Feasibility of measuring the QGP speed of sound from ultracentral heavy-ion collisions

Jean-François Paquet

June 29, 2026

VANDERBILT UNIVERSITY

5th International Workshop on Emergent QCD Collectivity Across Scales: From Small System Collisions to Jets @ Rice Global Paris Center

Analytical insights into the interplay of momentum, multiplicity, and the speed of sound in heavy-ion collisions

Gabriel Soares Rocha [1,*], Lorenzo Gavassino [2,†], Mayank Singh [1,‡], and Jean-François Paquet [1,2,§]

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Phys. Rev. C **110**, 034913 – **Published 26 September, 2024**

Feasibility of measuring the speed of sound of the quark-gluon plasma from the multiplicity and mean p_T of ultracentral heavy-ion collisions

Lorenzo Gavassino [1,*], Henry Hirvonen [1,2,†], Jean-Francois Paquet [2,1,‡], Mayank Singh [2,§], and Gabriel Soares Rocha [3,2,∥]

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Phys. Rev. C **112**, 054903 – **Published 17 November, 2025**

Lorenzo Gavassino

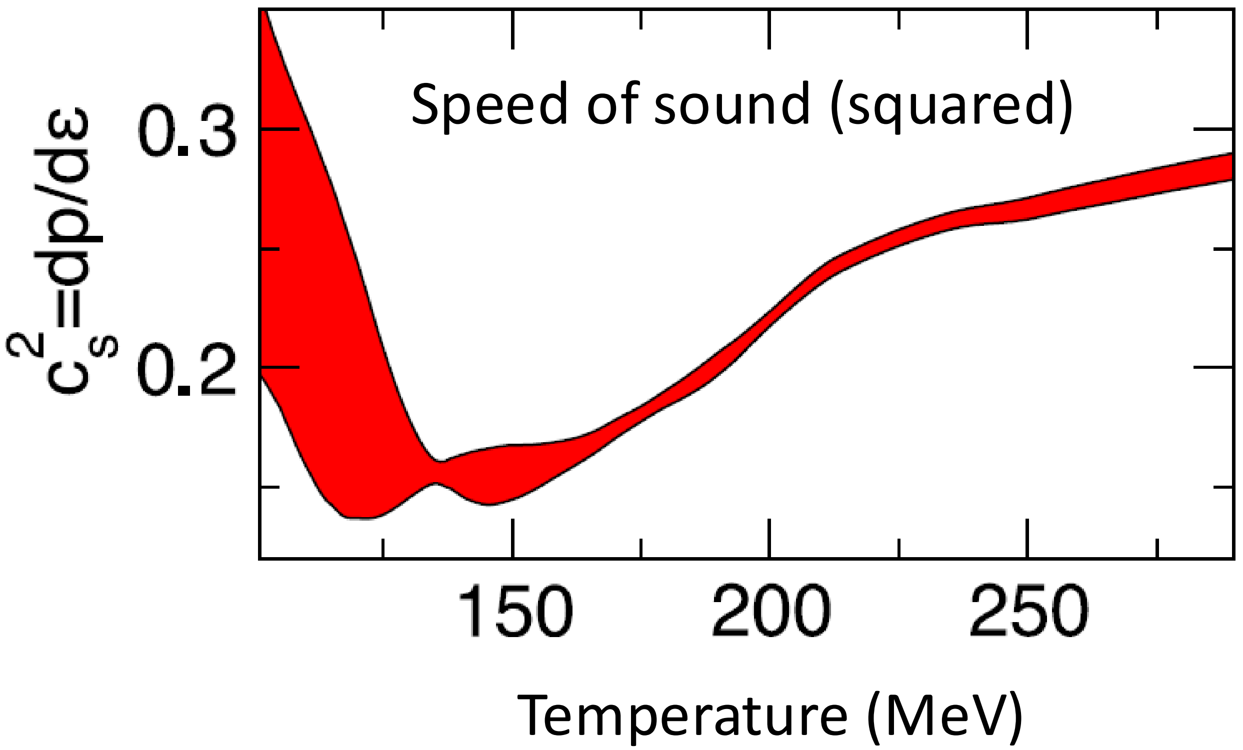
Gabriel Soares Rocha

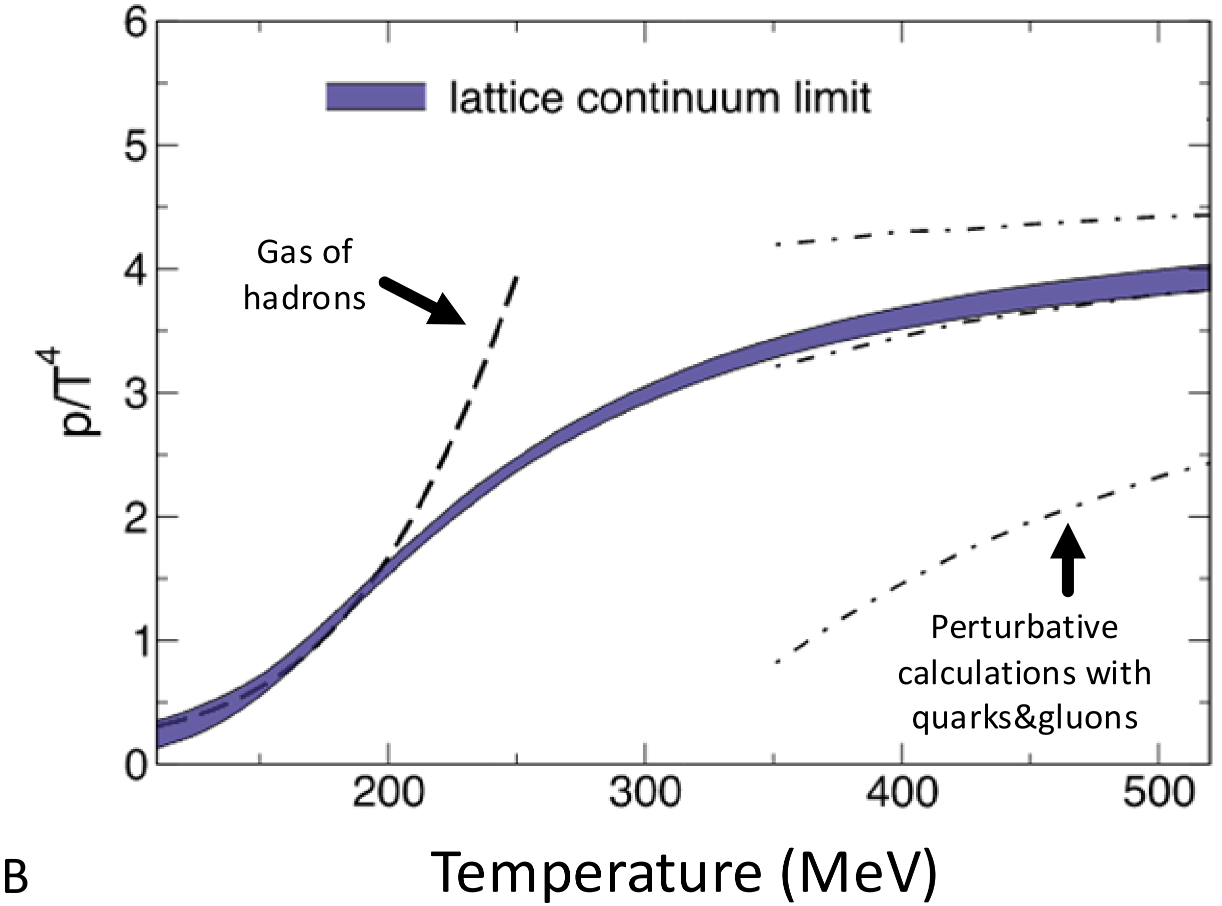
Henry Hirvonen

Mayank Singh

QCD thermodynamics: the equation of state

- Equation of state calculated from first principles (finite-temperature lattice QCD)



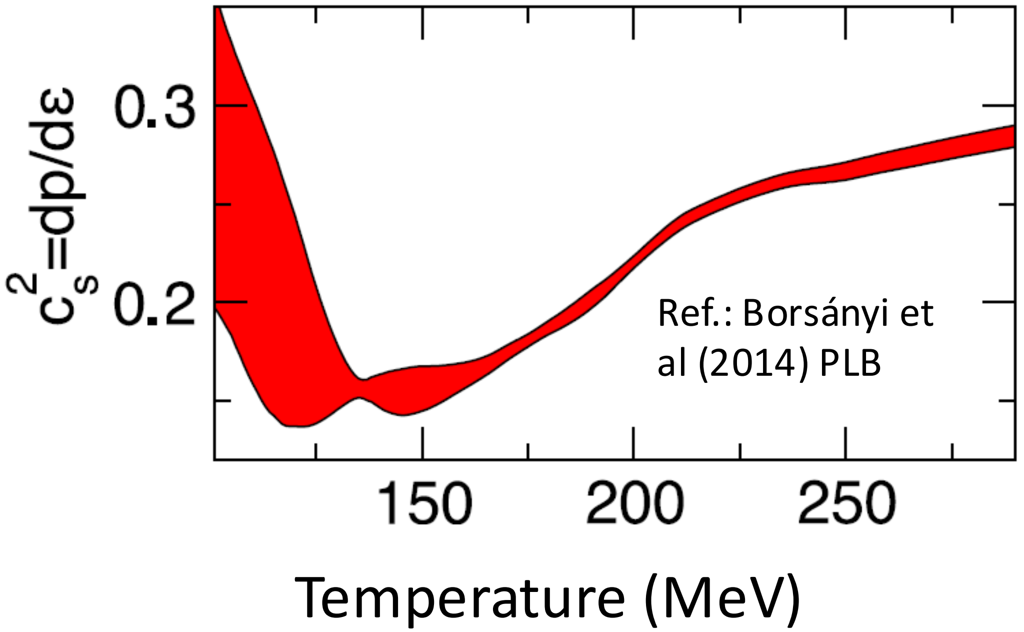


Ref.: Borsányi et al (2014) PLB

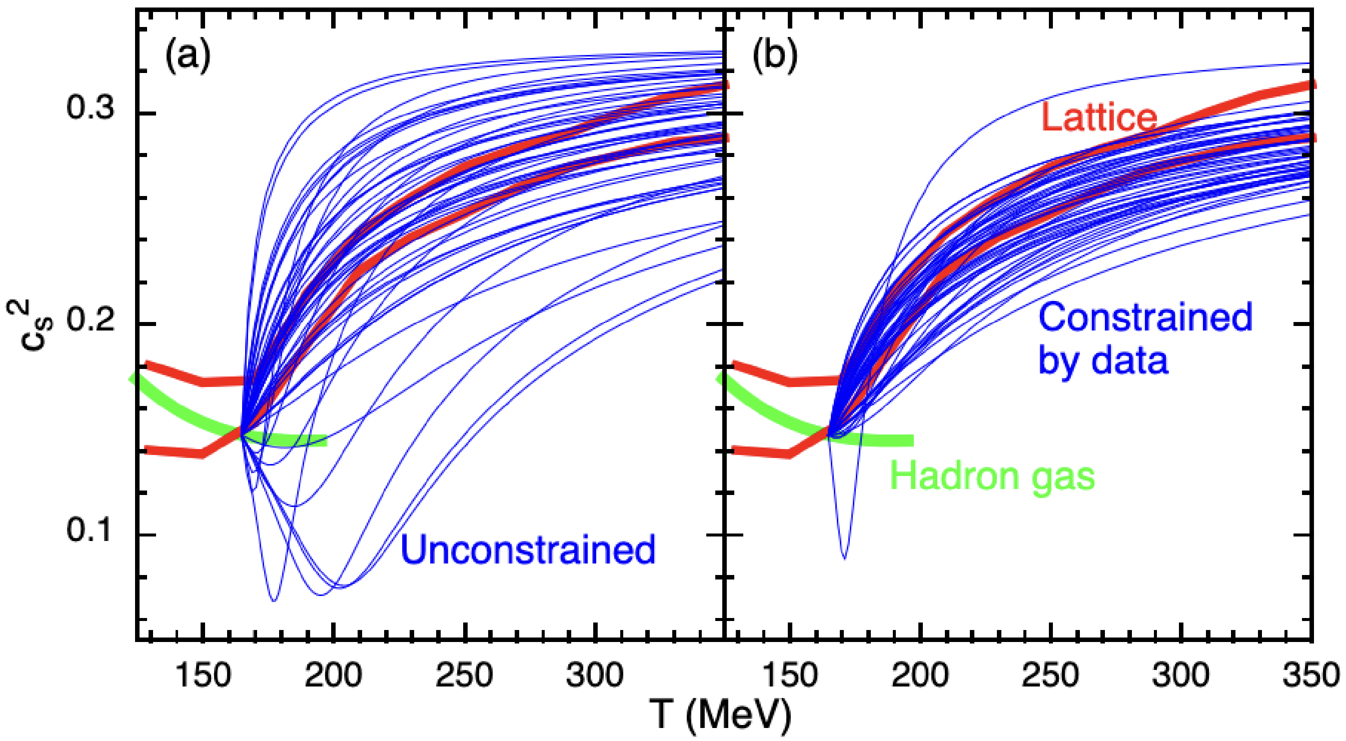
The QCD equation of state

Pratt et al PRL 114, 202301 (2015)
Sangaline, Pratt PRC 93,024908(2016)

Speed of sound (squared)
from lattice QCD



Speed of sound (squared)
from Bayesian analysis of soft hadrons



Why isn't the QCD equation of state always extracted?

Equation of state enters here

"Energy deposition" → **Early dynamics** → **Hydrodynamics** → **Hadronic transport** → **Measurements**

"Energy deposition"
- Parametric models (e.g. Glauber, Trento)
- Effective QCD field theory (IP-Glasma)

Early dynamics
- Effective QCD kinetic theory (e.g. KoMPoST)
- Holography

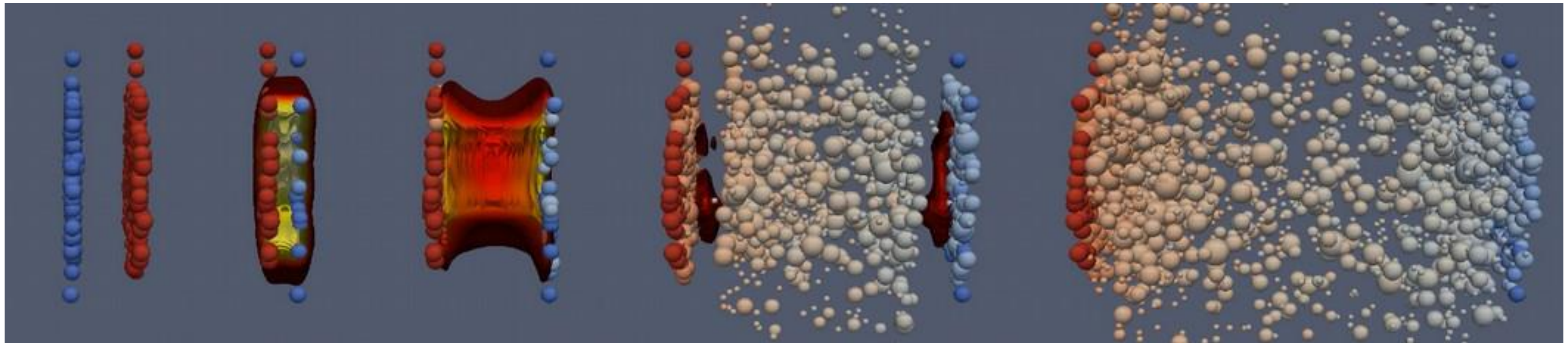
Hydrodynamics

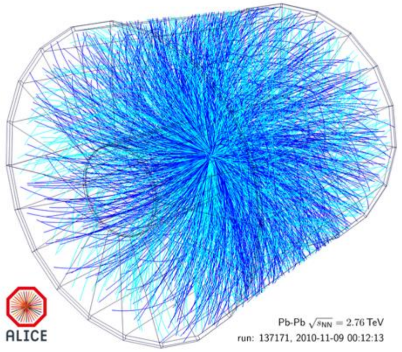
Relativistic viscous (Israel-Stewart type) hydrodynamics

Hadronic transport

Relativistic hadronic transport

Measurements





Why isn't the QCD equation of state always extracted?

Interface with hydrodynamics (e.g. initial time)

Equation of state enters here

Interface with hydrodynamics (e.g. "particlization" temperature)

"Energy deposition" → **Early dynamics** → **Hydrodynamics** → **Hadronic transport** → **Measurements**

"Energy deposition":
- Parametric models (e.g. Glauber, Trento)
- Effective QCD field theory (IP-Glasma)

Early dynamics:
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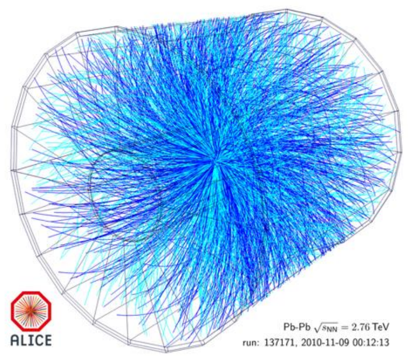
Hydrodynamics:
Relativistic viscous (Israel-Stewart type) hydrodynamics

Hadronic transport:
Relativistic hadronic transport

Measurements:



Initial condition parameters (nucleon width, fluctuations, …)

Shear and bulk viscosity, relaxation times, …

- Local equilibrium is $\pm$ assumed in hydrodynamics, hence lattice equation of state generally considered a safe "prior"

Why isn't the QCD equation of state always extracted?

"Energy deposition" → **Early dynamics** → **Hydrodynamics** → **Hadronic transport** → **Measurements**

- Parametric models (e.g. Glauber, Trento)
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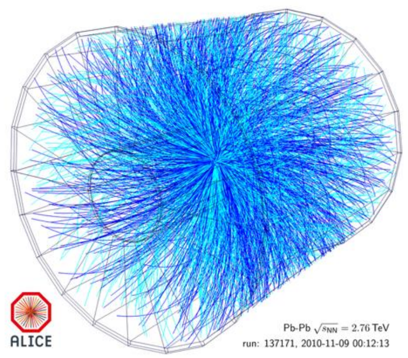
Relativistic viscous (Israel-Stewart type) hydrodynamics

Relativistic hadronic transport



"Pion multiplicity"(Initial condition parameters, Equation of state vs T, Shear&bulk viscosity vs T, …) $\leftrightarrow$ Measurement

"Kaon multiplicity"(Initial condition parameters, Equation of state vs T, Shear&bulk viscosity vs T, …) $\leftrightarrow$ Measurement

… …

"Pion $\langle p_T \rangle$"(Initial conditions parameter, Equation of state vs T, Shear&bulk viscosity vs T, …) $\leftrightarrow$ Measurement

Mapping QCD equation of state to data

"Energy deposition" → Early dynamics → Hydrodynamics → Hadronic transport → **Measurements**

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$$\overrightarrow{Model} = \begin{pmatrix} \text{Model prediction for } \mathrm{dN}_{\pi^+}/\mathrm{dy} \\ \text{Model prediction for } \mathrm{dN}_{K^+}/\mathrm{dy} \\ \text{Model prediction } \ldots \end{pmatrix}; \quad \overrightarrow{params} = \begin{pmatrix} \text{Model parameter 1} \\ \text{Model parameter 2} \\ \text{Model parameter } \ldots \end{pmatrix}; \quad \overrightarrow{data} = \begin{pmatrix} \text{Data for } \mathrm{dN}_{\pi^+}/\mathrm{dy} \\ \text{Data for } \mathrm{dN}_{K^+}/\mathrm{dy} \\ \text{Data for } \ldots \end{pmatrix};$$

$$\boxed{\overrightarrow{Model}(\overrightarrow{params}) = \overrightarrow{data}}$$

Mapping QCD equation of state to data

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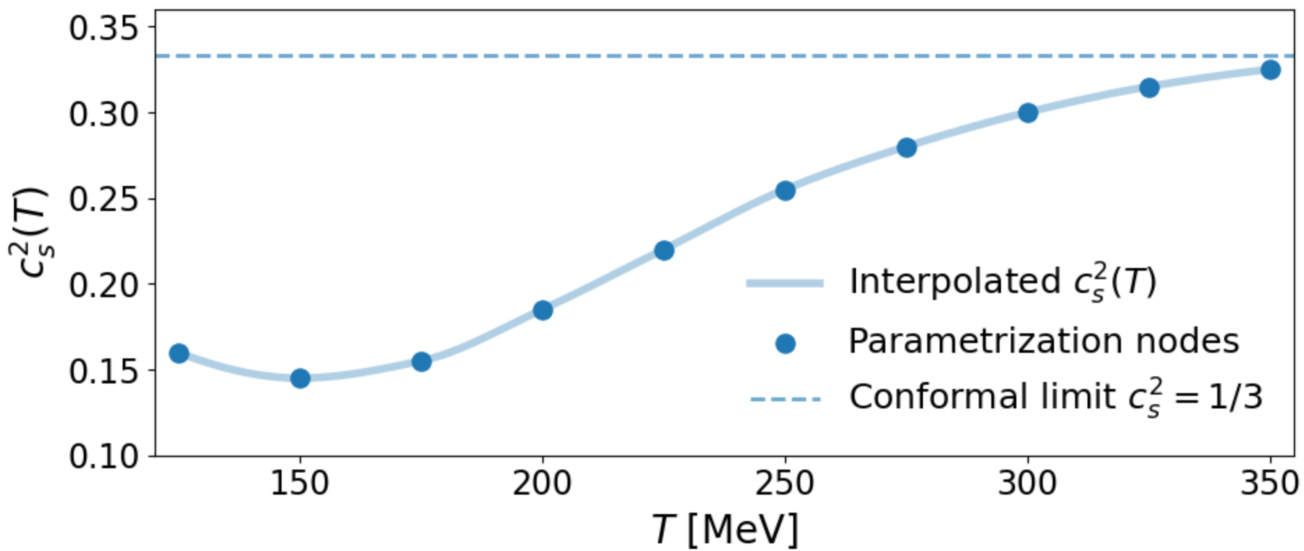

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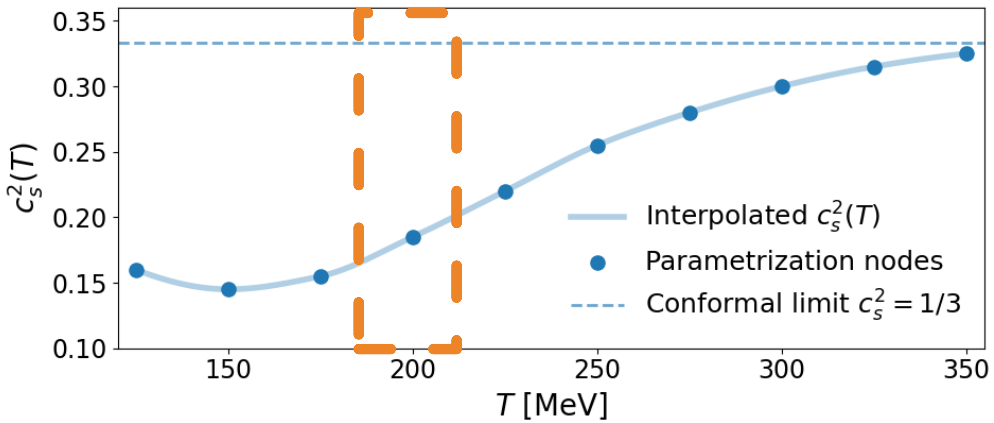
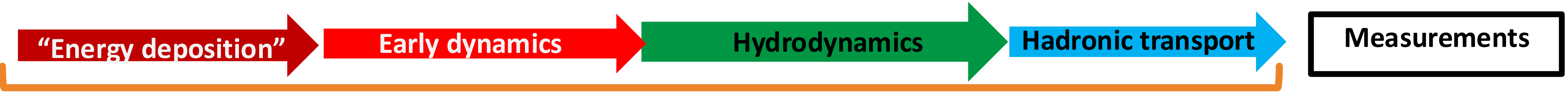
$$\overrightarrow{Model}(\overrightarrow{params}) = \overrightarrow{data} \Rightarrow \overrightarrow{params} = \overrightarrow{Model}^{-1}(\overrightarrow{data})$$

e.g.

