Critical dynamics of a scalar field near four spatial dimensions

Laura Batini
Institute for Theoretical Physics, ETH Zürich

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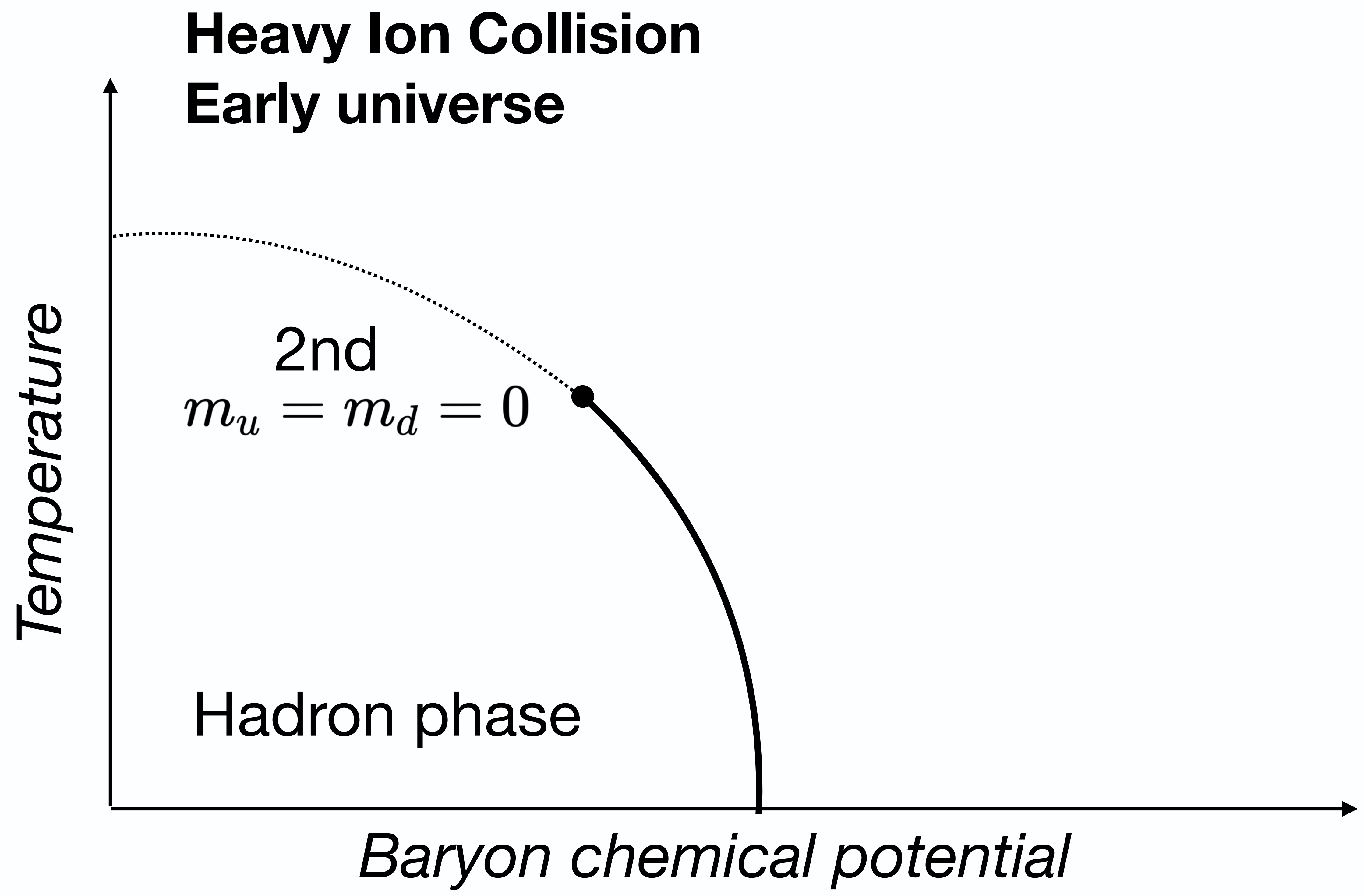
Based on: LB and E. Grossi *arXiv:2608.07292*

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

Motivation

- Dynamics of critical phenomena is a central problem across high-energy and condensed-matter physics

Florio, Grossi, Soloviev and Teaney, PRD **105** (2022) 054512
Rajagopal & Wilczek, Nucl. Phys. B **399** (1993) 395
Son & Stephanov, PRD **70** (2004) 056001



Critical dynamics

Approaching the critical point critical slow dynamics: $\tau \sim \xi^z$

Dynamic critical exponent

Correlation length

Critical dynamics

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Hohenberg, Halperin, *Rev. Mod. Phys.* **49**, 435 (1977)

Theory of dynamic critical phenomena

P. C. Hohenberg

Bell Laboratories, Murray Hill, New Jersey 07974
and Physik Department, Technische Universität München, 8046, Garching, W. Germany

B. I. Halperin*

Department of Physics, Harvard University, Cambridge, Mass. 02138

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Order parameter

Conserved quantities

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Slow variables for chiral phase transition

- Meson field — **Order parameter**
- Chiral charge
- Energy
- Momentum

$\left.\begin{array}{l}\text{Chiral charge}\\ \text{Energy}\\ \text{Momentum}\end{array}\right\}$ **Conserved quantities**

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Slow variables for chiral phase transition

- Meson field — **Order parameter**
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Chiral charge, Energy, Momentum: **Conserved quantities**

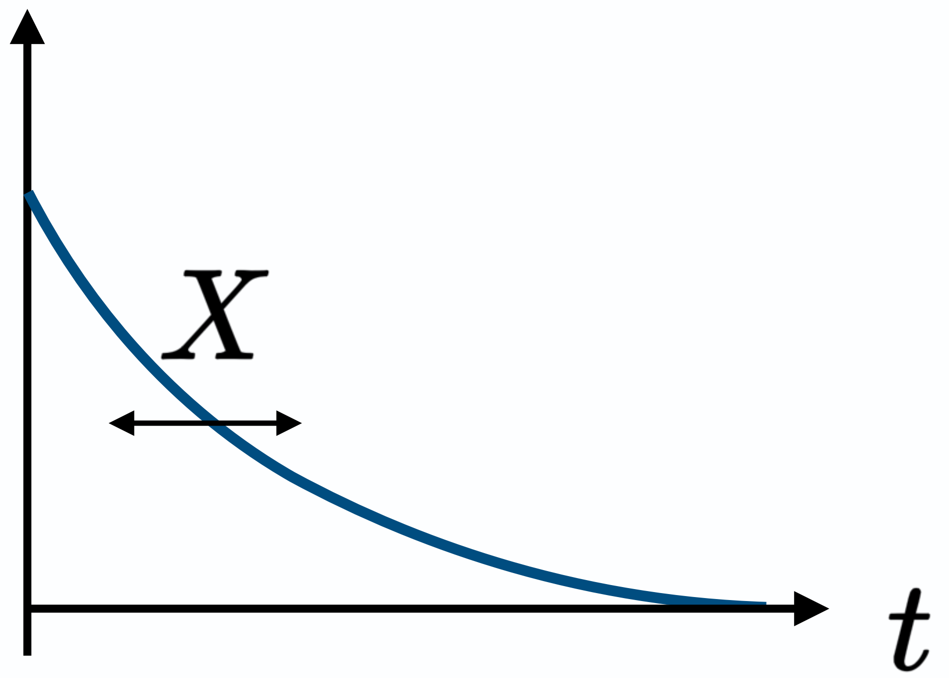
In this work: **Purely dissipative non-conserved order parameter (Model A)**

