Collective flow and hydrodynamization as a function of system size from kinetic theory simulations

Clemens Werthmann
Ghent University

GHENT UNIVERSITY
Funded by the European Union
European Research Council
Established by the European Commission

Do small systems hydrodynamize?

Cons:

Effective field theory of $T^{\mu\nu}$ for long wavelength dynamics close to equilibrium

- gradient expansion: Knudsen number $\mathrm{Kn} \sim \ell_{\mathrm{mfp}}/L$
- expansion around equilibrium: inverse Reynolds number $\mathrm{Re}^{-1} \sim |\pi_{\mu\nu}|/P$

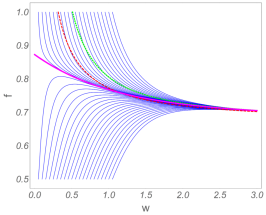
Denicol, Niemi, Molnar, Rischke, PRD 85, 114047

Transport coefficients computed from perturbations to equilibrium ("Green-Kubo relations")

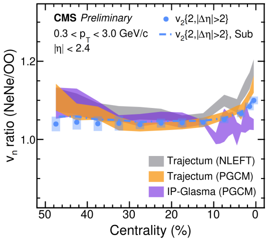
$$\eta = \frac{1}{T}\int_0^\infty \mathrm{d}t \int \mathrm{d}^3x \, \langle \pi^{xy}(x,t)\pi^{xy}(0,t)\rangle_{\mathrm{eq}}$$

Pros:

Early universal dynamics through "hydrodynamic attractor"
Heller, Spaliński, PRL 115, 072501



Accurate predictions for light ions
Giacalone et al., PRL 135 1 012302CMS-PAS-HIN-25-009



Caveats of early hydrodynamic description

Transport coefficients:

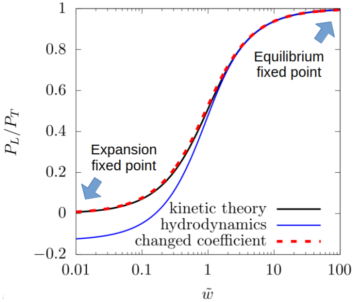
Expansion driven memory loss brings universal dynamics.

But: attractor does not speed up equilibration!

- early time part of attractor adjusted by changing only one second order coefficient Jean-Paul Blaizot, Li Yan, PRC 104 (2021) 5, 055201
- transport coefficients can differ in non-equilibrium:
  $\eta/s < 1/4\pi$ in holography M.F. Wondrak, M. Kaminski, M. Bleicher, PLB 811, 135973

$\Rightarrow$ Optimizing transport coefficients to describe data:

pheno model, not consistent effective field theory!



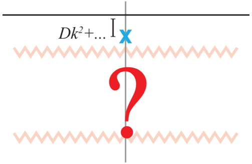
Initial state:

$$v_2 = \kappa \epsilon_2 \,, \qquad \frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}\eta} \approx \# \cdot \int_{\mathbf{x}_\perp} (e\tau^\alpha)_0{}^\gamma$$

When adjusting initial state parameters to match final state observables, may cover inaccuracies in modeling of dynamical response ($\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$-estimate: Giuliano Giacalone et al., PRL 123, 262301)

What is hydrodynamization?

Decay of non-hydrodynamic modes

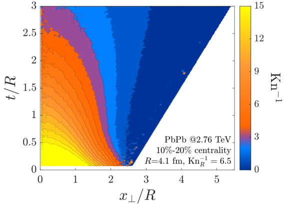


Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

Match with constitutive relations

$$T^{\mu\nu} \stackrel{?}{=} (e+P)u^\mu u^\nu - Pg^{\mu\nu}$$
$$(-\eta\sigma^{\mu\nu} + \ldots)$$

Small Kn, Re^{-1}

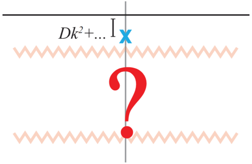


V. Nugara, V. Greco, S. Plumari, EPJC 85 (2025) 3, 311

- isotropization in anisotropic hydro
- depleting corona fraction in core-corona model
- approaching flow velocities in multi-fluid hydro
- ...

What is hydrodynamization?