$$c_s^2(T = 200\ MeV) = \text{function}(\overrightarrow{data})$$



Mapping QCD equation of state to data



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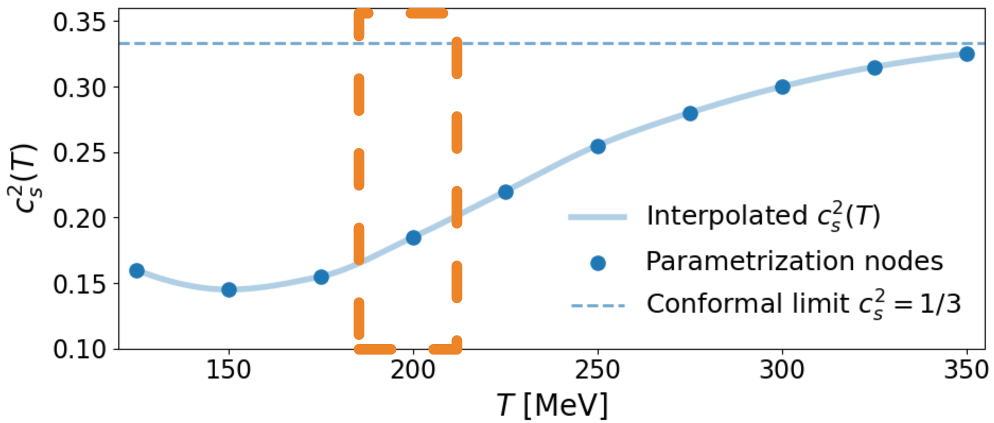
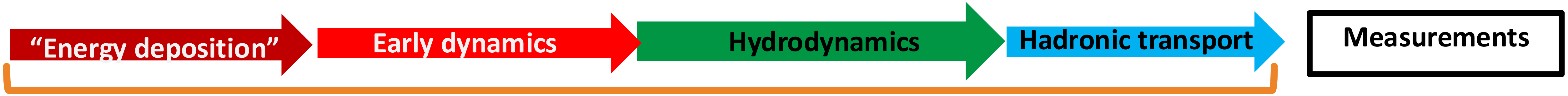
$$\text{e.g. } c_s^2(T = 200\ MeV) = \text{function}(\overrightarrow{data})$$

- Model is non-linear:
  Associating a measurement to single parameter is challenging



Mapping QCD equation of state to data



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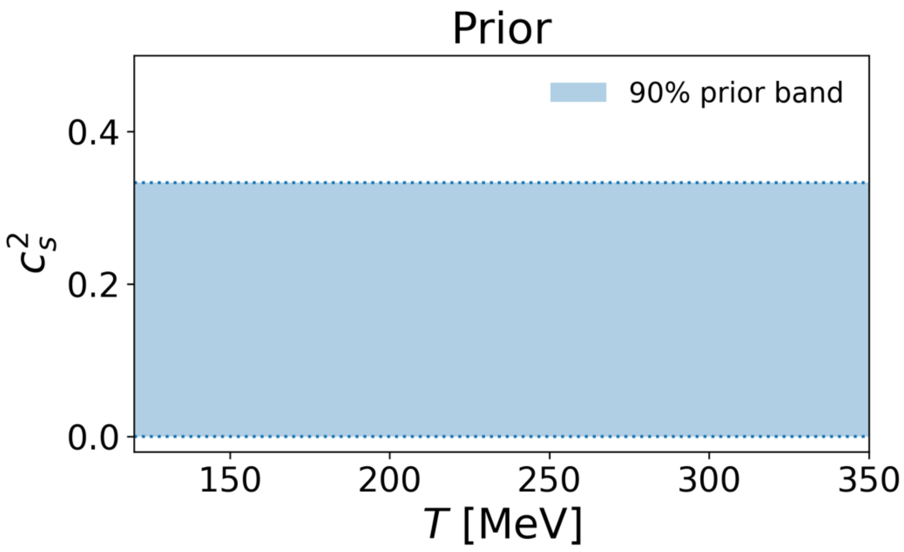
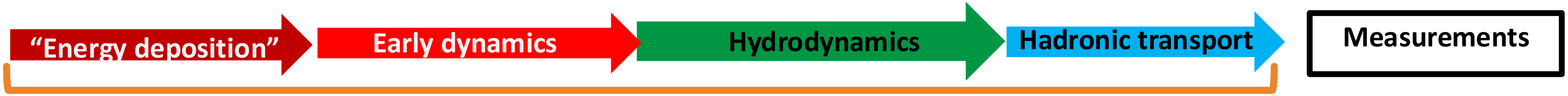


Can a small data set constrain strongly the equation of state?

Mapping QCD equation of state to data



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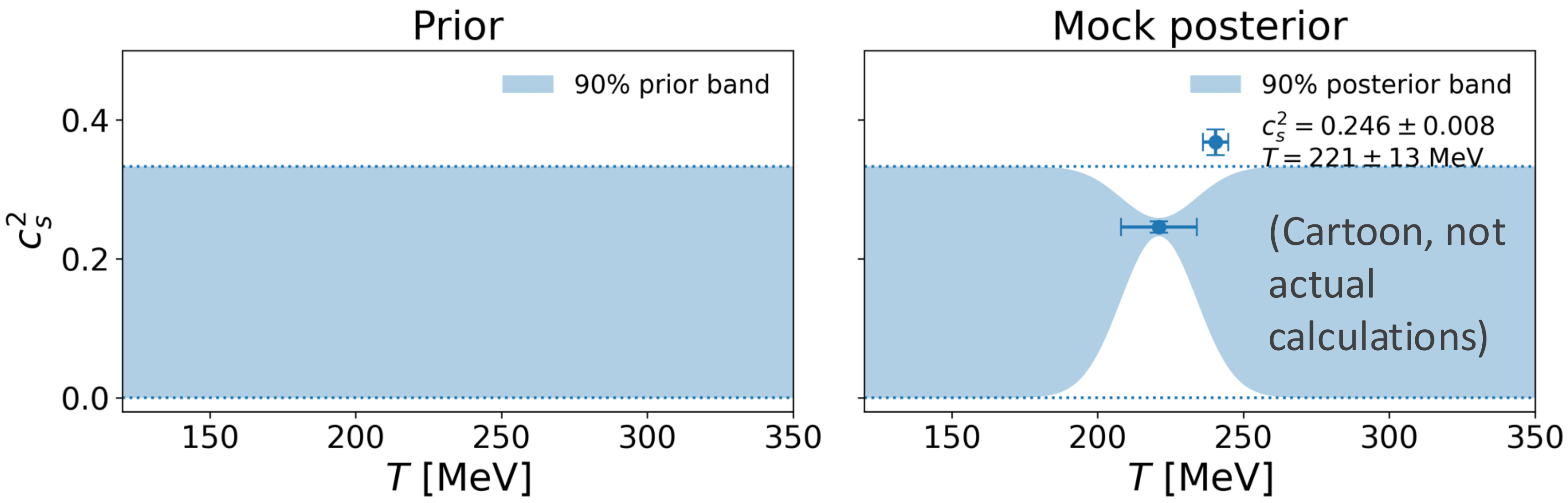


To my knowledge, this has not been tried yet

Some results from my group

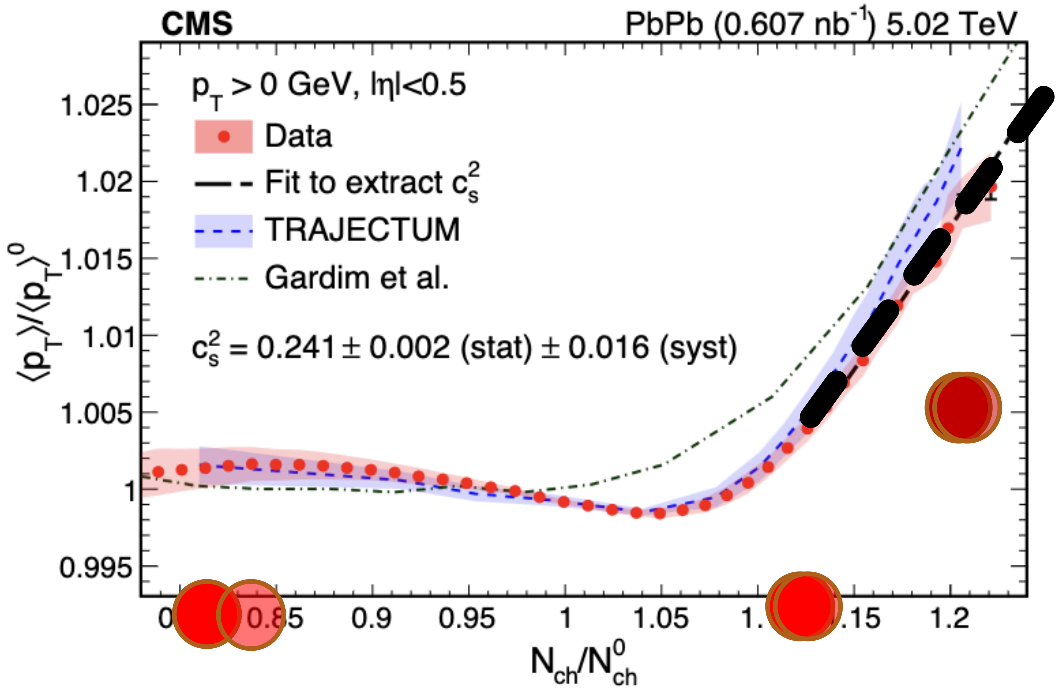
Interpretation of effective temperature T_{eff}, and assumption on the smoothness of the speed of sound

Measuring the equation of state

Thermodynamics of hot strong-interaction matter from ultrarelativistic nuclear collisions #7

Fernando G. Gardim (Alfenas Fed. U., Pocos de Caldas and IPhT, Saclay), Giuliano Giacalone (IPhT, Saclay), Matthew Luzum (Sao Paulo U.), Jean-Yves Ollitrault (IPhT, Saclay) (Aug 26, 2019)

Published in: *Nature Phys.* 16 (2020) 6, 615-619 • e-Print: 1908.09728 [nucl-th]



Ref.: CMS Collaboration, Rept.Prog.Phys. 87 (2024)

$$\Rightarrow \frac{d\ln\langle p_T\rangle}{d\ln dN_{ch}/d\eta} = c_s^2\left(T_{\mathrm{eff}} \approx \frac{\langle p_T\rangle}{3}\right)$$

This relation can be tested in any hydrodynamic model, with different assumptions (different initial conditions, …)

Why do we assume that effective temperature T_{eff} is proportional to $\langle p_T \rangle$?

Why do we assume that effective temperature T_{eff} is proportional to $\langle p_T \rangle$?
$\Rightarrow$ Can verify in simulations

Why do we assume that effective temperature T_{eff} is proportional to $\langle p_T \rangle$?
⇒ Can verify in simulations

In simulation, all the energy and entropy flowing out of the system can be computed:

$$E = \int_\Sigma \mathrm{d}^3\sigma_\mu T^{t\mu}, \quad S = \int_\Sigma \mathrm{d}^3\sigma_\mu s u^\mu,$$

T_{eff} defined by solving

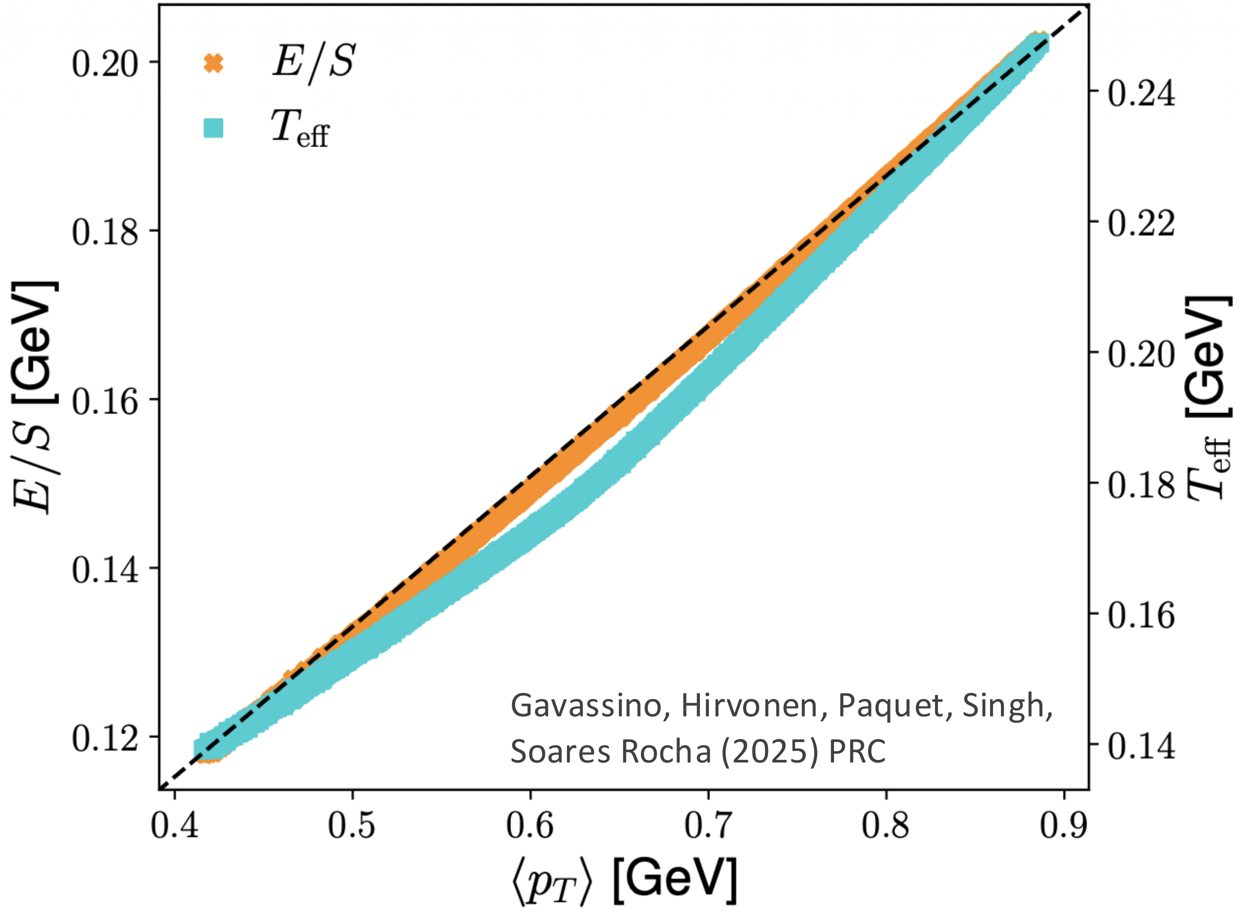
$$E \equiv \varepsilon_{lattice}(T_{\text{eff}})V_{\text{eff}}$$
$$S \equiv s_{lattice}(T_{\text{eff}})V_{\text{eff}}$$

Ref.: Gardim, Giacalone, Luzum, Ollitrault Nature Phys. 16, 615 (2020)

$\varepsilon_{lattice}(T)$ and $s_{lattice}(T)$ are the energy density ε and the entropy density s from the lattice QCD equation of state, as a function of the temperature T

What is $\langle p_T \rangle$ proportional to in simulations?

E and S are the energy and entropy flowing out of the system



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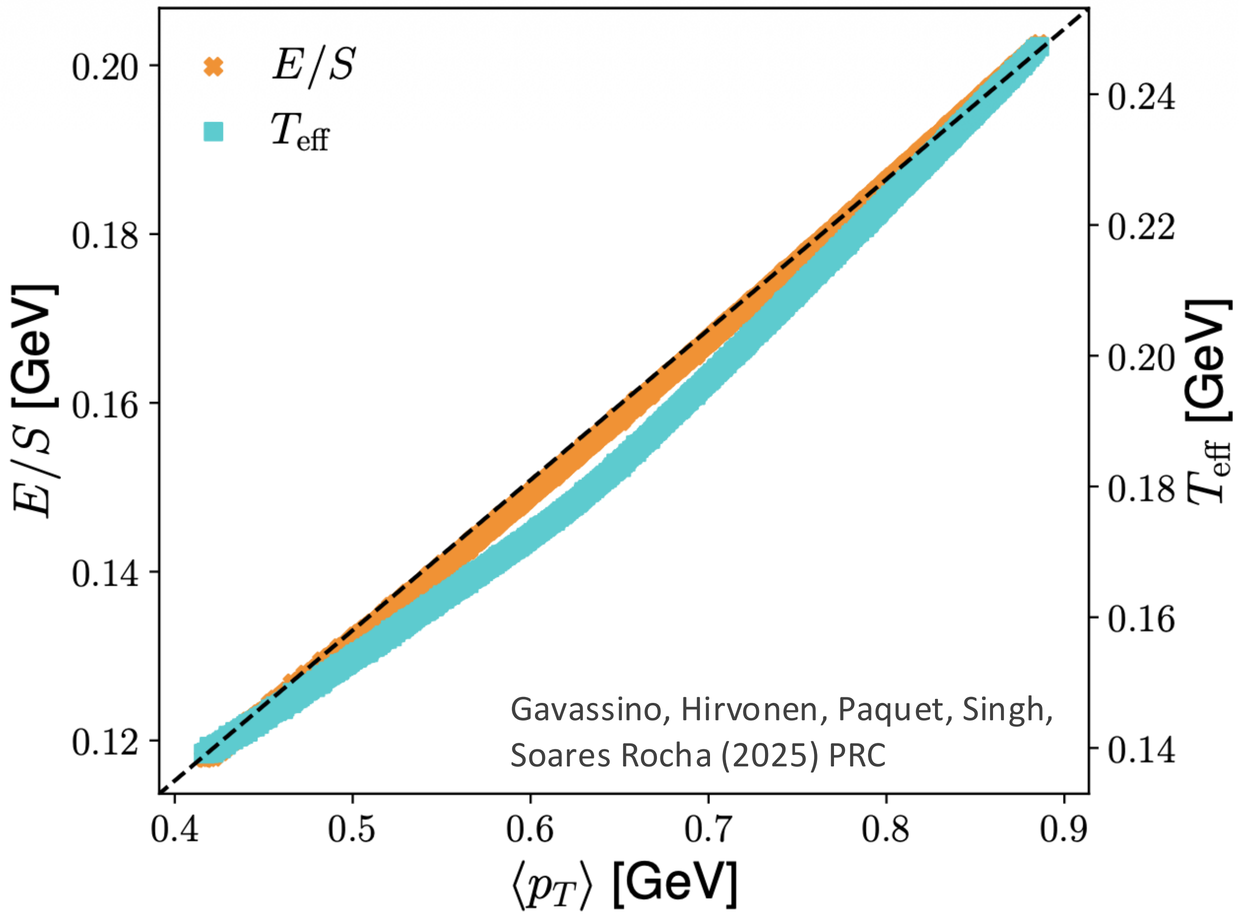


T_{eff} defined by solving

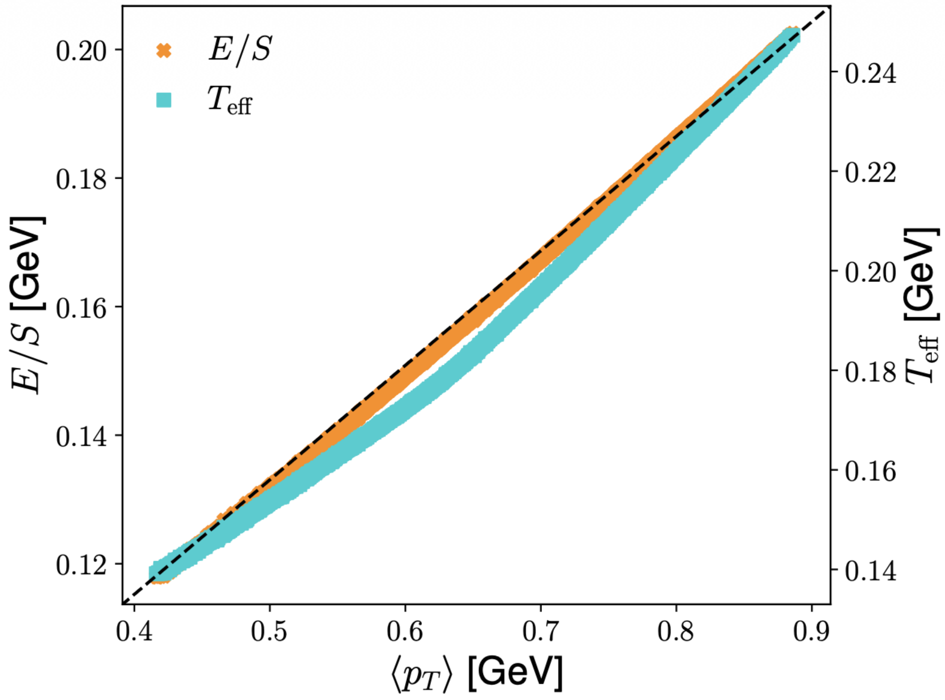
$$E \equiv \varepsilon_{lattice}(T_{\mathrm{eff}})V_{\mathrm{eff}}$$
$$S \equiv s_{lattice}(T_{\mathrm{eff}})V_{\mathrm{eff}}$$

Both E/S and T_{eff} are strongly correlated to the mean p_T

What is $\langle p_T \rangle$ proportional to (in simulations)?

