Renormalizability and UV behaviour of 5D gauge theories

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[arXiv:2601.00453]

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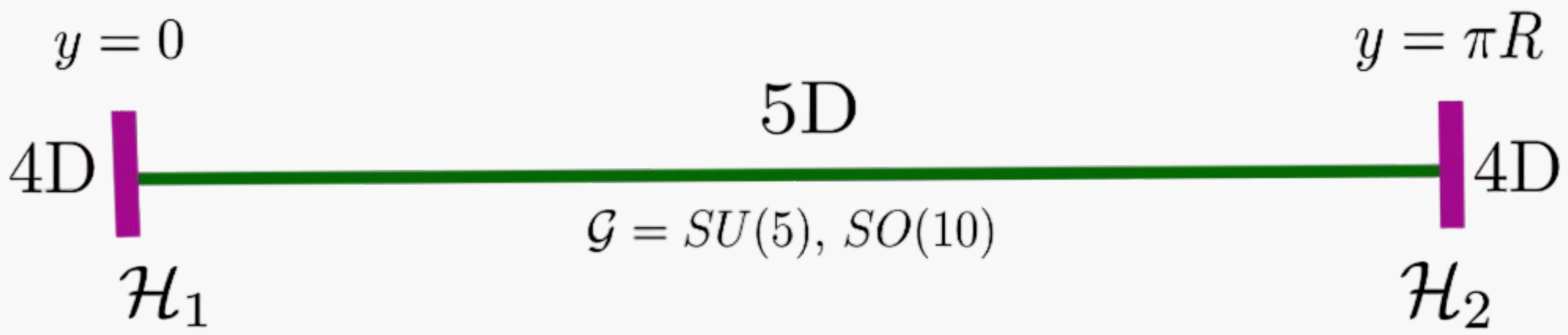
Outline

- Motivation: the asymptotic GUT example
- RGEs: from 4D to 5D
- Renormalization of 5D gauge theories (one loop analysis)
- Conclusions

Asymptotic GUTs

- **5D models**: one extra dimension [1] compactified on $K = S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$



- Symmetry is broken via orbifolding ($\mathcal{G} \to \mathcal{H}_i$ on each boundary [2])

$$\mathcal{G}_{4\mathrm{D}} \equiv \mathcal{H}_i \cap \mathcal{H}_j \supset \mathcal{G}_{\mathrm{SM}}$$

remnant 4D theory

[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)
[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Asymptotic GUTs

orbifold

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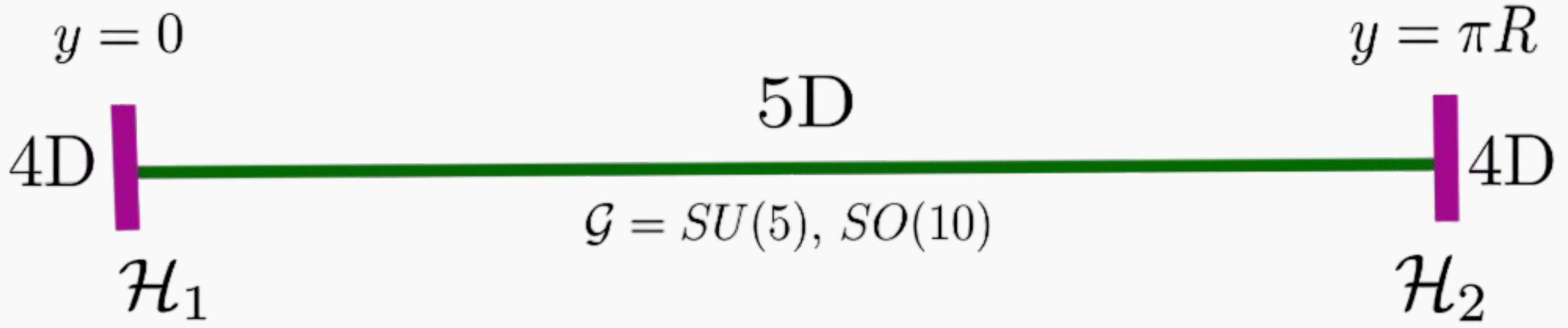

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Kaluza Klein decomposition

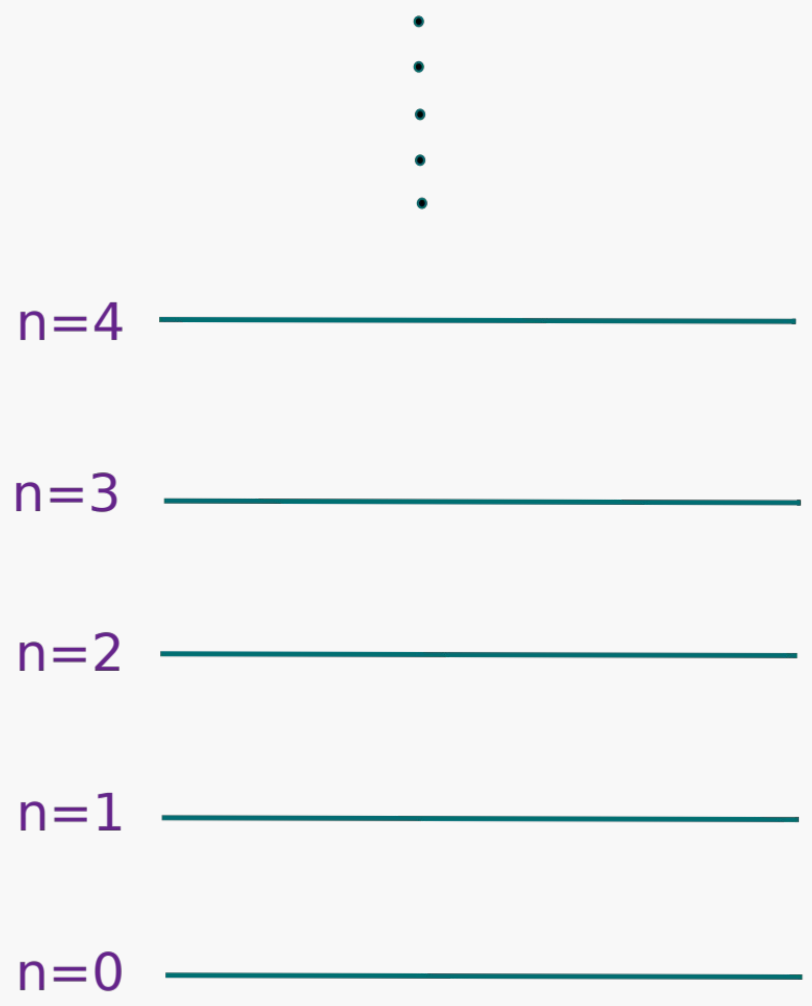
$$\Phi(x^\mu, y) = \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu)\cos(ny/R) + \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu)\sin(ny/R)$$

