RG fixed points from Outer Automorphisms & Goofy transformations

Andreas Trautner

based on:

arXiv:2505.00099 JHEP 10 (2025) 051
arXiv:2508.02646 PLB 873 (2026) 140190
arXiv:2603.12318 (submitted to PLB) w/ Thede De Boer

13th International Conference on the
Exact Renormalization Group (ERG2026)
University of Sussex
01.09.26

Supported by grants 2023.06787.CEECIND, 2024.01362.CERN, UIDP/00777/2020, UIDB/00777/2020

Outline

- Outer automorphisms are sufficient conditions for RG fixed points.
- What is a goofy transformation?

  $\longrightarrow$ Definition and examples
- RG stability of goofy trafos even if explicitly broken.

  $\longrightarrow$ Soft breaking by kinetic terms
- Comments and conclusion.

Outs and the RG flow

New general insight about the structure of quantum corrections: [De Boer, AT 2603.12318]

Existence of a consistent <u>Outer Automorphism</u> (Out) of a QFT

$$\Longrightarrow$$

Existence of a Renormalization Group (RG) fixed point (hyperplane, separatrix).

- This statement does not require perturbation theory.
- Enables an exact, non-perturbative computation of RG fixed points.
- This statement applies to *"regular"* as well as *"goofy"* outer automorphisms.

This constitutes a formalization and generalization of
't Hooft's technical naturalness argument.

['t Hooft '79]

RG stable parameter relations and symmetry

Why do symmetries lead to RG stable parameter relations?

$$\text{Preserved symmetry} \overset{(?)}{\Longleftrightarrow} \text{RG stable parameter relations.}$$

RG stable parameter relations and symmetry

Why do symmetries lead to RG stable parameter relations?

Preserved symmetry $\stackrel{(?)}{\Longleftrightarrow}$ RG stable parameter relations.

This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \mu \frac{\mathrm{d}\, g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

Proof of this (for all possible symmetries) has never been given to the best of my knowledge.

RG stable parameter relations and symmetry

Why do symmetries lead to RG stable parameter relations?

$$\text{Preserved symmetry} \overset{(?)}{\Longleftrightarrow} \text{RG stable parameter relations.}$$

This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \mu \frac{\mathrm{d}\, g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

We have given a proof of this, incl. generalization to goofy transformations [De Boer, AT 2603.12318].

RG stable parameter relations and symmetry

Why do symmetries lead to RG stable parameter relations?

Preserved symmetry $\overset{(?)}{\Longleftrightarrow}$ RG stable parameter relations.

This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \mu \frac{\mathrm{d}\, g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

We have given a proof of this, incl. generalization to goofy transformations [De Boer, AT 2603.12318].

In short: *Prospective* symmetries act as *outer automorphisms*, therefore, are equivalent to a (linear) map within the parameter space of a theory. [Fallbacher, AT '15]

RG stable parameter relations and symmetry

Why do symmetries lead to RG stable parameter relations?

Preserved symmetry $\overset{(?)}{\Longleftrightarrow}$ RG stable parameter relations.

This is a folk wisdom, based on ’t Hooft’s technical naturalness argument: ['t Hooft ’79]

$$\beta_g \;\equiv\; \mu\frac{\mathrm{d}\,g}{\mathrm{d}\mu} \;\propto\; g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

We have given a proof of this, incl. generalization to goofy transformations [De Boer, AT 2603.12318].

In short: *Prospective* symmetries act as *outer automorphisms*, therefore, are equivalent to a (linear) map within the parameter space of a theory. [Fallbacher, AT ’15]

Schematically: Lagrangian in terms of G-invariant operators $\mathcal{O}_n[\phi]$ and couplings λ_n,

$$\mathcal{L}[\phi,\lambda_n] = \sum_n \lambda_n \mathcal{O}_n[\phi]\;, \qquad \text{invariant under } G \qquad \phi_{\boldsymbol{r}} \mapsto \rho_{\boldsymbol{r}}(g)\phi_{\boldsymbol{r}}\;, \quad \forall g \in G\;.$$

Action of Outs: On fields $\phi_{\boldsymbol{r}} \mapsto U\phi'_{\boldsymbol{r}'}$ (with consistency condition $\rho_{\boldsymbol{r}}(u(g)) = U\rho_{\boldsymbol{r}'}(g)U^{-1}$, $\forall\, g \in G$)

On $\mathcal{L}[\phi,\lambda_n]$, Out trafo acts like $\vec{\lambda} \mapsto F(\vec{\lambda}) = \mathcal{U}\vec{\lambda}$.

$\Rightarrow$ Couplings transform covariantly under Outs! (Can be viewed as “spurions”)

RG stable parameter relations and symmetry

Why do symmetries lead to RG stable parameter relations?

Preserved symmetry $\overset{(?)}{\Longleftrightarrow}$ RG stable parameter relations.

This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \mu \frac{\mathrm{d}\,g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

We have given a proof of this, incl. generalization to goofy transformations [De Boer, AT 2603.12318].

In short: *Prospective* symmetries act as *outer automorphisms*, therefore, are equivalent to a (linear) map within the parameter space of a theory. [Fallbacher, AT '15]

Couplings trafo covariantly under Outs.
$\Rightarrow$ β-functions trafo covariantly.
$\Rightarrow$ Non-perturbative constraints on $\beta_{\vec{\lambda}}$, allow to compute RG fixed points.
[AT 2505.00099], [De Boer, AT 2603.12318].

$$\beta_{\vec{\lambda}} \equiv \mu \frac{\mathrm{d}\,\vec{\lambda}}{\mathrm{d}\mu} = \vec{f}(\lambda_1, \lambda_2, \dots)\,,$$

$$\boxed{\mathcal{U}\beta_{\vec{\lambda}} \overset{!}{=} \vec{f}\Big|_{\vec{\lambda}\to\mathcal{U}\vec{\lambda}}\,.}$$

(For *goofy*: $\vec{\lambda}$ includes WFR coefficients.)

"Symmetries of β-functions $\geq$ symmetries of theory."

Intuitive understanding based on Standard Model CP transformation

- CP is an existing Out of the Standard Model, mapping all irreps $\boldsymbol{r} \leftrightarrow \boldsymbol{r}^*$.
- CP-Out trafo acts on fields, schematically: $\phi(x,t) \mapsto U\phi^*(-x,t)$,
- **Equivalently**, CP trafo acts as mapping in parameter space: $\delta_{\mathrm{CKM}} \equiv \delta \mapsto -\delta$.
- Parameters trafo covariantly $\Longrightarrow$ β-functions trafo covariantly!