Critical fluctuations can relax or propagate

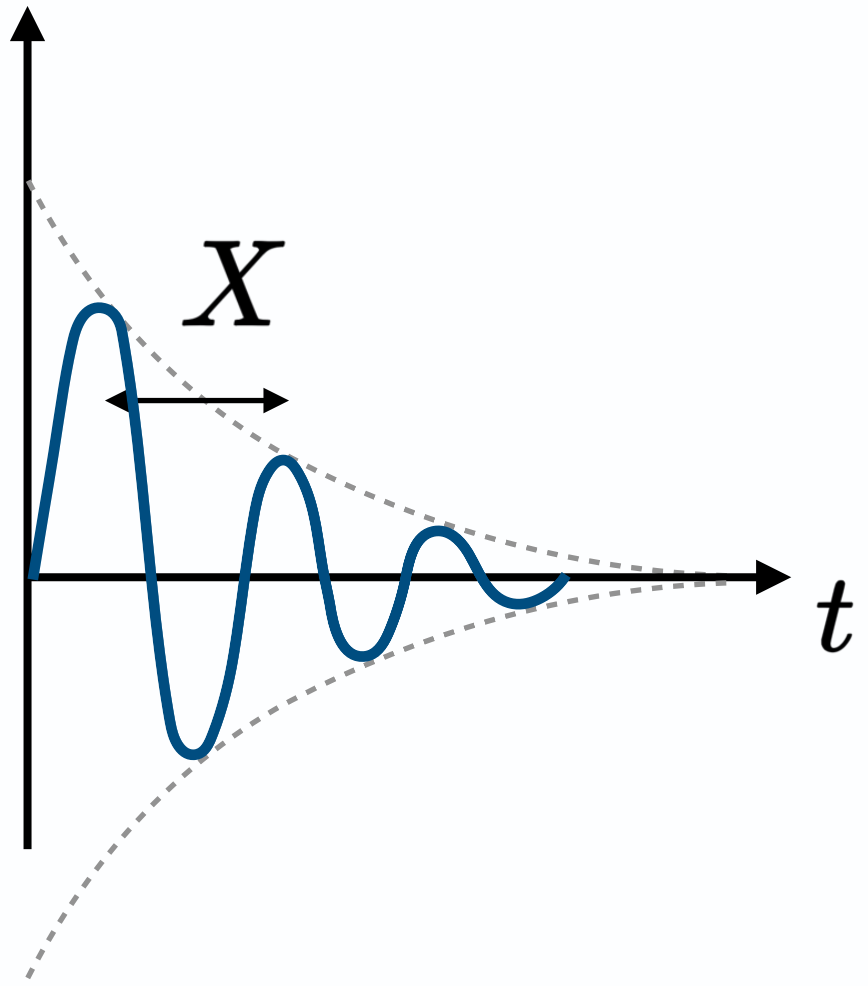
Two types of **slow modes** for slow variables

Roth & von Smekal, JHEP 10 (2023); Roth et al., PRD 111 (2025)

- **Relaxational mode** $\sim \exp\left(-Xt\right)$



- **Propagating mode** $\sim \exp\left(-Xt \pm i\Omega t\right)$



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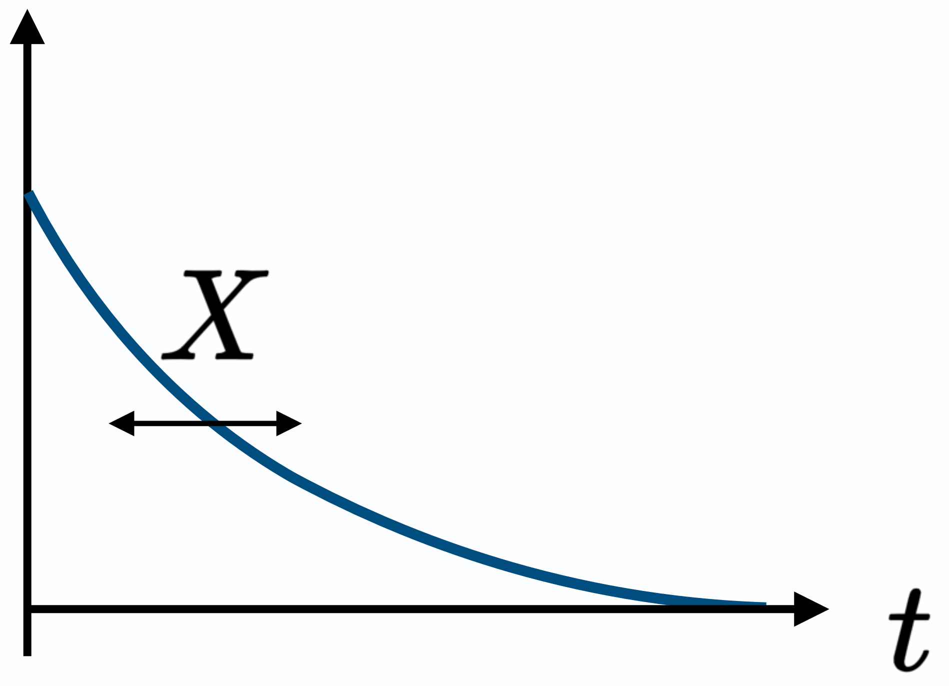

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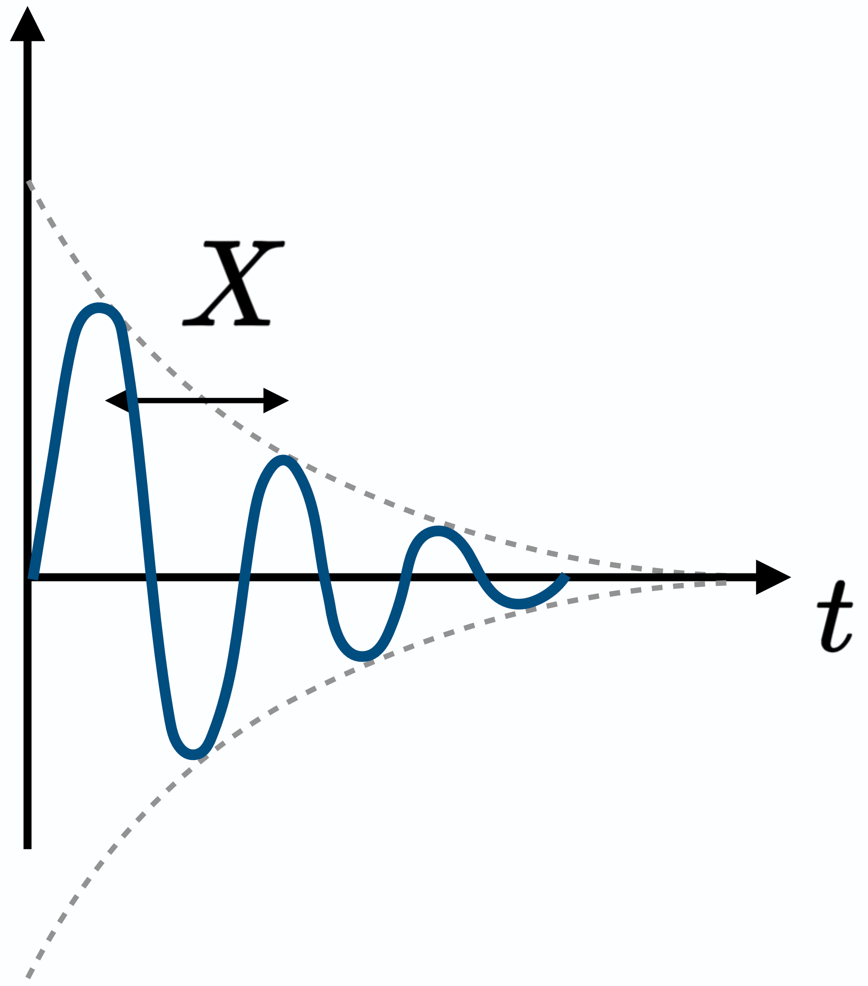

