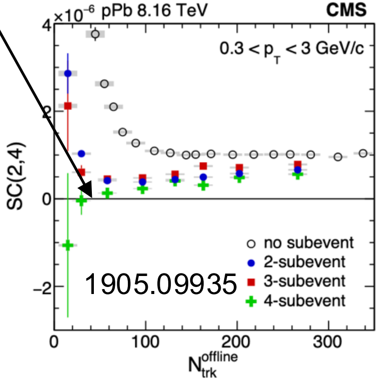




SC(2,4)

SC(2,3)





Extend subtraction to m-particle correlations

- Extends along the five nonflow sub methods based on scaling: $1/N^{m-1}$.
- Nonflow estimator: correlators containing v_1 is dominated by nonflow (due to the sign change around $<p_T>$, genuine p_T-integrated v_1 signal should be small).

$$\langle v_1^2\rangle \sim 1/N \quad \langle v_1^2 \delta p_T\rangle \sim 1/N^2 \qquad c_1\{4\} = 2\langle v_1^2\rangle^2 - \langle v_1^4\rangle \sim 1/N^3$$

$$\langle V_1^2 V_2^*\rangle \sim 1/N^2 \quad \langle V_1 V_2 V_3^*\rangle \sim 1/N^2 \qquad sc\{1,n\} = \langle v_1^2 v_n^2\rangle - \langle v_1^2\rangle\langle v_n^2\rangle \sim 1/N^3$$

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- Method1: nonflow scales as $1/N^{m-1}$:

$$\left\langle v_n^2\right\rangle = \left\langle v_n^2\right\rangle^{\mathrm{obs}} - N_{2,\mathrm{pp}}/N_2 \left\langle v_n^2\right\rangle_{\mathrm{pp}}^{\mathrm{obs}},$$

$$\left\langle v_n^2\, \delta p_{\mathrm{T}}\right\rangle = \left\langle v_n^2\, \delta p_{\mathrm{T}}\right\rangle^{\mathrm{obs}} - (N_{3,\mathrm{pp}}/N_3)^2 \left\langle v_n^2\, \delta p_{\mathrm{T}}\right\rangle_{\mathrm{pp}}^{\mathrm{obs}},$$

$$c_n\{4\} = c_n\{4\}^{\mathrm{obs}} - (N_{4,\mathrm{pp}}/N_4)^3\, c_n\{4\}_{\mathrm{pp}}^{\mathrm{obs}}.$$

Extend subtraction to multi-particle correlations

- Method2a: using native estimators

$$c_n\{4\} = c_n\{4\}^{\rm obs} - \frac{c_1\{4\}^{\rm obs}}{c_1\{4\}^{\rm obs}_{\rm LM}} c_n\{4\}^{\rm obs}_{\rm LM}$$

$$\langle v_n^2 \rangle = \langle v_n^2 \rangle^{\rm obs} - \frac{\langle v_1^2 \rangle^{\rm obs}}{\langle v_1^2 \rangle^{\rm obs}_{\rm LM}} \langle v_n^2 \rangle^{\rm obs}_{\rm LM} \qquad \langle v_n^2 \delta p_T \rangle = \langle v_n^2 \delta p_T \rangle^{\rm obs} - \frac{\langle v_1^2 \delta p_T \rangle^{\rm obs}}{\langle v_1^2 \delta p_T \rangle^{\rm obs}_{\rm LM}} \langle v_n^2 \delta p_T \rangle^{\rm obs}_{\rm LM}$$

- Method2b: using estimators of same order

$$\langle V_2^2 V_4^* \rangle = \langle V_2^2 V_4^* \rangle^{\rm obs} - \frac{\langle V_1^2 V_2^* \rangle^{\rm obs}}{\langle V_1^2 V_2^* \rangle^{\rm obs}_{\rm LM}} \langle V_2^2 V_4^* \rangle^{\rm obs}_{\rm LM} \qquad \langle V_2 V_3 V_5^* \rangle = \langle V_2 V_3 V_5^* \rangle^{\rm obs} - \frac{\langle V_1 V_2 V_3^* \rangle^{\rm obs}}{\langle V_1 V_2 V_3^* \rangle^{\rm obs}_{\rm LM}} \langle V_2 V_3 V_5^* \rangle^{\rm obs}_{\rm LM}$$

- Method2c: using mixed-order estimators

$$\langle v_n^2 \delta p_T \rangle = \langle v_n^2 \delta p_T \rangle^{\rm obs} - \frac{\left(\langle v_1^2 \rangle^{\rm obs}\right)^2}{\left(\langle v_1^2 \rangle^{\rm obs}_{\rm LM}\right)^2} \langle v_n^2 \delta p_T \rangle^{\rm obs}_{\rm LM} \qquad \langle v_n^2 \rangle = \langle v_n^2 \rangle^{\rm obs} - \left(\frac{\langle v_1^2 \delta p_{\rm T} \rangle^{\rm obs}}{\langle v_1^2 \delta p_{\rm T} \rangle^{\rm obs}_{\rm pp}}\right)^{1/2} \langle v_n^2 \rangle^{\rm obs}_{\rm pp}$$

Extend subtraction to multi-particle correlations

- Method2a: using native estimators

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$$\langle v_n^2\rangle = \langle v_n^2\rangle^{\text{obs}} - \frac{\langle v_1^2\rangle^{\text{obs}}}{\langle v_1^2\rangle^{\text{obs}}_{\text{LM}}} \langle v_n^2\rangle^{\text{obs}}_{\text{LM}} \qquad \langle v_n^2\delta p_T\rangle = \langle v_n^2\delta p_T\rangle^{\text{obs}} - \frac{\langle v_1^2\delta p_T\rangle^{\text{obs}}}{\langle v_1^2\delta p_T\rangle^{\text{obs}}_{\text{LM}}} \langle v_n^2\delta p_T\rangle^{\text{obs}}_{\text{LM}}$$