Any 5D field

$$\Phi_{5\mathrm{D}}$$

(scalar, fermion, gauge boson)

Infinite tower of 4D fields



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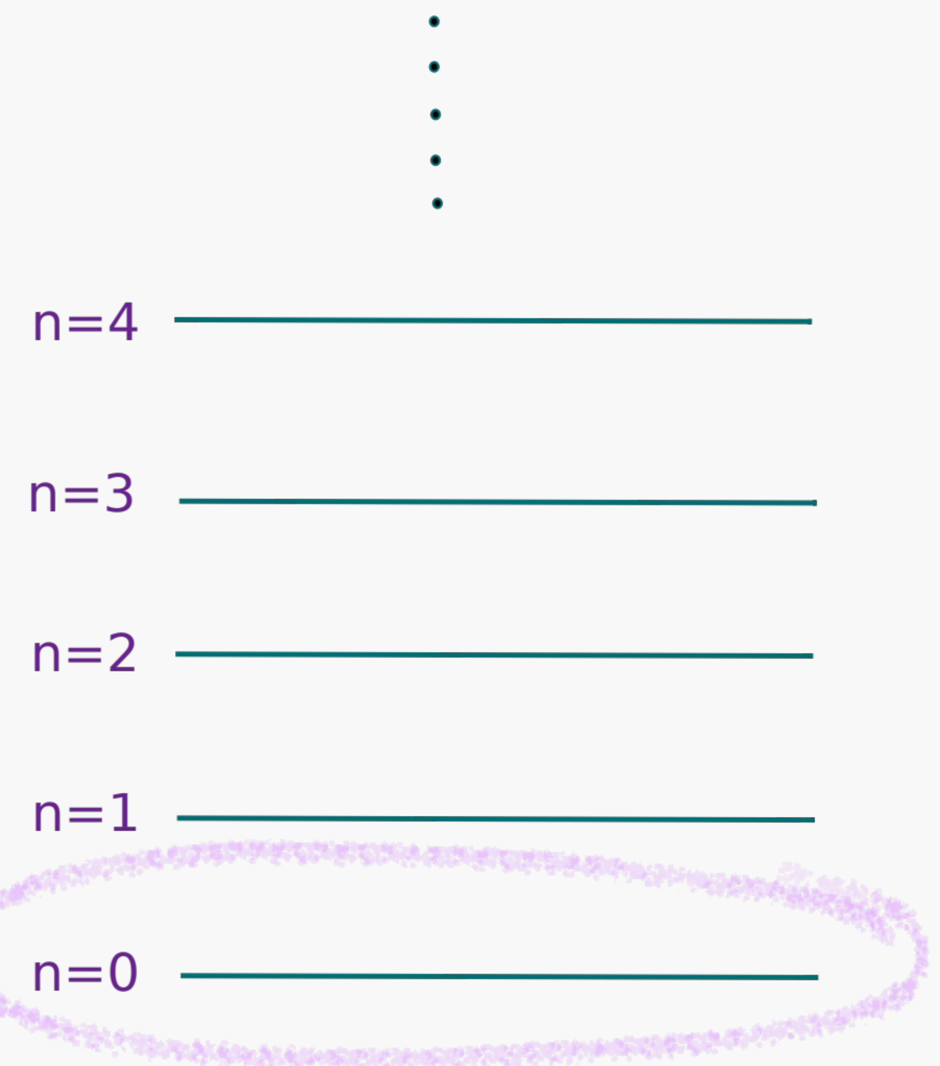


SM particles

Why?