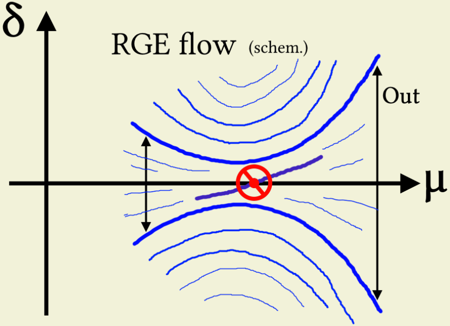
This implies **to all order**:

$$\beta_\delta \equiv \mu\frac{\mathrm{d}\,\delta}{\mathrm{d}\mu} \overset{!}{=} \delta \times f_{\mathrm{CP-inv.}}(\delta^2, \lambda, \dots)\ ,$$

$\Rightarrow$ RG fixed point: $\beta_\delta = 0$ @ $\delta = 0$.

(@$\delta = 0$, CP Out joins symmetry)

Note: generally, Trafo $\neq$ Symmetry



This argument generalizes to all* Outs incl. *goofy* transformations. [De Boer, AT 2603.12318]

What is a *goofy* transformation?

Note: This is a very young topic, understanding this for yourself may be rewarding.

Complete list of existing literature on goofy transformations:

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO $\rightarrow$ GOOFy**
- [Haber and Ferreira 2502.11011]
- [AT 2505.00099], [De Boer, AT 2603.12318]
- [Ferreira, Grzadkowski, Ogreid 2506.21145],
- [De Boer, Görtz, Incrocci 2507.22111], [AT 2508.02646]
- [Kuncinas, Osland, Rebelo 2512.07657], [de Medeiros Varzielas, Kuncinas 2608.05304]
- [Grzadkowski, Ogreid 2602.20849]
- [Ferreira 2604.07458]
- [Pilaftsis 2605.18341]
- And very brief remarks in: [Cao, Cheng, Xu 2305.12764v2 (of 3/2024)], [Pilaftsis 2408.04511]

The first ever Goofy transformation

Goofy transformations have first been discovered in the 2HDM,

$m_{11,22},\ \lambda_{1,2,3,4} \in \mathbb{R}$
$m_{12},\ \lambda_{5,6,7} \in \mathbb{C}$

$$
\begin{aligned}
V(\Phi^*,\Phi) \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& - \left\{ m_{12}^2 \Phi_1^\dagger \Phi_2 \right\} + \left\{ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 \right\} + \text{h.c.}.
\end{aligned}
$$

FGOO≡[Ferreira, Grzadkowski, Ogreid, Osland '23] found the parameter relations

$$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,.$$

- These are renormalization group (RG) stable *to all orders*.
- Do **not** correspond to any of the previously known *regular* 2HDM symmetries.
- Correspond to invariance of $V(\Phi^*,\Phi)$ under trafo:

$$\underline{\boldsymbol{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\boldsymbol{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

Applied to canonical (gauge-)kinetic terms, this transformation maps

$$\mathcal{K} \;=\; (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) \;\mapsto\; -\mathcal{K}.$$

$\Rightarrow$ This trafo cannot be an *exact* symmetry in a *dynamical* theory ($\mathcal{K} \neq 0$).

But how can it then support the RG fixed relations above?

Goofy transformations have first been discovered in the 2HDM.

[Ferreira, Grzadkowski, Ogreid, Osland '23]

And subsequently many new (goofy) RG fixed points of 2HDM identified.

[Trautner '25]

But the 2HDM is unnecessarily complicated
for understanding the nature of goofy transformations.

My definition of a Goofy transformation

Any theory:

$$\mathcal{L}[\phi] \; = \; \mathcal{K}_{\text{(kinetic)}}[\phi] \; - \; \mathcal{V}_{\text{(potential)}}[\phi] \; .$$

- How come, when talking about symmetry transformations, we usually only consider transforming/constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under <u>*all possible*</u> sym. transformations...

My definition of a Goofy transformation

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(\text{kinetic})}[\phi] \;-\; \mathcal{V}_{(\text{potential})}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider transforming/constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...
  (e.g. $\mathcal{K}[\phi] = \mathcal{K}_{ij}\;\partial_\mu\phi_i\partial^\mu\phi_j$) . . . but this is just wrong!

My definition of a Goofy transformation

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(\mathrm{kinetic})}[\phi] \;-\; \mathcal{V}_{(\mathrm{potential})}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider transforming/constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under <u>*all possible*</u> sym. transformations...
  (e.g. $\mathcal{K}[\phi] = \mathcal{K}_{ij}\,\partial_\mu\phi_i\partial^\mu\phi_j$) . . . but this is just wrong!

My defining criterion:

Regular transformations act <u>**trivially**</u> on kinetic term.	$\longleftrightarrow$	***Goofy*** transformations act <u>**non-trivially**</u> on kinetic term.

My definition of a Goofy transformation

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{\text{(kinetic)}}[\phi] \;-\; \mathcal{V}_{\text{(potential)}}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider transforming/constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...
  (e.g. $\mathcal{K}[\phi] = \mathcal{K}_{ij}\;\partial_\mu\phi_i\partial^\mu\phi_j$) . . . but this is just wrong!

My defining criterion:

Regular transformations act **trivially** on kinetic term.	$\longleftrightarrow$	***Goofy*** transformations act **non-trivially** on kinetic term.

Crucial: **Transformation $\neq$ Symmetry.**

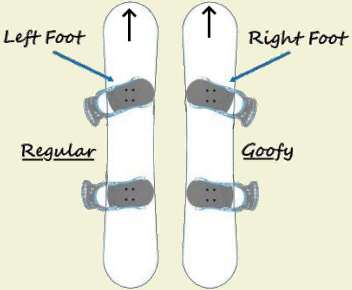
- Goofy trafo cannot be an *exact* symmetry in a *dynamical* theory ($\mathcal{K}\neq 0$).
- But trafos (regular/goofy) can be physically important, even if explicitly broken.
- Concept is so fundamental, it might as well matter for classical mechanics *etc.*

On the naming: Regular vs. Goofy

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO $\rightarrow$ GOOFy**

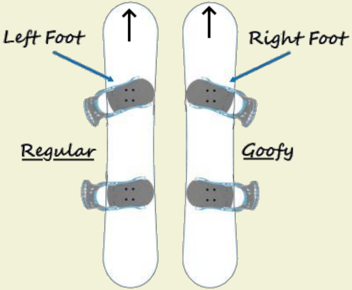
On the naming: Regular vs. Goofy

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410]   **FGOO $\to$ GOOFy**
- But also: in Surfing, Skateboarding, Snowboarding, ...



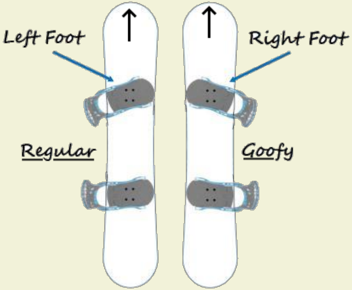
On the naming: Regular vs. Goofy

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410]   **FGOO $\rightarrow$ GOOFy**
- But also: in Surfing, Skateboarding, Snowboarding, . . .