Both can appear in microscopic or effective theories: **relativistic scalar theory, …**

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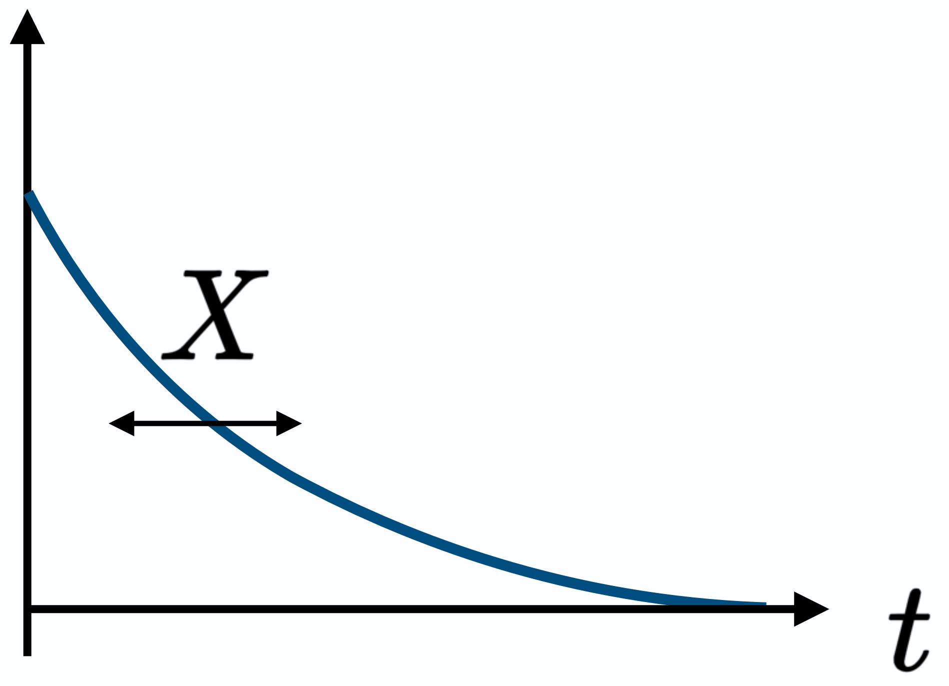

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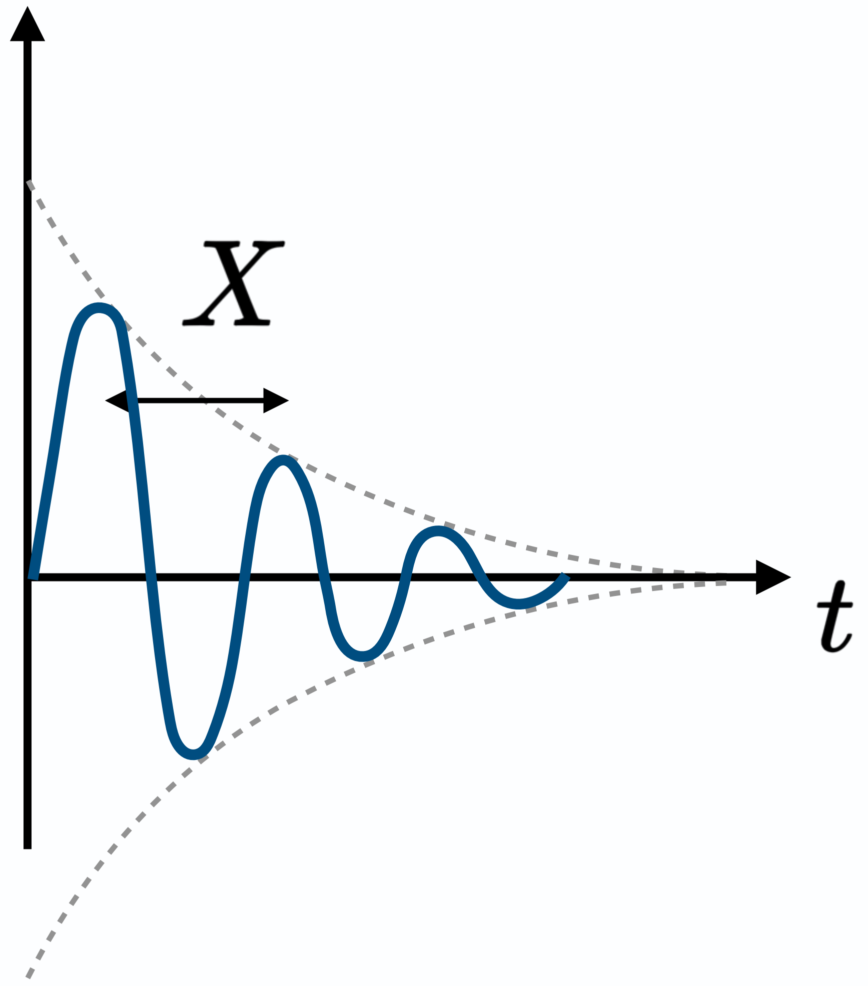


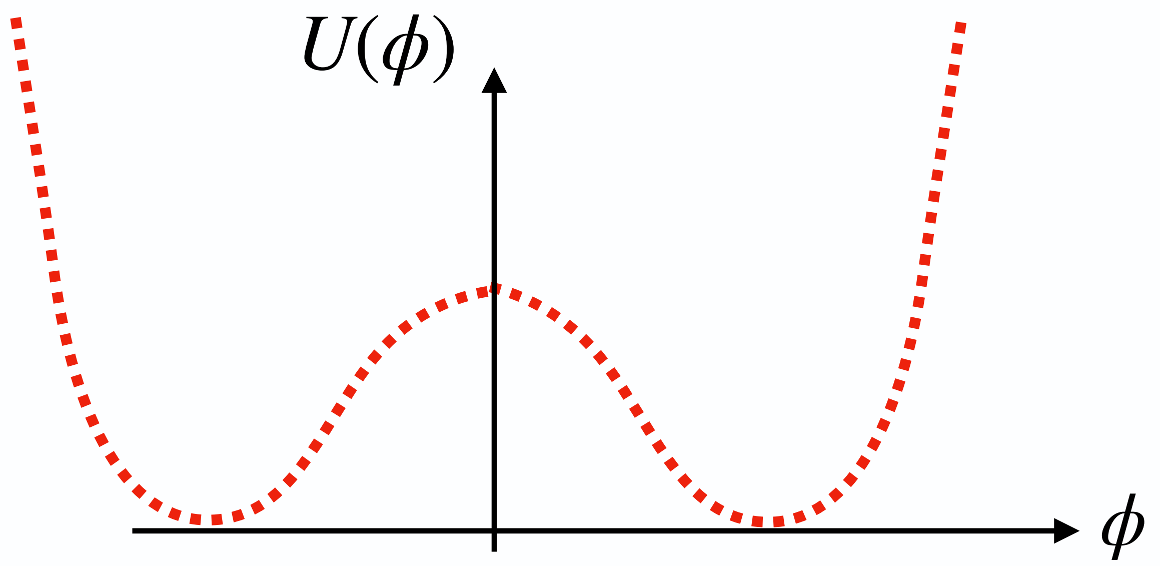
Both can appear in microscopic or effective theories: **relativistic scalar theory, …**

Which behaviour controls the infrared when both are present?