- no proton decay: lower GUT scale
- less parameters/smaller representations
- hierarchy problem (?)

interesting playground for
beyond the Standard Model
physics

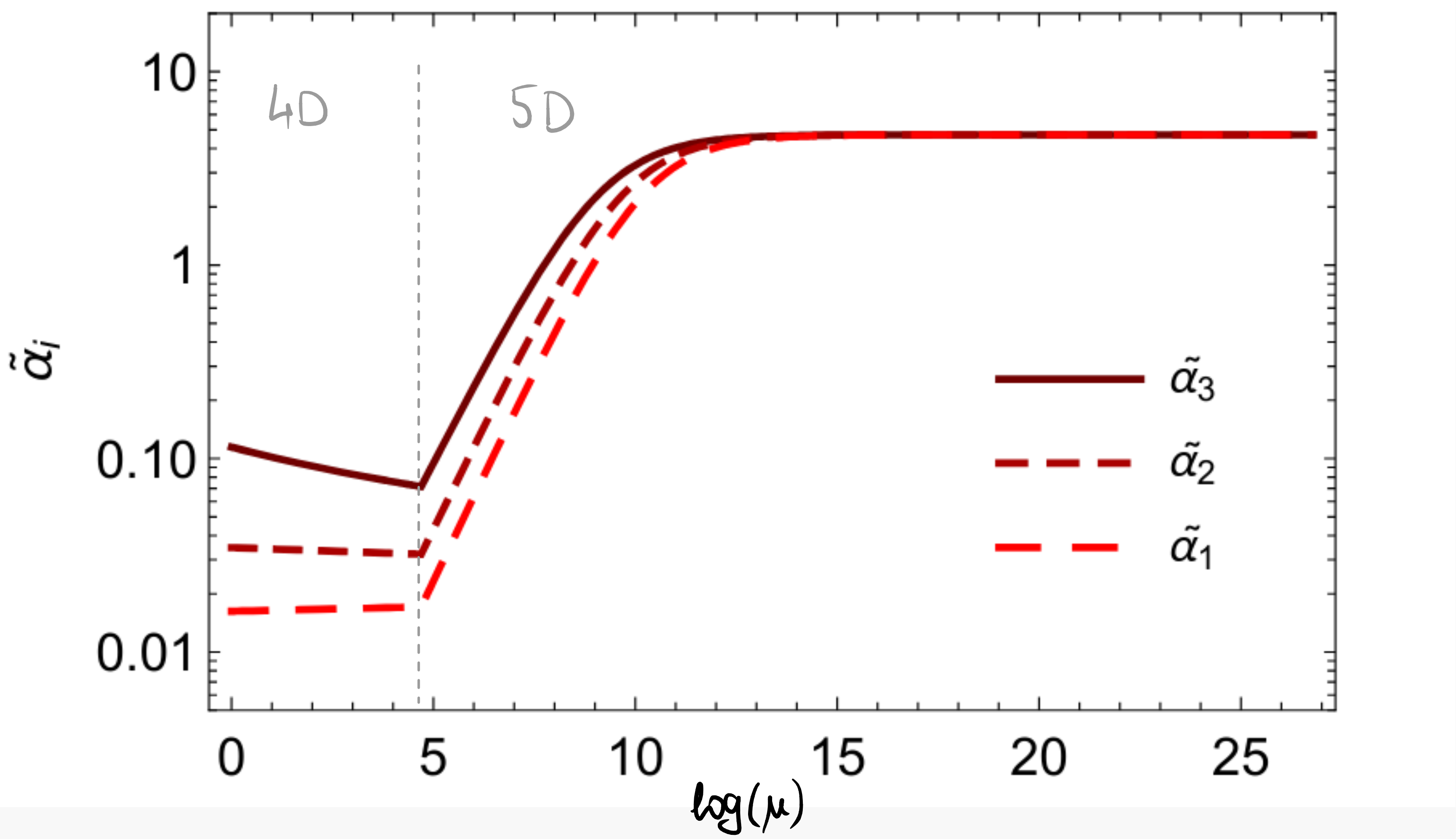
RGEs: from 4D to 5D

- 4D: logarithmic running $$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3$$

- 5D: power-law running [2]

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t) - 1) b^i_5 g_i^3$$

where $S(t) = \begin{cases} \mu\ \mathrm{R}{=}M_Z\ \mathrm{R}\ e^t, & \text{for } \mu \geq 1/R \\ 1, & \text{otherwise} \end{cases}$



