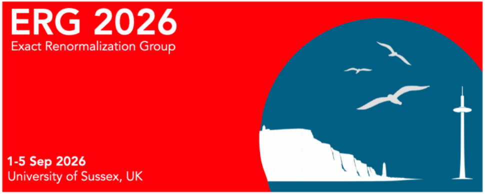

Wave interactions in Astrophysical* Plasmas

LAMPS

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4th September 2026

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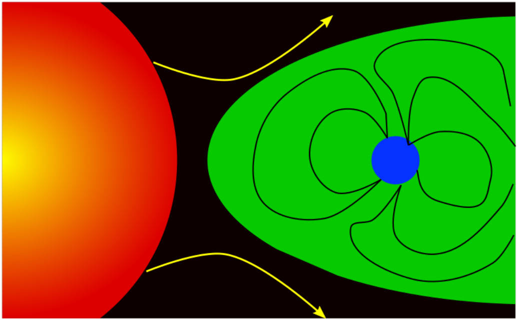

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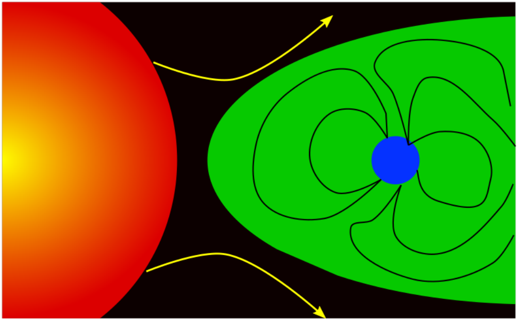

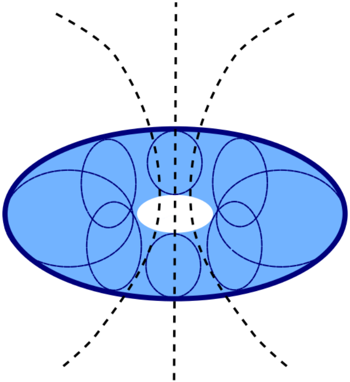

Fusion plasmas

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ex3: Goldreich Sridhar (GD)

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Anisotropy

ex4: Balescu's model

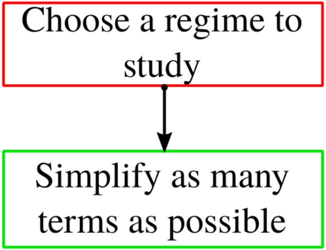
8 types of modes

In practice

In practice

Choose a regime to study

In practice



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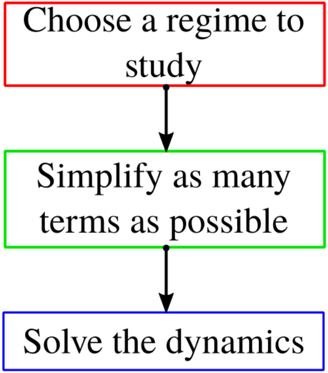
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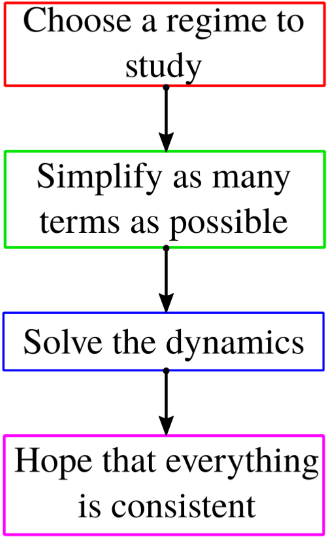
In practice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The bigger picture

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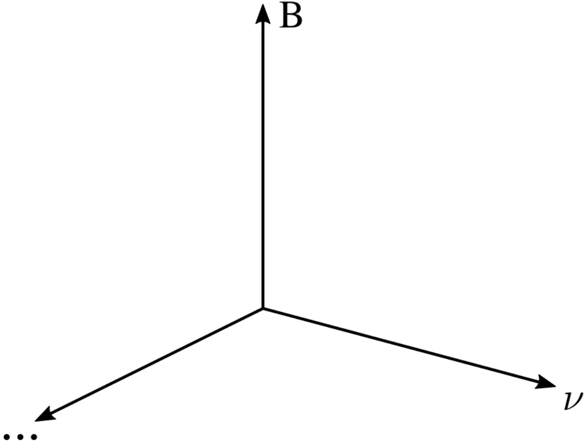
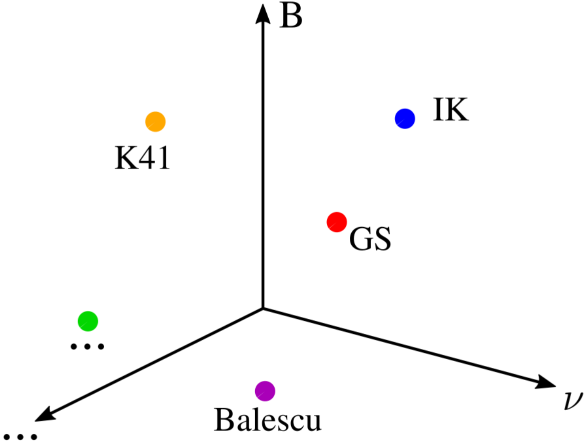


Parameter space

The bigger picture



Parameter space

How to build a dynamical model compatible with the various regimes?

The MHD equations

The MHD equations

$$\rho(\partial_t u_\alpha + u_\beta \partial_\beta u_\alpha) + \partial_\alpha P - \epsilon_{\alpha\beta\gamma} j_\beta B_\gamma - \nu \Delta u_\alpha = 0$$

Navier-Stokes

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Maxwell

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Maximal degree of symmetry

Contracting the equations

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Advection

Viscosity

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A little bit of generalisation...

$$\big(\partial_t + (\Sigma_1 \cdot \Phi)_\beta \partial_\beta\big)\Phi_\alpha + (\Psi_0)_\beta \partial_\beta (\Sigma_3 \cdot \Phi)_\alpha - \nu_+ \partial^2 \Phi_\alpha - \nu_- \partial^2 (\Sigma_1 \cdot \Phi)_\alpha + \partial_\alpha P = f_\alpha$$

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Application both to astrophysics and fusion plasmas

How multiple scaling regimes can be generated?

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$$\boxed{\text{The only possible scaling is K41}}$$

The tensorial structure does matter!

- Propagator inversion in the Φ sector:

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