Langevin equation

Zinn-Justin, *Quantum Field Theory and Critical Phenomena*

Statics: Landau-Ginzburg



Propagation

$$\boxed{\frac{1}{c^2}\partial_t^2\phi} + \boxed{X\partial_t\phi} = -\frac{\delta\mathcal{H}}{\delta\phi} + \boxed{\xi}$$

Dissipation & Noise

$$\langle \xi\xi' \rangle = 2XT\delta$$

Langevin equation

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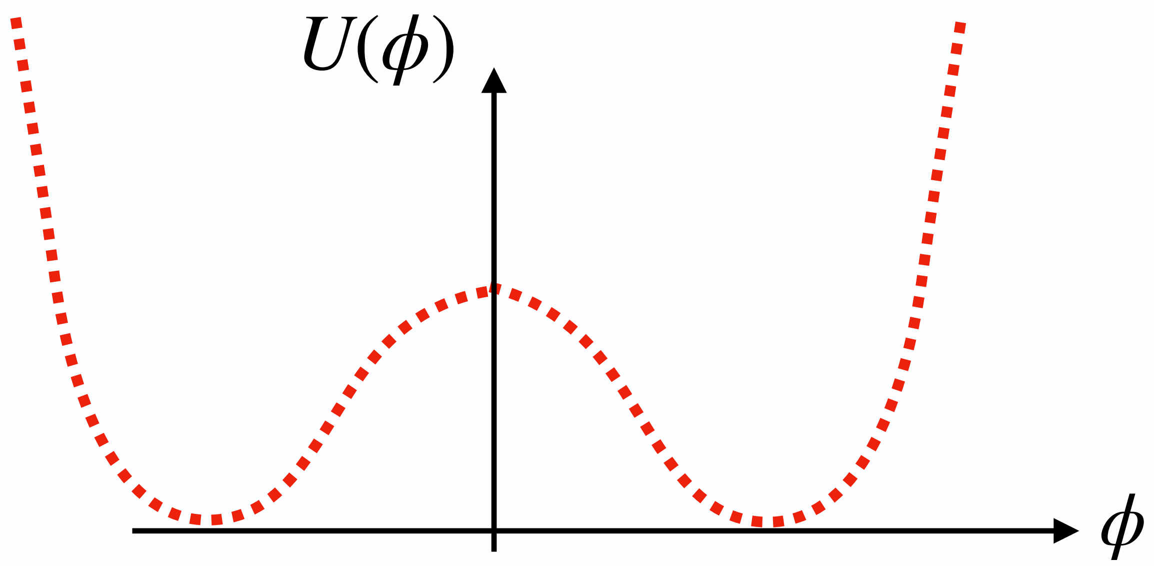

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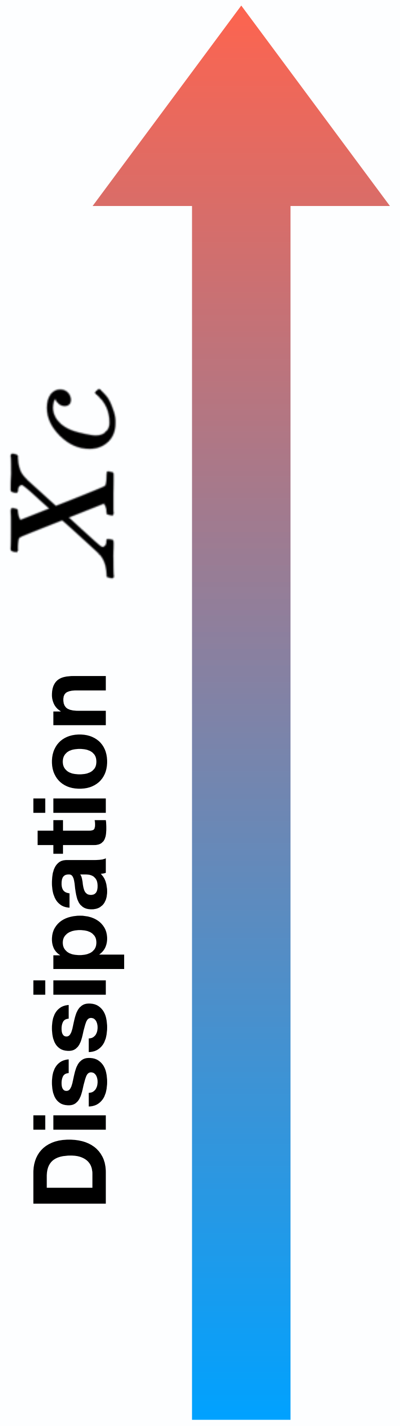

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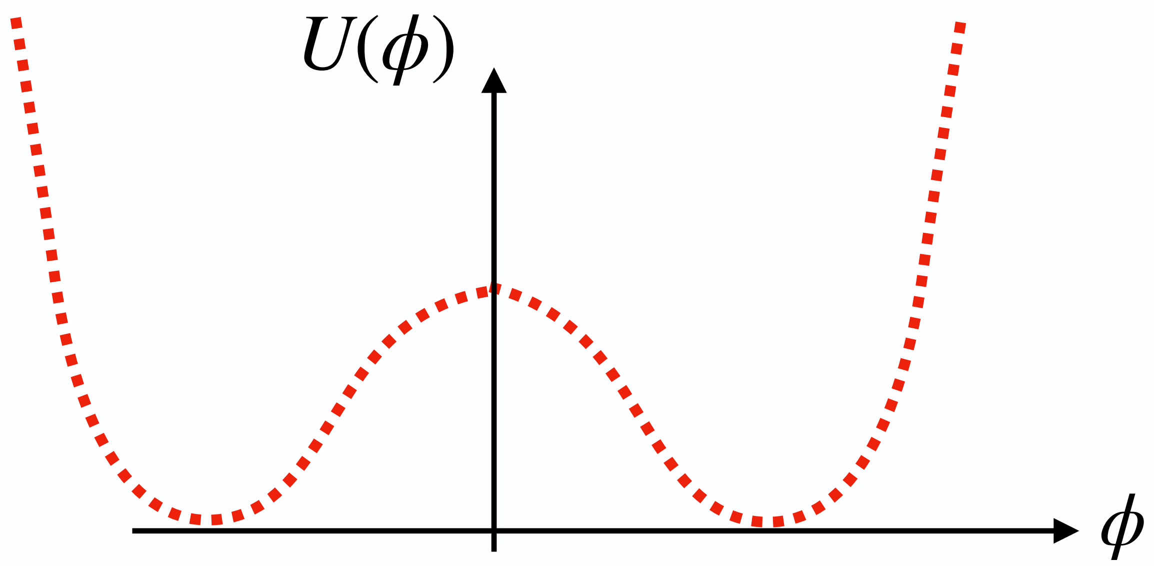