If $\langle p_T \rangle \propto E/S$

$$\Rightarrow \frac{d\ \ln\langle p_T \rangle}{d\ \ln(dN/dy)} \sim \frac{P}{\varepsilon}$$



If $\langle p_T \rangle \propto T_{eff}$

$$\Rightarrow \frac{d\ \ln\langle p_T \rangle}{d\ \ln(dN/dy)} \sim c_s^2$$

What is extracted (c_s^2 or P/ε) depends on what $\langle p_T \rangle$ is assumed to be correlated with

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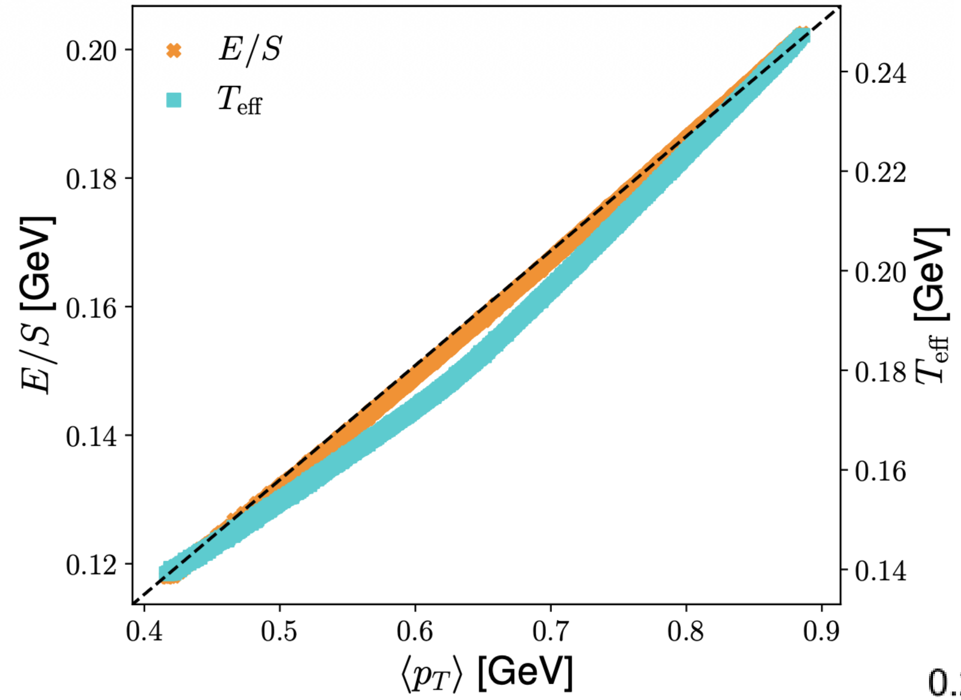

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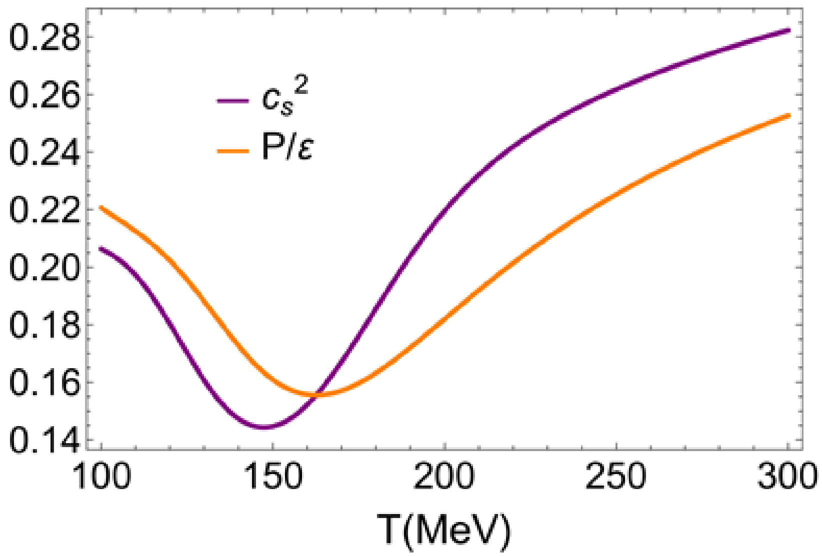
What is extracted (c_s^2 or P/ε) depends on what $\langle p_T \rangle$ is assumed to be correlated with

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

$$\frac{P(\varepsilon) - P(\varepsilon_{ref})}{\varepsilon - \varepsilon_{ref}} = \frac{1}{\varepsilon - \varepsilon_{ref}} \int_{\varepsilon_{ref}}^{\varepsilon} d\ \varepsilon'\ c_s^2(\varepsilon')$$



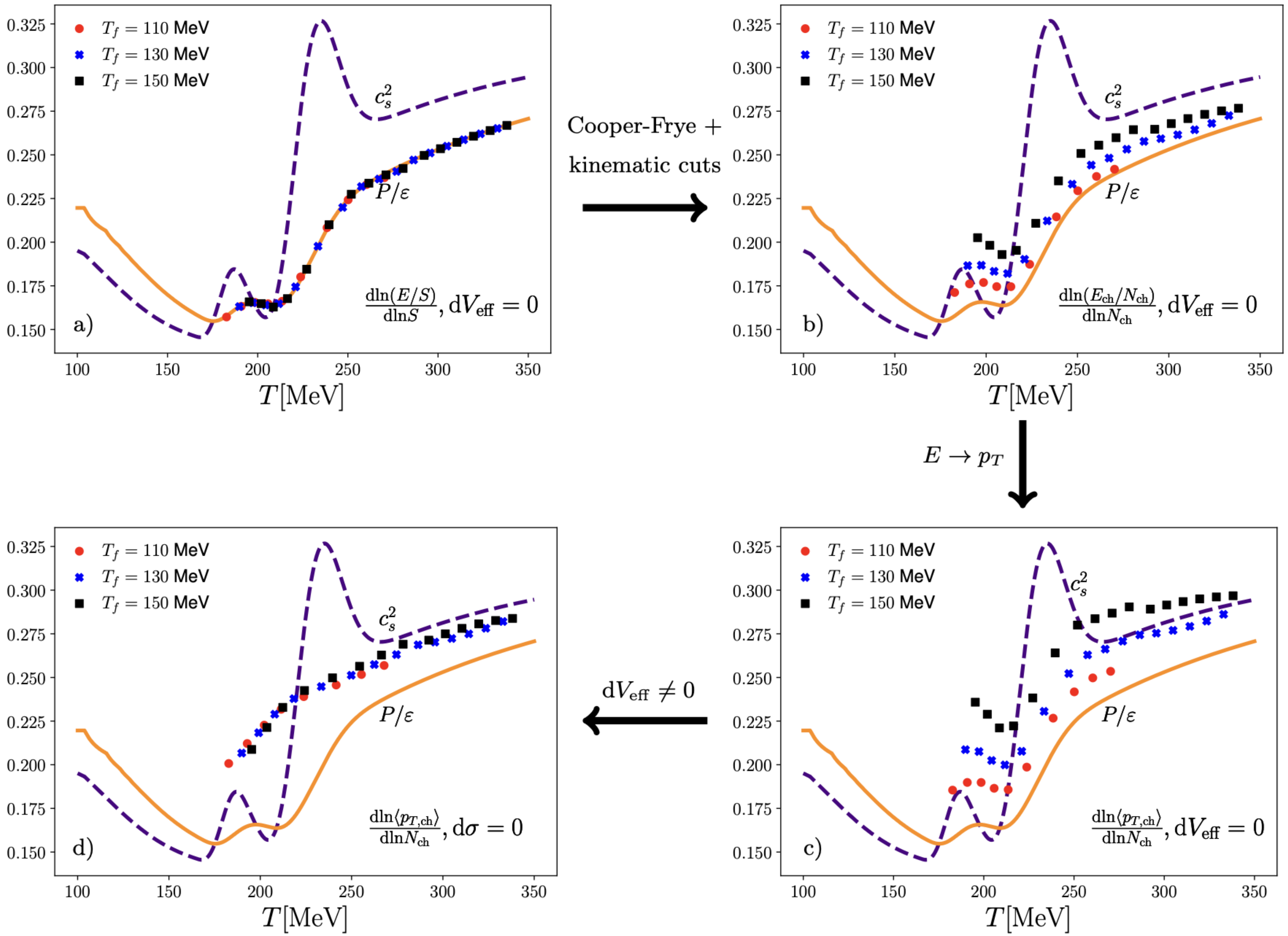
arXiv > hep-ph > arXiv:2503.20765

High Energy Physics – Phenomenology

[Submitted on 26 Mar 2025 (v1), last revised 8 Oct 2025 (this version, v4)]

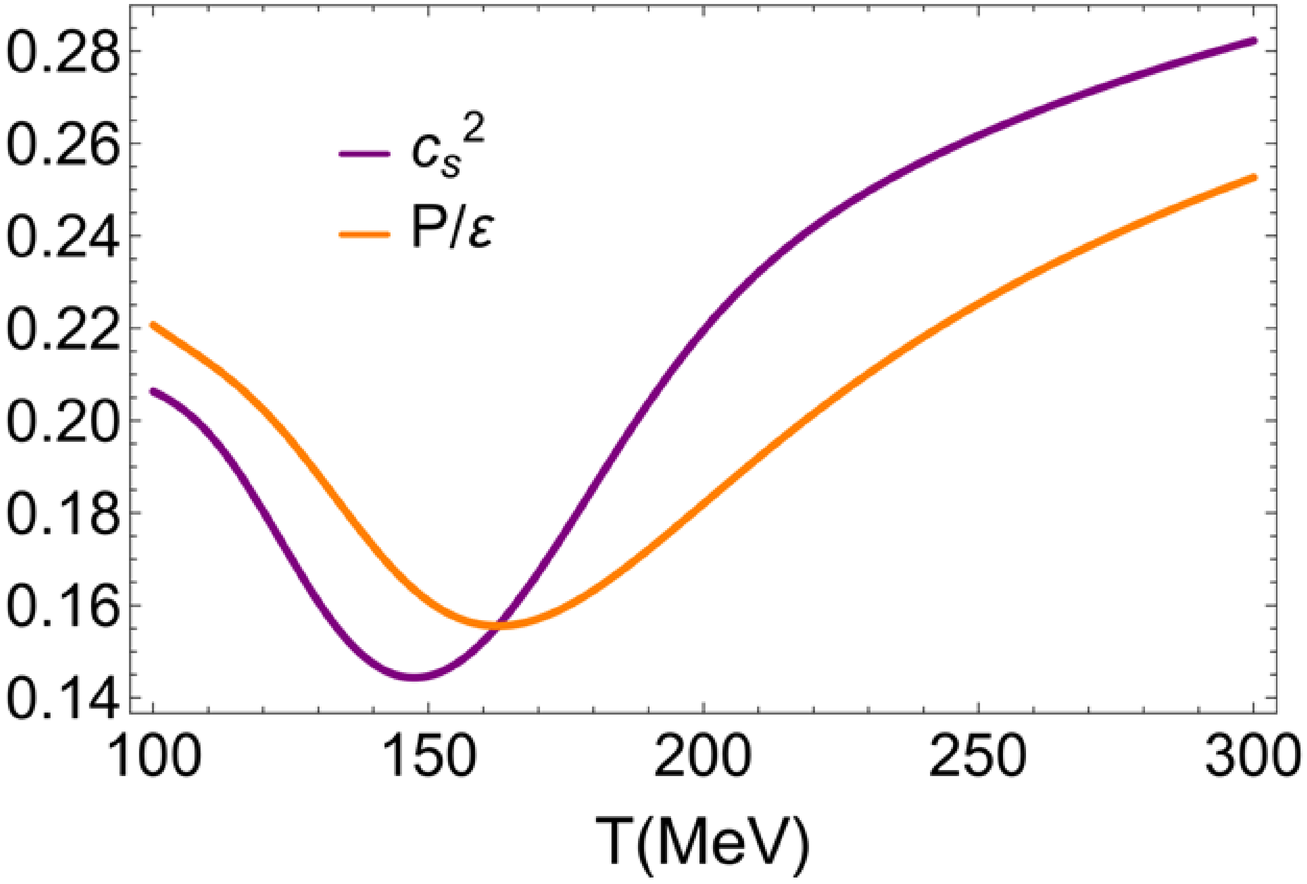
Feasibility of measuring the speed of sound of the quark-gluon plasma from the multiplicity and mean p_T of ultracentral heavy-ion collisions

Lorenzo Gavassino, Henry Hirvonen, Jean-François Paquet, Mayank Singh, Gabriel Soares Rocha



Numerical results using QCD equation of state

- Try different initial temperatures T_0 and different particlization temperatures T_f in numerical simulations
  $\Rightarrow$ Yields different T_{eff}
- Use lattice QCD equation of state
- Compare $\frac{d\ \ln\langle p_{T,ch}\rangle}{d\ \ln(dN_{ch}/d\eta)}$ to speed of sound



More details: Smooth Gaussian initial conditions with ideal hydrodynamics, Cooper-Frye and hadronic decays

Numerical results using QCD equation of state

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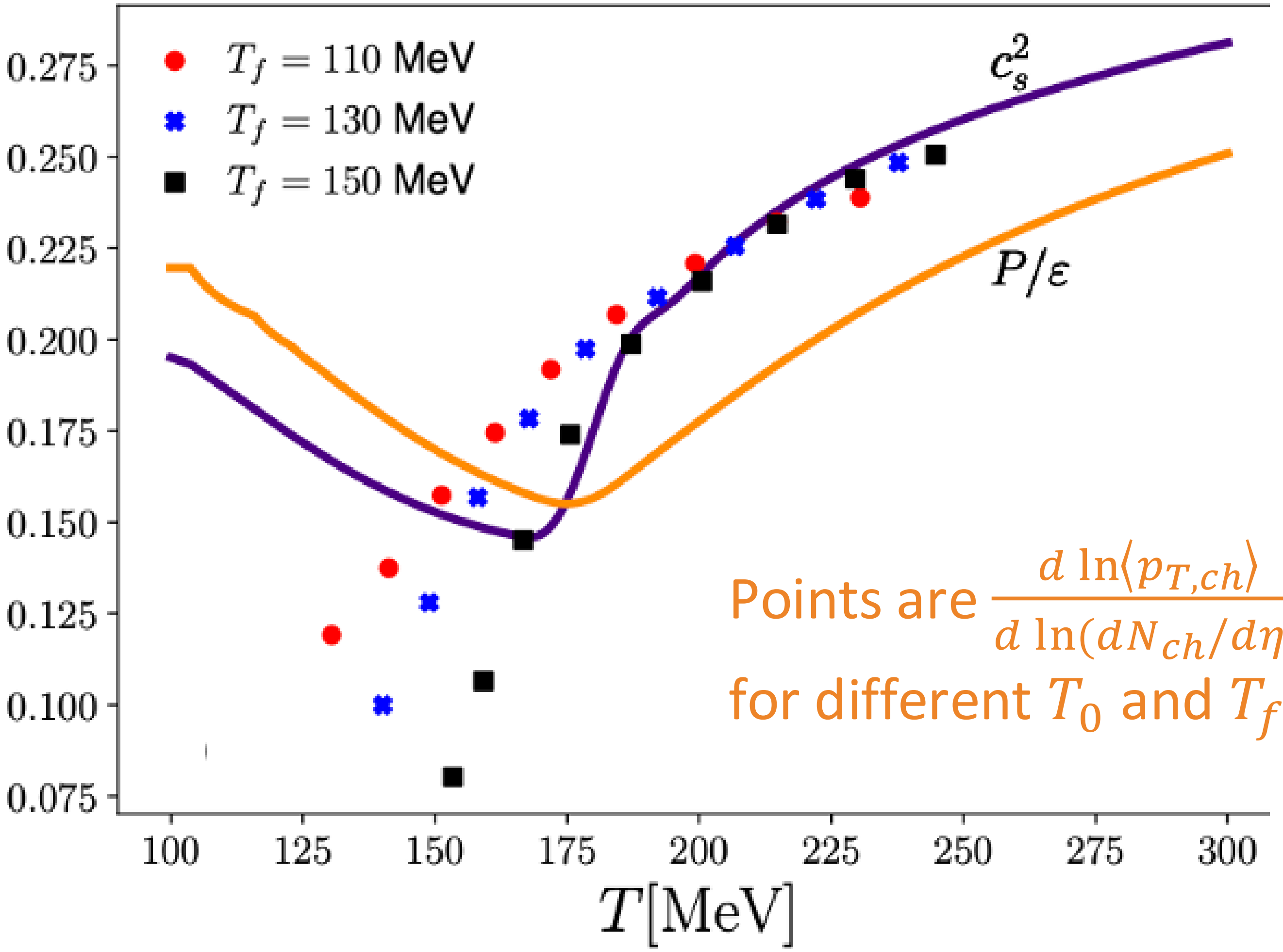

Points are $\frac{d\,\ln\langle p_{T,ch}\rangle}{d\,\ln(dN_{ch}/d\eta)}$ for different T_0 and T_f

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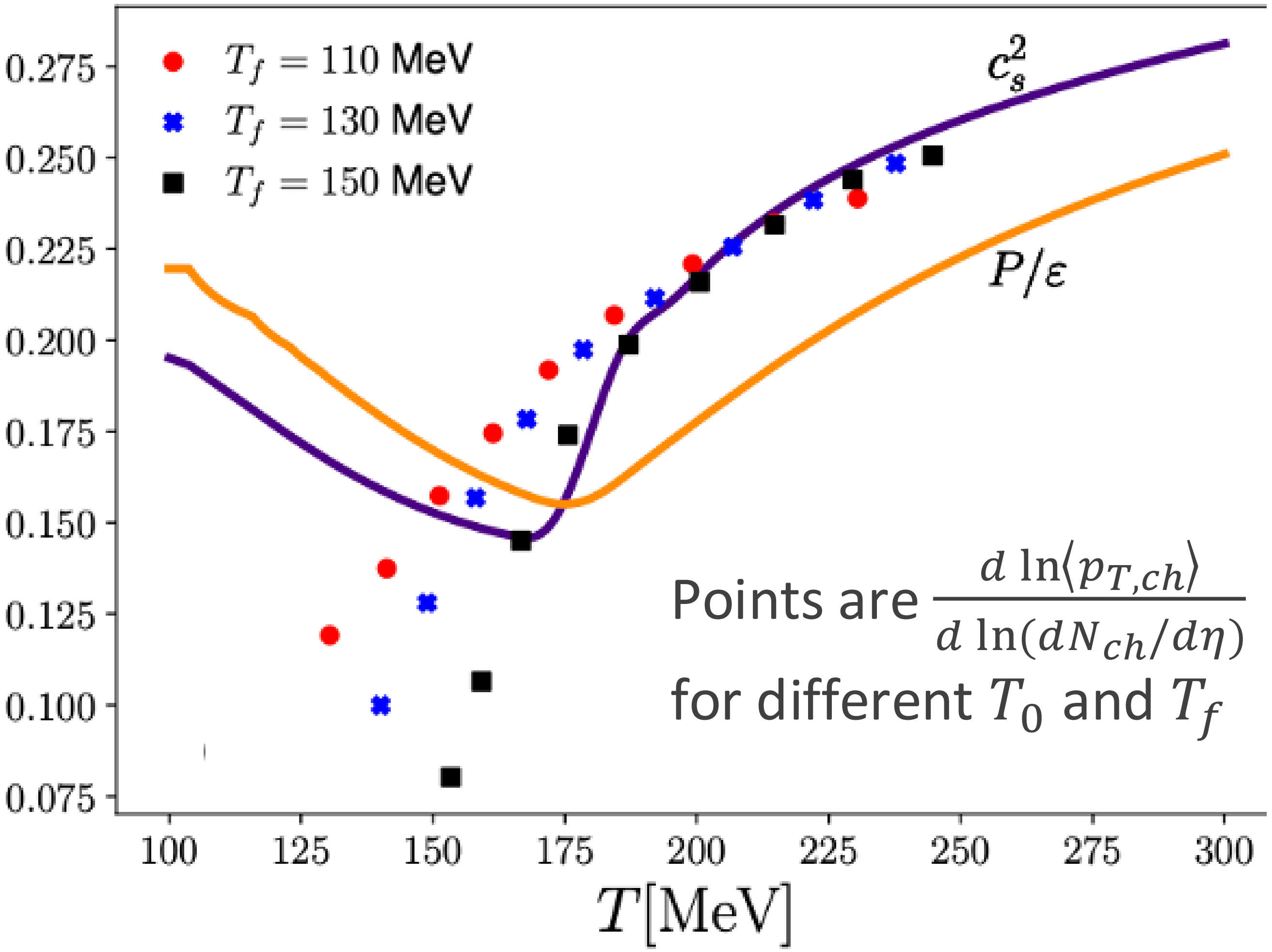
Be careful with small systems or low $\sqrt{s_{NN}}$ that have low initial temperature T_0

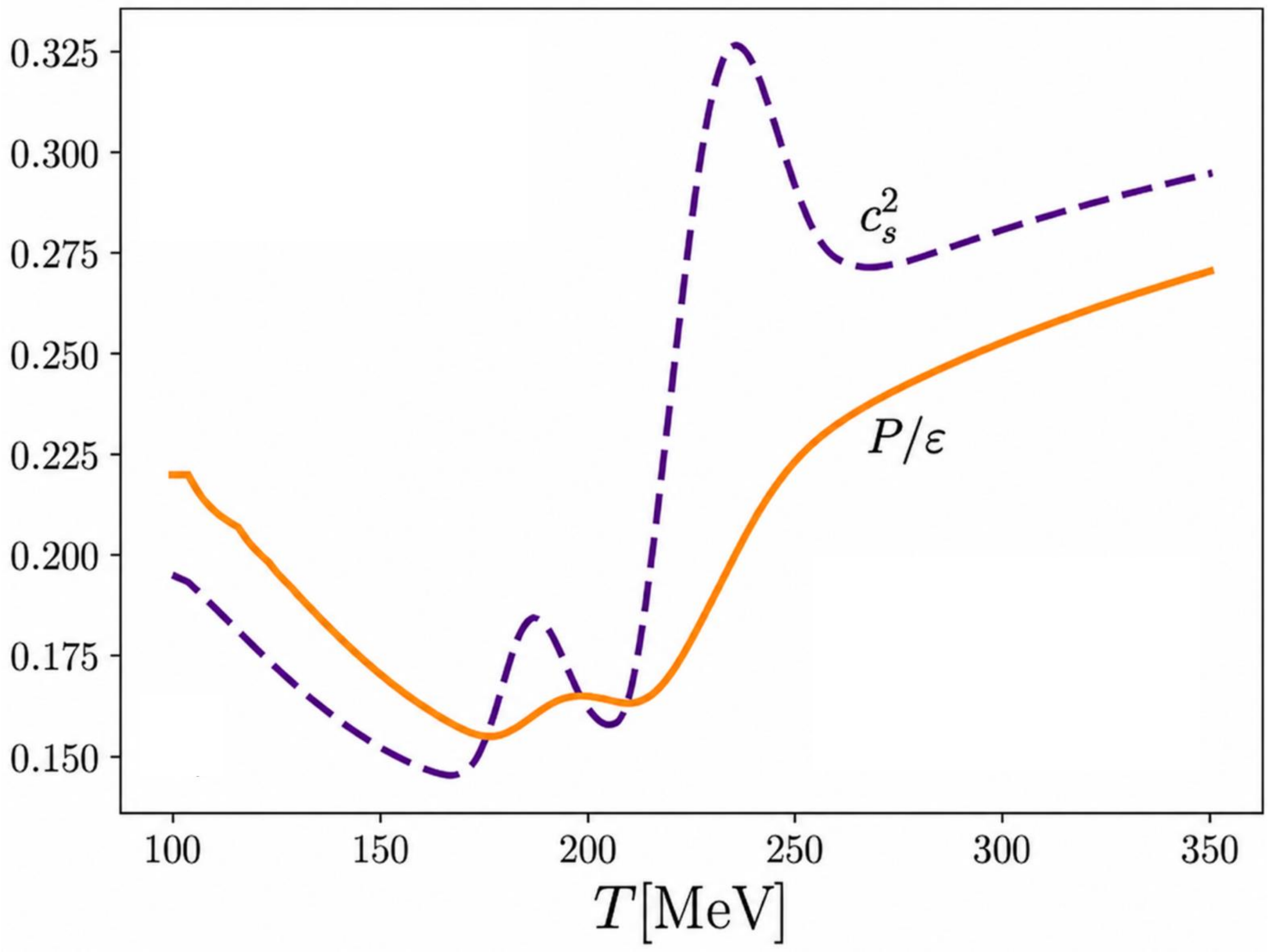
Agreement at high temperature



Points are $\frac{d\ \ln\langle p_{T,ch}\rangle}{d\ \ln(dN_{ch}/d\eta)}$ for different T_0 and T_f

Numerical results using modified equation of state

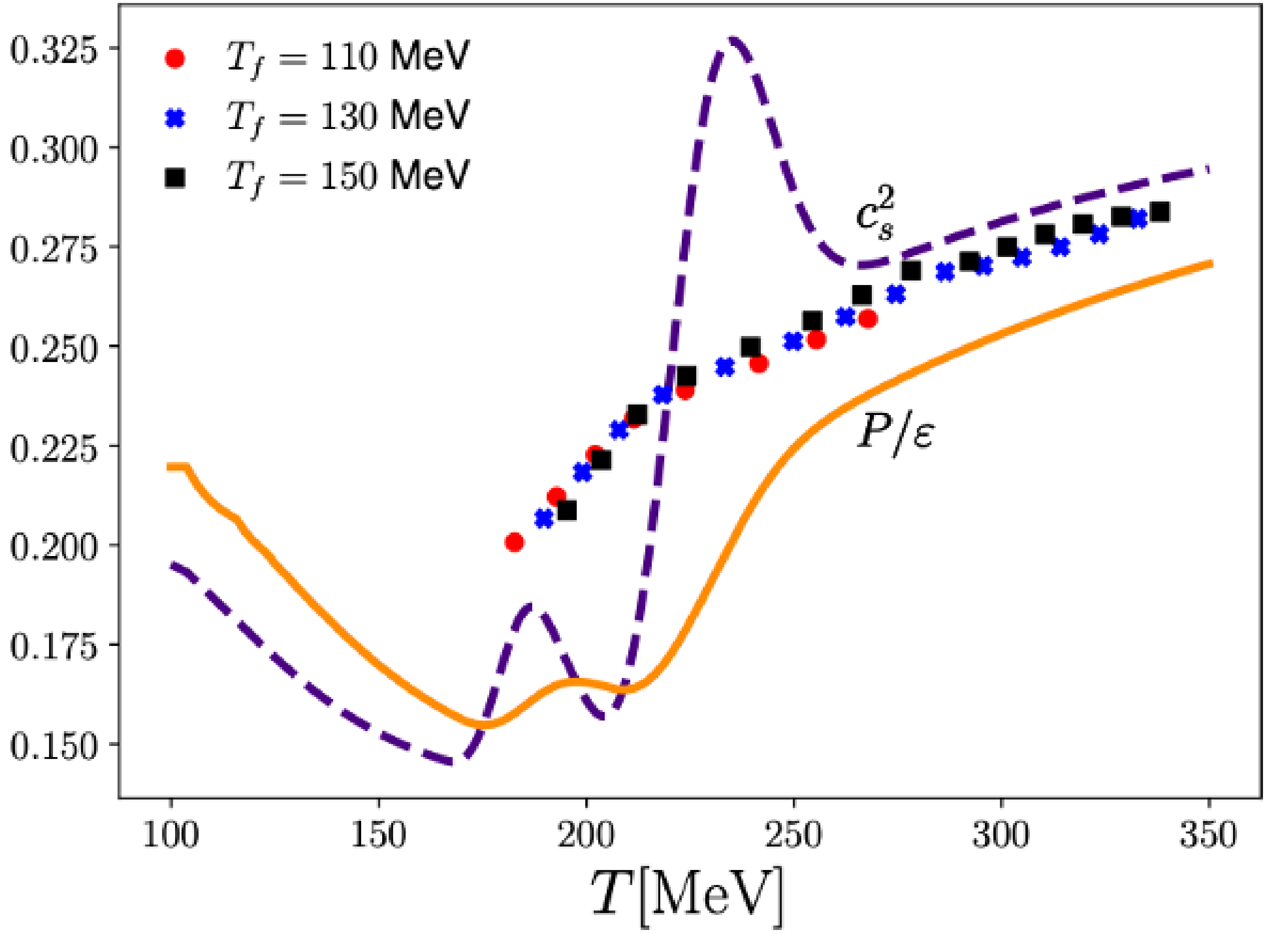
- Try different initial temperatures T_0 and different particlization temperatures T_f in numerical simulations
  $\Rightarrow$ Yields different T_{eff}
- Use **modified** equation of state
- Compare $\frac{d\,\ln\langle p_{T,ch}\rangle}{d\,\ln(dN_{ch}/d\eta)}$ to **modified** speed of sound