Decay of non-hydrodynamic modes

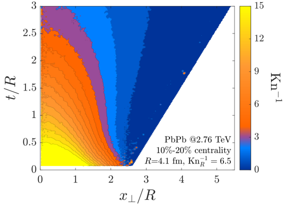


Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

Match with constitutive relations

$$T^{\mu\nu} \stackrel{?}{=} (e+P)u^\mu u^\nu - Pg^{\mu\nu}$$
$$(-\eta\sigma^{\mu\nu} + \dots)$$

Small Kn, Re^{-1}



V. Nugara, V. Greco, S. Plumari, EPJC 85 (2025) 3, 311

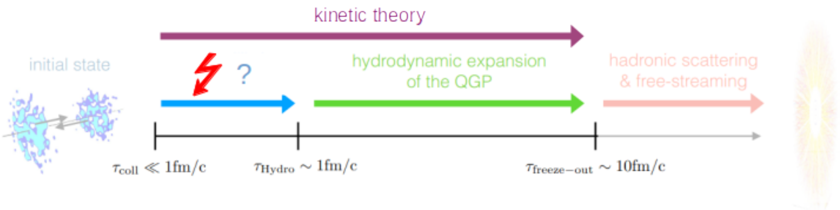
- isotropization in anisotropic hydro
- depleting corona fraction in core-corona model
- approaching flow velocities in multi-fluid hydro
- ...

Common ground:

- hydrodynamization is a smooth transition
- applicability of hydro $\widehat{=}$ microscopic details have become irrelevant

Assessing applicability of hydro by comparing flow

Philosophy: Won't get precise answer anyways, so let's find a practical one
⇒ **When does hydro accurately reproduce flow of kinetic theory?**



- need formulation of kinetic theory that is applicable in dense limit
- how to match the two descriptions?
  1. Hybrid: kinetic theory → hydro
  2. counteract pre-equilibrium difference

Dynamical descriptions

kinetic theory in conformal relaxation time approximation

$$p^\mu \partial_\mu f = -\frac{f - f_{\mathrm{eq}}}{\tau_R}\,, \quad \tau_R = 5T^{-1}\frac{\eta}{s}$$

- some microscopic info "integrated out"
  $\Rightarrow$ no diluteness requirement
- dynamics depends only on opacity $\hat{\gamma}$
  Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{1}{a\pi}R\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

total interactions $\hat{\gamma} \sim \mathrm{Kn}^{-1}$: depends on shear viscosity, transverse size and energy scale

Dynamical descriptions

kinetic theory in conformal relaxation time approximation

$$p^\mu \partial_\mu f = -\frac{f - f_{\text{eq}}}{\tau_R}\,, \quad \tau_R = 5T^{-1}\frac{\eta}{s}$$

- some microscopic info "integrated out"
  $\Rightarrow$ no diluteness requirement
- dynamics depends only on opacity $\hat{\gamma}$
  Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{1}{a\pi}R\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

total interactions $\hat{\gamma} \sim \mathrm{Kn}^{-1}$: depends on shear viscosity, transverse size and energy scale

viscous hydrodynamics:

- conformal equation of state $\mathcal{E} = 3P = aT^4$
- transport coefficients from RTA
- elliptic flow of energy ε_p: extracted directly from hydro, no need for particlization

$$\varepsilon_p = \frac{\int_{\mathbf{x}_\perp} T^{xx} - T^{yy} + 2iT^{xy}}{\int_{\mathbf{x}_\perp} T^{xx} + T^{yy}}$$

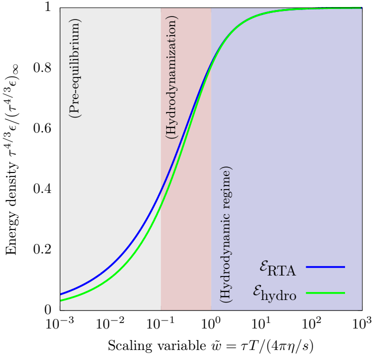
Inhomogeneous cooling and "scaled hydro"

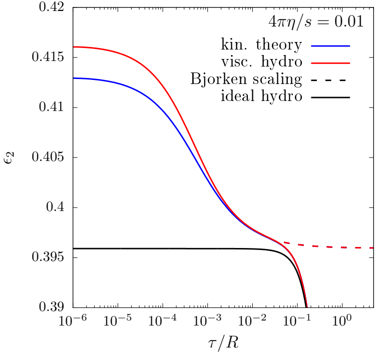
$\tau \ll \langle \nabla_\perp e \rangle$: system undergoes local Bjorken flow at each point in transverse plane