- Method2b: using estimators of same order

$$\langle V_2^2 V_4^*\rangle = \langle V_2^2 V_4^*\rangle^{\text{obs}} - \frac{\langle V_1^2 V_2^*\rangle^{\text{obs}}}{\langle V_1^2 V_2^*\rangle^{\text{obs}}_{\text{LM}}} \langle V_2^2 V_4^*\rangle^{\text{obs}}_{\text{LM}} \qquad \langle V_2 V_3 V_5^*\rangle = \langle V_2 V_3 V_5^*\rangle^{\text{obs}} - \frac{\langle V_1 V_2 V_3^*\rangle^{\text{obs}}}{\langle V_1 V_2 V_3^*\rangle^{\text{obs}}_{\text{LM}}} \langle V_2 V_3 V_5^*\rangle^{\text{obs}}_{\text{LM}}$$

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$$\langle v_n^2\delta p_T\rangle = \langle v_n^2\delta p_T\rangle^{\text{obs}} - \frac{\left(\langle v_1^2\rangle^{\text{obs}}\right)^2}{\left(\langle v_1^2\rangle^{\text{obs}}_{\text{LM}}\right)^2} \langle v_n^2\delta p_T\rangle^{\text{obs}}_{\text{LM}} \qquad \langle v_n^2\rangle = \langle v_n^2\rangle^{\text{obs}} - \left(\frac{\langle v_1^2\delta p_{\text{T}}\rangle^{\text{obs}}}{\langle v_1^2\delta p_{\text{T}}\rangle^{\text{obs}}_{\text{pp}}}\right)^{1/2} \langle v_n^2\rangle^{\text{obs}}_{\text{pp}}$$

Caveat:

- Nonflow subtraction on cumulants not moments (avoid flow-nonflow mixing).
- Proliferation of methods, what is the best method for each observable?
  →Need a detailed HIJING study to check the residual subtractions
- Account for selection bias on N

Multiplicity weight

Jiangyong Jia

Plot the scaled observables $\mathcal{O}\{m\}^{1/(m-1)} \sim 1/N \quad \mathcal{O} = 1/N, \langle v_2^2\rangle, \langle v_2^2 \delta p_{\mathrm{T}}\rangle, \langle v_1^2\rangle, \langle v_1^2 \delta p_{\mathrm{T}}\rangle, c_2\{4\}$

2PC scales with 1/N, so we should plot at the event with the similar level of non-flow

$$\frac{1}{N_2} = \left\langle \frac{1}{N} \right\rangle = \frac{\sum_i 1/N_i \, N_i(N_i-1)}{\sum_i N_i(N_i-1)} \qquad N_2 \approx \frac{\sum_i N_i^2}{\sum_i N_i}$$

m-particle correlation scales with $1/N^{m-1}$, so we should plot at the x-axis value N_m.

$$\frac{1}{N_m^{m-1}} = \left\langle \frac{1}{N^{m-1}} \right\rangle = \frac{\sum_i 1/N_i^{m-1} \times N_i\,(N_i-1)\ldots(N_i-m+1)}{\sum_i N_i\,(N_i-1)\ldots(N_i-m+1)} \qquad N_m^{m-1} = \frac{\sum_i N_i\,(N_i-1)\ldots(N_i-m+1)}{\sum_i N_i} \approx \frac{\sum_i N_i^m}{\sum_i N_i}$$

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2PC scales with 1/N, so we should plot at the event with the similar level of non-flow

$$\frac{1}{N_2} = \left\langle \frac{1}{N} \right\rangle = \frac{\sum_i 1/N_i \; N_i(N_i-1)}{\sum_i N_i(N_i-1)} \qquad N_2 \approx \frac{\sum_i N_i^2}{\sum_i N_i}$$

m-particle correlation scales with $1/N^{m-1}$, so we should plot at the x-axis value N_m.

$$\frac{1}{N_m^{m-1}} = \left\langle \frac{1}{N^{m-1}} \right\rangle = \frac{\sum_i 1/N_i^{m-1} \times N_i\,(N_i-1)\ldots(N_i-m+1)}{\sum_i N_i\,(N_i-1)\ldots(N_i-m+1)} \qquad N_m^{m-1} = \frac{\sum_i N_i\,(N_i-1)\ldots(N_i-m+1)}{\sum_i N_i} \approx \frac{\sum_i N_i^m}{\sum_i N_i}$$

Why they show up like this: consider N particles from N_s sources with k particles in each source

$$N_i = N_{s,i} k$$

$$\mathcal{O}\{m\} = \frac{\sum_i N_s k(k-1)\ldots(k-m+1)\hat{o}}{\sum_i N_i\,(N_i-1)\ldots(N_i-m+1)} \approx \frac{\sum_i N_{s,i} k_m \hat{o}}{\sum_i N_i^m} = \frac{\sum_i N_{s,i} k_m \hat{o}}{\sum_i N_{s,i}^m k^m}$$

hence $$N_m^{m-1}\mathcal{O}\{m\} = \frac{\sum_i N_i^m}{\sum_i N_i}\mathcal{O}\{m\} = \frac{\sum_i N_i^m}{\sum_i N_i}\frac{\sum_i N_{s,i} k_m \hat{o}}{\sum_i N_i^m} = \frac{\sum_i N_{s,i} k_m \hat{o}}{\sum_i N_{s,i} k} \sim \mathrm{const}$$

General formula for nonflow subtraction (using pp)

Jiangyong Jia

- Expected scaling behavior $N_m \mathcal{O}\{m\}^{1/(m-1)} \approx N_{m,\mathrm{pp}} \mathcal{O}\{m\}_{\mathrm{pp}}^{1/(m-1)}$

- Use ε (contains v_1) to estimate the scale factor for nonflow

$$\mathcal{O}\{m\} = \mathcal{O}\{m\}^{\mathrm{obs}} - f_{\mathcal{O}}^{\mathcal{E}} \times \mathcal{O}\{m\}_{\mathrm{pp}}^{\mathrm{obs}} \qquad f_{\mathcal{O}}^{\mathcal{E}} = \left(\left(\frac{\mathcal{E}\{k\}^{\mathrm{obs}}}{\mathcal{E}\{k\}_{\mathrm{pp}}^{\mathrm{obs}}} \right)^{1/(k-1)} \underbrace{\frac{N_{m,\mathrm{pp}}}{N_{k,\mathrm{pp}}} \cdot \frac{N_k}{N_m}}_{W_{m,k}} \right)^{m-1}$$