Numerical results using modified equation of state

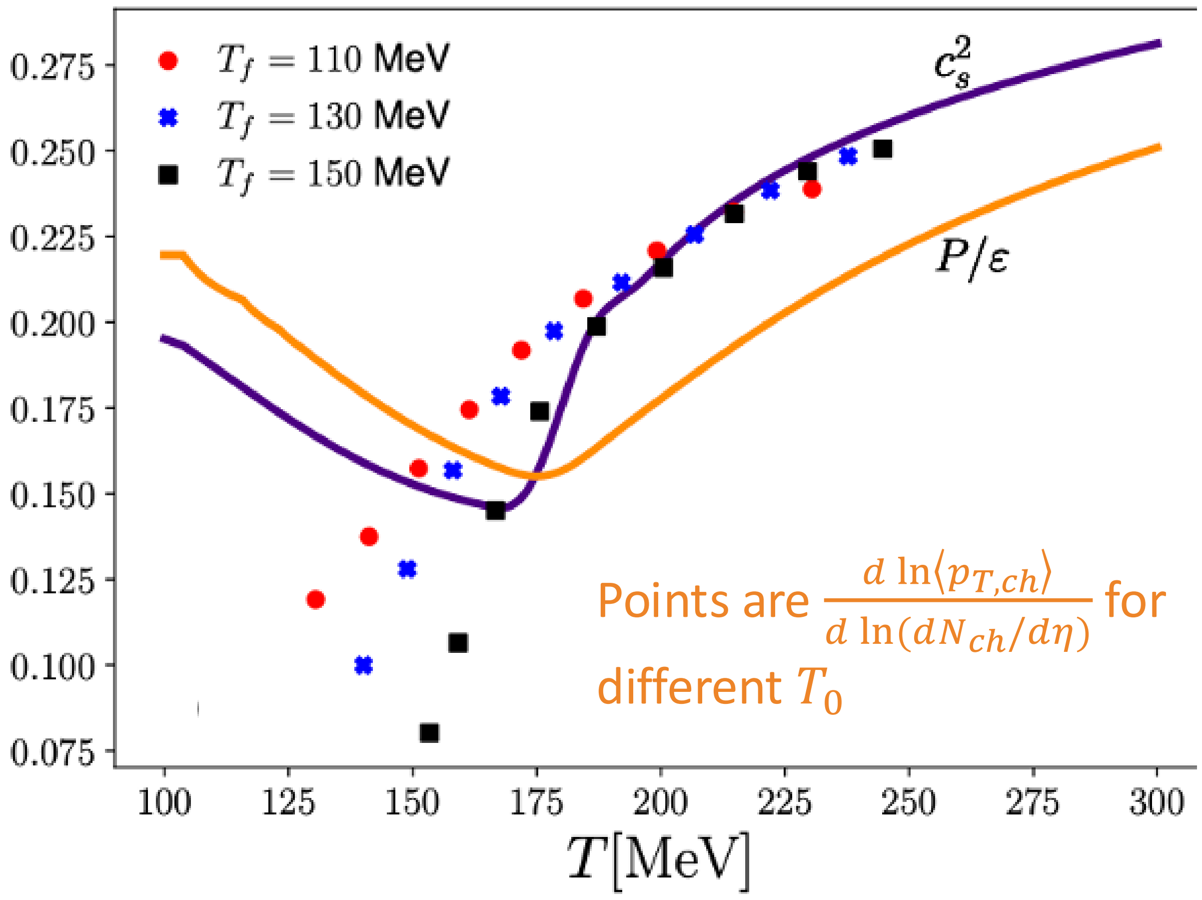
- Try different initial temperatures T_0 and different particlization temperatures T_f in numerical simulations
  $\Rightarrow$ Yields different T_{eff}
- Use **modified** equation of state
- Compare $\frac{d\ \ln\langle p_{T,ch}\rangle}{d\ \ln(dN_{ch}/d\eta)}$ to **modified** speed of sound

Does not track c_s^2, at least where it changes rapidly

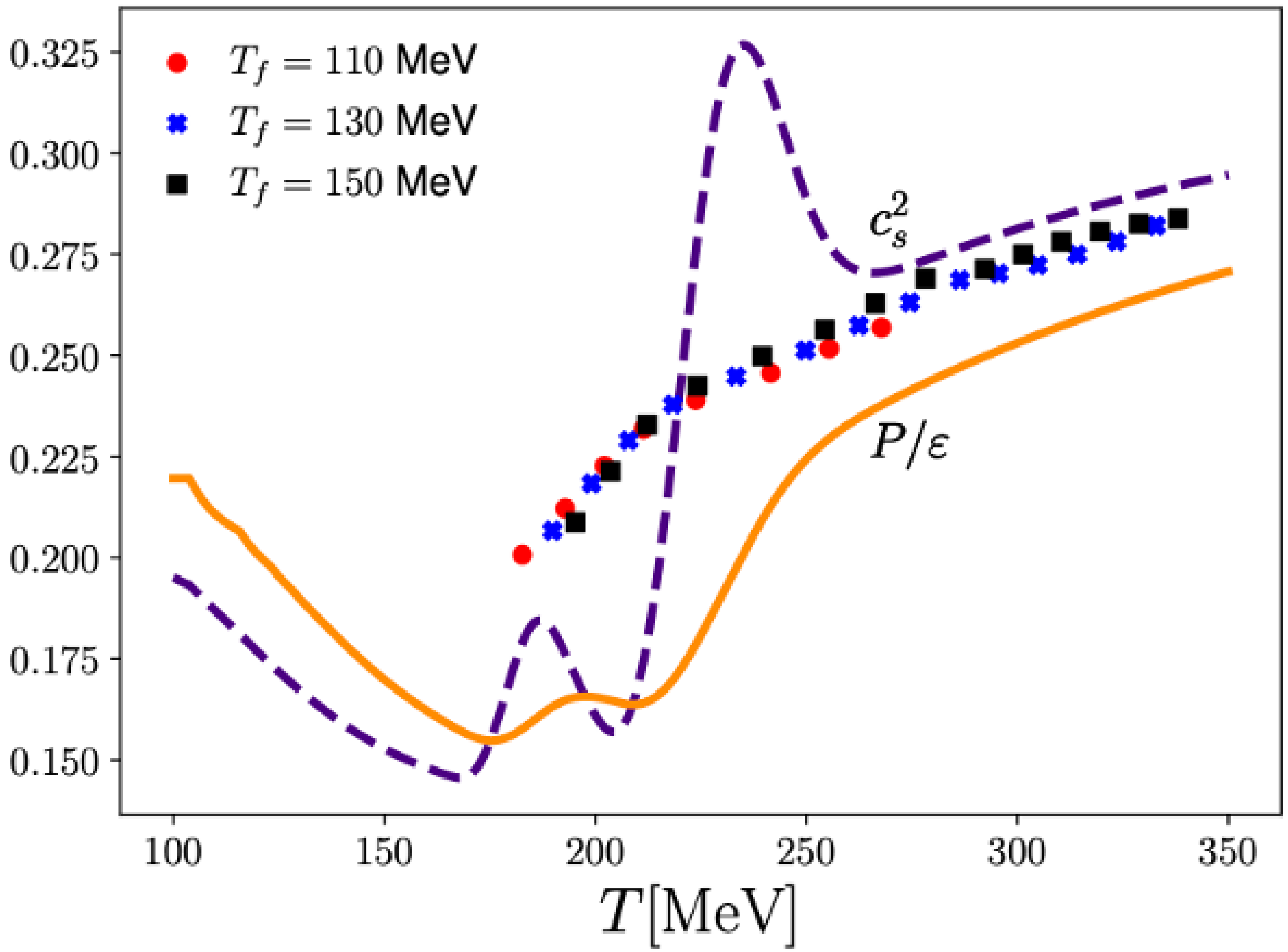


Numerical results

QCD equation of state



Modified equation of state



Some results from my group

Question:
How can an observable only depend on the equation of state?

e.g. "Pion multiplicity"(Initial condition parameters, Equation of state vs T, Shear&bulk viscosity vs T, …)

Possibility:
- Observable independent of initial conditions (or other parameters).
- Observable dependent on well-understood features of initial conditions (and independent of poorly-understood features)

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Question:
How can an observable only depend on the equation of state?

e.g. "Pion multiplicity"(Initial condition parameters, Equation of state vs T, Shear&bulk viscosity vs T, …)

Possibility:
- **Observable independent of initial conditions (or other parameters). How? Under what conditions?**
- Observable dependent on well-understood features of initial conditions (and independent of poorly-understood features)

Measuring the equation of state

Thermodynamics of hot strong-interaction matter from ultrarelativistic nuclear collisions #7

Fernando G. Gardim (Alfenas Fed. U., Pocos de Caldas and IPhT, Saclay), Giuliano Giacalone (IPhT, Saclay), Matthew Luzum (Sao Paulo U.), Jean-Yves Ollitrault (IPhT, Saclay) (Aug 26, 2019)

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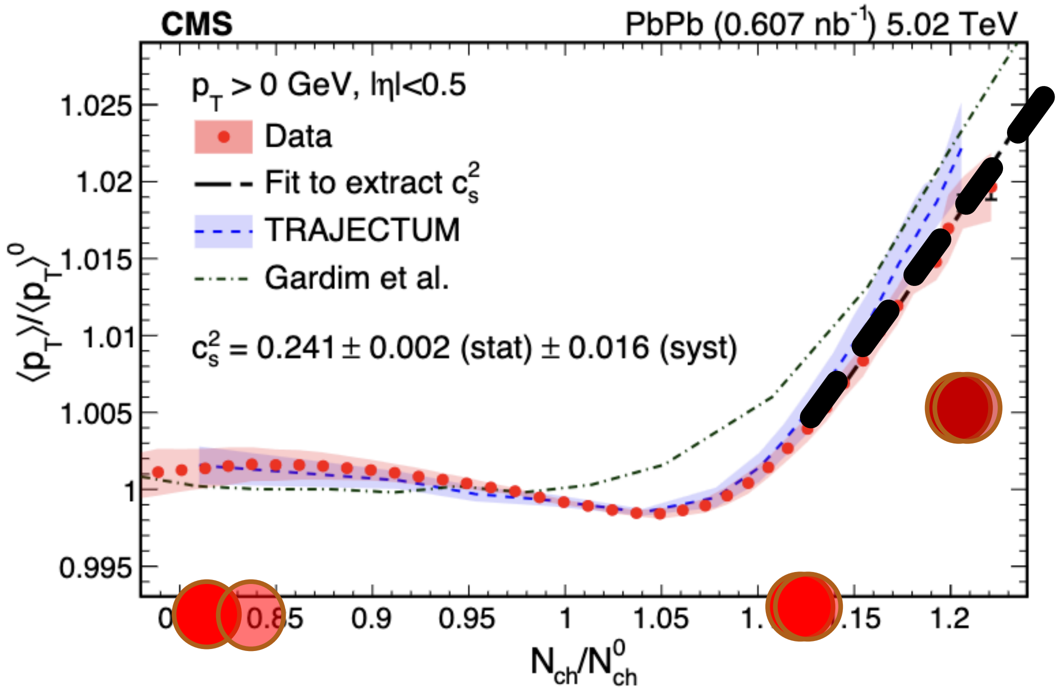

Ref.: CMS Collaboration, Rept.Prog.Phys. 87 (2024)

$$\Rightarrow \frac{d \ln \langle p_T \rangle}{d \ln dN_{ch}/d\eta} = c_s^2 \left(T \approx \frac{\langle p_T \rangle}{3} \right)$$

This relation can be tested in any hydrodynamic model, with different assumptions (different initial conditions, …)

Question:
How can an observable only depend on the equation of state?
(i.e. how can it forget about initial conditions?)

- Suppose we have:
  - a plasma in perfect local equilibrium
    Assumption 1: no viscosity

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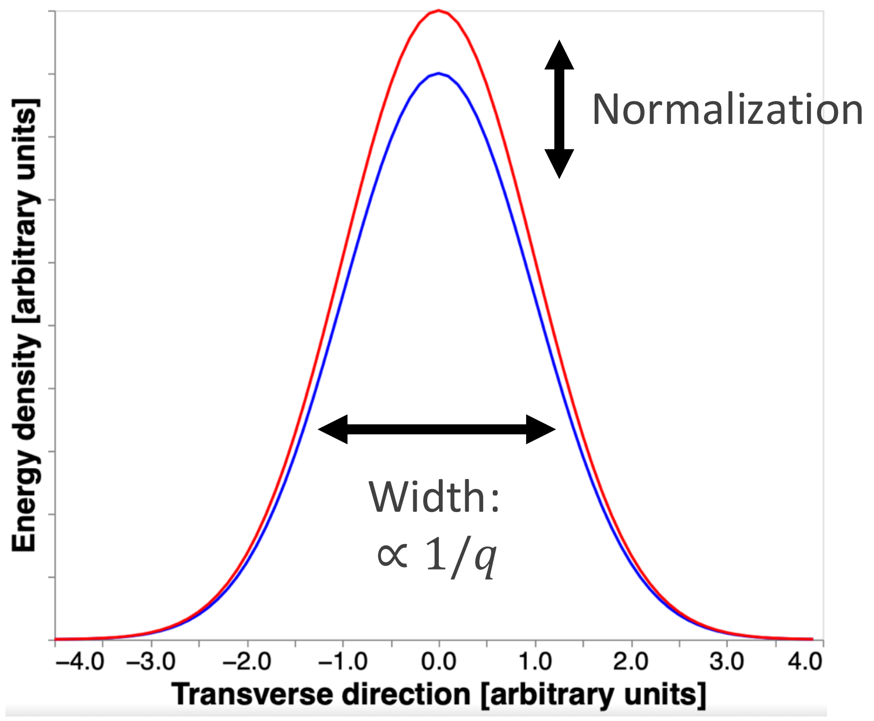
    Assumption 2:

Initial conditions of hydrodynamics at $\tau = \tau_0$



More details: Gubser ideal hydrodynamics

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    Assumption 3: no kinematic cuts

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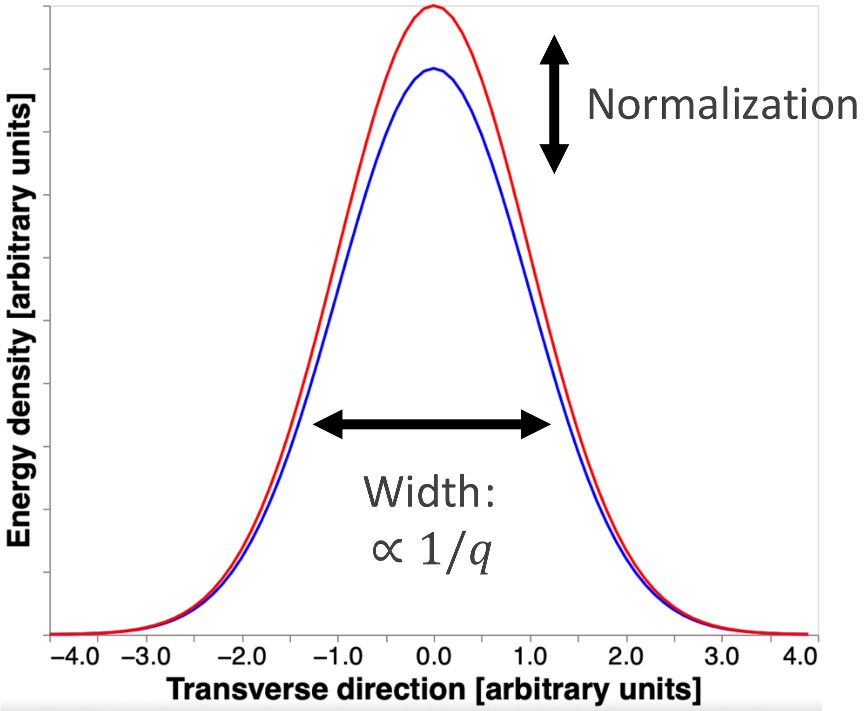

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    Assumption 2:
  - We can measure all particles
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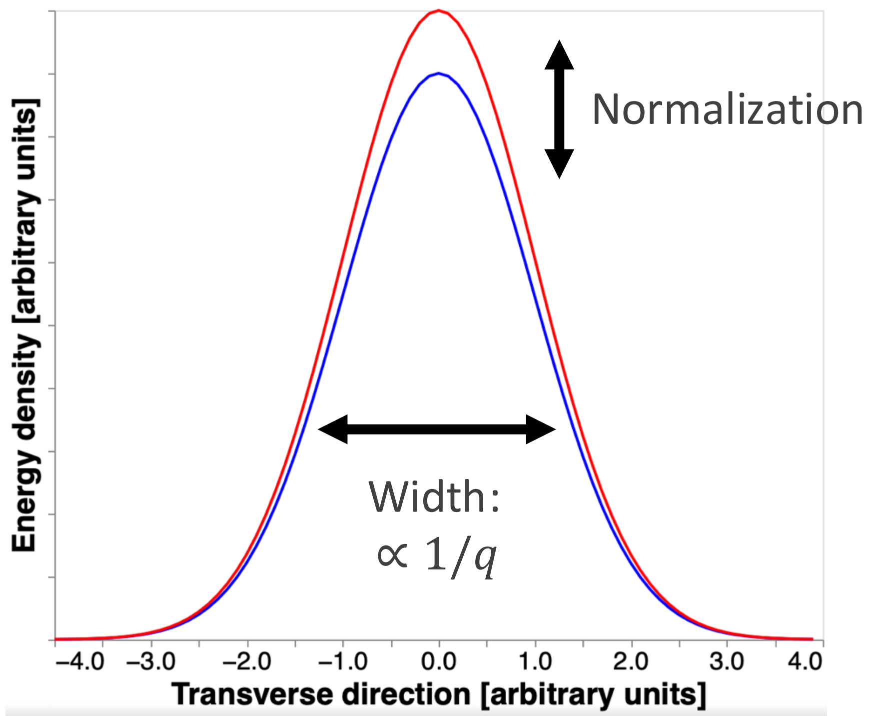
  - c_s^2 constant, no need to define T_{eff}
    - Assumption 4: $c_s^2 = 1/3$

Initial conditions of hydrodynamics at $\tau = \tau_0$



More details: Gubser ideal hydrodynamics, and Cooper-Frye with a massless boson

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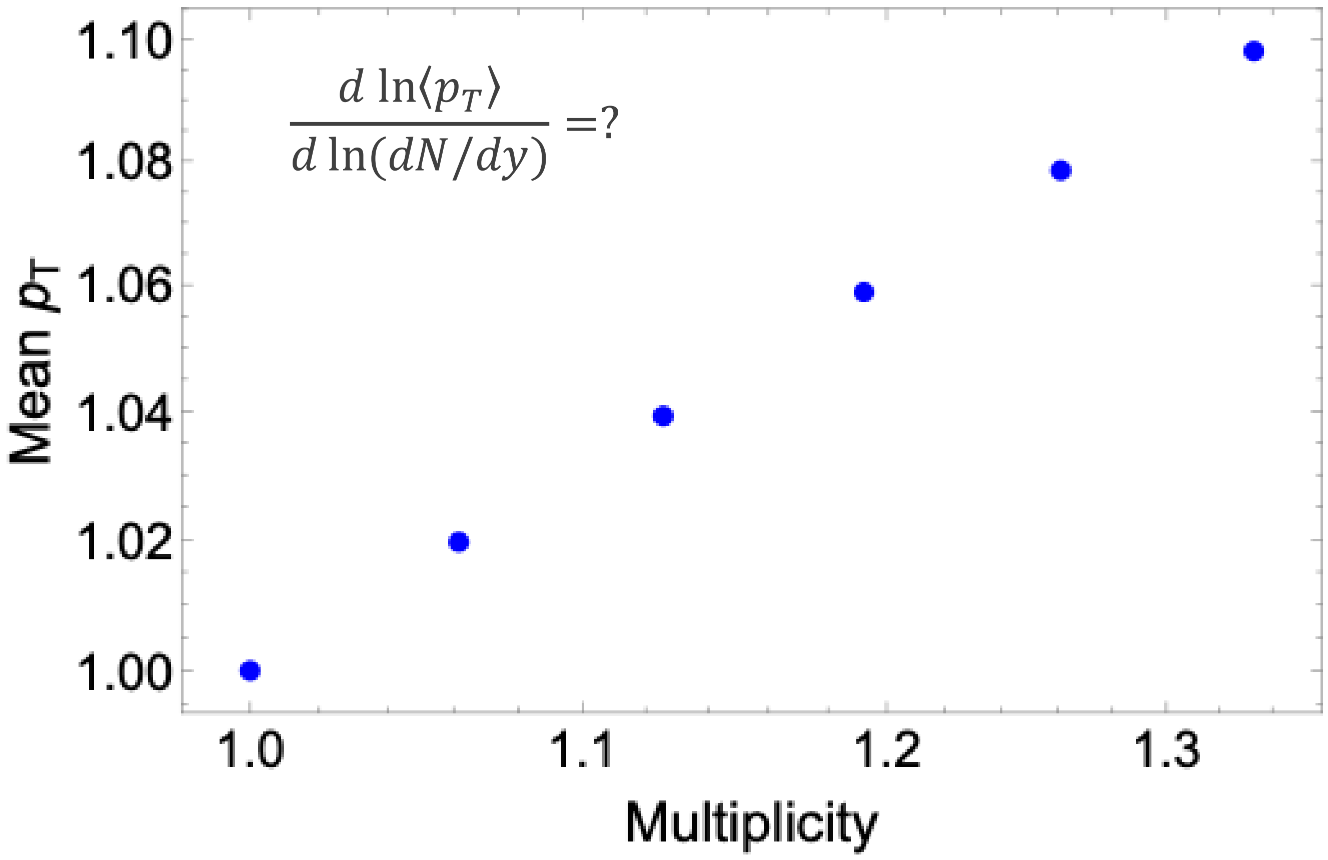