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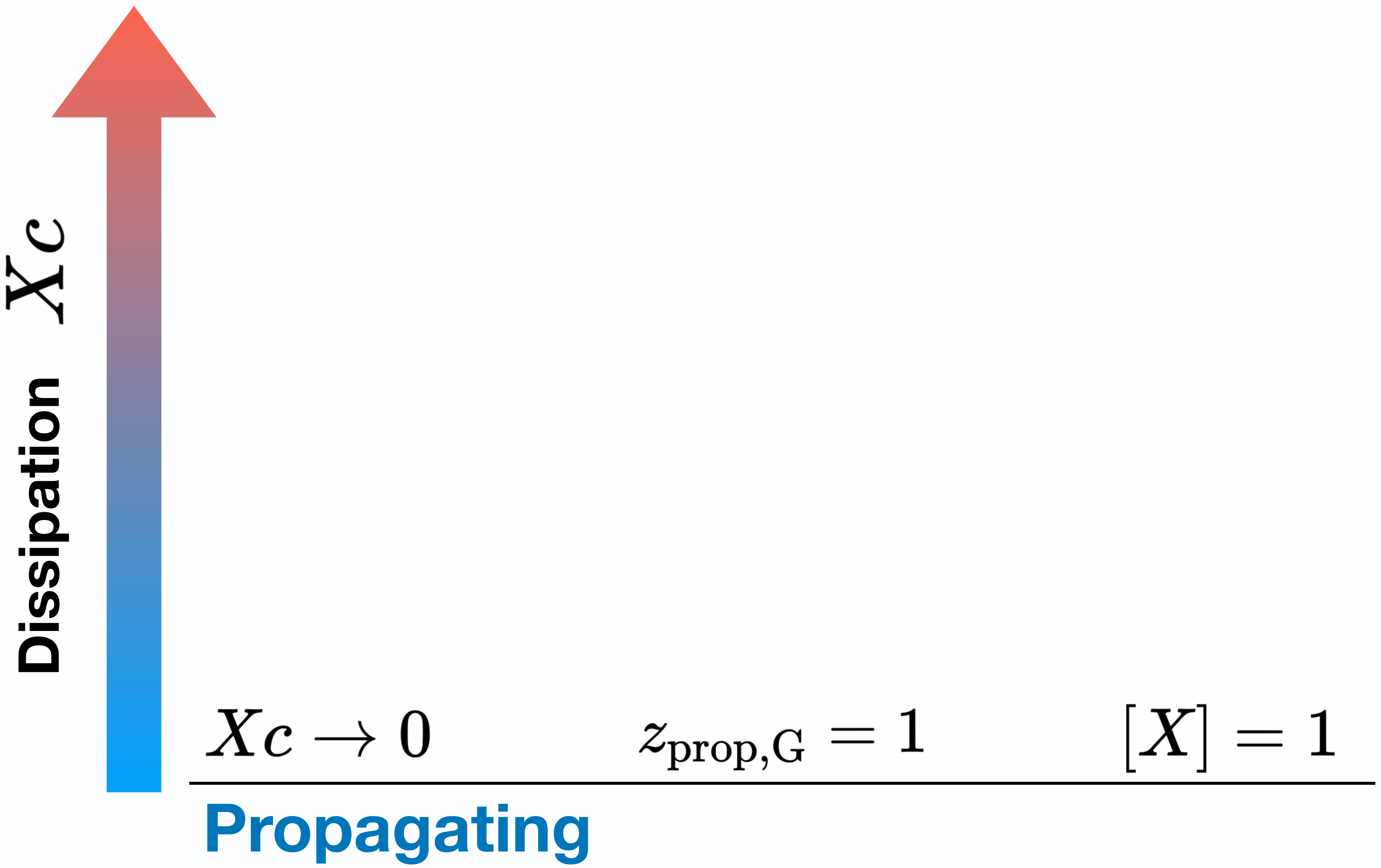

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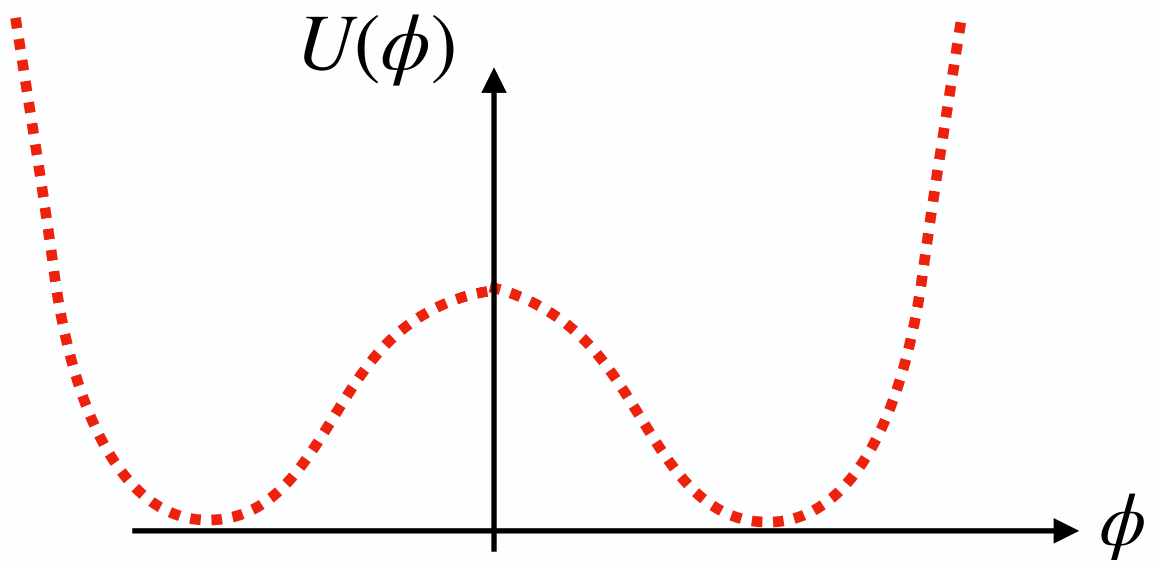

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$Xc \to \infty$ $\quad z_{\mathrm{od,G}} = 2$ $\quad [c^{-2}] = -2$