On the naming: Regular vs. Goofy

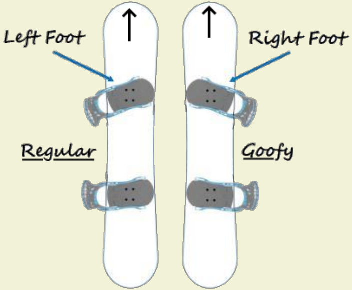
- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO $\rightarrow$ GOOFy**
- But also: in Surfing, Skateboarding, Snowboarding, ...



- Coincidentally, **Goofy** is 2023 German “Youth word of the year”. [Langenscheidt]

On the naming: Regular vs. Goofy

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO → GOOFy**
- But also: in Surfing, Skateboarding, Snowboarding, . . .



- Coincidentally, **Goofy** is 2023 German “Youth word of the year”. [Langenscheidt]

  e.g. 2012: YOLO, . . . , 2020: lost, 2021: cringe, . . . , 2023: goofy

RG stability of (goofy) parameter relations

A central question on goofy transformations:

How can it be that "goofy" parameter relations can be all-order RG stable,
even if the corresponding goofy transformation is explicitly broken
by the (gauge-)kinetic terms $\mathcal{K}$ (WFR coefficients)?

Simple example model: (Goofy) Outs and the RG flow

Outs and the RG flow

Consider a QFT of real scalar fields ϕ_1, $\phi_2 \in \mathbb{R}$, invariant under transformation:

$$\vec{\phi} := \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \mapsto s \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \; t \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \text{with} \qquad s := \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \; t := \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}.$$

$$\mathcal{L} \;=\; \left(\partial_\mu \phi_1\right)^2 + \left(\partial_\mu \phi_2\right)^2 + m \left(\phi_1^2 + \phi_2^2\right) + \boldsymbol{\lambda} \left(\phi_1^4 + \phi_2^4\right) + 2\boldsymbol{\lambda_p}\, \phi_1^2 \phi_2^2 \;.$$

Outs and the RG flow

Consider a QFT of real scalar fields ϕ_1, $\phi_2 \in \mathbb{R}$, invariant under transformation:

$$\vec{\phi} := \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \mapsto s \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \; t \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad \text{with} \quad s := \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \; t := \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}.$$

$$\mathcal{L} \;=\; (\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2 + m\left(\phi_1^2 + \phi_2^2\right) + \boldsymbol{\lambda}\left(\phi_1^4 + \phi_2^4\right) + 2\boldsymbol{\lambda_p}\, \phi_1^2 \phi_2^2 \,.$$

Known: Three RG fixed “points” with “emergent symmetries”:
[Iliopoulos, Nanopoulos, Tomaras ’80],. . . , [Berezhiani, Giambattista, Maiezza, Kobakhidze 2406.13575]

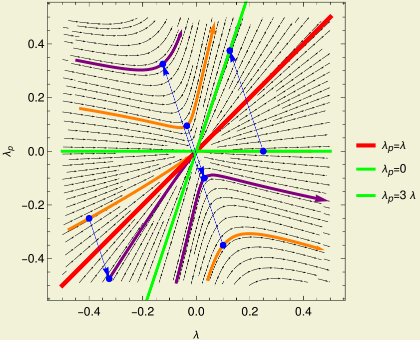
$(i)\ \boldsymbol{\lambda = \lambda_p}$, $\quad (ii)\ \boldsymbol{\lambda_p = 0}$, $\quad (iii)\ \boldsymbol{\lambda_p = 3\lambda}$.

Proven by explicitly computing β-functions and solving for RG flow in perturbation theory.

$$\beta_\lambda = \frac{72\lambda^2 + 8\lambda_p^2}{16\pi^2} - \frac{3264\lambda^3 + 320\lambda\lambda_p^2 + 256\lambda_p^3}{256\pi^4} + \ldots,$$

$$\beta_{\lambda_p} = \frac{48\lambda\lambda_p + 32\lambda_p^2}{16\pi^2} - \frac{960\lambda^2\lambda_p + 2304\lambda\lambda_p^2 + 576\lambda_p^3}{256\pi^4} + \ldots$$

e.g. PyR@TE, RGBeta [Sartore, Schienbein, Poole, Thomsen, Steudtner, . . .]



Outs and the RG flow

Look at it our way:

[De Boer, AT 2603.12318]

$$\mathcal{L} \;=\; \boldsymbol{\kappa_1}\left(\partial_\mu\phi_1\right)^2 + \boldsymbol{\kappa_2}\left(\partial_\mu\phi_2\right)^2 + m\left(\phi_1^2+\phi_2^2\right) + \overbrace{\lambda\left(\phi_1^4+\phi_2^4\right) + 2\lambda_p\,\phi_1^2\phi_2^2}^{V_4:=}\;.$$

Outs and the RG flow

Look at it our way:

[De Boer, AT 2603.12318]

$$\mathcal{L} \;=\; \boldsymbol{\kappa_1}\left(\partial_\mu\phi_1\right)^2 + \boldsymbol{\kappa_2}\left(\partial_\mu\phi_2\right)^2 + m\left(\phi_1^2+\phi_2^2\right) + \overbrace{\lambda\left(\phi_1^4+\phi_2^4\right) + 2\lambda_p\,\phi_1^2\phi_2^2}^{V_4:=}\;.$$

1. In addition to $t=\sigma_1$, $s=\sigma_3$, there is an accidental *goofy* symmetry of of potential V_4:

$$\vec{\phi}\mapsto v\,\vec{\phi}\quad\text{with}\quad v=\begin{pmatrix} & -\mathrm{i}\\ \mathrm{i} & \end{pmatrix}=\sigma_2.$$

Full symmetry group of V_4:

$$P=\langle\sigma_1,\sigma_2,\sigma_3\rangle\qquad,\quad \mathrm{Out}(P)=D_{12}.$$

Outs and the RG flow

Look at it our way:

[De Boer, AT 2603.12318]

$$\mathcal{L} \;=\; \boldsymbol{\kappa_1}\,(\partial_\mu\phi_1)^2 + \boldsymbol{\kappa_2}\,(\partial_\mu\phi_2)^2 + m\,(\phi_1^2+\phi_2^2) + \overbrace{\lambda\,(\phi_1^4+\phi_2^4) + 2\lambda_p\,\phi_1^2\phi_2^2}^{V_4:=}\;.$$

1. In addition to $t=\sigma_1$, $s=\sigma_3$, there is an accidental *goofy* symmetry of of potential V_4:

$$\vec\phi \mapsto v\,\vec\phi \quad \text{with} \quad v = \begin{pmatrix} & -\mathrm{i} \\ \mathrm{i} & \end{pmatrix} = \sigma_2.$$