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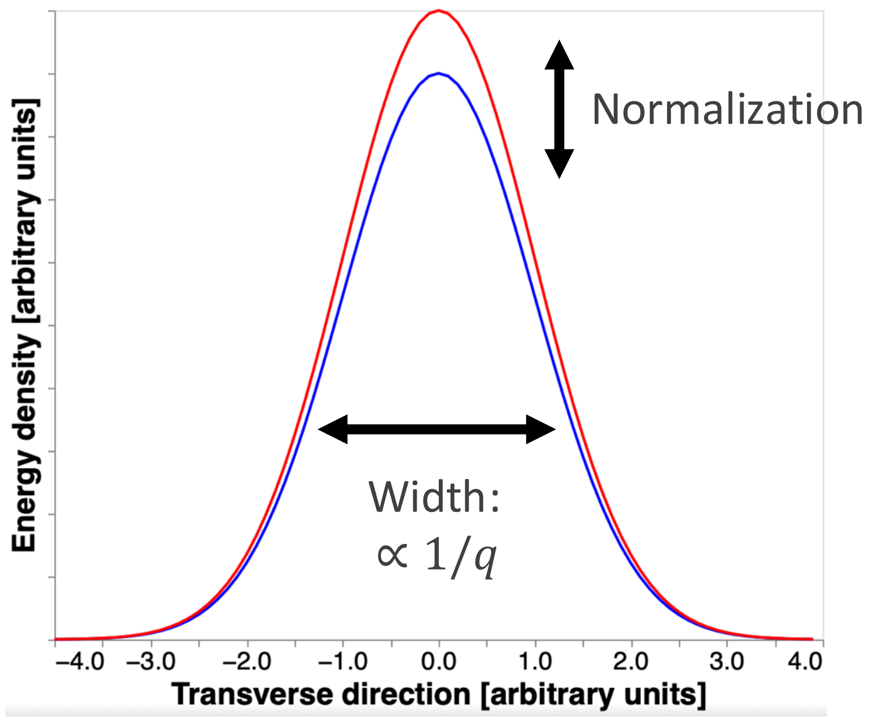

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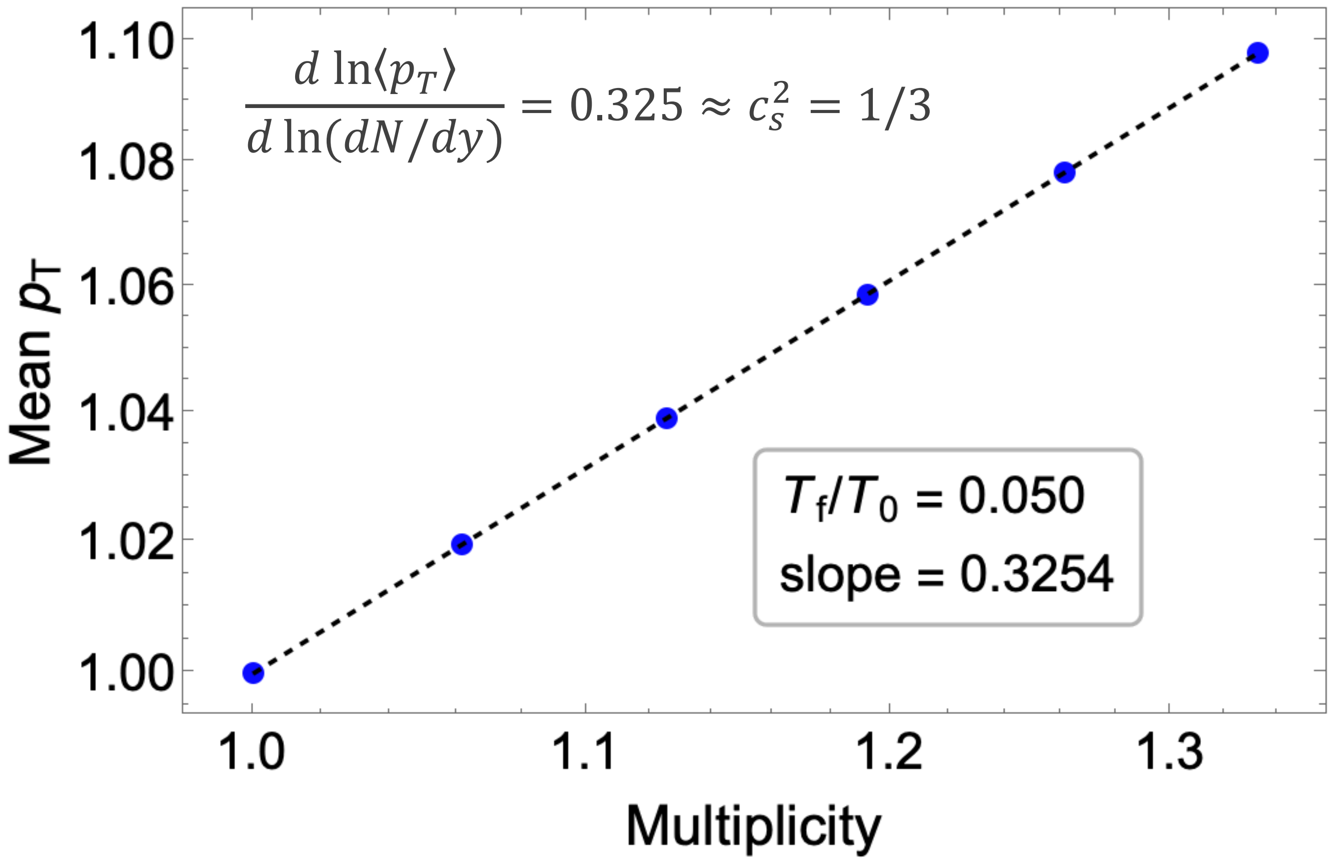

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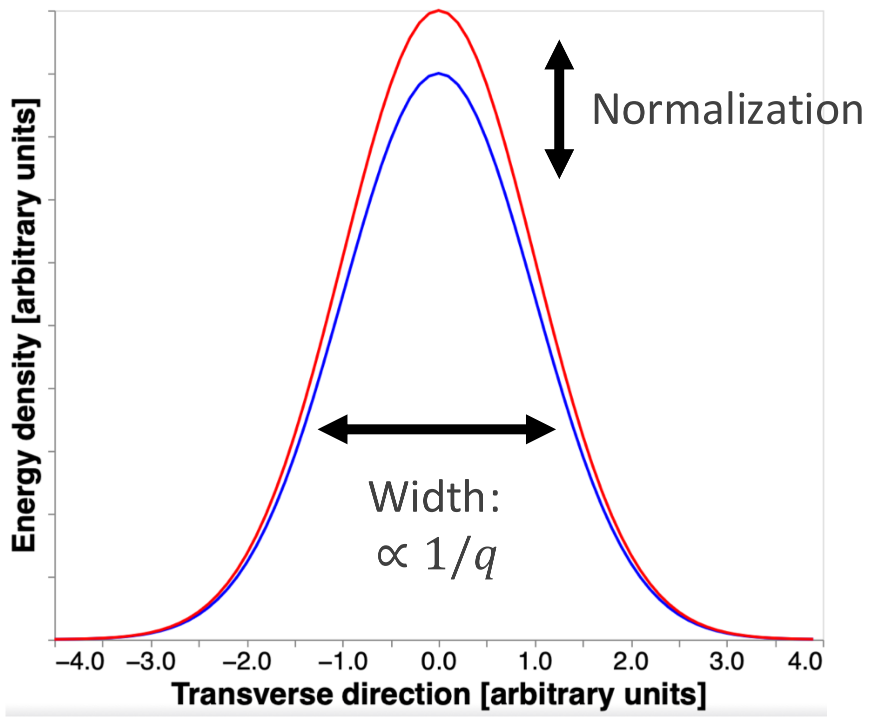

In heavy-ion collisions, $T_f \approx 0.15$ GeV, and $T_0 \approx 0.3 - 0.7$ GeV (depends on $\sqrt{s_{NN}}$ and nuclei)

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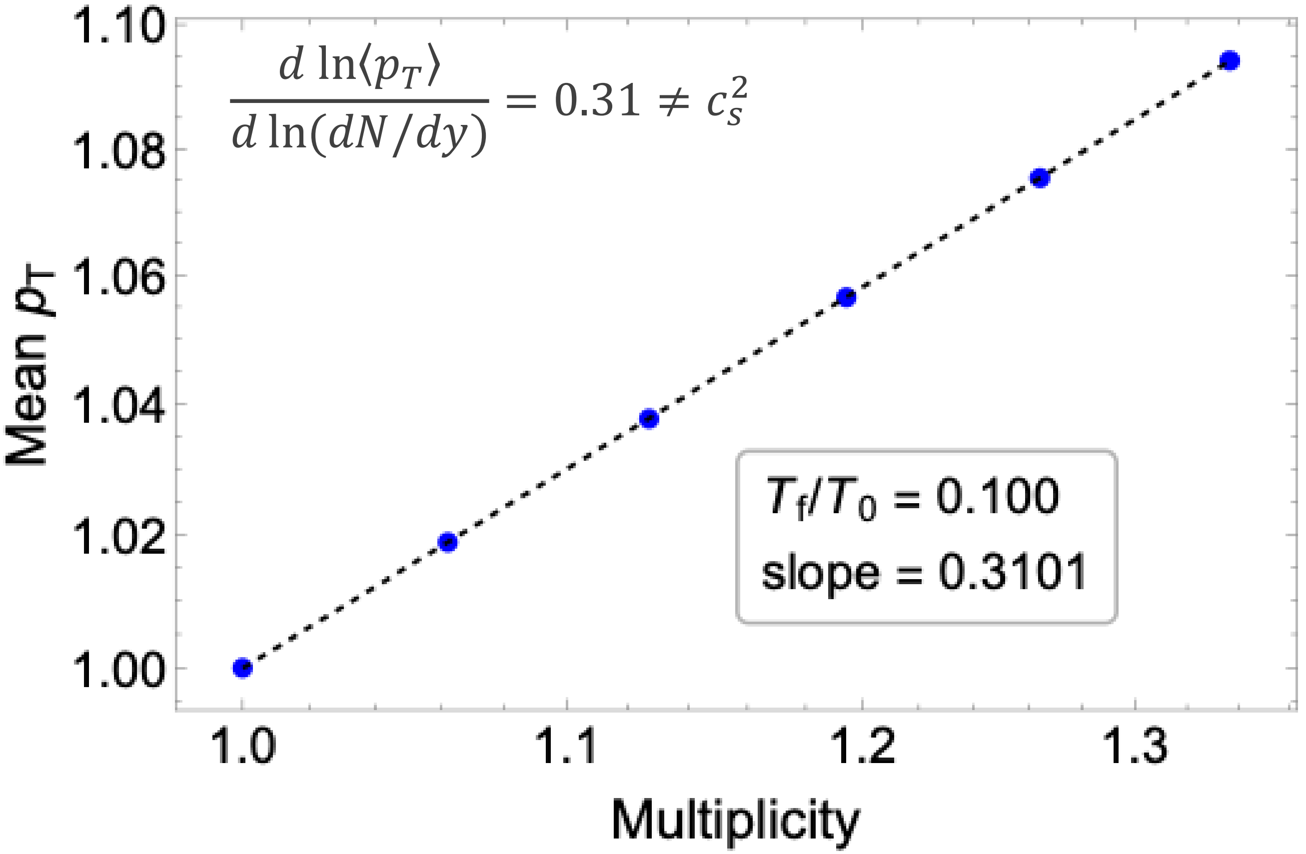

Slope changes as initial temperature T_0 decreases

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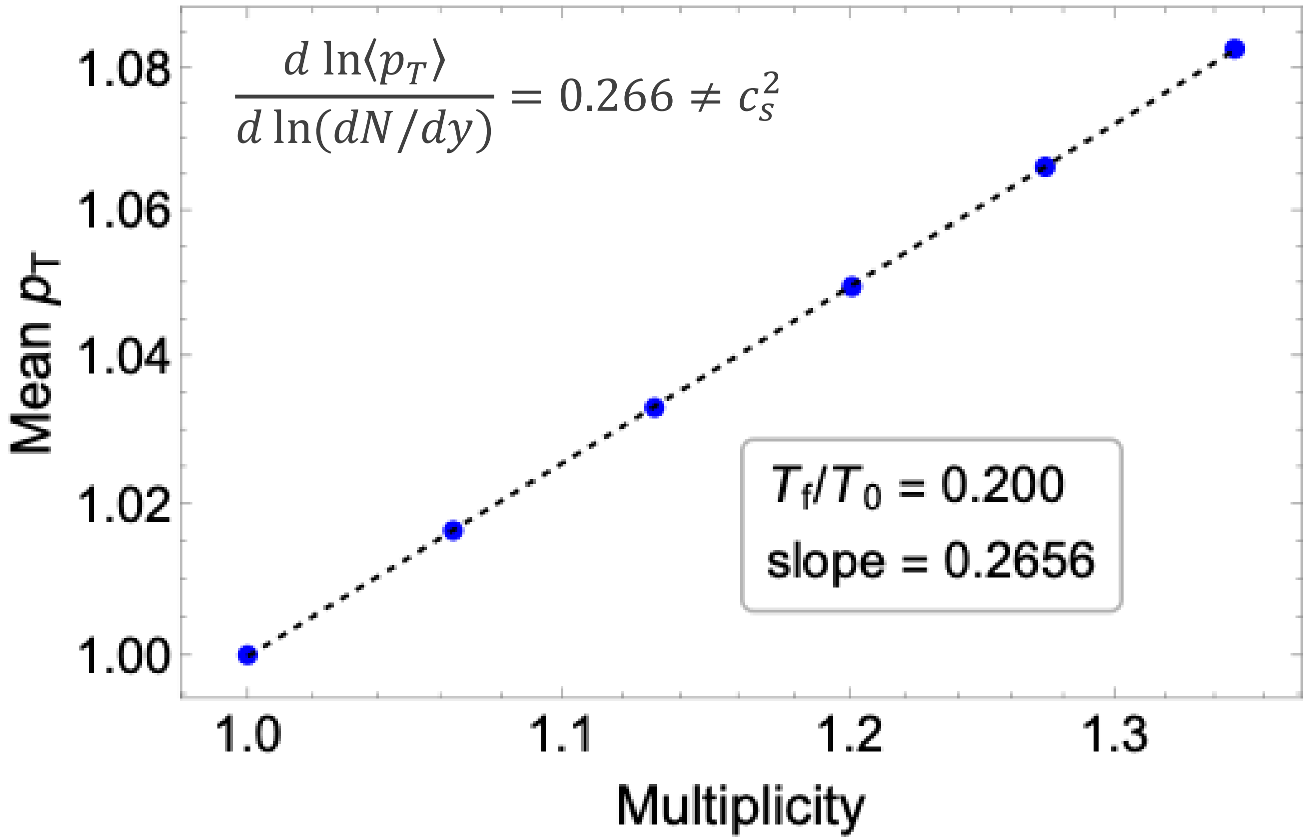

Slope differs more from c_s^2 as the system's initial temperature T_0 decreases

Question:
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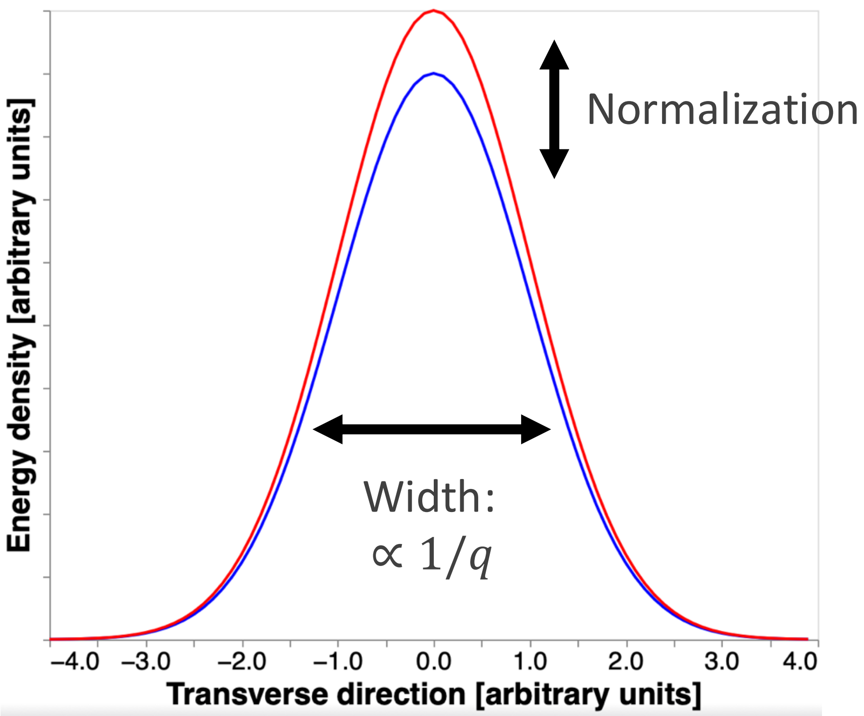
$$\frac{d\ln\langle p_T\rangle}{d\ln(dN/dy)} = \frac{1}{3} - \frac{2}{3}\frac{1}{(1+q^2\tau_0^2)}\left(\frac{T_f}{T_0}\right)^{3/2}$$

Dependence on initial conditions (initial time τ_0 and initial inverse width q)

Initial conditions of hydrodynamics at $\tau = \tau_0$



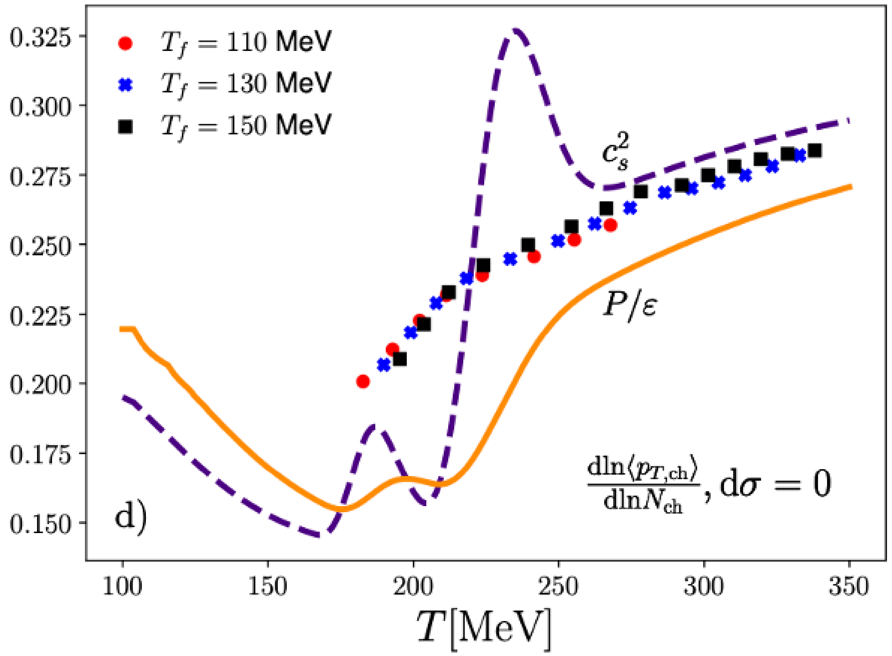
Observables need time to forget about initial conditions

Be careful with small systems or low $\sqrt{s_{NN}}$ that have low initial temperature T_0

Summary and outlook

Summary (I)

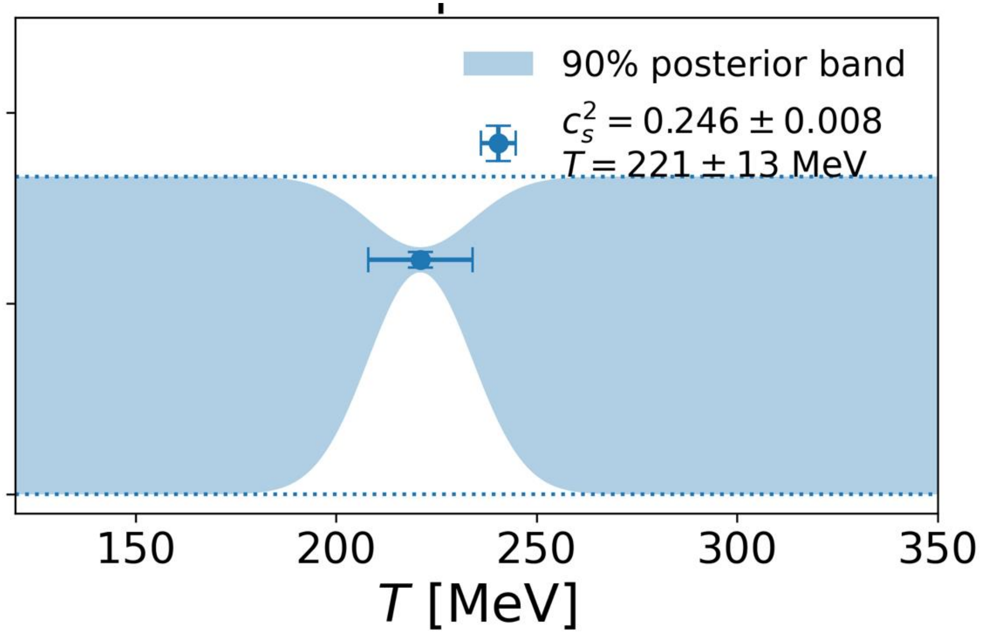
- Strong constraints on equation of state likely requires forgetting about many features of the initial conditions
  - Example with simpler system: $$\frac{d\ln\langle p_T\rangle}{d\ln(dN/dy)} = \frac{1}{3} - \frac{2}{3}\frac{1}{(1+q^2\tau_0^2)}\left(\frac{T_f}{T_0}\right)^{3/2}$$
  - Likely a challenge for low $\sqrt{s_{NN}}$ or smaller systems
- What is $\langle p_T\rangle$ related to?
  Consequence: is it P/ε or c_s^2 that is being measured?
- Need to assume the speed of sound varies slowly?



Summary (II)

- Distinction between "independent measurement" and "distillation of model" not clear-cut because prior assumptions from model generally need to be used

- How to reconcile Bayesian analyses with the idea that a small number of measurements can constrain equation of state at a specific temperature?



Questions

Backup

Ultracentral slope and theory

- How can we connect thermodynamic fields with global observables?
- N_{ch}, $<p_T>$: two thermodynamic proxies

Highly non-trivial

Gardim, Giacalone, Luzum, Ollitrault Nature Phys. 16, 615 (2020)

$$\frac{d\ln\langle p_{T,\mathrm{ch}}\rangle}{d\ln N_{\mathrm{ch}}}$$

$$\langle p_T\rangle \longleftrightarrow T \qquad N_{\mathrm{ch}} \longleftrightarrow s(T)$$

$$\frac{d\ln T}{d\ln s} = c_s^2$$

$$\neq$$

$$\langle p_T\rangle \longleftrightarrow E_{\mathrm{tot}}/S_{\mathrm{tot}} \qquad N_{\mathrm{ch}} \longleftrightarrow s(T)$$

$$\frac{d\ln(\varepsilon/s)}{d\ln s} = \frac{P}{\varepsilon}$$

Gavassino, Hirvonen, Paquet, Singh, GSR 2503.20765 (2025)

Mean p_T as a proxy of?