- Multiplicity weight bias depends on the order of correlation $W_{m,k} \equiv \frac{N_{m,\mathrm{pp}}}{N_{k,\mathrm{pp}}} \cdot \frac{N_k}{N_m} \approx \frac{N_{m,\mathrm{pp}}}{N_{k,\mathrm{pp}}}$

- We can extend to the template method

$$\mathcal{O}\{m\}^{\mathrm{tmp}} = \frac{\mathcal{O}\{m\}^{\mathrm{obs}} - f_{\mathcal{O}}^{\mathcal{E}} \mathcal{O}\{m\}_{\mathrm{pp}}^{\mathrm{obs}}}{1 - f_{\mathcal{O}}^{\mathcal{E}}}$$

- as well as the improved template method

$$\mathcal{O}\{m\} = \mathcal{O}^{\mathrm{sub}}\{m\}^{\mathrm{tmp}} - f_{\mathcal{O},\,\mathrm{nextLM}}^{\mathcal{E}} \left(\mathcal{O}^{\mathrm{sub}}\{m\}^{\mathrm{tmp}} - \mathcal{O}^{\mathrm{sub}}\{m\}_{\mathrm{nextLM}}^{\mathrm{tmp}} \right)$$

Dataset and analysis

- 5.36 TeV: pp and O+O; 200 GeV: pp, O+O and dAu.
  - 2000M events are generated for each datasets
- Selection: 0.2-2 GeV, $|\eta|<2.5$ (LHC), $|\eta|<1.5$ (RHIC)
  - Number of particles denoted as N.
- Two subevents: $\eta_a<-0.5$, $\eta_b>0.5$.
- We focus on $\langle v_2^2\rangle$, $\langle v_2^2\delta p_T\rangle$, and $c_2\{4\}$

Stress test the subtraction method

$$\mathcal{O}\{m\} = \mathcal{O}\{m\}^{\rm obs} - f_{\mathcal{O}}^{\mathcal{E}} \times \mathcal{O}\{m\}^{\rm obs}_{\rm pp}$$

$$f_{\mathcal{O}}^{\mathcal{E}} = \left(\left(\frac{\mathcal{E}\{k\}^{\rm obs}}{\mathcal{E}\{k\}^{\rm obs}_{\rm pp}} \right)^{1/(k-1)} \underbrace{\frac{N_{m,\rm pp}}{N_{k,\rm pp}} \cdot \frac{N_k}{N_m}}_{W_{m,k}} \right)^{m-1}$$

Three choices of reference ε

Three observables $\mathcal{O}\{m\}$

$\mathcal{O}\{m\}$	$\mathcal{E} = 1/N$	$\langle v_1^2\rangle$	$\langle v_1^2\delta p_{\rm T}\rangle_{\rm ling}$
$\langle v_2^2\rangle$	$\frac{N_{2,\rm pp}}{N_2}$	$\frac{\langle v_1^2\rangle^{\rm obs}}{\langle v_1^2\rangle^{\rm obs}_{\rm pp}}$	$\left(\frac{\langle v_1^2\,\delta p_{\rm T}\rangle^{\rm obs}}{\langle v_1^2\,\delta p_{\rm T}\rangle^{\rm obs}_{\rm pp}}\right)^{1/2} W_{2,3}$
$\langle v_2^2\,\delta p_{\rm T}\rangle$	$\frac{N^2_{3,\rm pp}}{N^2_3}$	$\left(\frac{\langle v_1^2\rangle^{\rm obs}}{\langle v_1^2\rangle^{\rm obs}_{\rm pp}} W_{3,2}\right)^2$	$\frac{\langle v_1^2\,\delta p_{\rm T}\rangle^{\rm obs}}{\langle v_1^2\,\delta p_{\rm T}\rangle^{\rm obs}_{\rm pp}}$
$c_2\{4\}$	$\frac{N^3_{4,\rm pp}}{N^3_4}$	$\left(\frac{\langle v_1^2\rangle^{\rm obs}}{\langle v_1^2\rangle^{\rm obs}_{\rm pp}} W_{4,2}\right)^3$	$\left(\frac{\langle v_1^2\,\delta p_{\rm T}\rangle^{\rm obs}}{\langle v_1^2\,\delta p_{\rm T}\rangle^{\rm obs}_{\rm pp}}\right)^{3/2} W^3_{4,3}$

Scaling behavior in standard method

Plot the scaled observables $\mathcal{O}\{m\}^{1/(m-1)} \sim 1/N$

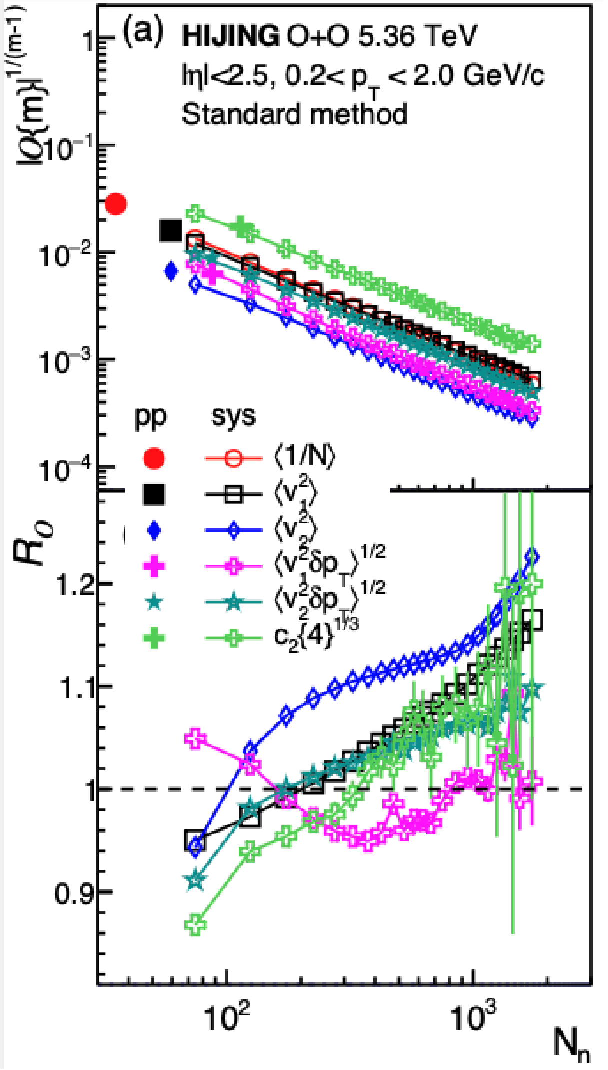
$$\mathcal{O} = \langle v_2^2 \rangle, \langle v_2^2 \delta p_{\mathrm{T}} \rangle, c_2\{4\}, \quad 1/N, \langle v_1^2 \rangle, \langle v_1^2 \delta p_{\mathrm{T}} \rangle$$