G. Cacciapaglia ,et.al., 104, PRD 2021

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RGEs: from 4D to 5D

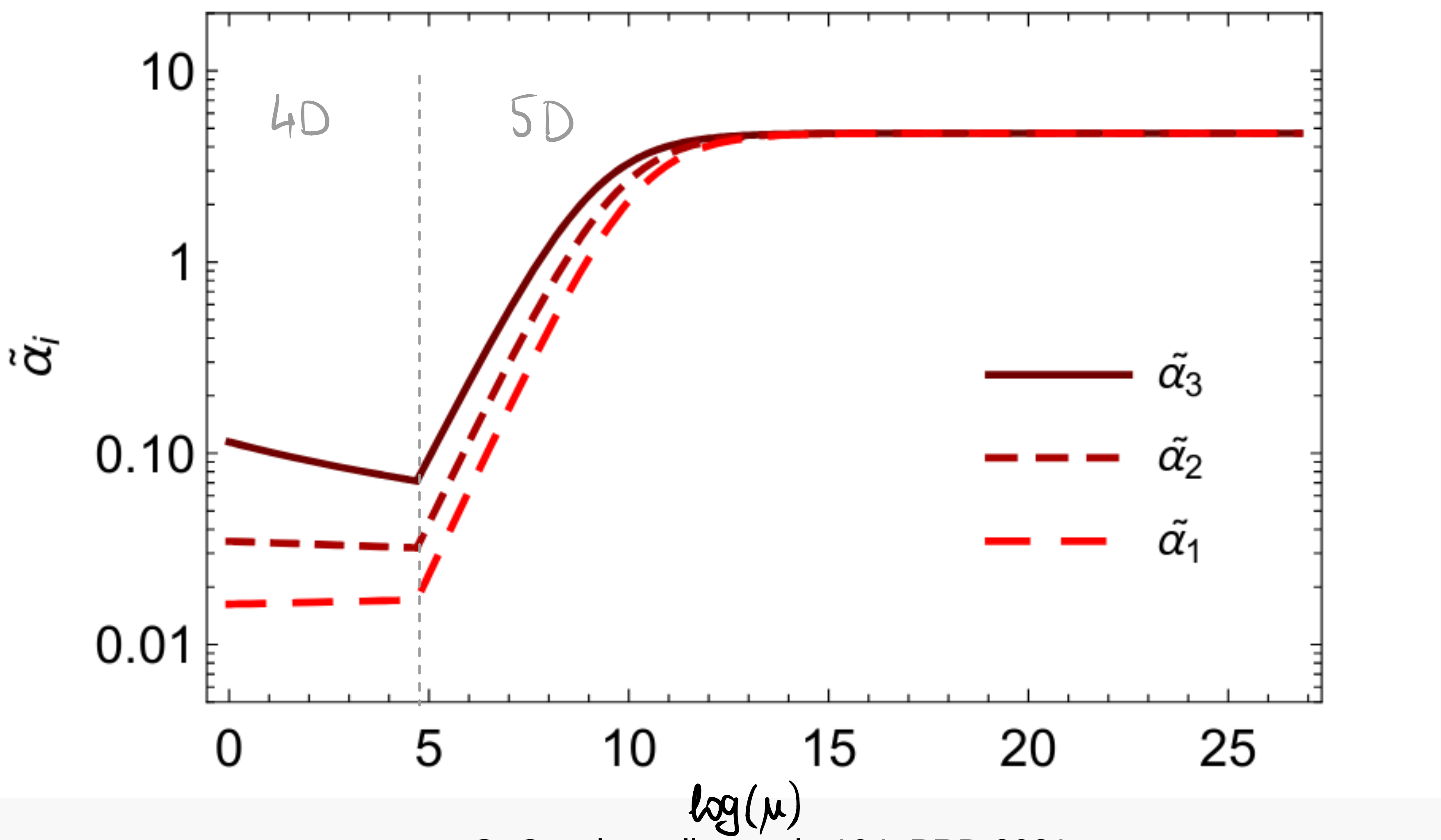
- 5D: couplings flow asymptotically towards a UV fixed point

It's existence $\Longleftrightarrow$ good behavior in the UV

- gauge couplings

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t)-1) b^i_5 g_i^3$$

Fixed point exists for $b^i_5 \leq 0$



G. Cacciapaglia ,et.al., 104, PRD 2021

RGEs: from 4D to 5D

- Yukawa couplings

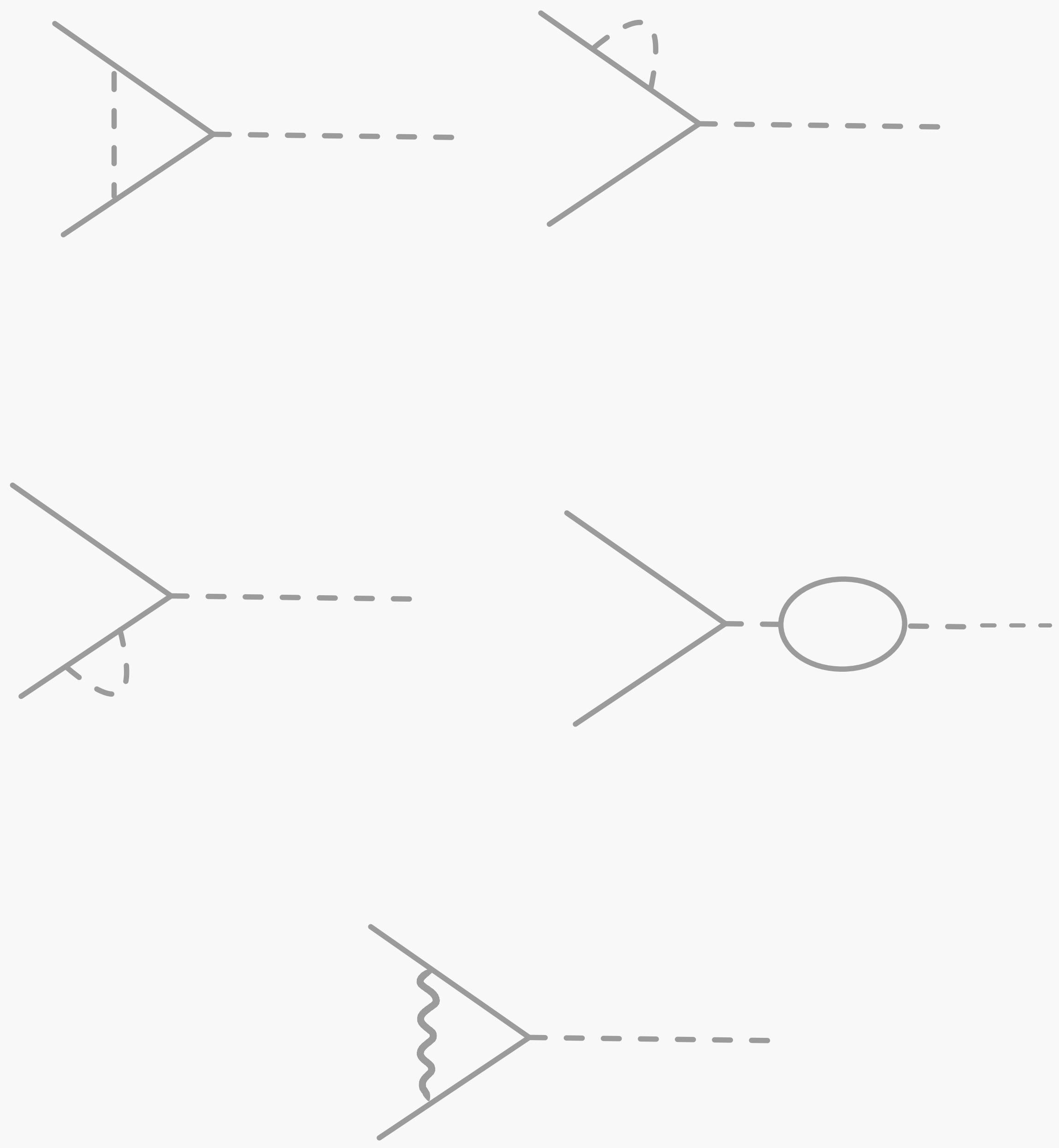
$$16\pi^2 \frac{dy}{dt} = \beta^y_{SM} + (S(t)-1)\beta^y_5$$