Gabriel Soares Rocha, IS2025

- Inspiration: particles at mid-rapidity

$\langle p_T \rangle \sim E_{tot}/N_{tot} \sim 3T_{eff}$

van Hove, Phys. Lett. B 118, 138 (1982)
Gardim, Giacalone, Luzum, Ollitrault Nature Phys. 16, 615 (2020)

$\langle p_T \rangle \sim E_{tot}/N_{tot} \sim E_{tot}/S_{tot}$

Gavassino, Hirvonen, Paquet, Singh, GSR 2503.20765 (2025)

- Validation in simulations:

$$\langle p_T \rangle / T_{\rm eff} \approx$$

3.07 (Pb+Pb, $\sqrt{s_{NN}}$=5.02 TeV)

Gardim et al Nature Phys. 16, 615 (2020)

3.05 (p+Pb, $\sqrt{s_{NN}}$=5.02 TeV)

Gardim, Krupczak, da Silva PRC 109, 014904 (2024)

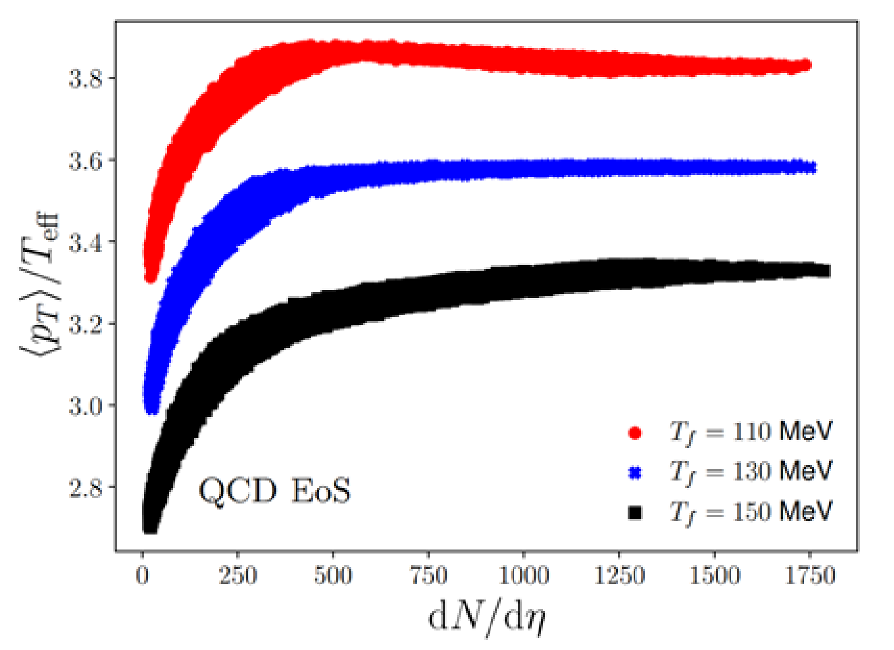
2.22 ~ 2.8 ($\sqrt{s_{NN}}$=5.02 TeV)

2.16 ~ 2.58 ($\sqrt{s_{NN}}$=8.16 TeV)

Mu, Sun, Yan, Huang arXiV:2501.02777 [nucl-th]

2.45 (p+Pb, $\sqrt{s_{NN}}$=5.02, 8.16 TeV)

CMS-PAS-HIN-25-001



Sensitivity to T_{switch}; model dependence – constrain T_{switch} w/ data

$$\langle p_T \rangle \; vs \; T_{eff}$$

Feasibility of measuring the speed of sound of the quark-gluon plasma from the multiplicity and mean p_T of ultracentral heavy-ion collisions

Lorenzo Gavassino [1,*], Henry Hirvonen [1,2,†], Jean-Francois Paquet [2,1,‡], Mayank Singh [2,§], and Gabriel Soares Rocha [3,2,ı]

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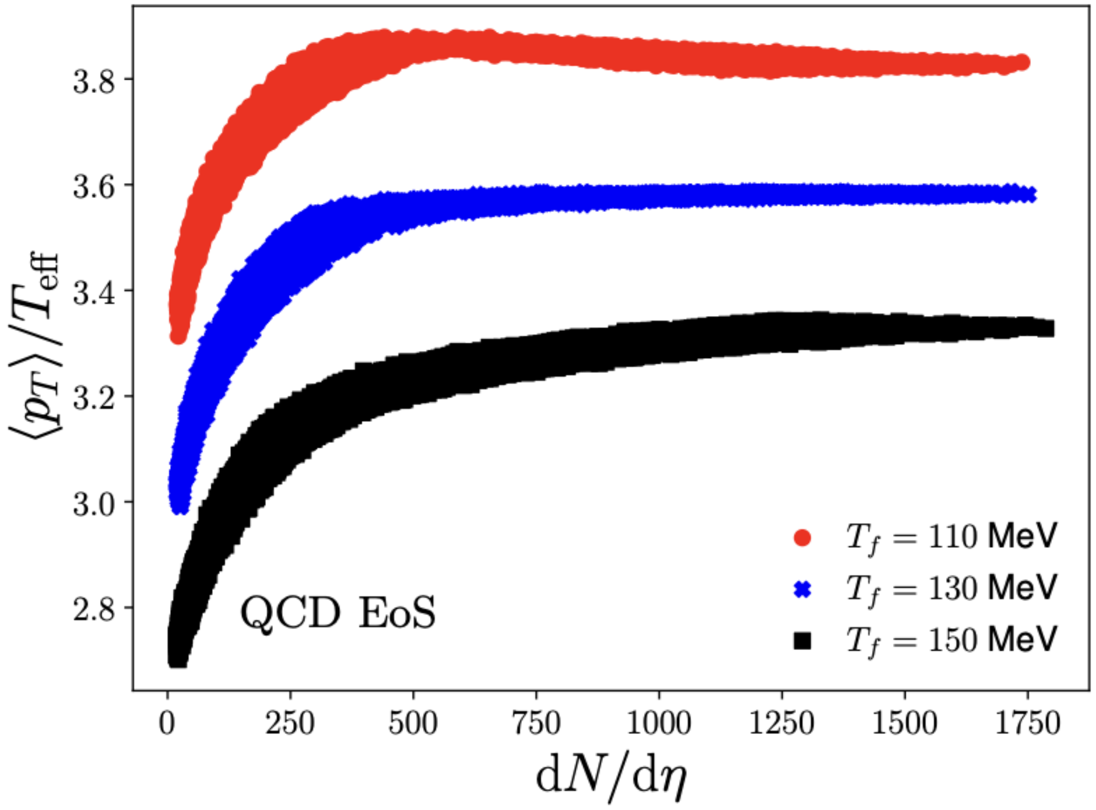
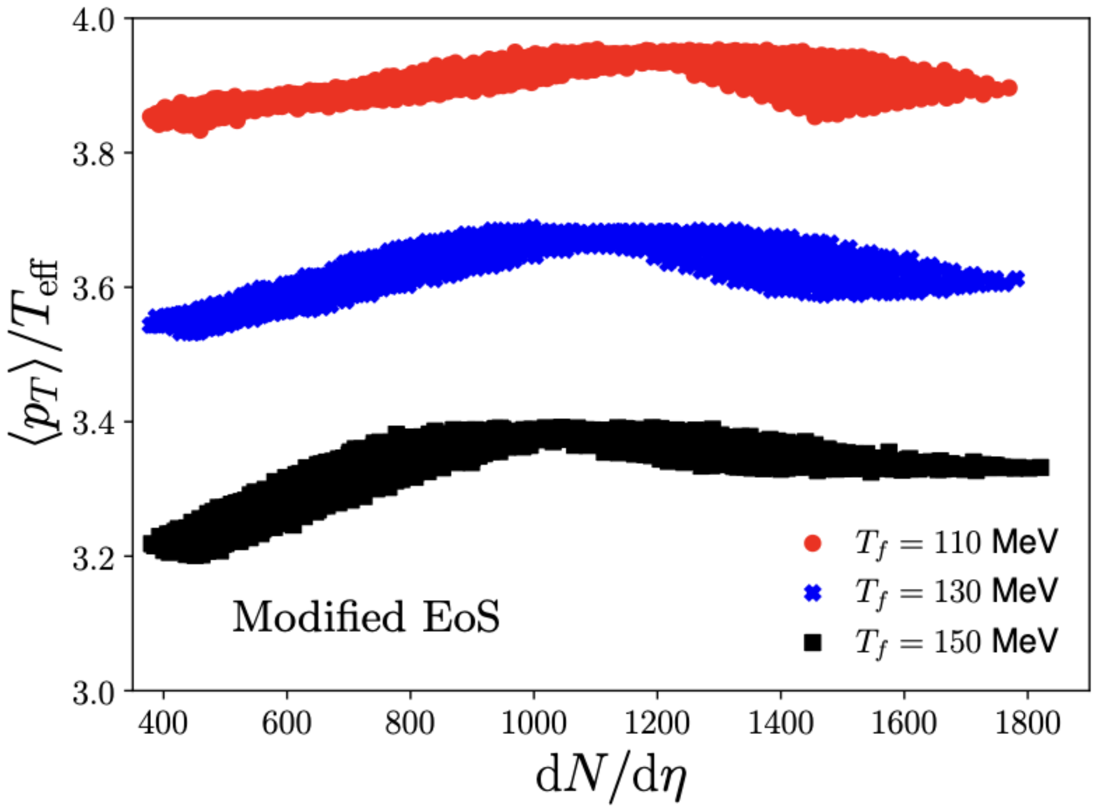

QCD EoS
$T_f = 110$ MeV
$T_f = 130$ MeV
$T_f = 150$ MeV
$\langle p_T \rangle / T_{\mathrm{eff}}$
$\mathrm{d}N/\mathrm{d}\eta$
3.8
3.6
3.4
3.2
3.0
2.8
0
250
500
750
1000
1250
1500
1750


Modified EoS
$T_f = 110$ MeV
$T_f = 130$ MeV
$T_f = 150$ MeV
$\langle p_T \rangle / T_{\mathrm{eff}}$
$\mathrm{d}N/\mathrm{d}\eta$
4.0
3.8
3.6
3.4
3.2
3.0
400
600
800
1000
1200
1400
1600
1800

$$P/\varepsilon \text{ vs } c_s^2$$

$$dV_{eff} = 0 \text{ vs } d\sigma = 0$$

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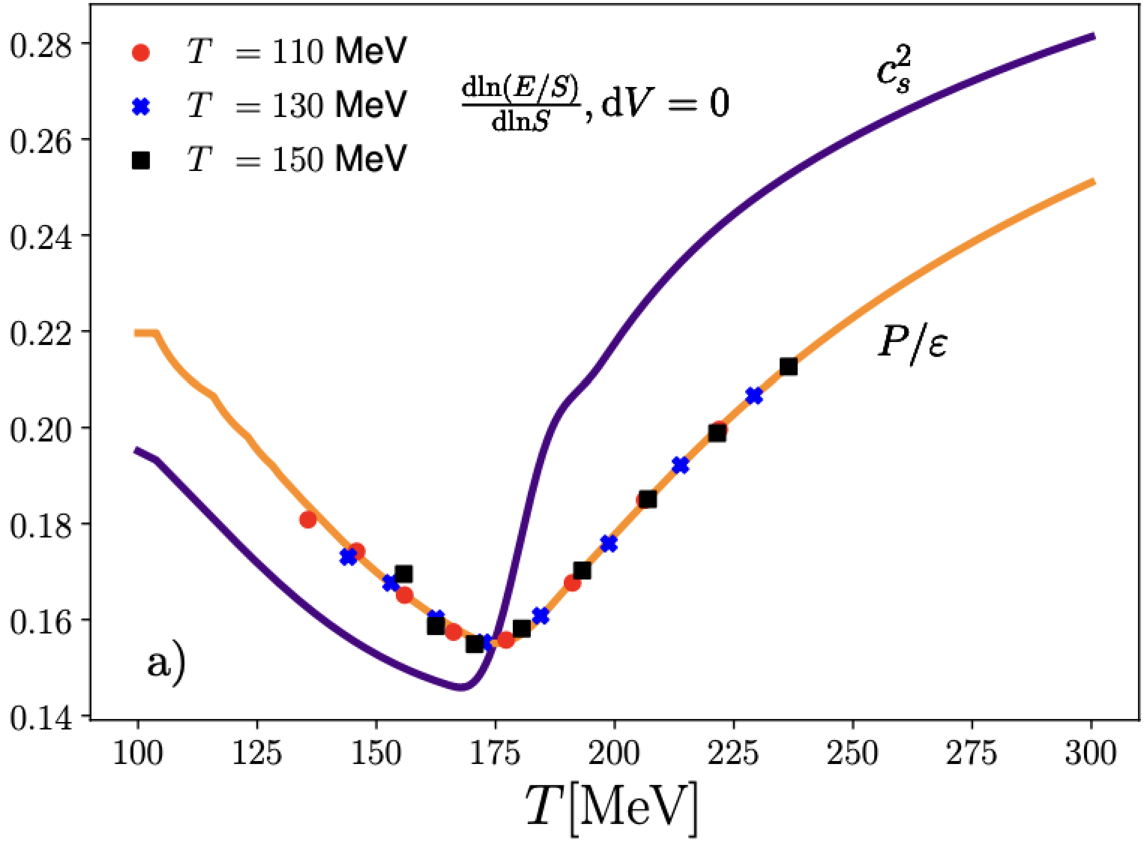
Numerical results with the QCD equation of state

- If we compute $\frac{d\ln\left(\frac{E}{S}\right)}{d\ln S}$ with $E = \int_\Sigma \mathrm{d}^3\sigma_\mu T^{t\mu},\ S = \int_\Sigma \mathrm{d}^3\sigma_\mu s u^\mu,$?



(by definition)

T_{eff} defined by

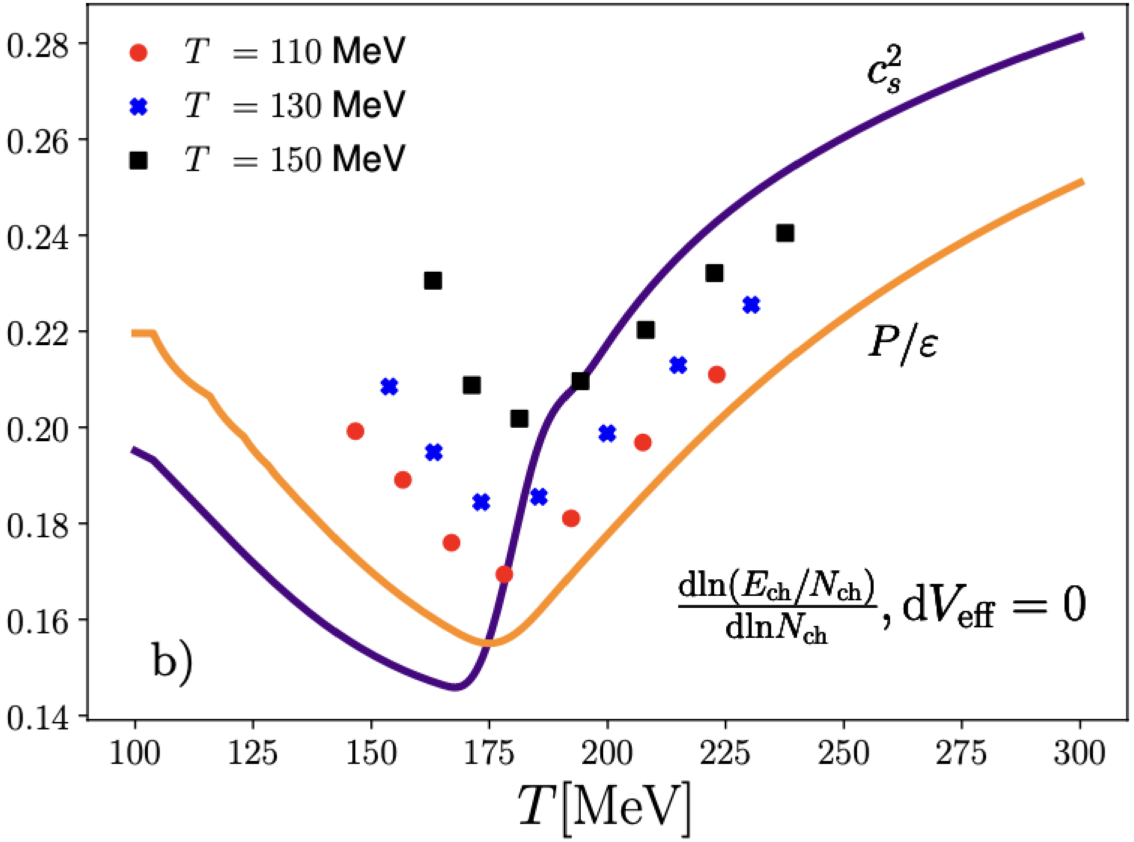
$$E \equiv \varepsilon(T_{\text{eff}})V_{\text{eff}},$$
$$S \equiv s(T_{\text{eff}})V_{\text{eff}}.$$

Numerical results with the QCD equation of state

- Replacing the energy/entropy on the hypersurface by the charged hadron energy/multiplicity

Cooper-Frye + kinematic cuts

$\longrightarrow$



$$E = \int_\Sigma \mathrm{d}^3\sigma_\mu T^{t\mu},$$

$$S = \int_\Sigma \mathrm{d}^3\sigma_\mu s u^\mu,$$

T_{eff} defined by

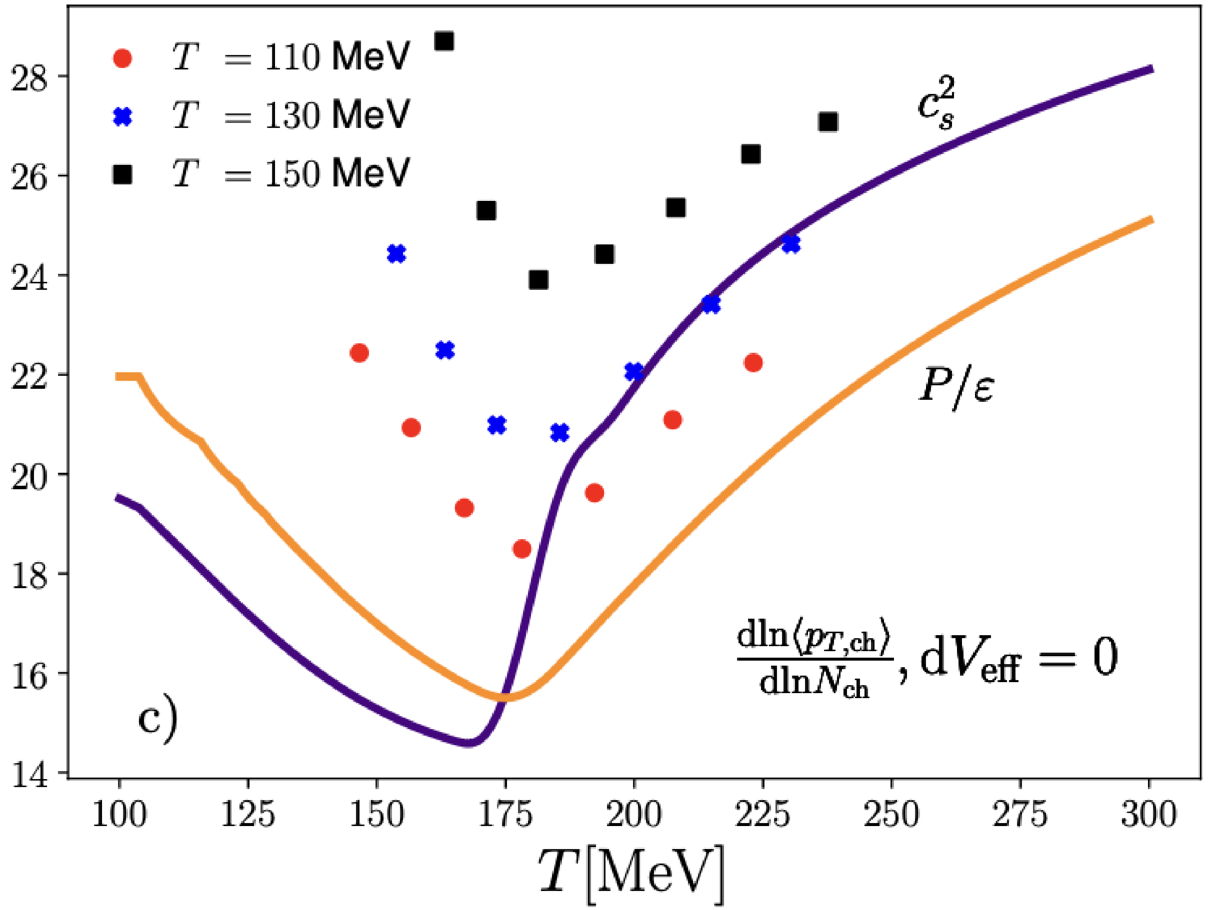
$$E \equiv \varepsilon(T_{\mathrm{eff}})V_{\mathrm{eff}},$$

$$S \equiv s(T_{\mathrm{eff}})V_{\mathrm{eff}}.$$

Numerical results with the QCD equation of state

- Replacing the energy of charged hadrons E_{ch} by their mean $\langle p\_T \rangle$

$E_{ch} \rightarrow \langle p_T \rangle$



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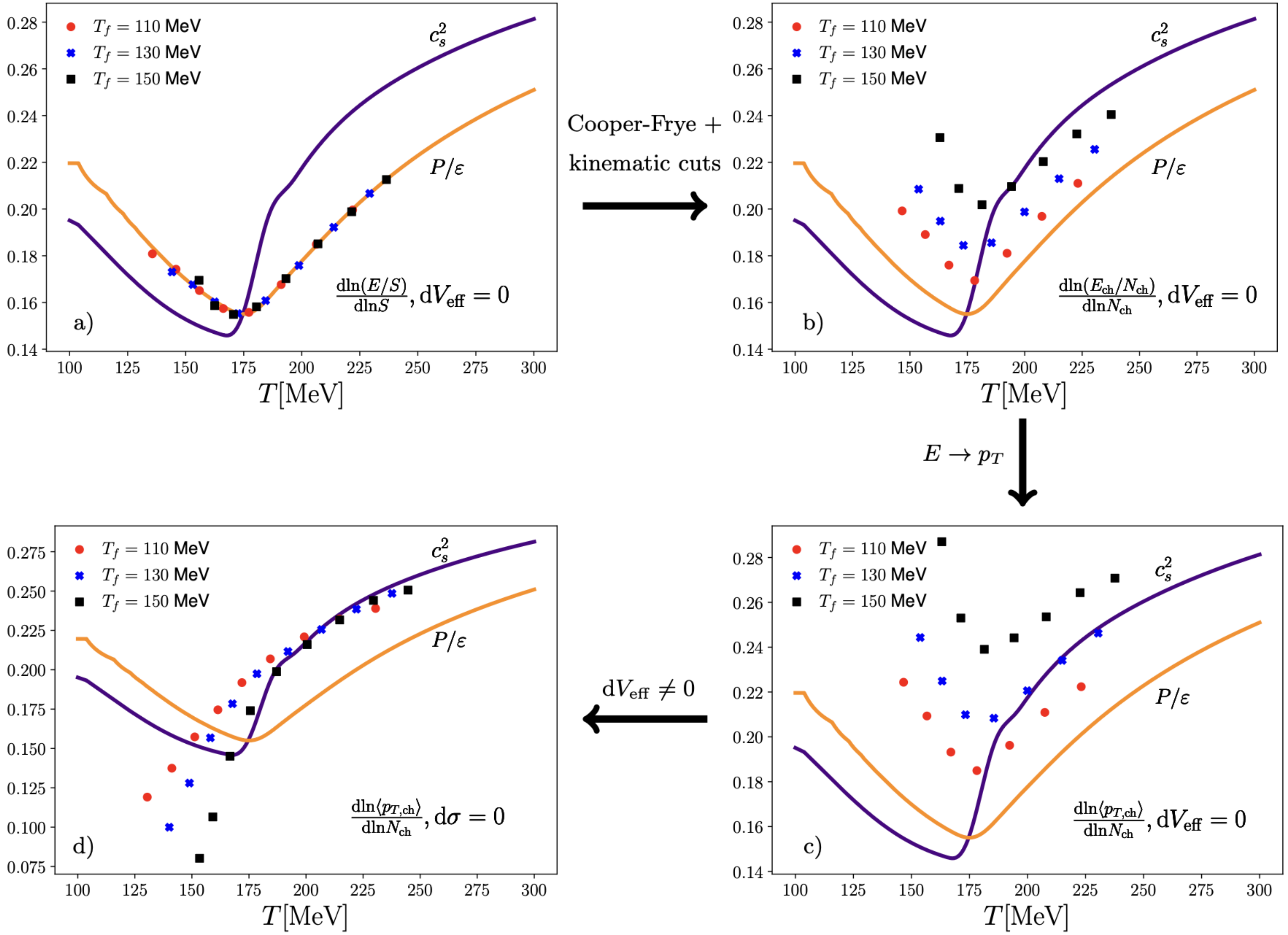
Matters if $d\,V_{eff} = 0$ (from T_{eff} and V_{eff} definition)

or

$d\sigma = 0$ (geometric transverse width fixed)