Since pp collision has a broad range in N, value of N_m where we plot the result depends on the observable $\mathcal{O}$ (solid symbols).

$$N_m \mathcal{O}\{m\}^{1/(m-1)} \approx N_{m,\mathrm{pp}} \mathcal{O}\{m\}_{\mathrm{pp}}^{1/(m-1)}$$

$$\Rightarrow R_{\mathcal{O}} \equiv \frac{N_m}{N_{m,\mathrm{pp}}} \left(\frac{\mathcal{O}\{m\}}{\mathcal{O}\{m\}_{\mathrm{pp}}} \right)^{1/(m-1)} \approx 1$$

Indeed, the $R_{\mathcal{O}}$ cluster around 1, here plotted for the standard method, >70% non-flow subtracted!



Scaling behavior in two subevent method

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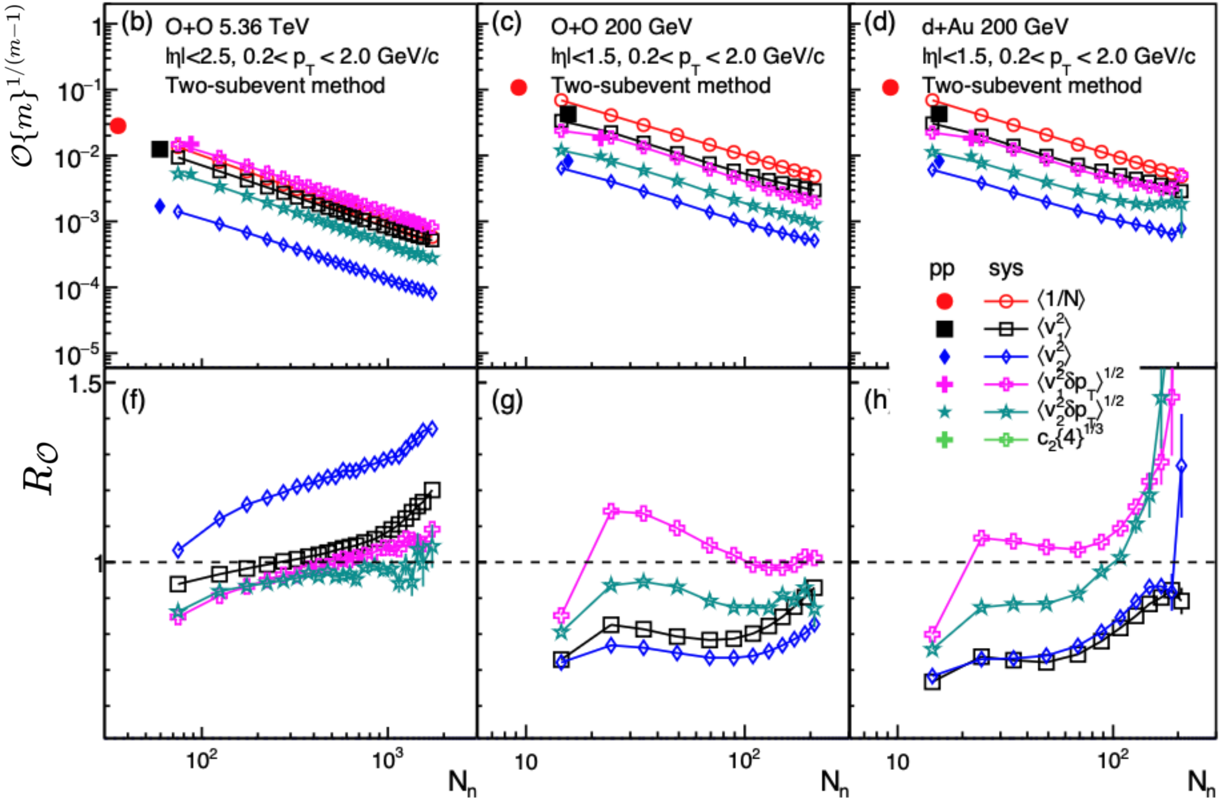
2sub-event results behaves like standard method.

The nonflow has somewhat different centrality dependence, but also generally within 20% from 1

Deviation from 1 reflects residual nonflow based on 1/N scaling.

It is clear some other reference observables may scale better, so need to explore other subtraction methods.

$$\mathcal{E} = 1/N \qquad \langle v_1^2 \rangle \qquad \langle v_1^2 \delta p_{\mathrm{T}} \rangle$$