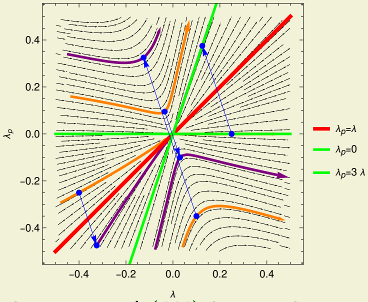
Full symmetry group of V_4:

$$P = \langle \sigma_1, \sigma_2, \sigma_3 \rangle \quad , \quad \mathrm{Out}(P) = D_{12}.$$

2. The full Out group is generated by:

$$\color{blue}{u}\color{black}{} : \vec\phi \mapsto \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{\vec\phi}{\sqrt{2}} \qquad \curvearrowright \begin{pmatrix} \lambda \\ \lambda_p \end{pmatrix} \stackrel{u}{\longmapsto} \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} \lambda \\ \lambda_p \end{pmatrix}$$

$$w : \vec\phi \mapsto \begin{pmatrix} 1 & \\ & \mathrm{i} \end{pmatrix} \vec\phi \qquad \curvearrowright \begin{pmatrix} \lambda \\ \lambda_p \\ \boldsymbol{\kappa_2} \end{pmatrix} \stackrel{w}{\longmapsto} \begin{pmatrix} \lambda \\ -\lambda_p \\ -\boldsymbol{\kappa_2} \end{pmatrix}$$



u, w and uwu explain all-order RG fixed points $\color{red}{(i)\ \boldsymbol{\lambda = \lambda_p}}$, $\color{green}{(ii)\ \boldsymbol{\lambda_p = 0}}$, and $\color{green}{(iii)\ \boldsymbol{\lambda_p = 3\lambda}}$.

This is despite the fact that goofy transformations are explicitly broken by kinetic terms!

Outs and the RG flow

Consider e.g. goofy Out transformation w: $\lambda_p \mapsto -\lambda_p$, $\kappa_2 \mapsto -\kappa_2$. (Corresponding to F.P. $(ii) : \lambda_p = 0$)

$\Longrightarrow$ Non-perturbative, all-order exact constraint on the system of β-functions:

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} \beta_\lambda \\ \beta_{\lambda_p} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \beta_\lambda(\lambda, \lambda_p, \kappa_2, \ldots) \\ \beta_{\lambda_p}(\lambda, \lambda_p, \kappa_2, \ldots) \end{pmatrix} \Bigg|_{\lambda_p \mapsto -\lambda_p,\, \boldsymbol{\kappa_2 \mapsto -\kappa_2}}$$

This imposes an all-order constraint on $\beta_{\vec{\lambda}}$. This implies that there is an expansion

$$\Longrightarrow \quad \beta_{\lambda_p} = \lambda_p \times f_+(\lambda, \lambda_p^2, \kappa_2^2) + \boldsymbol{\kappa_2} \times g_+(\lambda, \lambda_p^2, \kappa_2^2)\ ,$$

Outs and the RG flow

Consider e.g. goofy Out transformation w: $\lambda_p \mapsto -\lambda_p$, $\kappa_2 \mapsto -\kappa_2$. (Corresponding to F.P. $(ii):\lambda_p = 0$)

$\Longrightarrow$ Non-perturbative, all-order exact constraint on the system of β-functions:

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} \beta_\lambda \\ \beta_{\lambda_p} \end{pmatrix} \stackrel{!}{=} \left.\begin{pmatrix} \beta_\lambda(\lambda, \lambda_p, \kappa_2, \dots) \\ \beta_{\lambda_p}(\lambda, \lambda_p, \kappa_2, \dots) \end{pmatrix}\right|_{\lambda_p \mapsto -\lambda_p,\, \boldsymbol{\kappa_2 \mapsto -\kappa_2}}$$

This imposes an all-order constraint on $\beta_{\vec{\lambda}}$. This implies that there is an expansion

$$\Longrightarrow \quad \beta_{\lambda_p} = \lambda_p \times f_+(\lambda, \lambda_p^2, \kappa_2^2) + \boldsymbol{\kappa_2} \times g_+(\lambda, \lambda_p^2, \kappa_2^2)\ ,$$

Crucial: here, $g_+ \propto \lambda_p^2 \implies \beta_{\lambda_p}\big|_{\lambda_p=0} = 0$, even if $\kappa_2 \neq 0$.

The explicit breaking by the kinetic terms is soft!

Outs and the RG flow

Consider e.g. goofy Out transformation w: $\lambda_p \mapsto -\lambda_p$, $\kappa_2 \mapsto -\kappa_2$. (Corresponding to F.P. $(ii) : \lambda_p = 0$)

$\Longrightarrow$ Non-perturbative, all-order exact constraint on the system of β-functions:

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} \beta_\lambda \\ \beta_{\lambda_p} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \beta_\lambda(\lambda, \lambda_p, \kappa_2, \dots) \\ \beta_{\lambda_p}(\lambda, \lambda_p, \kappa_2, \dots) \end{pmatrix} \Bigg|_{\lambda_p \mapsto -\lambda_p,\, \boldsymbol{\kappa_2 \mapsto -\kappa_2}}$$

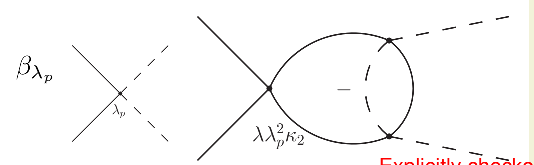
This imposes an all-order constraint on $\beta_{\vec{\lambda}}$. This implies that there is an expansion

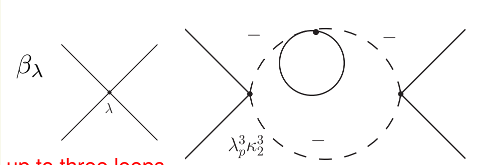
$$\Longrightarrow \quad \beta_{\lambda_p} = \lambda_p \times f_+(\lambda, \lambda_p^2, \kappa_2^2) + \boldsymbol{\kappa_2} \times g_+(\lambda, \lambda_p^2, \kappa_2^2) \ ,$$

Crucial: here, $g_+ \propto \lambda_p^2 \implies \beta_{\lambda_p}\big|_{\lambda_p=0} = 0$, even if $\kappa_2 \neq 0$.

The explicit breaking by the kinetic terms is soft!

The kinetic term sign flip is crucial(!) for structure of β-functions e.g. @ two loops:





Explicitly checked up to three loops.