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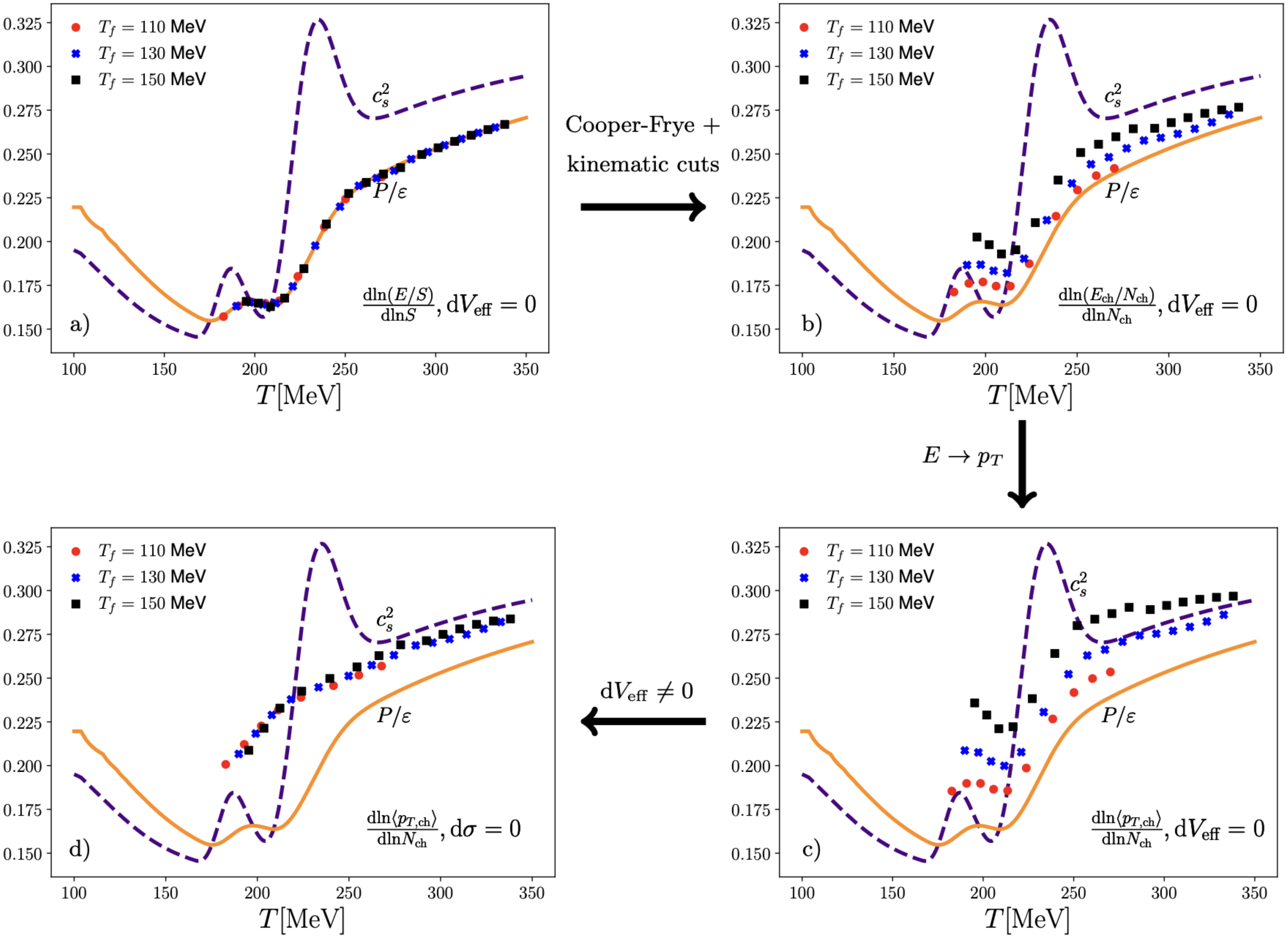

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$d\sigma = 0$ (geometric transverse width fixed)

Constant speed of sound results

Analytical insights into the interplay of momentum, multiplicity, and the speed of sound in heavy-ion collisions

Gabriel Soares Rocha [1,*], Lorenzo Gavassino [2,†], Mayank Singh [1,‡], and Jean-François Paquet [1,2,§]

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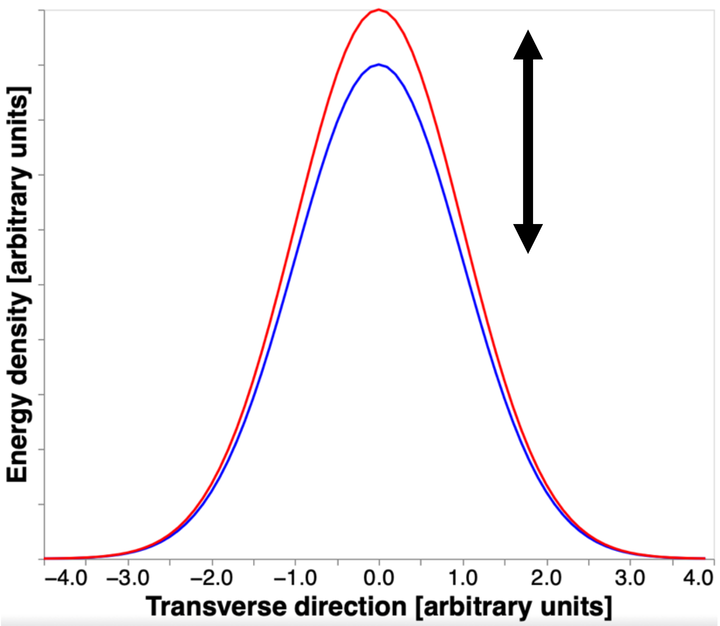
Phys. Rev. C **110**, 034913 – **Published 26 September, 2024**

A simplified model of ultracentral collisions

- Energy density scaled by an overall constant, but shape remains the same



- Assume a constant speed of sound $P = c_s^2 \varepsilon$

$$s = A(1+c_s^2)T^{c_s^{-2}}, \quad \varepsilon = AT^{1+c_s^{-2}}, \quad P = c_s^2 A T^{1+c_s^{-2}},$$

- Assume inviscid relativistic fluid dynamics

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = \varepsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

$$\partial_\mu s^\mu = 0, \text{ where } s^\mu = s u^\mu,$$

- Family of solutions with same flow velocity profile and rescaled temperature profile

$$s^\mu = A(1+c_s^2)T^{c_s^{-2}} u^\mu,$$

$$T^{\mu\nu} = AT^{1+c_s^{-2}}\left[(1+c_s^2)u^\mu u^\nu - c_s^2 g^{\mu\nu}\right].$$

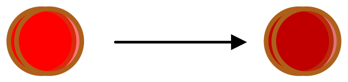
Measuring the equation of state: a thought experiment

- If the initial temperature of the system is scaled by a factor α
  - System's total entropy: $S_{tot} \propto \alpha^{c_s^{-2}}$
  - System's total energy: $E_{tot} \propto \alpha^{1+c_s^{-2}}$

$$\frac{d\ln S_{\mathrm{tot}}}{d\ln\alpha} = \frac{1}{c_s^2},$$

$$\frac{d\ln E_{\mathrm{tot}}}{d\ln\alpha} = 1 + \frac{1}{c_s^2}.$$

Scale temperature by α



$$\Rightarrow \quad c_s^2 = \frac{d\ln(E_{\mathrm{tot}}/N_{\mathrm{tot}})}{d\ln N_{\mathrm{tot}}}$$

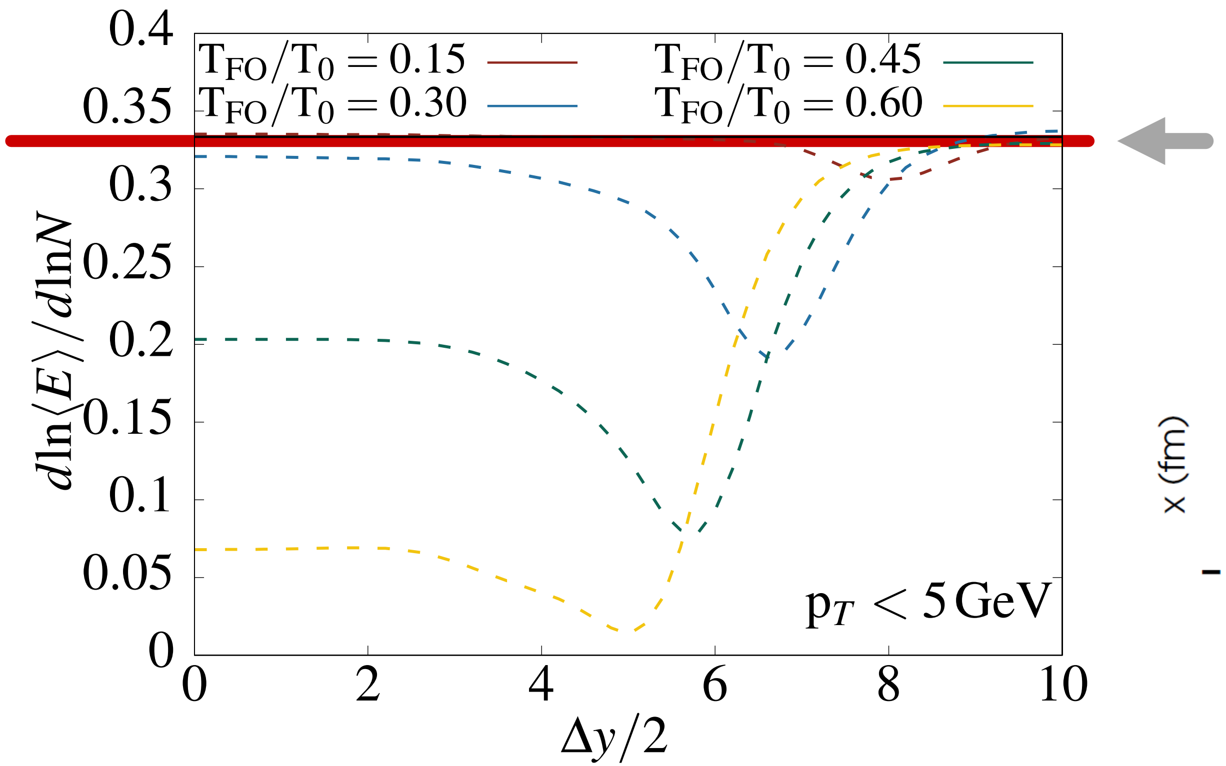
True if

- Initial temperature is uniformly rescaled
- Speed of sound is constant
- No significant effect of viscosity (entropy is constant)
- **<u>All particles can be measured (and only those part of the plasma)</u>**

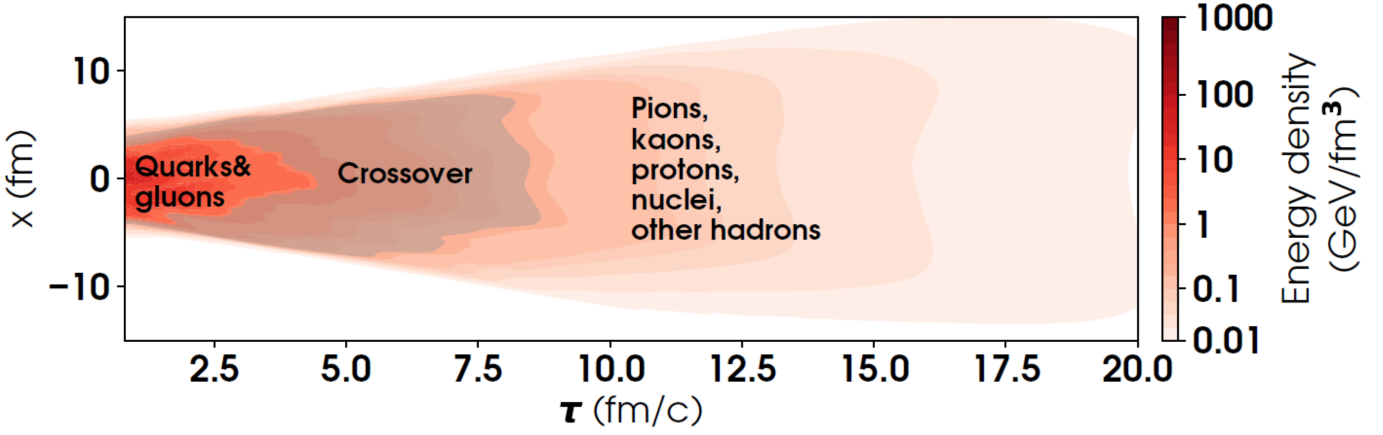
Numerical simulation results

- Optical Glauber initial conditions, with temperature rescaled & hydro with $c_s^2 = 1/3$
- Fluid converted into particles (Cooper-Frye) on hypersurface at $T = T_{FO}$

$$c_s^2 = \frac{d\ln(E_{\text{tot}}/N_{\text{tot}})}{d\ln N_{\text{tot}}}$$



$c_s^2 = 1/3$



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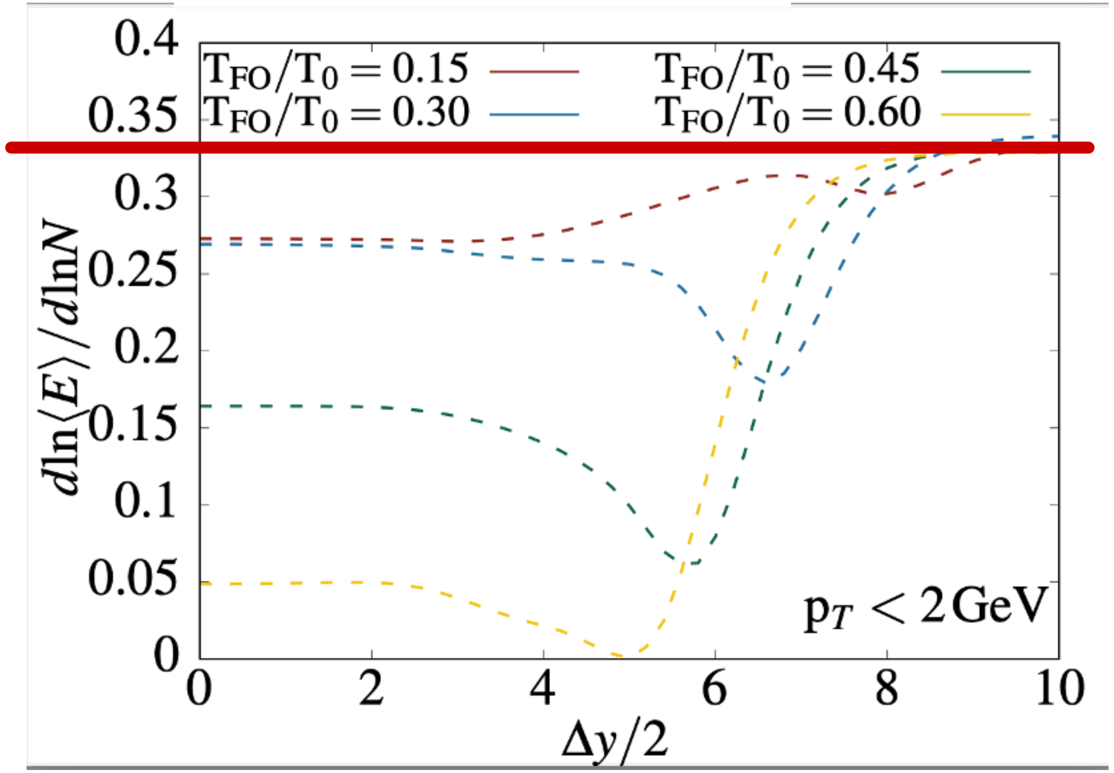

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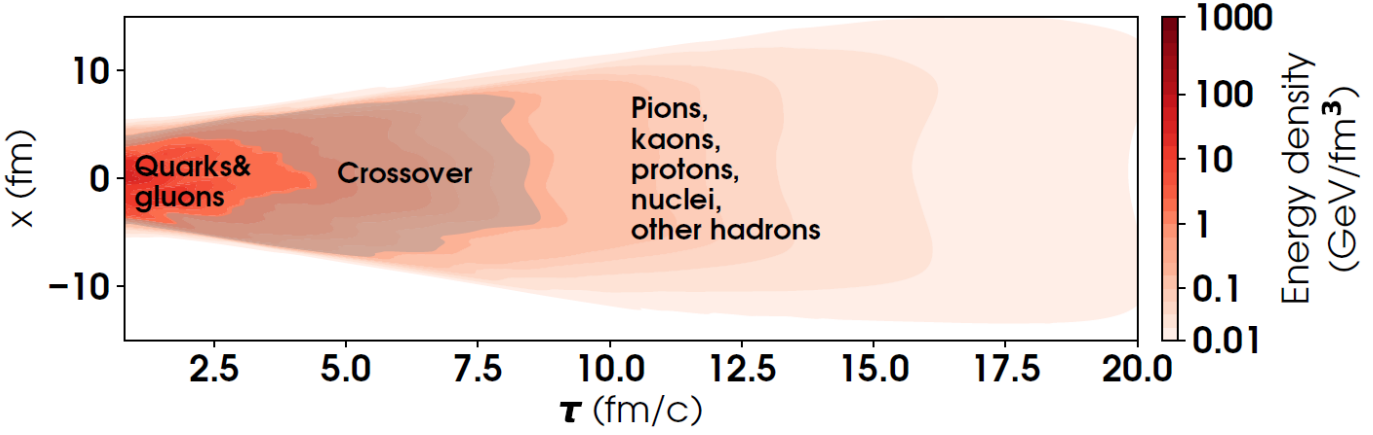
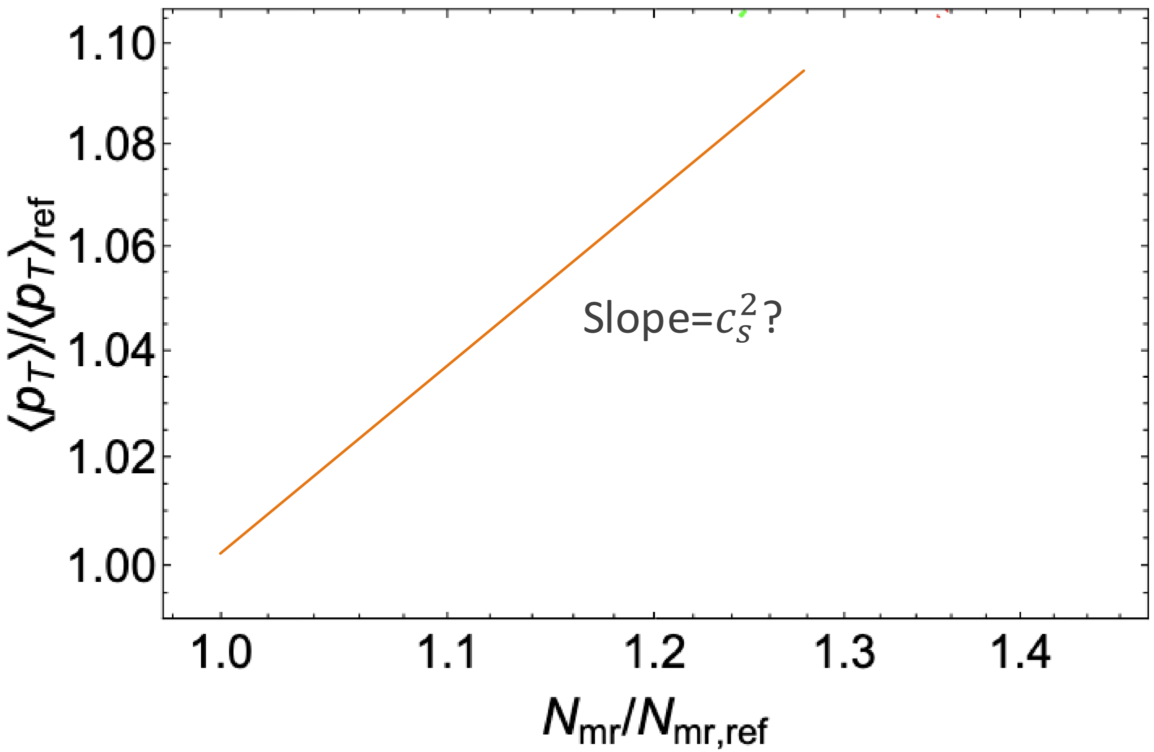


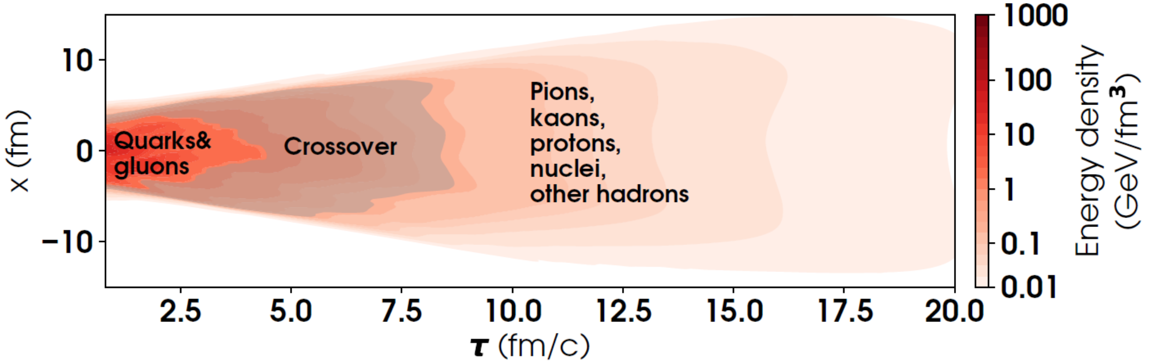
Focusing on mid-rapidity observables

$$c_s^2 = \frac{d\ln\left(\frac{E_{tot}}{N_{tot}}\right)}{d\ln N_{tot}} \rightarrow^{?} \frac{d\ln\langle p_T\rangle}{d\ln(dN/dy)}$$

Numerical test:

- Inviscid 1+1D hydro with Gaussian initial conditions
- **Constant speed of sound**
- Assume plasma converts to massless particles





$$N_{\rm mr} = \left.\frac{dN}{dy}\right|_{y=0}$$

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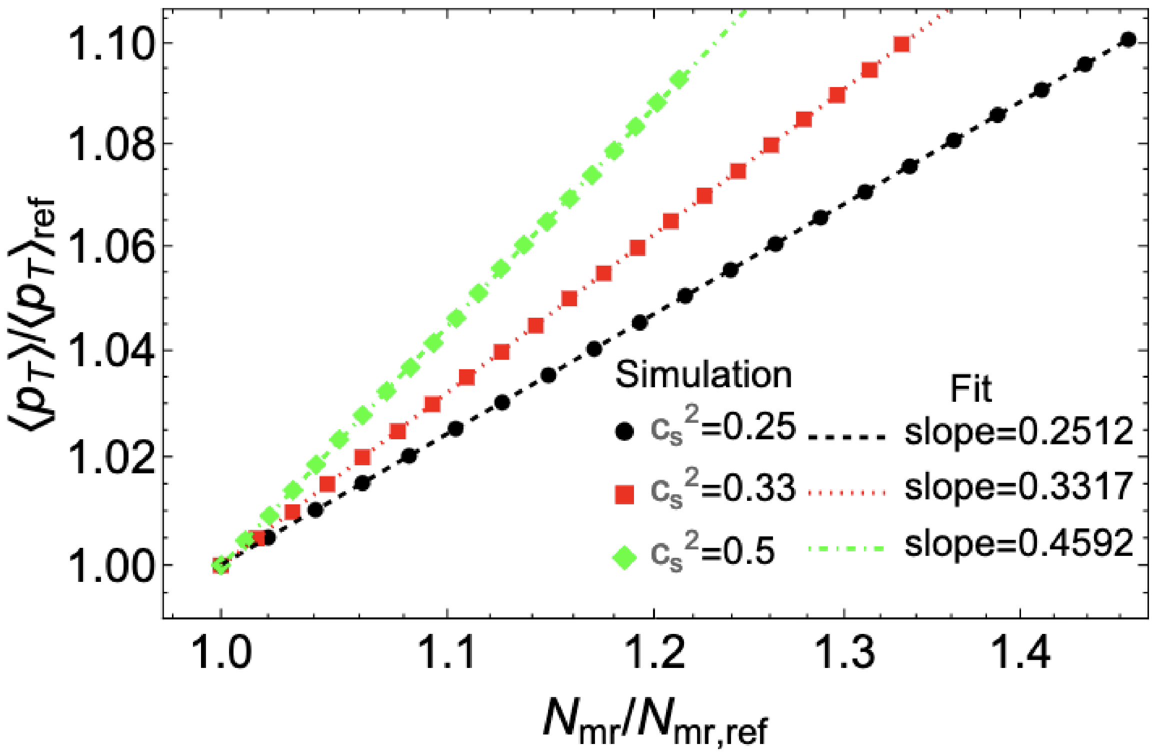