where the β-function is given by [3]

$$\beta_y = y(c_y y^2 - d_y g^2)$$

Fixed point $\tilde{\alpha}^*_y = \dfrac{d_y \tilde{\alpha}^*_g - 2\pi}{c_y}$ exists for $\mathrm{d}_y \tilde{\alpha}^*_g \geq 2\pi$

$$\tilde{\alpha}_{y(g)} = \alpha_{y(g)} \mu R$$



[3] G. Cacciapaglia, W. Isnard, R. Pasechnik, AP [arXiv:2601.00453]

RGEs: from 4D to 5D

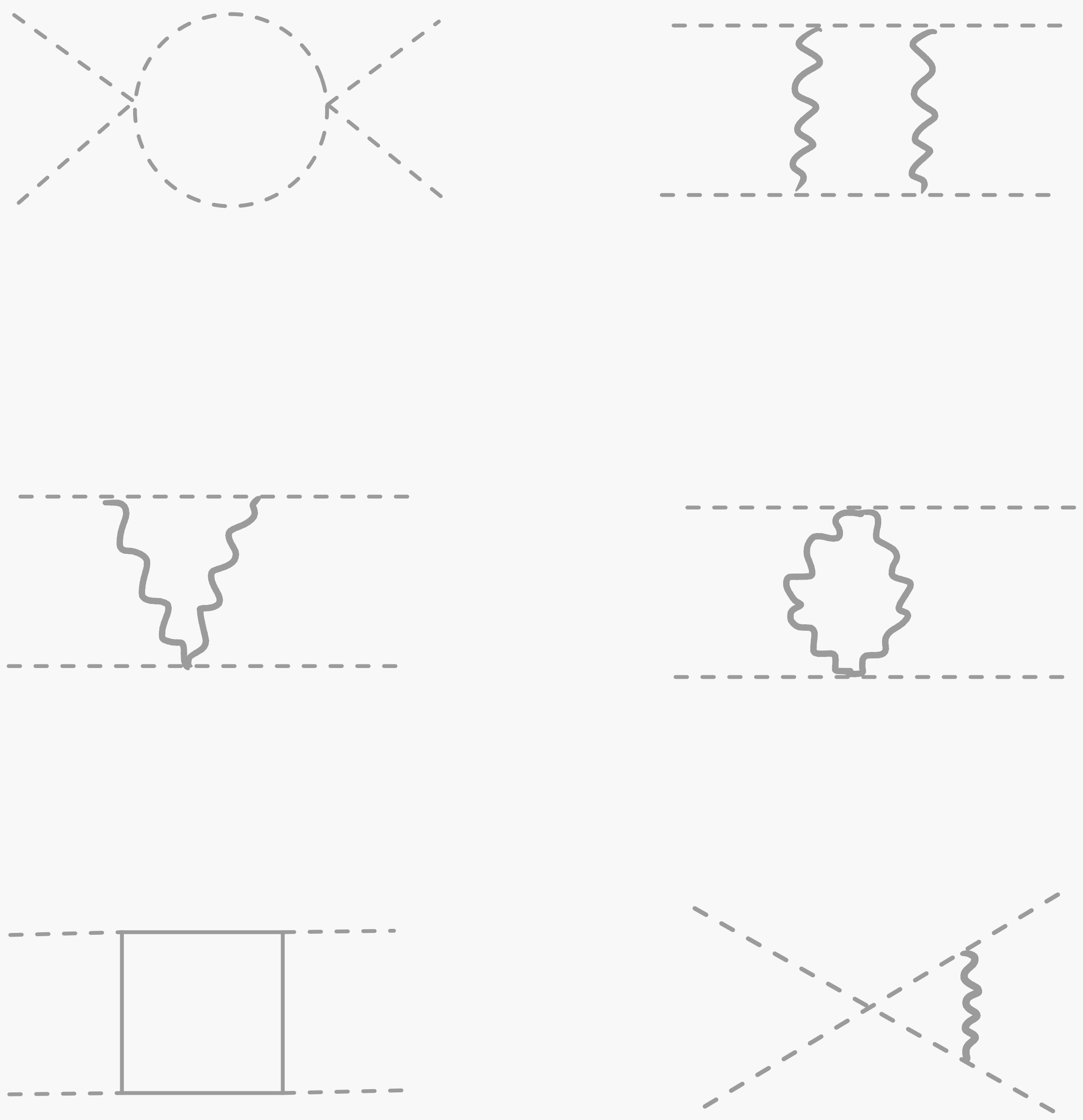
- Scalar couplings $\frac{1}{4!}\lambda\phi^4$

$$16\pi^2 \frac{d\lambda}{dt} = \beta^{\lambda}_{SM} + (S(t)-1)\beta^{\lambda}_{5}$$

where the β-function is given by [3]

$$\beta_\lambda = a_\lambda \lambda^2 - b_\lambda y^4 - c_\lambda \lambda g^2 + d_\lambda g^4 + e_\lambda \lambda y^2$$

Fixed point condition is more complicated...