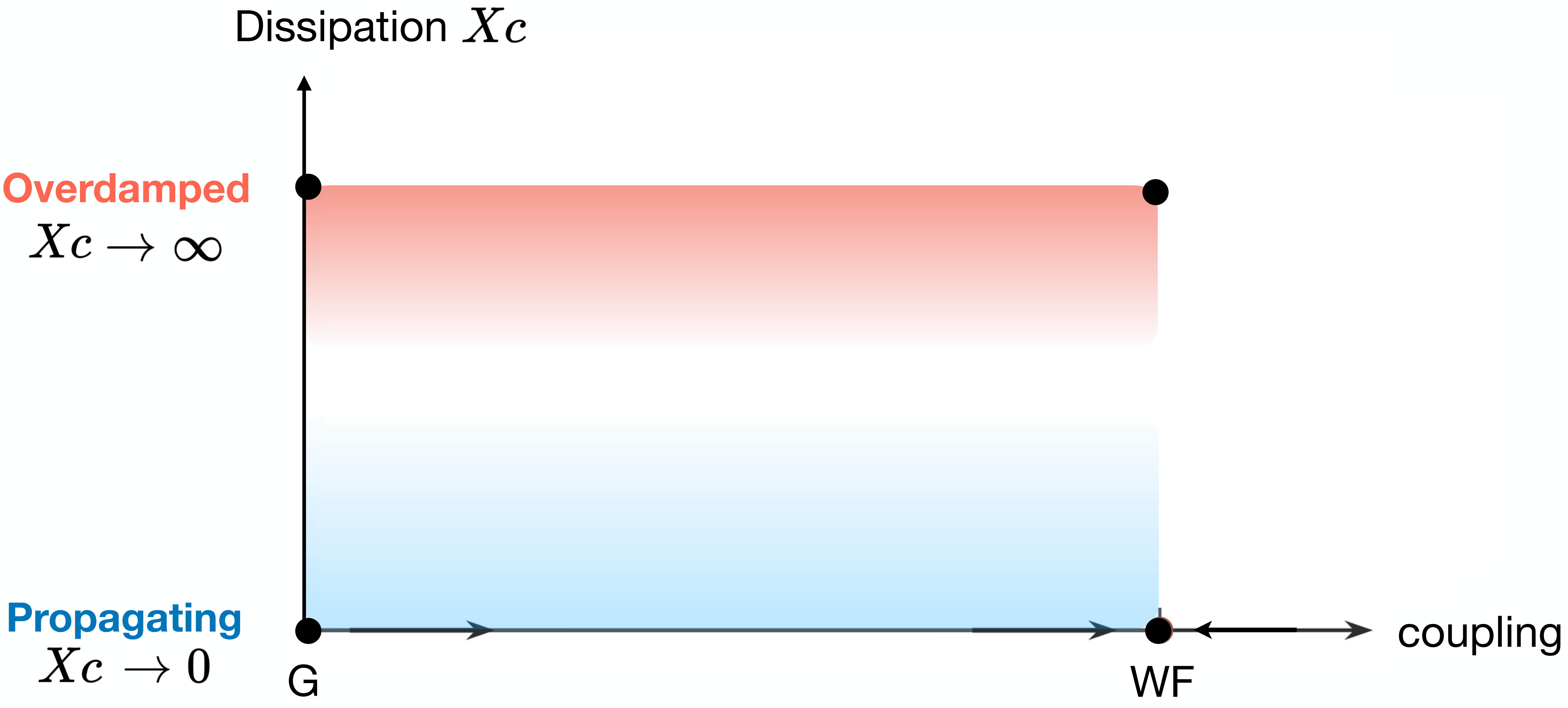
Overdamped (Model A)

Dissipation Xc

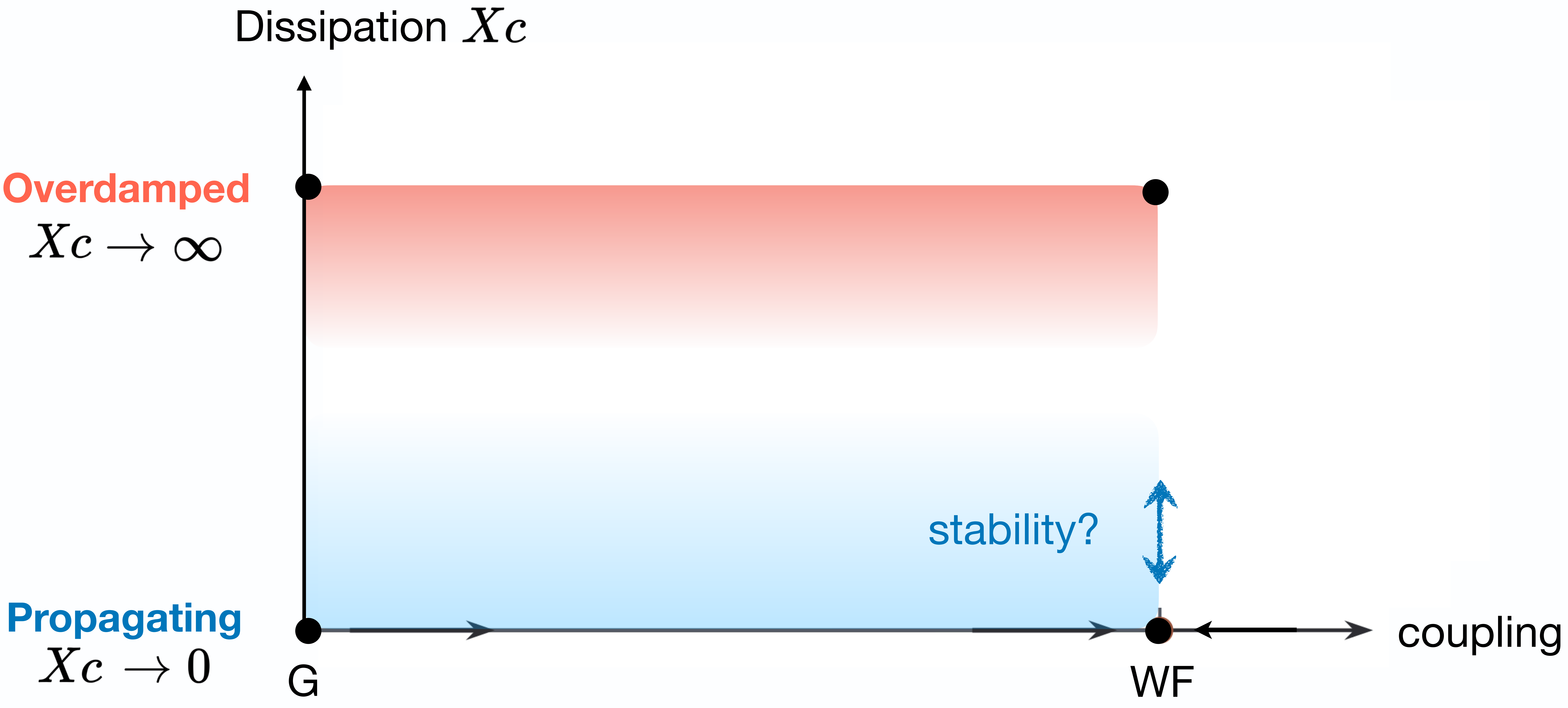
$Xc \to 0$ $\quad z_{\mathrm{prop,G}} = 1$ $\quad [X] = 1$