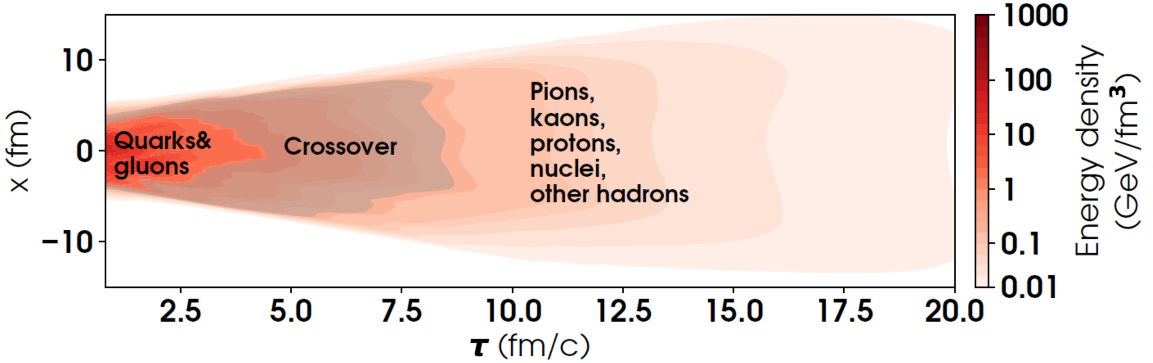

$$N_{\rm mr} = \left.\frac{dN}{dy}\right|_{y=0}$$

But this is if we wait a long time before stopping hydrodynamics

Freeze-out temperature (~ lifetime) dependence

$$c_s^2 = \frac{d\,\ln\left(\frac{E_{tot}}{N_{tot}}\right)}{d\,\ln N_{tot}} \rightarrow^{?} \frac{d\,\ln\langle p_T\rangle}{d\,\ln(dN/dy)}$$

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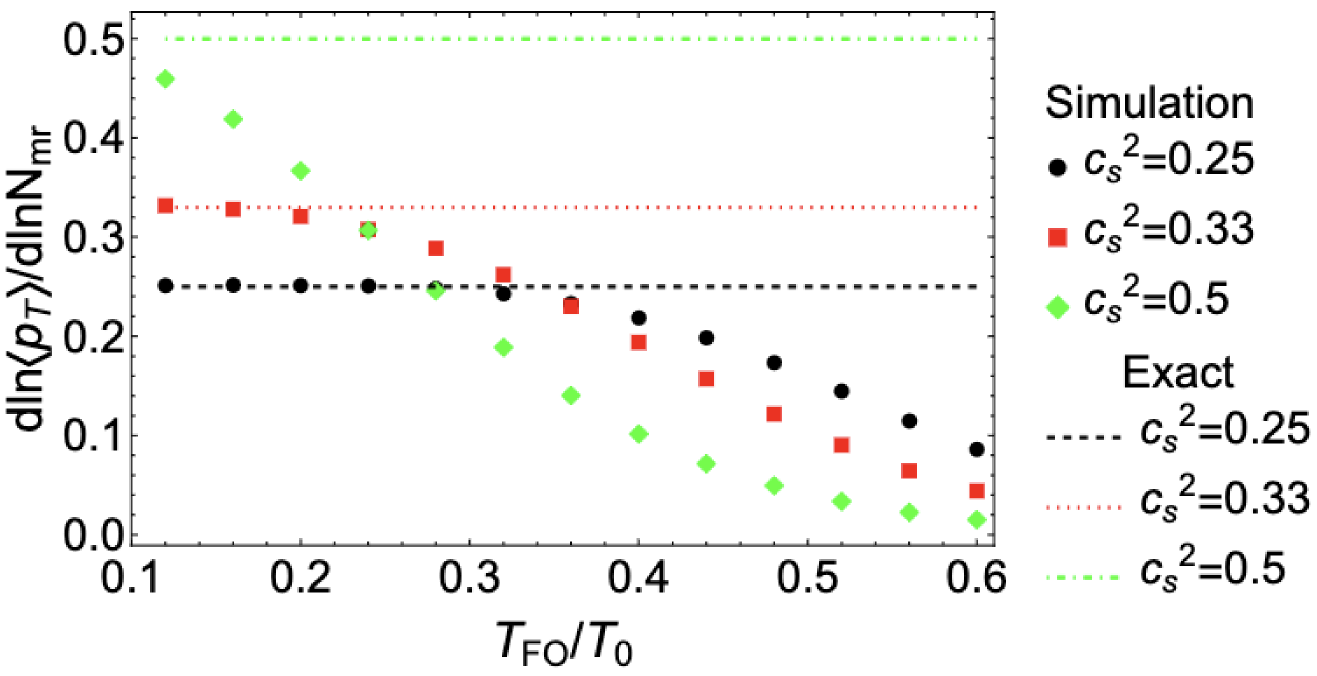

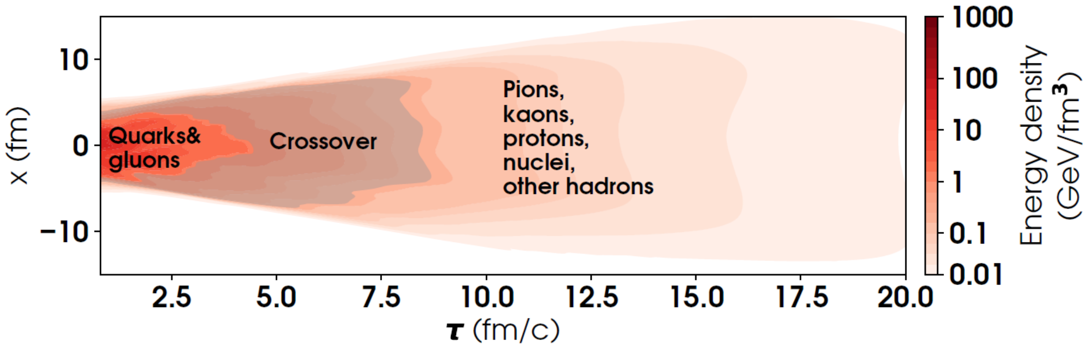

$$N_{\mathrm{mr}} = \left.\frac{dN}{dy}\right|_{y=0}$$

Freeze-out temperature (~ lifetime) dependence

$$c_s^2 = \frac{d\ln\left(\frac{E_{tot}}{N_{tot}}\right)}{d\ln N_{tot}} = \lim_{T_{FO}\to 0} \frac{d\ln\langle p_T\rangle}{d\ln(dN/dy)}$$

Numerical test:

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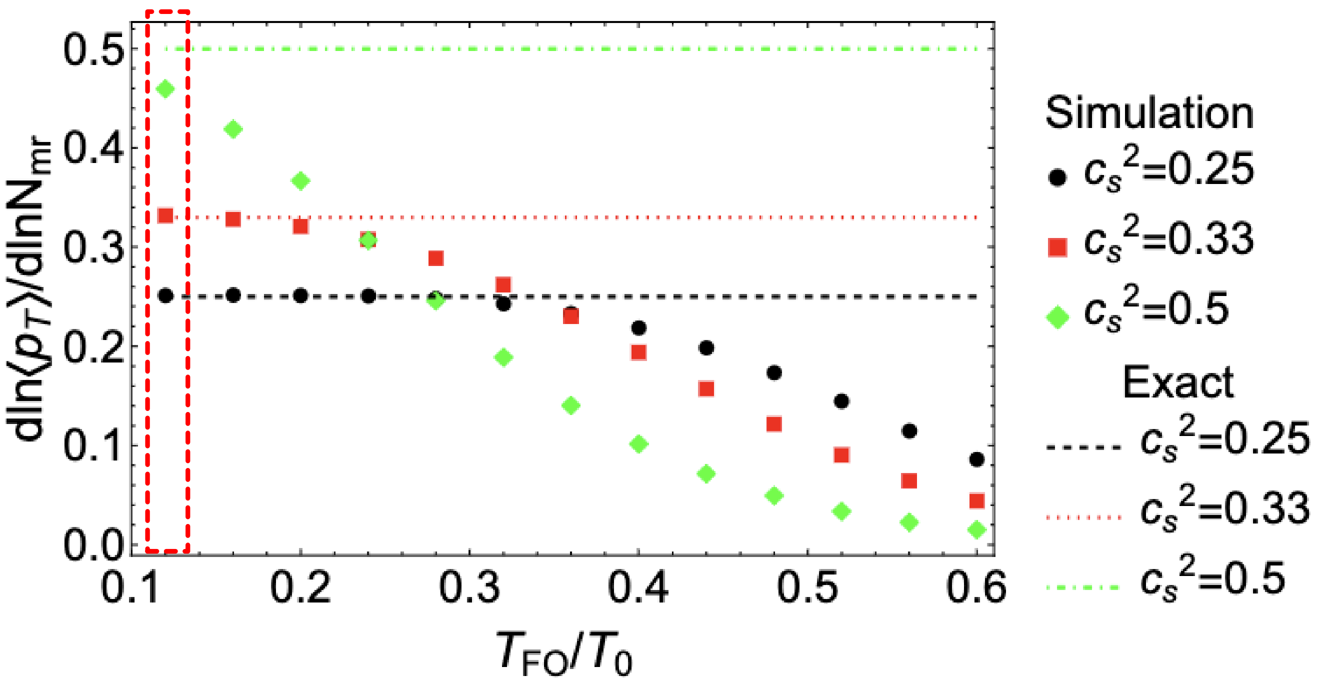
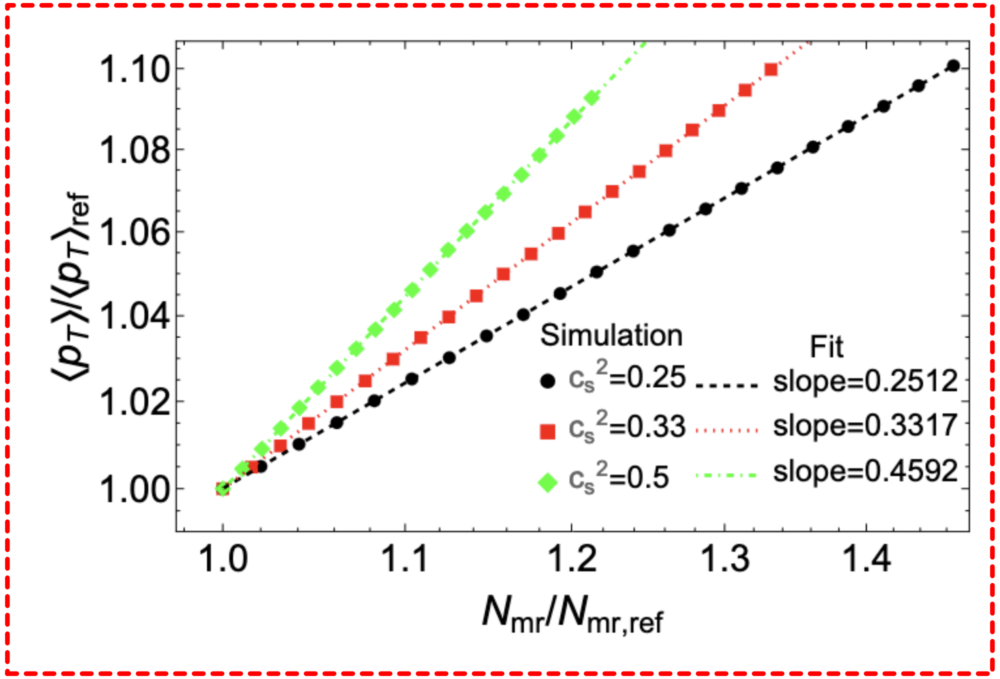
- **Constant speed of sound**
- Assume plasma converts to massless particles





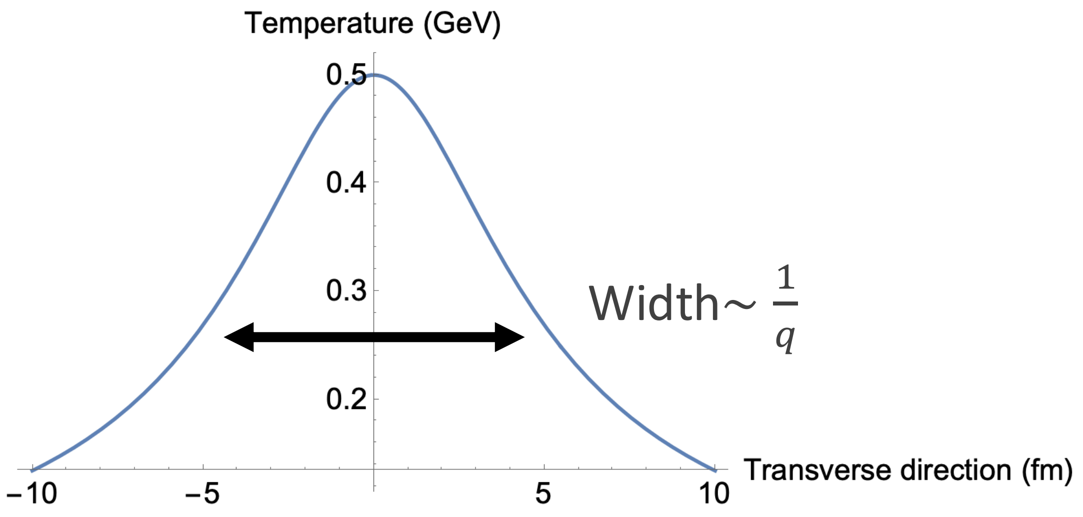
More insights for midrapidity measurements: analytical result with the Gubser solution

$$T(\tau,r)=\frac{\hat{T}_0(2q\tau)^{2/3}}{\tau\left(1+2q^2\left(\tau^2+r^2\right)+q^4\left(\tau^2-r^2\right)^2\right)^{1/3}}$$

$$u^r(\tau,r)=\frac{2rq^2\tau}{\sqrt{1+2q^2\left(\tau^2+r^2\right)+q^4\left(\tau^2-r^2\right)^2}}$$

$$\hat{T}_0=T_0\tau_0\left(\frac{1+q^2\tau_0^2}{2q\tau_0}\right)^{2/3}$$

Speed of sound is $c_s^2=1/3$



Measuring the equation of state: analytical result with the Gubser solution

- Given analytical temperature profile and flow velocity:

$$T(\tau, r) = \frac{\hat{T}_0 (2q\tau)^{2/3}}{\tau\left(1 + 2q^2\left(\tau^2 + r^2\right) + q^4\left(\tau^2 - r^2\right)^2\right)^{1/3}} \qquad u^r(\tau, r) = \frac{2rq^2\tau}{\sqrt{1 + 2q^2\left(\tau^2 + r^2\right) + q^4\left(\tau^2 - r^2\right)^2}}$$

- Can compute number of produced particles with Cooper-Frye:

$$E_p \frac{dN}{d^3p} = \frac{dN}{dy d^2 p_T} = \frac{g}{(2\pi)^3} \int_\Gamma \int_0^{2\pi} d\phi \int_{\mathbb{R}} d\eta_s \frac{\tau r [m_T \cosh(\eta_s - y) dr - p_T \cos\phi d\tau]}{\exp\{[m_T \cosh\kappa \cosh(\eta_s - y) - p_T \sinh\kappa \cos\phi]/T\} - 1},$$

$$\frac{dN}{dy} \equiv \int d^2 p_T \frac{dN}{dy d^2 p_T},$$

$$\frac{dN}{dy}\langle p_T \rangle \equiv \int d^2 p_T\, p_T \frac{dN}{dy d^2 p_T}.$$

Measuring the equation of state: analytical result with the Gubser solution

$$\hat{T}_0 = T_0\tau_0\left(\frac{1+q^2\tau_0^2}{2q\tau_0}\right)^{2/3}$$

- For massless bosons, we find

$$\left.\frac{dN}{dy}\right|_{y=0} \approx \frac{4\zeta(3)}{\pi}\hat{T}_0^3\left[1-\frac{1}{2}(q\tau_0)^{1/2}\left(\frac{T_{\mathrm{FO}}}{q\hat{T}_0}\right)^{3/2}\right]$$

$$\left.\frac{dN}{dy}\right|_{y=0}\cdot\langle p_T\rangle \approx \frac{4\pi^{7/2}q\Gamma\left(\frac{11}{6}\right)}{45\Gamma\left(\frac{7}{3}\right)}\hat{T}_0^4.$$

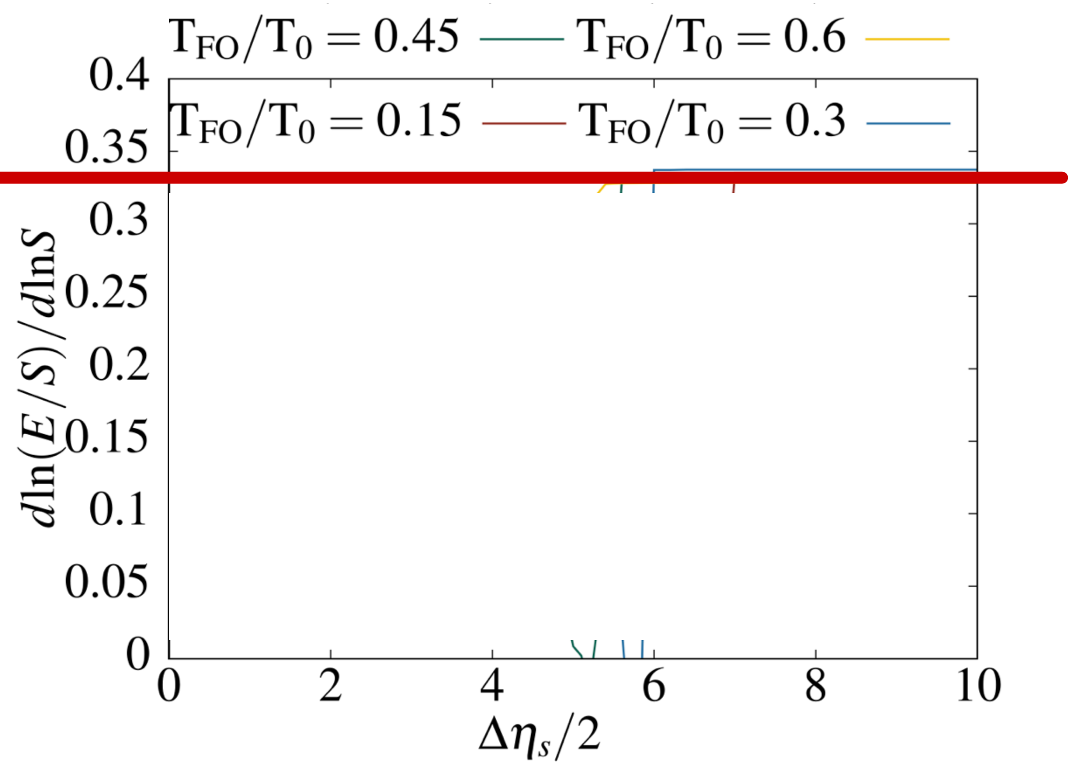
$$\frac{d\ln\langle p_T\rangle}{d\ln\left.\frac{dN}{dy}\right|_{y=0}} = \frac{d\ln\langle p_T\rangle/d\alpha}{d\ln\left.\frac{dN}{dy}\right|_{y=0}/d\alpha} \approx \frac{1}{3}-\frac{2}{3}\frac{1}{\boxed{\left(1+q^2\tau_0^2\right)}}\left(\frac{T_{\mathrm{FO}}}{T_0}\right)^{\boxed{3/2}},$$

Dependence on initial conditions (initial time τ_0 and initial inverse width q)

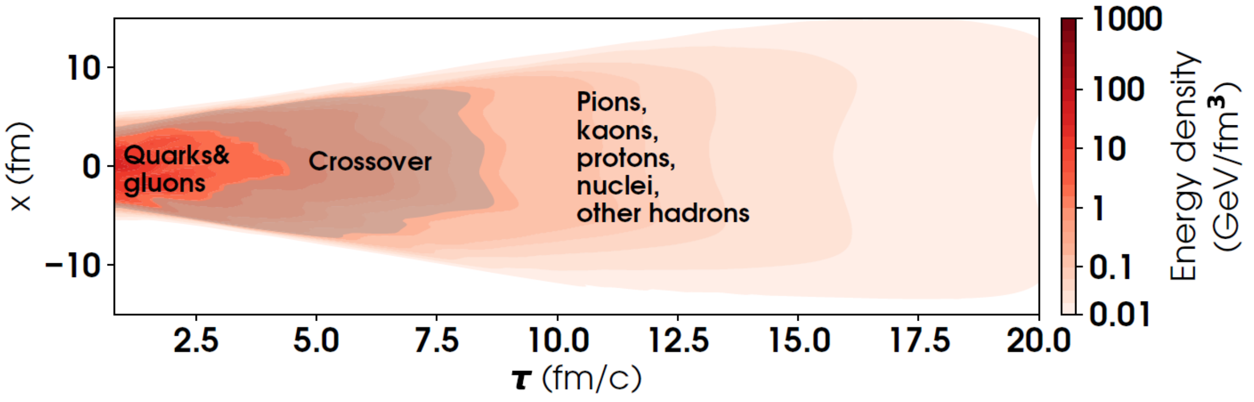
$$\boxed{c_s^2 = \frac{d\ln\left(\frac{E_{tot}}{N_{tot}}\right)}{d\ln N_{tot}} = \lim_{T_{FO}\to 0}\frac{d\ln\langle p_T\rangle}{d\ln(dN/dy)}}$$

Numerical simulation results

- Optical Glauber initial conditions, with temperature rescaled
- Inviscid relativistic hydrodynamics with $c_s^2 = 1/3$
- Compute the total energy and total entropy on hypersurface at $T = T_{FO}$



$$c_s^2 = \frac{d\ln(E_{\rm tot}/N_{\rm tot})}{d\ln N_{\rm tot}}$$



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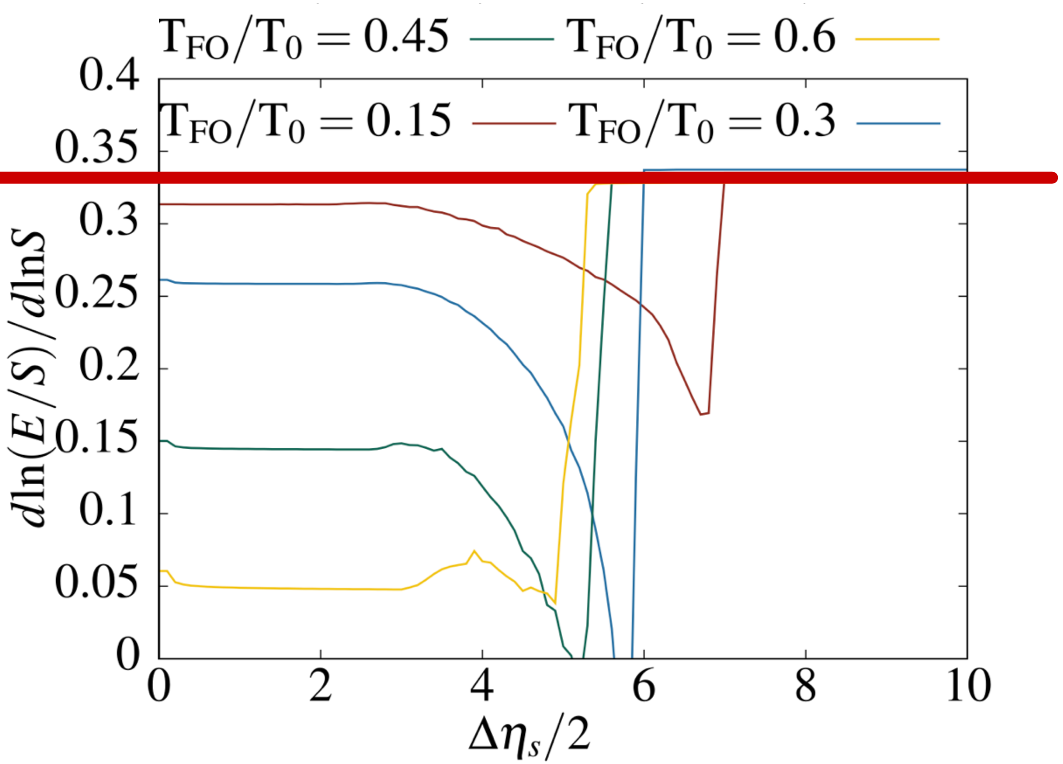

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