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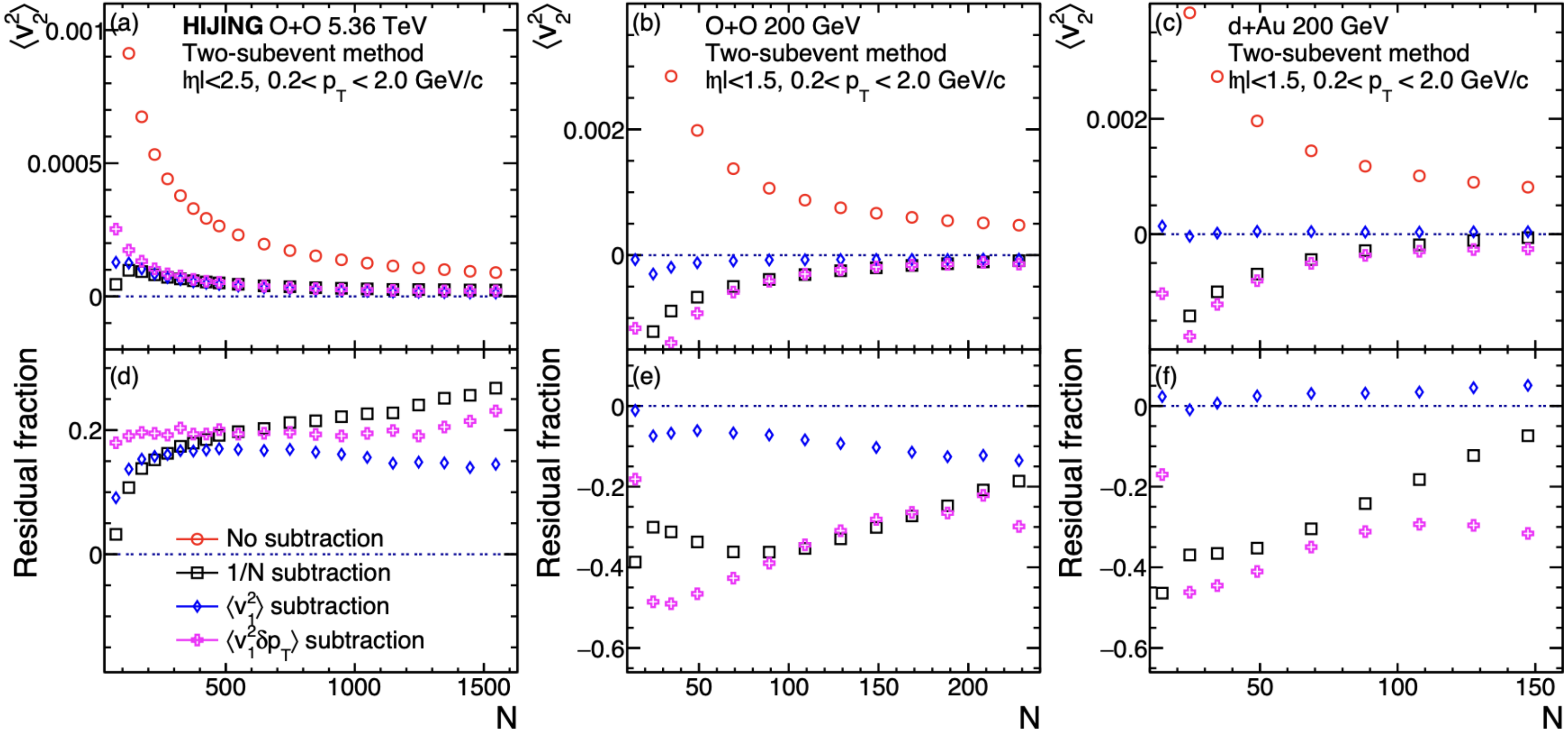
Result of subtraction for $\langle v_2^2 \rangle$

$\langle v_1^2 \rangle$ method consistently gives better result
$\langle v_1^2 \delta p_T \rangle$ scaling is worst and 1/N scaling is in between

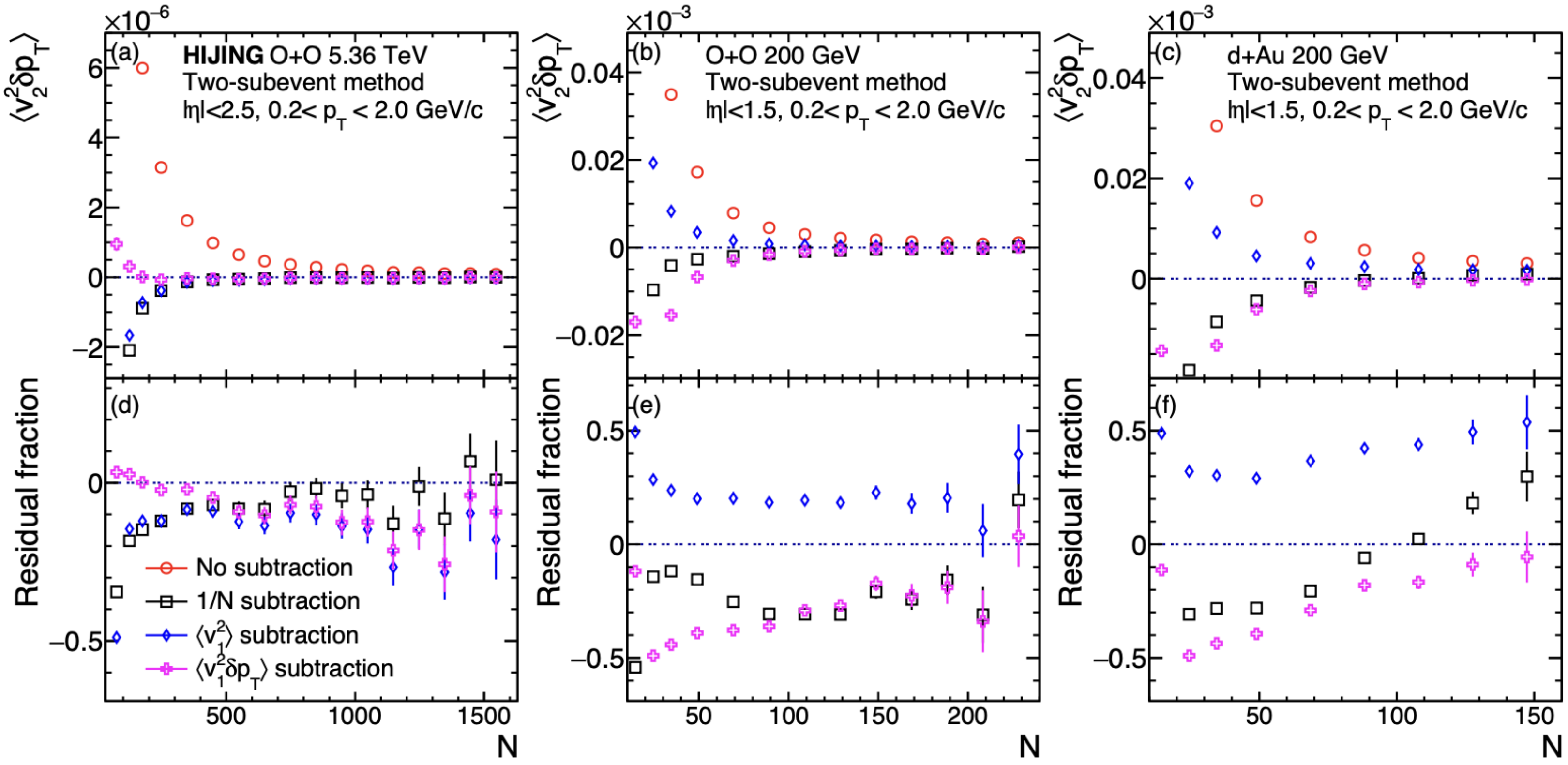
$$\frac{\mathcal{O}\{m\}}{\mathcal{O}\{m\}^{\mathrm{obs}}}$$