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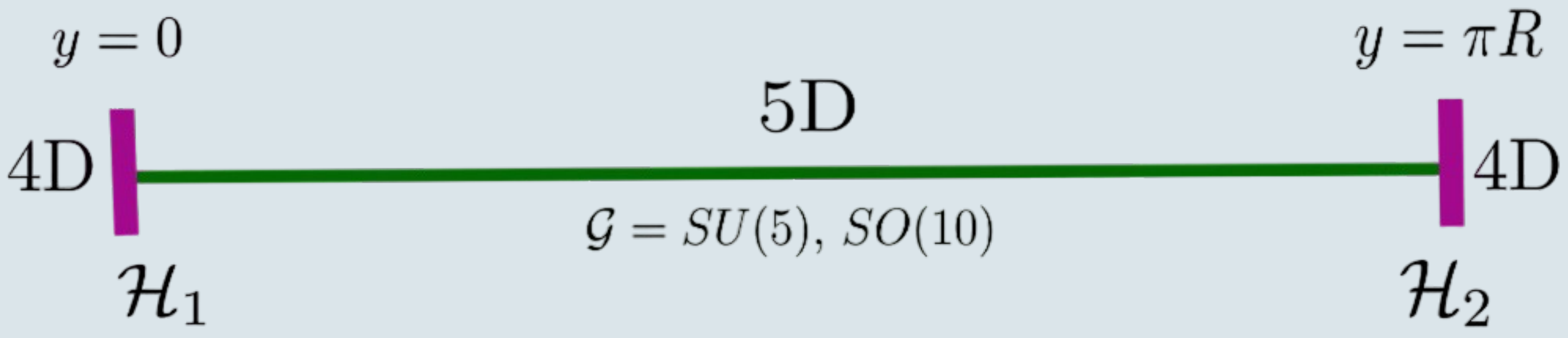
Assume a **theory** with couplings that flow asymptotically towards **fixed points**

+ the fixed points are perturbative

Can we **renormalize** the theory in the bulk and on the boundary?

(one loop)



Disclaimer!

What is a **renormalizable theory**?

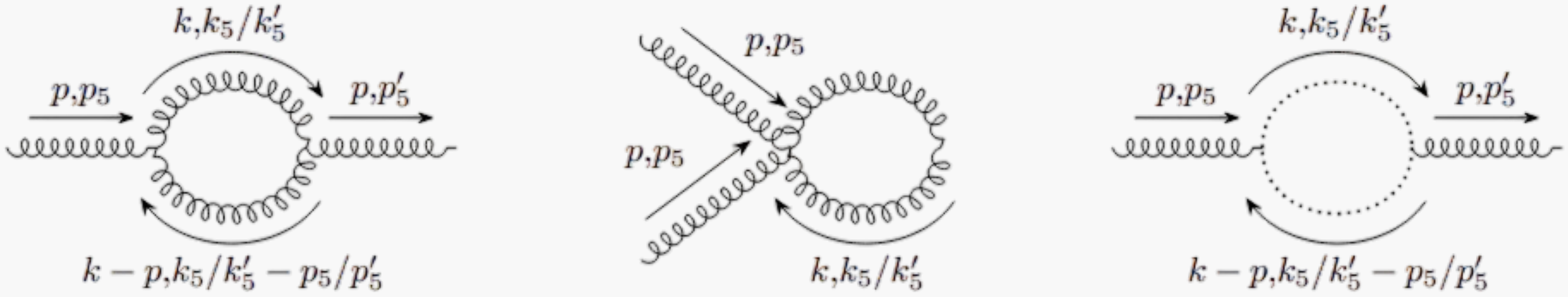
Couplings have perturbative **fixed points**	+	**Finite number of counterterms** to absorb the power law divergencies

Renormalization of 5D gauge theories

Common lore: **5D** gauge theories are **non-renormalizable** [4]

... but under certain conditions a fixed point exists → theories valid up to arbitrarily high energy scales [5]

Pure Yang Mills theory $$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F^{MN} F_{MN}$$



[4] H. Gies, Phys. Rev. D 68 (2003)

[5] T. Morris, JHEP 2005 (2005)

Renormalization of 5D gauge theories

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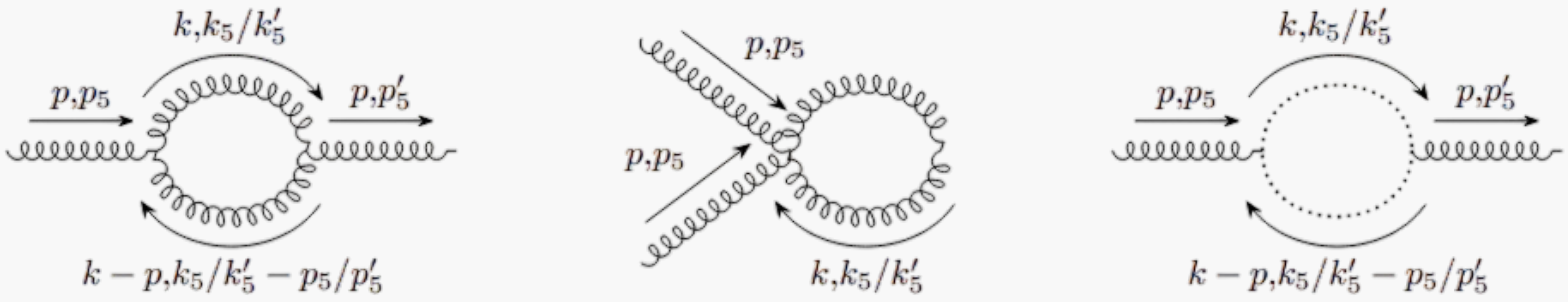


$$i\Sigma = \underbrace{[p_5 - \text{conserving terms}]}_{\text{bulk}} + \underbrace{[p_5 - \text{non-conserving terms}]}_{\text{boundary}}$$