$\Rightarrow$ "Attractor behaviour": quick convergence to universal evolution

early times $\epsilon \sim \tau^{-1} \longrightarrow$ equilibration $\tau_{\rm eq} \propto T^{-1}\eta/s \longrightarrow$ ideal hydro $\epsilon \sim \tau^{-4/3}$

$\Rightarrow$ hotter regions cool more, eccentricities change (amount differs between theories)



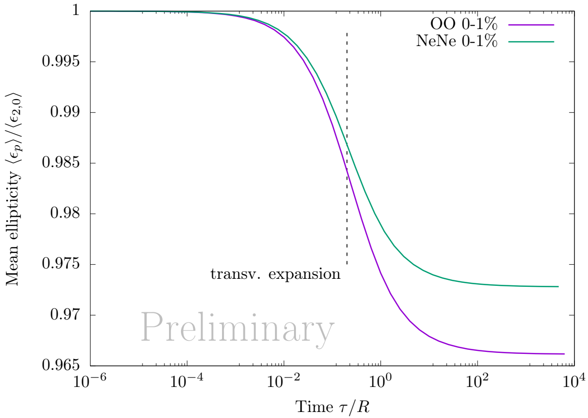


our hydro simulations: scaled initial condition of hydro in anticipation of different pre-equilibrium

Ambruș, Schlichting, CW, PRD 107 (2023) 094013

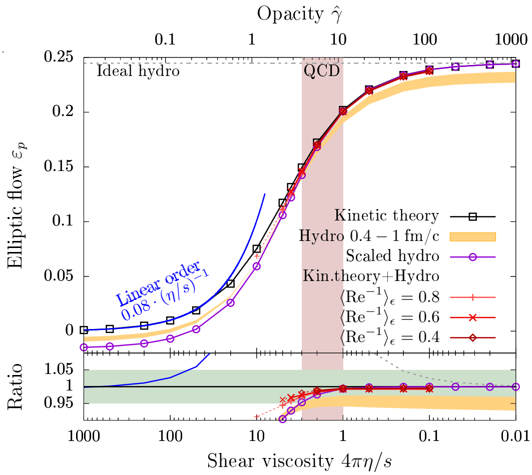
- eccentricity values at onset of transverse expansion determine final state flow
- typically changes eccentricities by $\mathcal{O}(1\%)$, but can be up to 10%

Early time eccentricity evolution in OO and NeNe



- OO: $\sim 3.4\%$, NeNe: $\sim 2.7\%$ total decrease
- when transverse expansion sets in, ϵ_2 has been reduced by a similar fraction $\sim 1.5\%$ in both systems $\Rightarrow$ good cancellation in ratio

Comparing hydro to kinetic theory (for now: average profile)



Victor E. Ambruș, Sören Schlichting, CW PRD 107 (2023) 094013

scaled hydro

- assumes equilibration before transverse expansion
- works for $\hat{\gamma} \gtrsim 4$

hybrid simulations

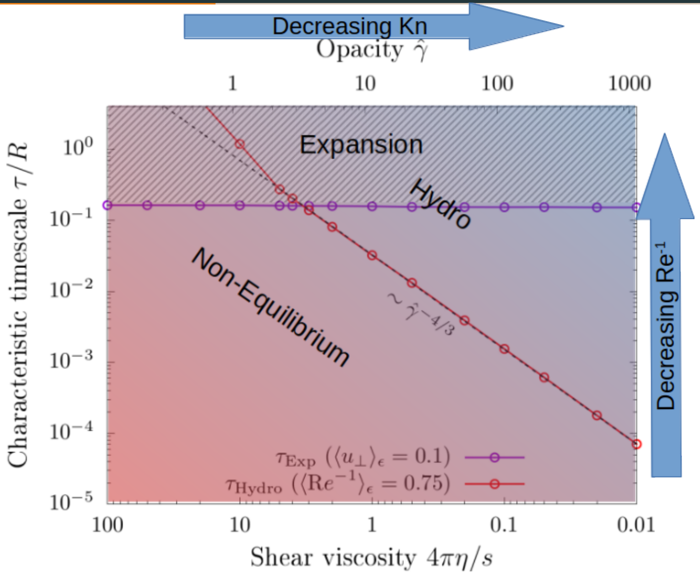
- later switch gives better agreement
- switching at fixed Re^{-1} gives good handle on accuracy