Result of subtraction for $\langle v_2^2 \delta p_T \rangle$

1/N scaling give better result
$\langle v_1^2 \rangle$ and $\langle v_1^2 \delta p_T \rangle$ scaling are comparable

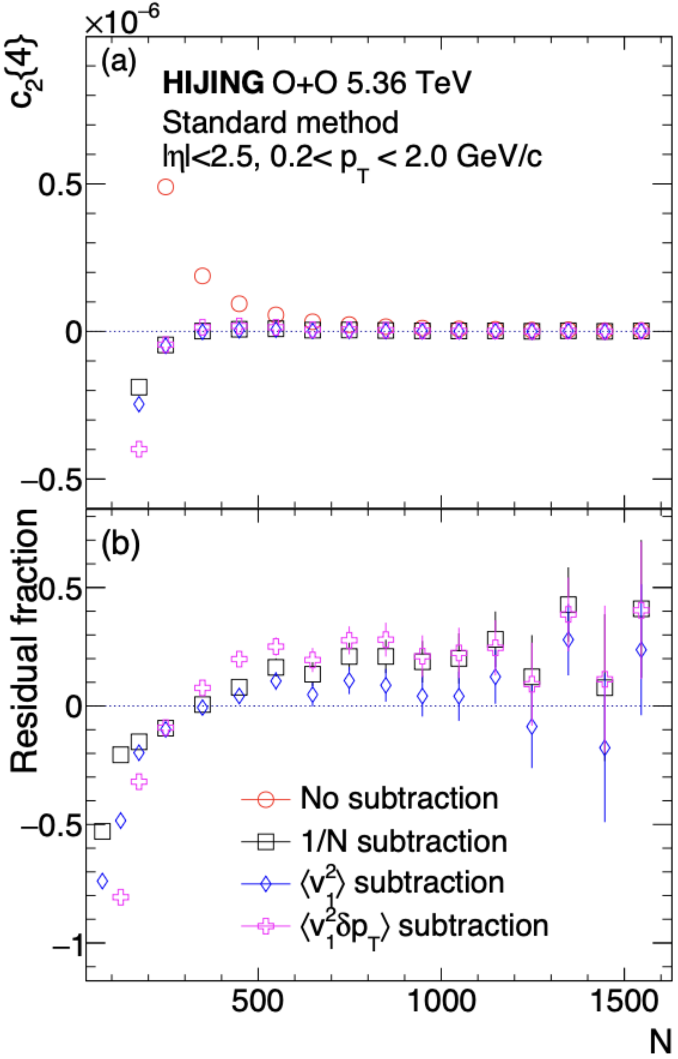
$$\frac{\mathcal{O}\{m\}}{\mathcal{O}\{m\}^{\text{obs}}}$$



Result of subtraction for $c_2\{4\}$

$$\frac{\mathcal{O}\{m\}}{\mathcal{O}\{m\}^{\mathrm{obs}}}$$

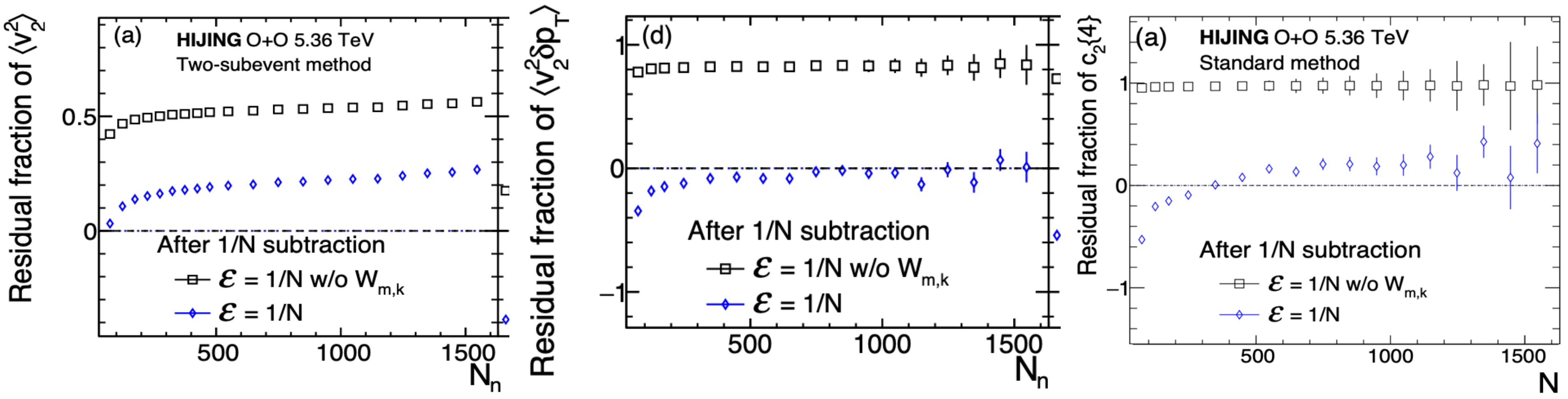
$\langle v_1^2 \rangle$ scaling works the best



Importance of using the correct weighting factor

Considering reweighted N : $N_m = \left(\frac{\sum_i N_i^m}{\sum_i N_i}\right)^{1/(m-1)}$ vs $\langle N \rangle$

$$\frac{\langle v_2^2\rangle_{\rm sub}}{\langle v_2^2\rangle} = 1 - \boxed{\frac{N_{2,\rm pp}}{N_2}}\frac{\langle v_2^2\rangle_{\rm pp}}{\langle v_2^2\rangle} \qquad \frac{\langle v_2^2\delta p_{\rm T}\rangle_{\rm sub}}{\langle v_2^2\delta p_{\rm T}\rangle} = 1 - \left(\boxed{\frac{N_{3,\rm pp}}{N_3}}\right)^2\frac{\langle v_2^2\delta p_{\rm T}\rangle_{\rm pp}}{\langle v_2^2\delta p_{\rm T}\rangle} \qquad \frac{v_2^4\{4\}_{\rm sub}}{v_2^4\{4\}} = 1 - \boxed{\left(\frac{N_{4,\rm pp}}{N_4}\right)^3}\frac{v_2^4\{4\}_{\rm pp}}{v_2^4\{4\}}$$



Again, considering reweighted N is essential!