Bulk: structure of divergencies same as in 4D (at one loop)

- Scaling changes from logarithmic 4D ($\sim \log \Lambda$) to 5D linear ($\sim \Lambda$)
- Same renormalization procedure as in 4D; **finite number of counterterms** are needed

Renormalization of 5D gauge theories

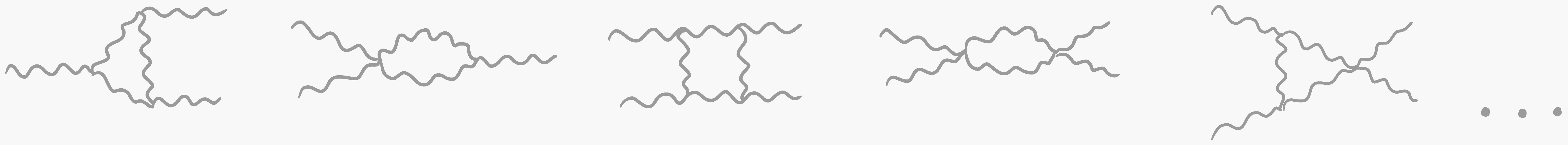
Pure Yang Mills theory $\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{MN} F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points
→ less straightforward than bulk

Two-point function: [3,6]

$$i\Sigma = \frac{g^2}{64\pi^2}\frac{1}{\epsilon}\left[(g_{\mu\nu} - p_\mu p_\nu)\left(\frac{11}{3} - (\xi - 1)\right) C(G) + g_{\mu\nu}\frac{p_5^2 + {p'_5}^2}{2}(4 + (\xi - 1))C(G)\right]$$

To reconstruct the full counterterm on the boundary we need **higher vertex corrections** as well (3-pt and 4-pt functions)



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[6] HC Cheng, et al., Phys. Rev. D, 66 (2002)

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{MN} F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points

- less straightforward than bulk
- divergencies can be absorbed by a **finite number of counterterms**
- **"magic gauge"** ($\xi = -3$): coefficients of the 2-,3- and 4-pt functions are the same

  localized counterterm can be unified into a single term $-\frac{1}{4} K \delta \left(F_{\mu\nu} F^{\mu\nu} \right)$

! Theory is renormalizable both in the bulk and on the boundary (at one loop)

(finite number of operators to cancel divergencies)

Renormalization of 5D gauge theories

Adding scalars and fermions

→ realistic models: go beyond pure Yang Mills [6,7]

$$\mathcal{L}_4 = -\frac{A}{4} F_{\mathcal{H}}^{\mu\nu} F_{\mathcal{H},\mu\nu} + B_1\; i\bar{\psi}\not{\mathcal{D}}\frac{1\pm\gamma_5}{2}\psi + B_2\left[(\partial_5\bar{\psi})\frac{1\pm\gamma_5}{2}\psi - \bar{\psi}\frac{1\mp\gamma_5}{2}(\partial_5\psi)\right] +$$
$$+C_1\;(\mathcal{D}_\mu\phi_+)^\dagger(\mathcal{D}^\mu\phi_+) + C_2\left[(\partial_5^2\phi_+^\dagger)\phi_+ + \phi_+^\dagger(\partial_5^2\phi_+)\right] + C_3\;(\partial_5\phi_-)^\dagger(\partial_5\phi_-) +$$
$$-m_{\rm loc}^2(\phi_+^\dagger\phi_+) - D\;(\phi_+^\dagger\phi_+)^2$$

Full set of boundary counterterms

[6] HC Cheng, et al., Phys. Rev. D, 66 (2002)

[7] H. Georgi, et al, Phys. Let. B 506 (2001)

Summary and conclusions

- **Naive power counting**: 5D gauge-Yukawa theory is **non-renormalizable**
- RGEs of couplings change in 5D: **power law running** (compared to 4D logarithmic)
- One loop analysis: finite number of counterterms are needed to absorb divergencies