Regime of applicability of hydrodynamics

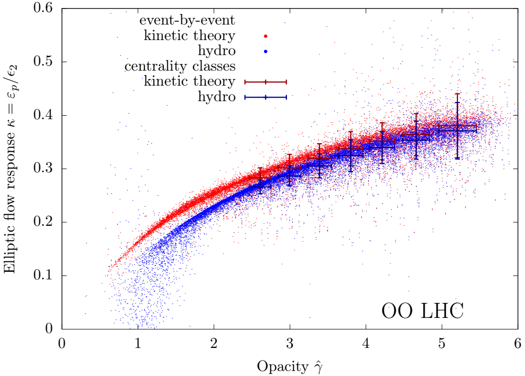
timescale of hydrodynamization:

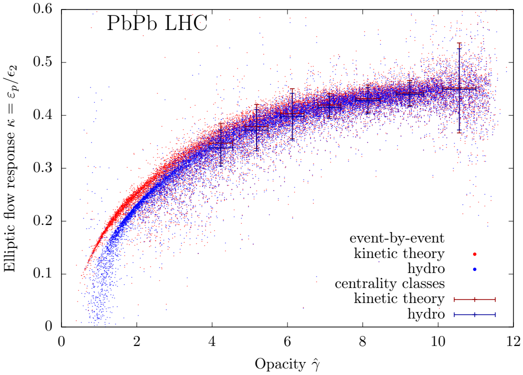
for $\hat{\gamma} \lesssim 3$ (central OO?), need non-equilibrium description of transverse expansion

Victor E. Ambruș, Sören Schlichting, CW PRL 130 (2023) 152301



Event-by-event Flow Responses



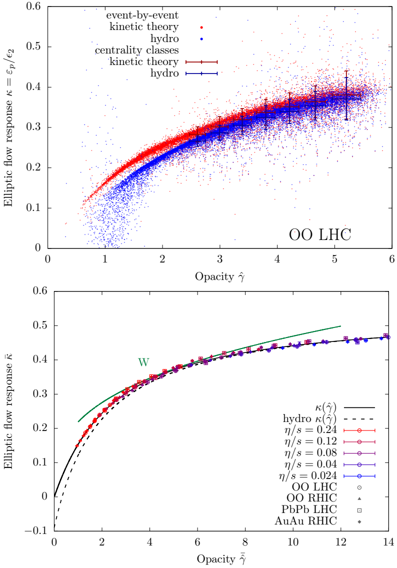


- still hydro $\rightarrow$ kinetic theory as $\hat{\gamma} \rightarrow \infty$
- some event-by-event spread, but mainly following universal curves

Victor E. Ambruș, Sören Schlichting, CW, PRD 111 (2025) 054024

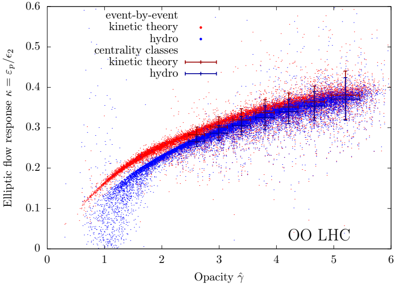
Universal flow response curve $v_2/\epsilon_2 = \kappa(\hat{\gamma})$