Comment on mass terms

$$\mathcal{L} \;=\; \kappa_1\left(\partial_\mu\phi_1\right)^2+\kappa_2\left(\partial_\mu\phi_2\right)^2+\boldsymbol{m_1}\phi_1^2+\boldsymbol{m_2}\phi_2^2+\lambda\left(\phi_1^4+\phi_2^4\right)+2\lambda_p\,\phi_1^2\phi_2^2\;.$$

- Goofy transformations unavoidably act non-trivially on mass terms.
- For w: $(m_1^2, m_2^2) \mapsto (m_1^2, -m_2^2)$

$\Rightarrow$ In the goofy-symmetric parameter space (and $\kappa_2 \neq 0$), the situation $m_1 \neq 0$, $m_2 = 0$ is stable under quantum corrections *to all order*!
(This is easy to digest here, as $\lambda_p = 0$ is a ϕ_1-ϕ_2-*decoupling* regime, but it holds more generally)

Goofy trafos can control mass parameters and scalar portals
$\curvearrowright$ give rise to radiatively stable mass gaps.

$\Rightarrow$ Goofy trafos can be used to address the EW hierarchy problem. [AT 2505.00099]

- Goofy Higgs & $\exists$Yukawas $\Rightarrow$ Goofy fermion trafos [de Boer, Goertz, Incrocci '25], [AT 2508.02646]
  $\Rightarrow$ Connection to flavor & CP.

In context of NHDM's, see [Kuncinas, Osland, Rebelo 2512.07657], [Ferreira 2604.07458], [de Medeiros Varzielas, Kuncinas 2608.05304]

More new goofy transformations in 2HDM

[AT '25]

Explicit action of flavor-/CP-type global-sign-flipping **Goofy** transformations

$$\vec{\Phi} \equiv \left(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*\right)^{\mathrm{T}} , \qquad \vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & -S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ -X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Goofy trafo.	parameter relations m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	accidental regular sym.
$\mathcal{P}_G$ $(\equiv \mathbb{1} \oplus -\mathbb{1})$	0	0	0								–
$\mathbb{Z}_{2,G}$ $(\equiv \sigma_3 \oplus -\sigma_3)$	0	0							0	0	–
$\mathrm{CP1}_G$	0	0	$-{m_{12}^2}^*$					λ_5^*	λ_6^*	λ_7^*	–
$\mathrm{CP2}_G$ (FGOO)		$-m_{11}^2$			λ_1					$-\lambda_6$	–
$\mathrm{U}(1)_G$	0	0	0					0	0	0	$\mathrm{U}(1)$
$\mathrm{CP3}_G$	0	0	$\boldsymbol{-{m_{12}^2}^*}$		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$	0	0	$\mathrm{U}(1)$
$\mathrm{SU}(2)_G$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0	$\mathrm{SU}(2)$
$\mathrm{CP2}_G^{\mathrm{soft}}$					λ_1					$-\lambda_6$	–

Parameter relations are RG stable to all orders in scalar and gauge quantum corrections.

These are *genuinely new* all-order goofy RG fixed points. (FGOO: $\mathrm{CP2}_G$ + all regular).

Conclusions

- Symmetry of β functions $\geq$ symmetry of action. [AT '25], [De Boer, AT 2603.12318]
  $\Rightarrow$ Non-perturbative Out-based computation of RG fixed points.
- Existence of an Out is a sufficient condition for an RG fixed point.
- *Goofy* transformations, by definition, do not leave invariant the kinetic terms.
- It is *mandatory* to include goofy transformations to understand *all* (partial) RG fixed points [RG fixed hyperplanes] of *any* QFT. [AT '25], [De Boer, AT 2603.12318]
- Explicit breaking of goofy trafos in kinetic terms **can** be soft, and parameter relations enforced by goofy trafos **can** be stable to all orders in RG evolution.
- Many of the most important puzzles in our theoretical understanding of Nature can be related to goofy transformations (e.g. EW hierarchy, flavor, ...)
- Parameter regions of *exact* goofy symmetry ($\kappa_i = 0$) are points where a QFT dynamically approaches a quasi-classical regime for **some** of the fields.
  $\rightarrow$ "dynamical classicalization"
- There are many goofy avenues worth exploring... (scaling is an Out!)

...I cannot think about QFT *without* goofy transformations anymore.

Thank You!

Image credits: PNGaaa.com, W a l t D i s n e y. ”Hawaiian Holiday” 1937

Backup slides

The first ever Goofy transformation

2HDM: Scalar doublets $\Phi_{a=1,2}(x)$ in $(\mathbf{2}, -1/2)$ of $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$.

The invariant scalar potential conventionally written as

$m_{11,22},\ \lambda_{1,2,3,4} \in \mathbb{R}$
$m_{12},\ \lambda_{5,6,7} \in \mathbb{C}$

$$
\begin{aligned}
V(\Phi^*, \Phi) \; = \; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& - \left\{ m_{12}^2 \Phi_1^\dagger \Phi_2 \right\} + \left\{ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 \right\} + \text{h.c.}.
\end{aligned}
$$

Community agreed, **all possible** (exact global) symmetries of 2HDM were known:
[Ivanov '06, '07], [Ferreira, Haber, Maniatis, Nachtmann, Silva '11], [Deshpande, Ma '78], [Ginzburg, Krawczyk '05], [Nishi '11], [Pilaftsis '12], ...

$$
\mathrm{CP1}\,, \quad \mathbb{Z}_2\,, \quad \mathrm{U}(1)\,, \quad \mathrm{CP2}\,, \quad \mathrm{CP3}\,, \quad \mathrm{SU}(2)\,.
$$

However, **FGOO**≡[Ferreira, Grzadkowski, Ogreid, Osland '23] found the new relations

$$
m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,.
$$

- These are renormalization group (RG) stable *to all orders*.
- Do **not** correspond to any of the known *regular* 2HDM symmetries.

The first ever Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring $\underline{\text{invariance of } V(\Phi^*, \Phi)}$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\Phi_1 \mapsto -\Phi_2^*\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \Phi_1^* \mapsto \Phi_2\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

The first ever Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\Phi_1 \mapsto -\Phi_2^*\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \Phi_1^* \mapsto \Phi_2\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

The first ever Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\underline{\boldsymbol{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\boldsymbol{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

The first ever Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\underline{\boldsymbol{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\boldsymbol{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

Very important because: Consider **canonical** (gauge-)kinetic terms

$$\mathcal{K} \;=\; (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)\;\;.$$

Applied to the **canonical** (gauge-)kinetic terms, this transformation maps

$$\mathcal{K} \;\mapsto\; -\mathcal{K}\,,$$

If imposed as an exact symmetry this would enforce $\mathcal{K} = 0$.

The first ever Goofy transformation

The relations $\quad m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \qquad (1)$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:
[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\underline{\boldsymbol{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\boldsymbol{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

Very important because: Consider **canonical** (gauge-)kinetic terms

$$\mathcal{K} \;=\; (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)\;\;.$$

Applied to the **canonical** (gauge-)kinetic terms, this transformation maps

$$\mathcal{K} \;\mapsto\; -\mathcal{K}\,,$$

If imposed as an exact symmetry this would enforce $\mathcal{K} = 0$.