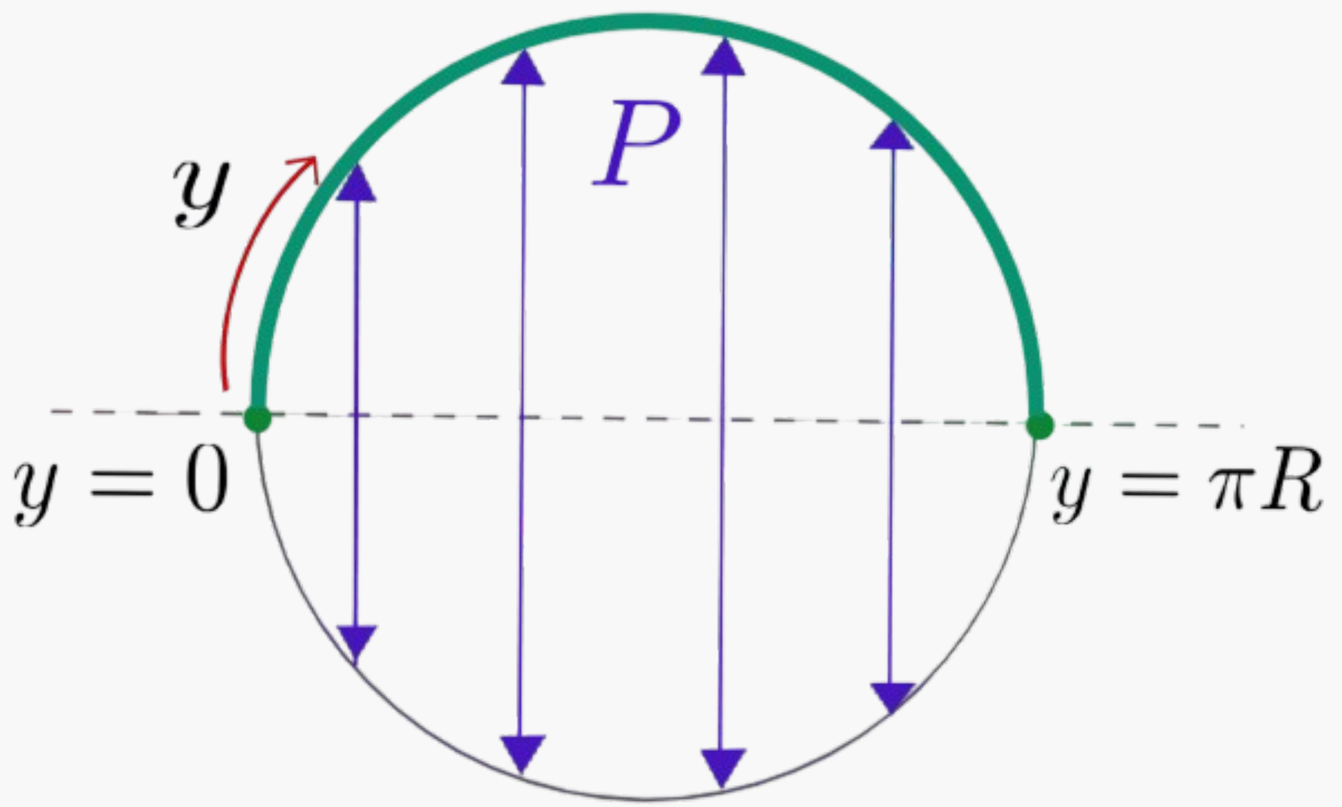
renormalizability

- Consistency requires the existence of UV fixed points: explicit conditions for their existence

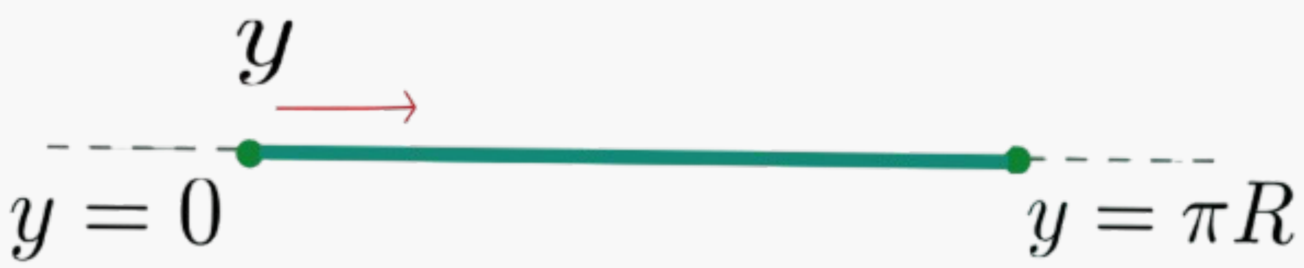
Next: understand higher loop behaviour

Backup slides

- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$.



- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$:



A generic d-dimensional loop in a gauge-Yukawa theory will look like:

$$I_d \sim \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{f(k^M, k^2)}{\Delta(k^2)} \; . \longrightarrow \boxed{I_d \sim \frac{\Omega_d}{(2\pi)^d} \int k^{d-1} \mathrm{d}k \; \frac{f}{\Delta(k^2)},}$$

$$f(k) \sim k^{2m}$$

$$\Delta(k^2) \sim k^{2n}$$

$n-m$	$d=$				Examples
	4	5	6	7	
1	Λ^2	Λ^3	Λ^4	Λ^5	Scalar masses
2	log	Λ	Λ^2	Λ^3	Coupling renormalization
3	finite	finite	log	Λ	Four-fermions, $(\Phi^\dagger\Phi)^3$
≥ 4	finite	finite	finite	finite	

Coefficients of the localized counterterms:

$$A = \frac{g_5^2}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \left[\frac{23}{3}(2C(\mathcal{H}) - C(\mathcal{G})) - \frac{1}{3}\left(\sum_{\text{even}} - \sum_{\text{odd}}\right) T(r_s)\right],$$

$$B_1 = \frac{1}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \left[7g_5^2 C(r) - \left(\sum_{\text{even}} - \sum_{\text{odd}}\right) y_5^2\right],$$

$$B_2 = \frac{1}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \left[g_5^2 C(r) - \left(\sum_{\text{even}} - \sum_{\text{odd}}\right) y_5^2\right],$$

$$C_1 = \frac{1}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \left[8g_5^2 T(r)\right],$$

$$C_2 = \frac{1}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \left[-2g_5^2 T(r) \mp \frac{\lambda_5}{2}\right],$$

$$C_3 = \frac{1}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \left[8g_5^2 T(r) \mp \frac{\lambda_5}{2}\right],$$