Implication of residual non-closure

subtraction in HIJING does not give perfect closure.

2312.07464

$$\frac{\mathcal{O}^{\mathrm{sub}}\{m\}}{\mathcal{O}\{m\}} = 1 - \left(\left(\frac{\mathcal{E}\{k\}}{\mathcal{E}\{k\}_{\mathrm{pp}}}\right)^{1/(k-1)} \frac{N_{m,\mathrm{pp}}}{N_{k,\mathrm{pp}}} \frac{N_k}{N_m}\right)^{m-1} \frac{\mathcal{O}\{m\}_{\mathrm{pp}}}{\mathcal{O}\{m\}} \neq 0$$

But we can introduce a correct factor K to enforce zero

$$0 = 1 - K_{\mathcal{O}}^{\mathcal{E}} \left(\left(\frac{\mathcal{E}\{k\}}{\mathcal{E}\{k\}_{\mathrm{pp}}}\right)^{1/(k-1)} \frac{N_{m,\mathrm{pp}}}{N_{k,\mathrm{pp}}} \frac{N_k}{N_m}\right)^{m-1} \frac{\mathcal{O}\{m\}_{\mathrm{pp}}}{\mathcal{O}\{m\}}$$

From HIJING

This factor $K_{\mathcal{E},\mathcal{O}}$ (N) can be applied to data to check consistency of results:

HIJING

$$\mathcal{O}\{m\} = \mathcal{O}\{m\}^{\mathrm{obs}} - K_{\mathcal{O}}^{\mathcal{E}} \times f_{\mathcal{O}}^{\mathcal{E}} \mathcal{O}\{m\}_{\mathrm{pp}}^{\mathrm{obs}} \qquad f_{\mathcal{O}}^{\mathcal{E}} = \left(\left(\frac{\mathcal{E}\{k\}^{\mathrm{obs}}}{\mathcal{E}\{k\}_{\mathrm{pp}}^{\mathrm{obs}}}\right)^{1/(k-1)} \underbrace{\frac{N_{m,\mathrm{pp}}}{N_{k,\mathrm{pp}}} \cdot \frac{N_k}{N_m}}_{W_{m,k}}\right)^{m-1}$$

Summary

- We developed a general nonflow subtraction strategy for m-particle correlations, inspired by approximate $1/N^{m-1}$ scaling behavior of nonflow, lead to a plethora of methods.
- Deviation from such scaling arise from selection bias, partially accounted in $\langle v_1^2 \rangle$ and $\langle v_1^2 \delta p_T \rangle$ method; Bias also arises from wide N range of reference, in part. for minbias pp.

- Estimate residual nonflow fraction in HIJING: ≲20-30% when converted to 2PC level

  → In real data: if flow signal is 30% of nonflow, the nonflow systematics can reach 100%.

- These methods are useful for understanding the nature of nonflow, and for evaluating the residual nonflow systematics in the subevent methods.
- But given the large number of nonflow subtraction methods, each observable-method combination need to be studied carefully in both models (HIJING, PYTHIA) and data

  → identify the best method for each observable.

Back up

Importance of using the correct weighting factor $\mathcal{E} = 1/N$

ong Jia

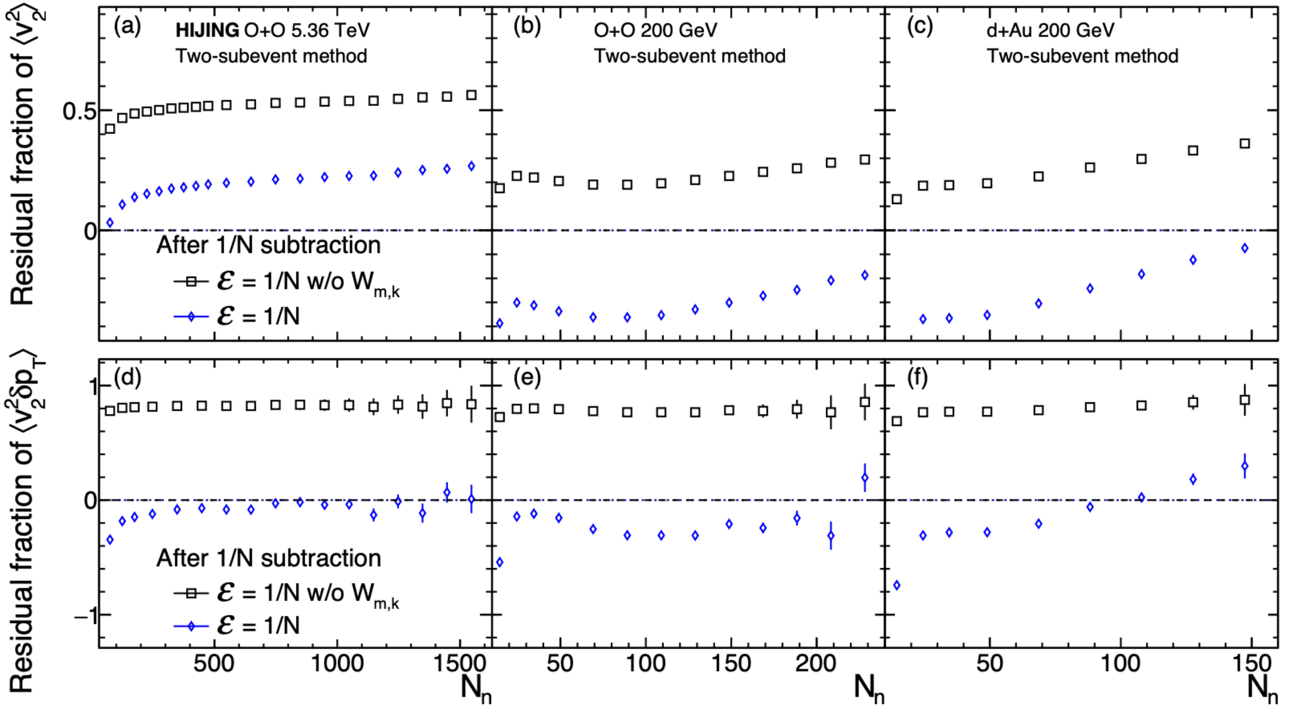
Considering reweighted N leads to improvement