$\Rightarrow$ This transformation cannot be an *exact* symmetry in a *dynamical* theory ($\mathcal{K} \neq 0$).
But how can relations (1) then be (RG) stable under quantum corrections?

Applications of Goofy transformations

- An entirely new class of possible transformations and associated RG stable fixed points have been missed so far in *all* QFTs, all models, ...
- Bare scalar mass terms $\phi^\dagger\phi$ explicitly break some goofy transformations (non-zero mass gaps are possible).
  $\Rightarrow$ Goofy symmetries can be instrumental in solving EW hierarchy problem.
  [AT '25](x2), [de Boer, Goertz, Incrocci '25]
- Imposing goofy trafo on SM Higgs does require goofy trafo of Fermion fields and reveals connection to flavor and CP violation. [AT 2508.02646]
- Goofy transformations can constrain non-canonical kinetic terms.

  In particular, the non-trivial Kähler potential of SUSY theories.
  see e.g. [Hiller, Schmaltz '01,'02]

  $\Rightarrow$ Possibility to remove a major roadblock for predictivity of many classes of flavor models (discrete symmetries, modular symmetries, ...)
  [Chen, Fallbacher, (Omura), Ratz, Staudt '12,'13], [Chen, Ramos-Sánchez, Ratz '19]

“Dynamical classicalization”

- If goofy relations are imposed only on V, the “couplings” of the kinetic terms (wave function renormalization coefficients) $\mathcal{K}_{ij}$ still run under RG.
- In this case, vanishing kinetic term(s) are RG fixed points (symmetry is enhanced there).
- Approaching the goofy points in RG flow, the theory approaches a regime where one or more of the fields become non-propagating
  “quasi-classical background” or “auxiliary” fields.

$\Rightarrow$ RG evolution *dynamically* approaches a quasi-classical regime!

- To fully explore this regime, and systematically explore the all-order constraints on WFR (anomalous dimensions), RGEs should be formulated starting from the most general possible basis.

$\curvearrowright$ Could track effect of goofy trafos on $\mathcal{K}_{ij}$ and get all-order constraints on WFR (anomalous dimensions), just as for the other couplings.

(supposedly, physically this should show up as a strong coupling regime in other couplings).

Comments on recent discussions

- **Spurion approach:** Our approach to constrain functional form of β functions is essentially the same as “spurion approach”. [Pilaftsis 2605.18341]

  BUT: Our treatment with Outs explains *why/how* the spurion approach works, and it allows to go beyond classic works, e.g. by explicitly including WFR $\mathcal{K}_{ij}$.
- **Goofy RG stability:** My explanation of goofy-stable RG fixed points does **not** require to **add** anything (fields, assumptions, requirements, ...). It is merely an explanation / understanding of the RG stability of goofy (and non-goofy) RG fixed parameter regions (using only the original theory).
- There is **no need** for a “wrong-signed” kinetic term (no ghosts or whatever...).
- **“Reduction of couplings”:** [Zimmermann ’85],...,[May, Mondragon, Patellis, Zoupanos ’23]

  RofC. is based on imposing assumptions on coupling regions. Looking at outer automorphisms is a way to find *good* relations that can be **all-order** stable (in contrast to ad-hoc assumptions which are not stable).

What is an outer automorphism?

Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathsf{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathsf{id}, \mathsf{a}, \mathsf{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \left(\text{think: } \mathsf{u}\,\mathsf{a}\,\mathsf{u}^{-1} = \mathsf{a}^2\right)$$

$\boldsymbol{Z_3}$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	ω	ω^2
$\mathbf{1}''$	1	ω^2	ω

$(\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3})$

What is an outer automorphism?

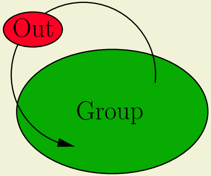
Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathsf{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathsf{id}, \mathsf{a}, \mathsf{a}^2\}$.
- Outer automorphism group (“Out”) of $\mathbb{Z}_3$: generated by

$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \big(\text{think: } \mathsf{u\,a\,u}^{-1} = \mathsf{a}^2\big)$$

$\boldsymbol{Z_3}$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1'}$	1	ω	ω^2
$\mathbf{1''}$	1	ω^2	ω

$(\omega := e^{2\pi\,i/3})$



What is an outer automorphism?

Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathsf{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathsf{id}, \mathsf{a}, \mathsf{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

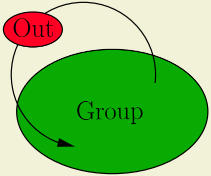
$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \left(\text{think: } \mathsf{u}\,\mathsf{a}\,\mathsf{u}^{-1} \;=\; \mathsf{a}^2\right)$$

Abstract: Out is a reshuffling of symmetry elements.

In words: Out is a **"symmetry of the symmetry"**.

$\boldsymbol{Z_3}$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1'}$	1	ω	ω^2
$\mathbf{1''}$	1	ω^2	ω

$(\omega := e^{2\pi\,\mathrm{i}/3})$



What is an outer automorphism?

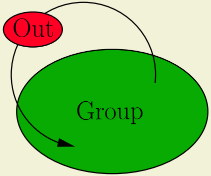
Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathrm{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathrm{id}, \mathsf{a}, \mathsf{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \left(\text{think: } \mathsf{u}\,\mathsf{a}\,\mathsf{u}^{-1} \;=\; \mathsf{a}^2\right)$$

$\boldsymbol{Z_3}$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	ω	ω^2
$\mathbf{1}''$	1	ω^2	ω

$(\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3})$



Abstract: Out is a reshuffling of symmetry elements.

In words: Out is a **"symmetry of the symmetry"**.

Concrete: Out is a 1:1 mapping of representations $\boldsymbol{r} \mapsto \boldsymbol{r}'$.
Comes with a transformation matrix U, which is given by

$$\boxed{U\rho_{\boldsymbol{r}'}(\mathrm{g})U^{-1} = \quad \rho_{\boldsymbol{r}}(u(\mathrm{g}))\;, \qquad \forall \mathrm{g} \in G\;.}$$

(consistency condition)

[Fallbacher, AT, '15]
[Holthausen, Lindner, Schmidt, '13]

- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$.
- $u\;:\;g \mapsto u(g)$: **outer** automorphism.
- U only unique up to inner automorphisms and multiplication by elements of group center.

What is an outer automorphism?

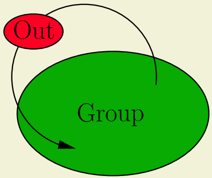
Example: $\mathbb{Z}_3$ symmetry, generated by $\mathrm{a}^3 = \mathrm{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathrm{id}, \mathrm{a}, \mathrm{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

$$u(\mathrm{a}) : \mathrm{a} \mapsto \mathrm{a}^2 . \quad \left(\text{think: } \mathrm{u}\,\mathrm{a}\,\mathrm{u}^{-1} = \mathrm{a}^2\right)$$

Z_3	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	ω	ω^2
$\mathbf{1}''$	1	ω^2	ω

$(\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3})$



Abstract: Out is a reshuffling of symmetry elements.