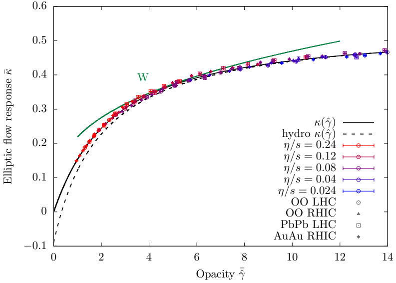
Hadronic collision simulations

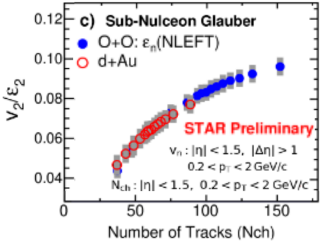


Universal flow response curve $v_2/\epsilon_2 = \kappa(\hat{\gamma})$

Hadronic collision simulation



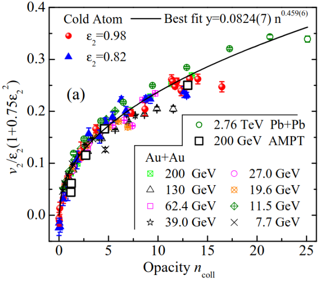




Experiment

Hadronic collisions

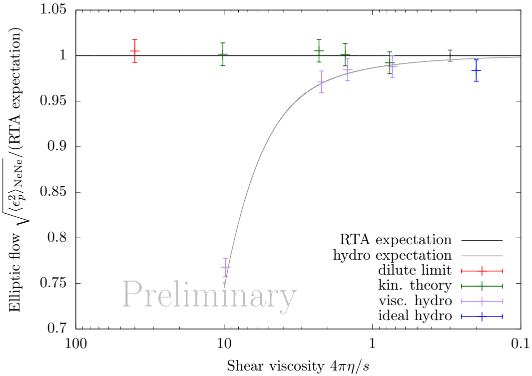
Andrew Tamis @ IS25

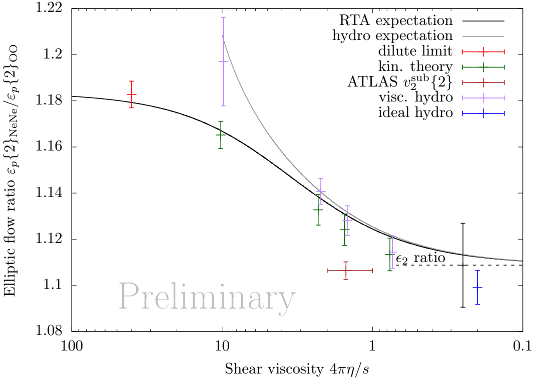


Cold atoms

Kei Li, Hong-Fang Song, Hao-Jie Xu, Yu-Liang Sun, Fuqiang Wang, arXiv:2405.02847

NeNe/OO flow ratio





reminder: $\hat{\gamma} \propto (\eta/s)^{-1}$

- flow from simulations matches expectation from response curves $\kappa(\hat{\gamma})$
- ratio modulation with opacity scale, because NeNe is bigger than OO
- taking the ratio reduces hydro error from few % to $< 1\%$.

V. Ambruș, G. Giacalone, CW, F. Zhou, WiP

Hydrodynamization Observable: Definition

measuring hydrodynamization

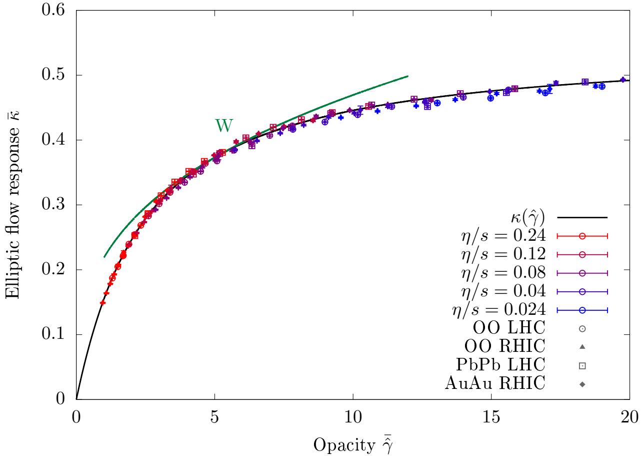
cancel geometry factors between similar systems:

$$\frac{\bar\kappa^{2k}_{\mathrm{RHIC}}}{\bar\kappa^{2k}_{\mathrm{LHC}}} \approx \frac{c_2^{\mathrm{RHIC}}\{2k\}}{c_2^{\mathrm{LHC}}\{2k\}}$$

$$\frac{\hat\gamma_{\mathrm{RHIC}}}{\hat\gamma_{\mathrm{LHC}}} \approx \left(\frac{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{RHIC}}}{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{LHC}}}\right)^{1/4}$$

to combine the two: log turns ratios into differences

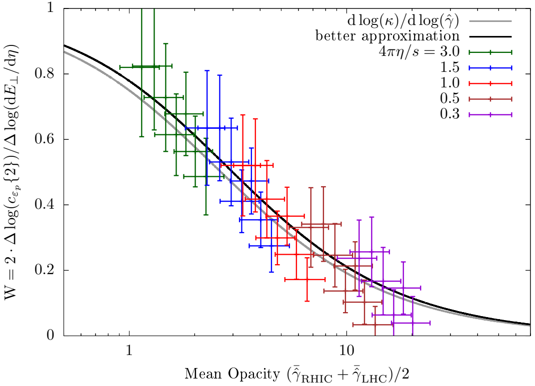
$$\mathrm{W} = \frac{2}{k}\frac{\Delta\log(c_2\{2k\})}{\Delta\log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat\gamma} \xrightarrow{\hat\gamma\to 0} 1 \qquad \xrightarrow{\hat\gamma\to\infty} 0$$