$$N_m = \left(\frac{\sum_i N_i^m}{\sum_i N_i}\right)^{1/(m-1)} \quad \text{vs} \quad \langle N \rangle$$

$$\frac{\langle v_2^2 \rangle_{\text{sub}}}{\langle v_2^2 \rangle} = 1 - \boxed{\frac{N_{2,\text{pp}}}{N_2}} \frac{\langle v_2^2 \rangle_{\text{pp}}}{\langle v_2^2 \rangle}$$

We know 1/N-scaling under-subtracts nonflow in data if not using N_{2pp}

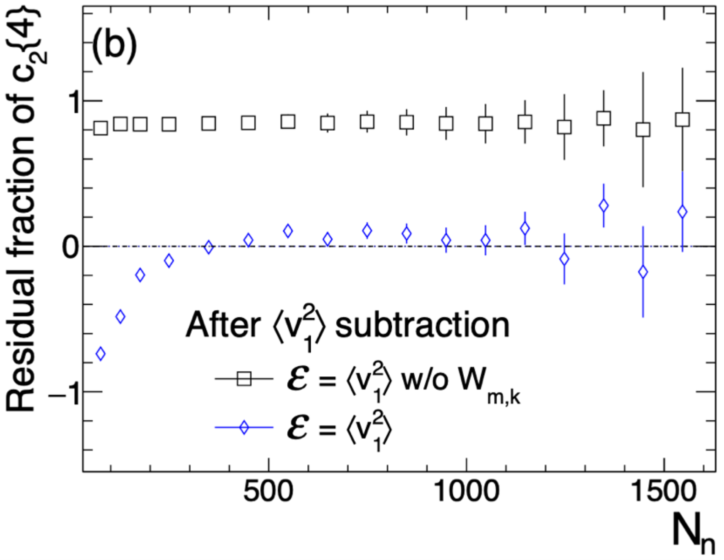
$$\frac{\langle v_2^2 \delta p_{\text{T}} \rangle_{\text{sub}}}{\langle v_2^2 \delta p_{\text{T}} \rangle} = 1 - \boxed{\left(\frac{N_{3,\text{pp}}}{N_3}\right)}^2 \frac{\langle v_2^2 \delta p_{\text{T}} \rangle_{\text{pp}}}{\langle v_2^2 \delta p_{\text{T}} \rangle}$$

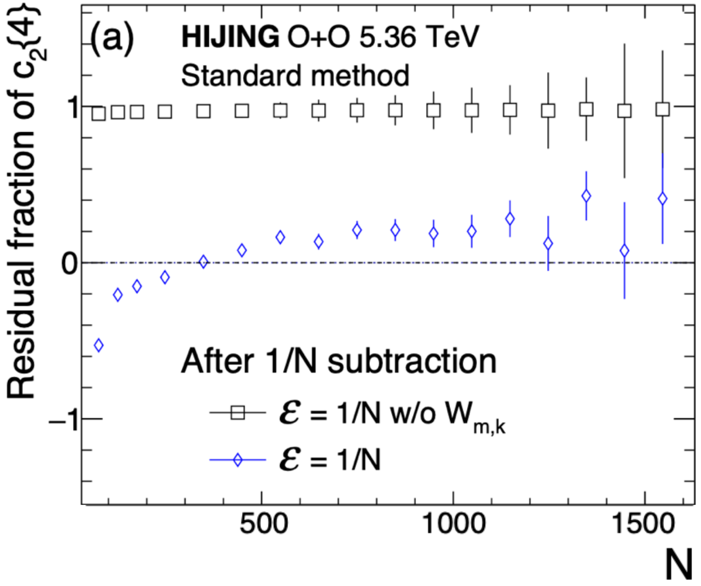


Importance of using the correct weighting factor

$$\frac{v_2^4\{4\}_{\rm sub}}{v_2^4\{4\}} = 1 - \left(\frac{\langle v_1^2\rangle}{\langle v_1^2\rangle_{\rm pp}} \boxed{\frac{N_{4,\rm pp}}{N_{2,\rm pp}}\frac{N_2}{N_4}}\right)^3 \frac{v_2^4\{4\}_{\rm pp}}{v_2^4\{4\}}$$

$$\frac{v_2^4\{4\}_{\rm sub}}{v_2^4\{4\}} = 1 - \boxed{\left(\frac{N_{4,\rm pp}}{N_4}\right)}^3 \frac{v_2^4\{4\}_{\rm pp}}{v_2^4\{4\}}$$





Again, considering reweighted N is essential!

Jiangyong Jia

Importance of using the correct weighting factor

Considering the reweighted N leads to improvement

$$\frac{\langle v_2^2\rangle_{\rm sub}}{\langle v_2^2\rangle} = 1 - \left(\frac{\langle v_1^2\delta p_{\rm T}\rangle}{\langle v_1^2\delta p_{\rm T}\rangle_{\rm pp}}\boxed{\frac{N_{2,\rm pp}}{N_{3,\rm pp}}}\frac{N_3}{N_2}\right)^{1/2}\frac{\langle v_2^2\rangle_{\rm pp}}{\langle v_2^2\rangle}$$

$$\frac{\langle v_2^2\delta p_{\rm T}\rangle_{\rm sub}}{\langle v_2^2\delta p_{\rm T}\rangle} = 1 - \left(\frac{\langle v_1^2\rangle}{\langle v_1^2\rangle_{\rm pp}}\boxed{\frac{N_{3,\rm pp}}{N_{2,\rm pp}}}\frac{N_2}{N_3}\right)^{2}\frac{\langle v_2^2\delta p_{\rm T}\rangle_{\rm pp}}{\langle v_2^2\delta p_{\rm T}\rangle}$$