In words: Out is a **"symmetry of the symmetry"**.

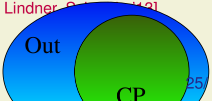
Concrete: Out is a 1:1 mapping of representations $\boldsymbol{r} \mapsto \boldsymbol{r}'$.
Comes with a transformation matrix U, which is given by

$$U \rho_{\boldsymbol{r}'}(\mathrm{g}) U^{-1} = \rho_{\boldsymbol{r}}(u(\mathrm{g})) \,, \qquad \forall \mathrm{g} \in G \,.$$

(consistency c

E.g.: Physical CP trafo
is a special case of this!
$\boldsymbol{r} \mapsto \boldsymbol{r}' = \boldsymbol{r}^*$

[Fallbacher, AT, '15]
sen, Lindner,



- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$.
- $u : g \mapsto u(g)$: **outer** automorphism.
- U only unique up to inner automorphisms and multiplication by elements of group center.

2HDM – Regular and global-sign-flipping goofy

Explicit action of global **regular** flavor- and CP-type transformations: [AT '25]

$$\vec{\Phi} \equiv \left(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*\right)^{\mathrm{T}} , \qquad \vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Flavor- and CP-type **Goofy versions** of these transformations:

$$\vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & -S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ -X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Note: all of these are "global-sign-flipping goofy transformations" $\mathcal{K} \mapsto -\mathcal{K}$.

Explicit choices for matrix generators of 2HDM transformations (ξ, ψ, $\theta \in \mathbb{R}$)

$$\mathbb{Z}_{2,(G)} : \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \sigma_3 , \qquad \mathrm{CP1}_{(G)} : \quad X = \mathbb{1}_2 ,$$

$$\mathrm{U(1)}_{(G)} : \quad S = \begin{pmatrix} e^{-i\xi} & 0 \\ 0 & e^{i\xi} \end{pmatrix} , \qquad \mathrm{CP2}_{(G)} : \quad X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv \varepsilon ,$$

$$\mathrm{SU(2)}_{(G)} : \quad S = \begin{pmatrix} e^{-i\xi}\cos\theta & -e^{-i\psi}\sin\theta \\ e^{i\psi}\sin\theta & e^{i\xi}\cos\theta \end{pmatrix} , \qquad \mathrm{CP3}_{(G)} : \quad X = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} .$$

2HDM – Relative-sign-flipping goofy transformations

We can also construct transformations that act with $\kappa_1 = -\kappa_2 = \pm 1$.
Action of goofy transformations with relative sign flip of the 2HDM kinetic terms in flavor- or CP-type subvariants:

$$\vec{\Phi} \mapsto \begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & B^* \end{pmatrix} \vec{\Phi}\,, \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & C \\ D^* & \mathbf{0} \end{pmatrix} \vec{\Phi}\,.$$

With examples of explicit matrix generators given by

$$\begin{aligned}
&\mathbb{Z}^-_{2,G}: && A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ B^* = \mathbb{1}, && C = D^* = 0\,, \\
&\mathrm{CP4}^-_G: && A = B^* = 0, && C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ D^* = \mathbb{1}\,. \\
&\mathbb{Z}^-_{4,G}: && A = B^* = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, && C = D^* = 0\,.
\end{aligned}$$

It turns out that none of these transformations is radiatively stable.
Reason: *The relative sign of the kinetic terms does enter the β functions of the 2HDM.*

RG stability of original FGOO transformation

FGOO 2HDM parameter relations: $m_{11}^2 = -m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7$.

- Consider e.g. $\beta_{\lambda_1-\lambda_2}$. Under FGOO trafo, must trafo like $(\lambda_1 - \lambda_2)$, i.e. as a $\mathbf{1}'$.
- Other $\mathbf{1}'$ covariants are $(\lambda_6 + \lambda_7)$ and κ_1, κ_2. ($m_{11}^2 + m_{22}^2$ has wrong mass dimension)

$\Rightarrow$ Beta function $\beta_{\lambda_1-\lambda_2}$ to all orders(!) schematically *can only* be given by

$$\beta_{\lambda_1-\lambda_2} = (\lambda_1 - \lambda_2)\, f_+(\lambda_i) + (\lambda_6 + \lambda_7)\, g_+(\lambda_i) + \overbrace{\kappa_1^n \kappa_2^m}^{n+m\,=\,\mathrm{odd}} h_+(\lambda_i)\,,$$

where f_+, g_+, h_+ are functions which are invariant under the trafo.

RG stability of original FGOO transformation

FGOO 2HDM parameter relations: $m_{11}^2 = -m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7$.

- Consider e.g. $\beta_{\lambda_1-\lambda_2}$. Under FGOO trafo, must trafo like $(\lambda_1 - \lambda_2)$, i.e. as a $\mathbf{1}'$.
- Other $\mathbf{1}'$ covariants are $(\lambda_6 + \lambda_7)$ and κ_1, κ_2. ($m_{11}^2 + m_{22}^2$ has wrong mass dimension)

$\Rightarrow$ Beta function $\beta_{\lambda_1-\lambda_2}$ to all orders(!) schematically <u>*can only*</u> be given by

$n+m=\text{odd}$

$$\beta_{\lambda_1-\lambda_2} = (\lambda_1 - \lambda_2)\, f_+(\lambda_i) + (\lambda_6 + \lambda_7)\, g_+(\lambda_i) + \kappa_1^n \kappa_2^m\, h_+(\lambda_i) \,,$$

where f_+, g_+, h_+ are functions which are invariant under the trafo.

- Turns out for 2HDM:
  The <u>global</u> sign of the kinetic term <u>does not</u> enter the β functions $\Leftrightarrow h_+ = 0$.
- Analogous arguments hold for $\beta_{m_{11}^2+m_{22}^2}$ and $\beta_{\lambda_6+\lambda_7}$.
- This shows that **if** FGOO relation is imposed, it is **not** violated in the RG flow.

$\curvearrowright$ Goofy symmetries are *explicitly* broken by $\mathcal{K} \neq 0$, but this breaking is **soft**!

Goofy transformations 101 – Goofy basis change

Consider multiplet of complex scalars $\phi_{a=1,\dots,N} \in \mathbb{C}$ and hermitean conjugate fields $\phi^\dagger_a$.