Propagating

Fixed-point structure



Fixed-point structure



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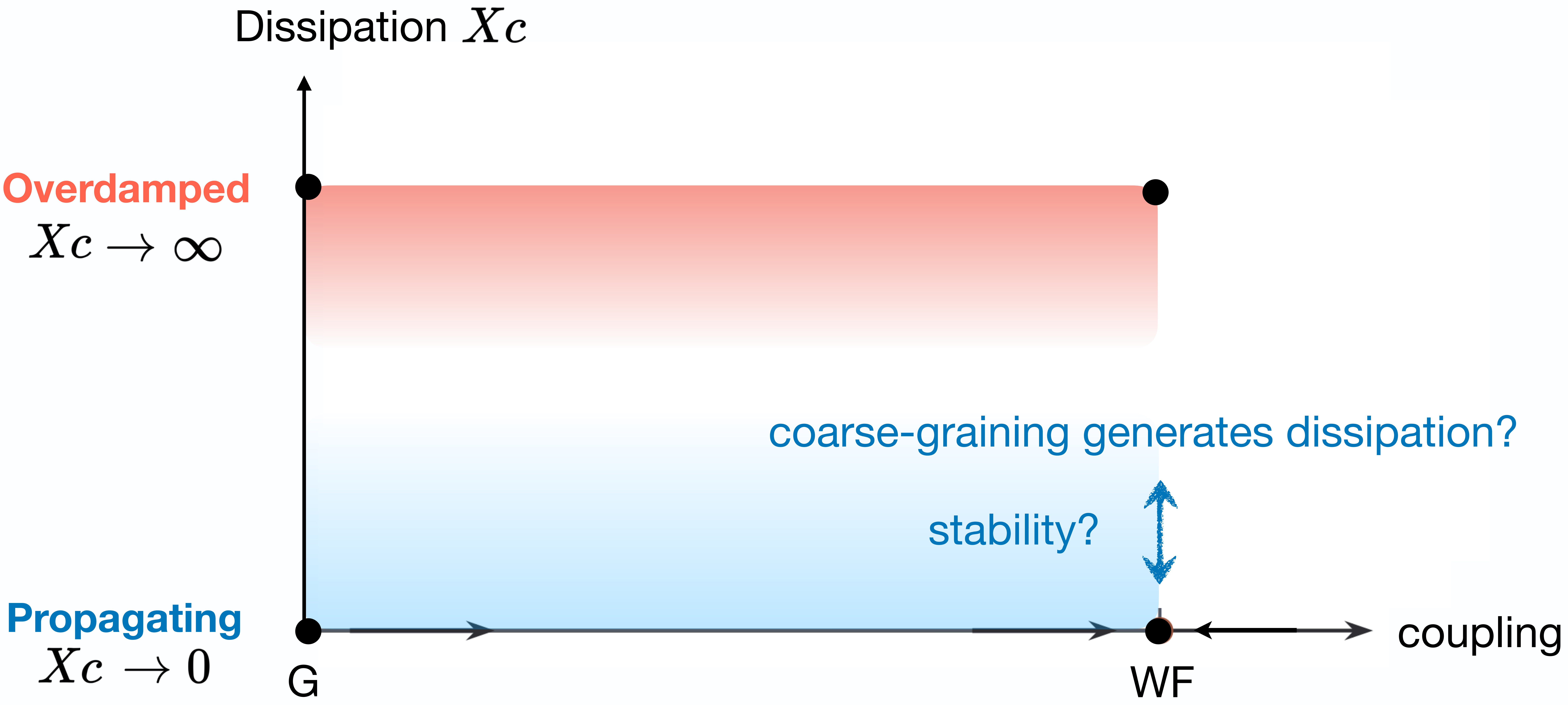

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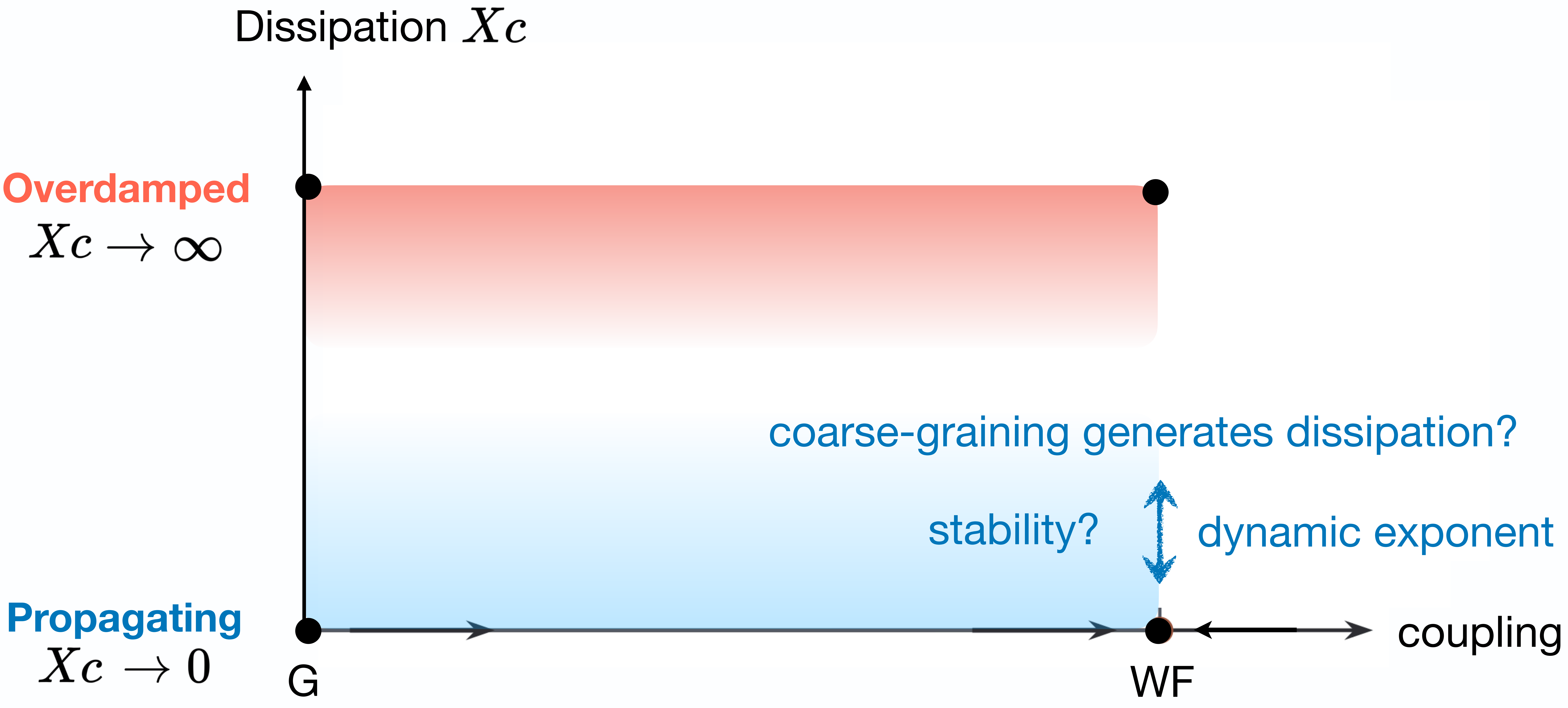

Superspace formulation

Canet, Chaté, and Delamotte, *J. Phys. A: Math. Theor.* **44**, 495001 (2011)
Sahu *et al.*, arXiv:2605.24029 (2026)

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- **Superfield:** $\Phi(t, \boldsymbol{x}, \theta, \bar{\theta}) = \phi(t, \boldsymbol{x}) + \theta\, \bar{c}(t, \boldsymbol{x}) + c(t, \boldsymbol{x})\, \bar{\theta} + \theta\bar{\theta}\, \phi_a(t, \boldsymbol{x})$