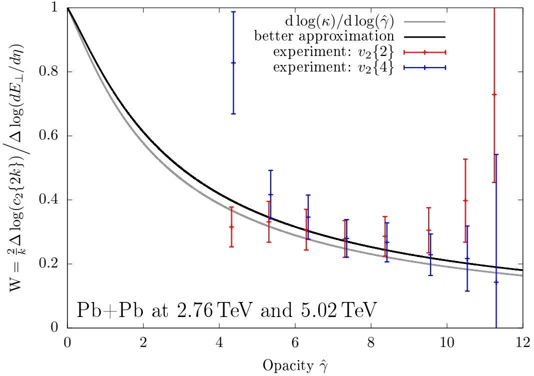
Hydrodynamization Observable: Proof of principle

crosscheck of W-observable:

with simulations



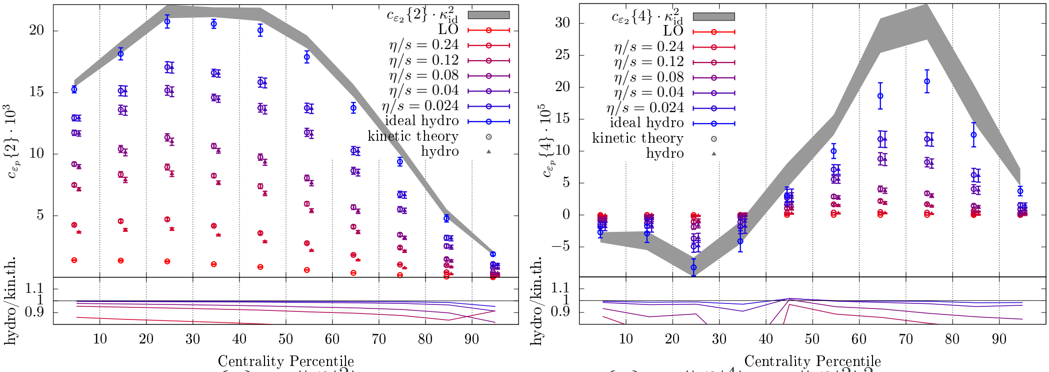
with experimental data



struggles: estimate of $\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}$; actually need flow of energy, not particle number

previous hydrodynamization criterion: $\hat{\gamma} \sim 3$ corresponds to $\mathrm{W} \sim 0.5$

Flow Cumulants



$$c_{\mathcal{O}}\{2\} = \langle |\mathcal{O}|^2 \rangle \qquad c_{\mathcal{O}}\{4\} = \langle |\mathcal{O}|^4 \rangle - 2\langle |\mathcal{O}|^2 \rangle^2$$

- $c_{\epsilon_p}\{2k\} \approx \kappa_{\rm id}^{2k} c_{\epsilon_2}\{2k\}$ in ideal hydro; opacity $\downarrow$: flow response $\downarrow$
- modulation of $c_{\epsilon_p}\{2k\}$ relative to $\propto c_{\epsilon_2}\{2k\}$ from centrality dependence of $\kappa(\hat{\gamma})$
- flow fluct. dominated by avg. response to geometry fluct $\langle (\epsilon_p)^n \rangle = \bar{\kappa}^n \langle (\epsilon_2)^n \rangle + \ldots$
- even “unorthodox” sign $(v_2\{4\} = \sqrt[4]{-c_{v_2}\{4\}})$ follows from initial state!

Summary

- accuracy of phenomenological hydro $\not\Rightarrow$ hydrodynamization
  - attractor $\not\Rightarrow$ faster equilibration
  - non-equilibrium transport coefficients
- flow responses follow universal curve $\kappa(\hat{\gamma})$
  - can help understand e.g. flow ratio NeNe/OO
- hydrodynamization observable measures strength of dynamical flow response independent of initial state:

$$\mathrm{W} = \frac{2}{k}\frac{\Delta\log(c_2\{2k\})}{\Delta\log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$$