Standard "canonical" kinetic term

$$\mathcal{K}[\phi,\phi^\dagger] \;=\; \partial_\mu \phi^\dagger \, \partial^\mu \phi \;=\; \partial_\mu \phi^\dagger_a \, \delta^{ab} \, \partial^\mu \phi_b \;.$$

Field space metric $\kappa^{ab}\phi^\dagger_a\phi_b$ is only unity, $\kappa^{ab} = \delta^{ab}$, in the **canonical basis.**
In any other basis $\kappa^{ab} \neq \delta^{ab}$. Recall that ϕ and $\phi^\dagger$ are independent degrees of freedom. In particular, we can always do a general *passive* **field redefinition**

$$\phi' \equiv V\phi \;, \qquad \phi'^\dagger \equiv \phi^\dagger U^\dagger \;.$$

Regular field redefinition: $U = V$, Goofy redefinition: $U \neq V$.
The kinetic term then reads

$$\mathcal{K}[\phi,\phi^\dagger] = \mathcal{K}[V^\dagger\phi', \phi'^\dagger U] = \partial_\mu \phi'^\dagger \left(U V^\dagger\right) \partial^\mu \phi' \;, \quad \Rightarrow \; \kappa = U V^\dagger \;.$$

This is nothing else than rotating the kinetic term **to** / **from** a non-canonical basis.

Note: this discussion holds for any term $(\phi^\dagger\phi)_{\mathbf{1}_0}$, (gauge-covariant) derivatives play no role here.

Goofy transformations 101 – Goofy basis change

We actually do this all the time in going to the canonical basis in the first place!
In the most general basis, the kinetic terms are

$$\mathcal{K} = \kappa^{ab}\,(\partial_\mu\phi_a)^\dagger(\partial^\mu\phi_b) \qquad \text{with} \qquad \kappa^{ab} = \kappa^{ba\,*}\ .$$

Wave function renormalization coefficients $\kappa^{ab} \equiv$ “couplings” of kinetic term.
Any hermitean matrix κ can be written as

$$\kappa \;=\; U^\dagger \begin{pmatrix} k_1 & 0 & \\ 0 & k_2 & \\ & & \ddots \end{pmatrix} U\,, \qquad (U \text{ unitary, eigenvalues } k_a \in \mathbb{R})$$

U's can be absorbed in $\phi^\dagger$, ϕ by a *regular* field redefinition.
Canonical diagonal entries are obtained by subsequent rescaling (no sum)

$$\phi_a = \sqrt{k_a}\,\phi'_a\,, \qquad \text{and} \qquad \phi^\dagger_a = \sqrt{k_a}\,\phi'^\dagger_a\ .$$

Note: if any one of the $k_a < 0$, this “rescaling” is a *goofy* transformation.

Goofy transformations $\mathbf{101}$ – Active goofy transformations

Goofy transformations as *active* transformations (we assume unitary A, and B)

$$\phi \mapsto A\phi\,, \qquad \phi^\dagger \mapsto \phi^\dagger B^\dagger\,.$$

For canonical kinetic term this corresponds to a mapping

$$\phi_a^\dagger\,\delta^{ab}\,\phi_b \mapsto \phi_a^\dagger (B^\dagger A)^{ab}\phi_b\,.$$

For the action to stay real valued, $(B^\dagger A)$ is required to be hermitean and unitary. Therefore, $(B^\dagger A)^2 = \mathbb{1}$ is of order 2. This means there is a basis in which

$$\sum_a \partial_\mu\phi_a^\dagger\,\partial^\mu\phi_a \quad \longmapsto \quad \sum_a \kappa_a\,\partial_\mu\phi_a^\dagger\,\partial^\mu\phi_a \qquad \text{with} \qquad \kappa_a = \pm 1 \quad \text{(uncorrelated signs)}\,.$$

In this basis goofy transformation corresponds to pure sign flips of kinetic terms.

Action of Outs on the action

Action $S = \int d^4x \, \mathcal{L}$, invariant under global symmetry G, fields ϕ_a transforming in irreps $\boldsymbol{r}_a$ of G. Matrix rep $\rho_{\boldsymbol{r}_i}(g)$. I.e. $\forall\, g \in G$

$$\phi_a \mapsto \rho_{\boldsymbol{r}_a}(g)\phi_a \; . \qquad \text{(no sum)}$$

Lagrangian in terms of G-invariant operators $\mathcal{O}_n[\phi_a]$ and couplings λ_n,

$$\mathcal{L}[\phi_a, \lambda_n] = \sum_n \lambda_n \mathcal{O}_n[\phi_a] \; .$$

For a general field redefinition $\tilde{\phi}_a := \mathcal{F}(\phi_a)$, consider

$$\mathcal{L}[\phi_a, \lambda_n] = \mathcal{L}[\mathcal{F}^{-1}(\tilde{\phi}_a), \lambda_n] =: \tilde{\mathcal{L}}[\tilde{\phi}_a, \lambda_n] = \sum_k \tilde{F}_k(\lambda_n)\tilde{\mathcal{O}}_k[\tilde{\phi}_a] \; .$$

In the last step we decomposed $\tilde{\mathcal{L}}$ in a similar way to $\mathcal{L}$ before. Crucially, only for Outs one has $\tilde{\phi}_a = U\phi'_a$, implying that under symmetry transformation the fields $\tilde{\phi}_a$ transforms as

$$\tilde{\phi}_a = U\phi'_a \mapsto U\rho_{\boldsymbol{r}'_a}(g)\phi'_a = \rho_{\boldsymbol{r}_a}(u(g))U\phi'_a = \rho_{\boldsymbol{r}_a}(u(g))\,\tilde{\phi}_a \; , \qquad \forall\, g \in G \; ,$$

i.e. in the $\rho_{\boldsymbol{r}_a}$ representations of a different group element. Since this has to hold for all group elements, the invariant operators in the decomposition of $\tilde{\mathcal{L}}$ are constrained in exactly the same way as the invariant operators in the decomposition of $\mathcal{L}$. Hence, for any operator $\tilde{\mathcal{O}}_k$ in (32) there must be an operator $\mathcal{O}_{n(k)}$ in (32) such that

$$\mathcal{O}_{n(k)}[\tilde{\phi}_a] = \tilde{\mathcal{O}}_k[\tilde{\phi}_a].$$

Therefore, we can write

$$\tilde{\mathcal{L}}[\tilde{\phi}_a, \lambda_n] = \sum_k \tilde{F}_k(\lambda_n)\tilde{\mathcal{O}}_k[\tilde{\phi}_a] = \sum_k \tilde{F}_k(\lambda_n)\mathcal{O}_{n(k)}[\tilde{\phi}_a] =: \sum_n F_n(\lambda_n)\mathcal{O}_n[\tilde{\phi}_a] = \mathcal{L}[\tilde{\phi}_a, F_n(\lambda_n)],$$

which implies that $\mathcal{L}[\phi_a, \lambda_n] = \mathcal{L}[\tilde{\phi}_a, F_n(\lambda)]$.