  Response field

  Classical field

  Ghost fields

  Grassmann coordinates

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  Response field

  Classical field Ghost fields Grassmann coordinates

- **Dynamic action:**

$$S[\phi,\phi_a,c,\bar c] = \int \left[\phi_a\left(\frac{1}{c^2}\partial_t^2\phi - \nabla^2\phi + U'(\phi)\right) + X\phi_a(\partial_t\phi - T\phi_a) - c\left(\frac{1}{c^2}\partial_t^2 + X\partial_t - \nabla^2 + U''(\phi)\right)\bar c\right]$$

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  Classical field — Ghost fields — Grassmann coordinates — Response field

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BRST + KMS allow supersymmetric form

Propagation **Dissipation** **Statics**

$$S[\Phi] = \int \mathrm{d}^{d+1}x\,\mathrm{d}\bar\theta\,\mathrm{d}\theta\left[\boxed{-\frac{1}{c^2}(D\bar D\Phi)(D\bar D\Phi)} + \boxed{X\,\bar D\Phi\,D\Phi} + \boxed{\frac{1}{2}(\boldsymbol{\nabla}\Phi)^2 + \frac{1}{2}m^2\Phi^2 + \frac{g}{4!}\Phi^4}\right]$$

Model A

$$\bar D = \frac{\partial}{\partial\bar\theta}, D = \frac{\partial}{\partial\theta} - \bar\theta\frac{\partial}{\partial t} \qquad \{D,\bar D\} = -\frac{\partial}{\partial t}$$

Two-point vertex

$$\Gamma^{(2)} = \Gamma^{(2)}_{0\text{loop}} + \Gamma^{(2)}_{1\text{loop}} + \Gamma^{(2)}_{2\text{loop}}$$

Two-point vertex

Overdamped (Model A)

$$\Gamma^{(2)}_{\text{tree,od}} = -2X\left(1 - \frac{\mathrm{i}\omega}{2} f(\theta,\bar{\theta})\right) + (\boldsymbol{p}^2 + m^2)h(\theta,\bar{\theta})$$

Propagating

$$\Gamma^{(2)}_{\text{tree,prop}} = \left(-\frac{\omega^2}{c^2} + \boldsymbol{p}^2 + m^2\right)h(\theta,\bar{\theta})$$

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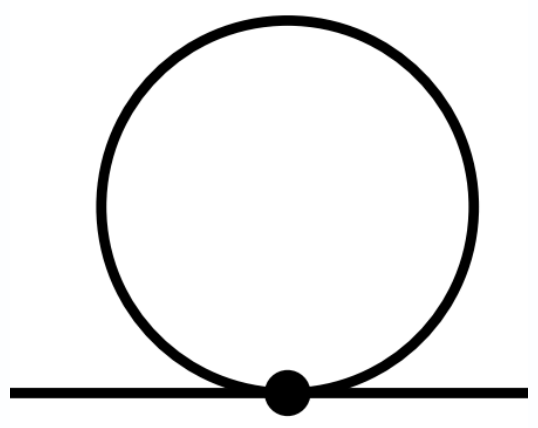
Tadpole



$$= \frac{g}{2} T_1 h(\theta,\bar{\theta})$$

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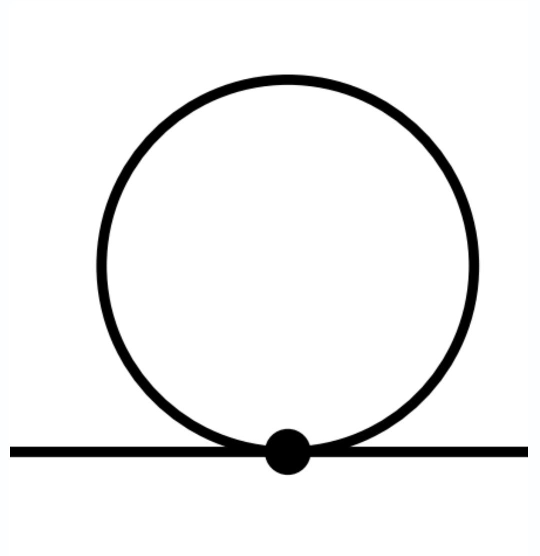
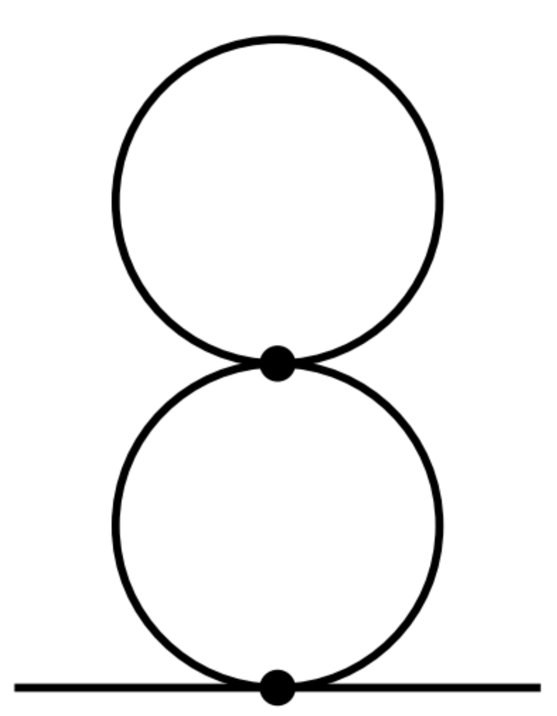


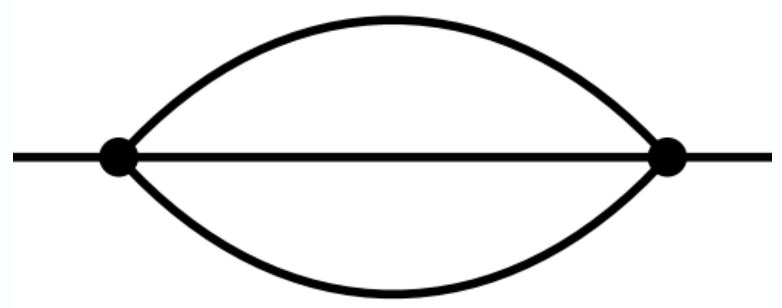
$$= \frac{g}{2}T_1 h(\theta,\bar\theta)$$

Double-bubble



$$= -\frac{g^2}{4}T_1T_2 h(\theta,\bar\theta)$$

Sunset



$$-g^2B(\omega,\boldsymbol{p})h(\theta,\bar\theta)$$

$$-g^2A(\omega,\boldsymbol{p})\left(1-\frac{i\omega}{2}f(\theta,\bar\theta)\right)$$

Two-point vertex

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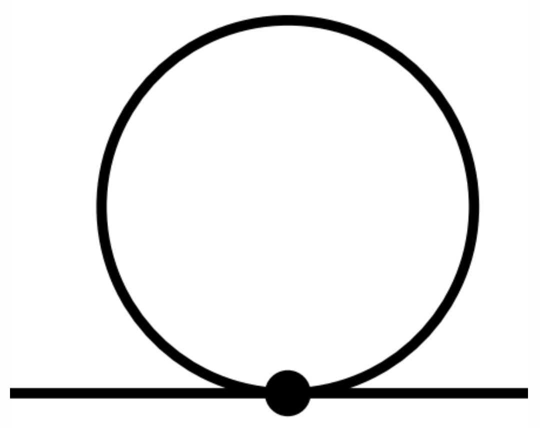

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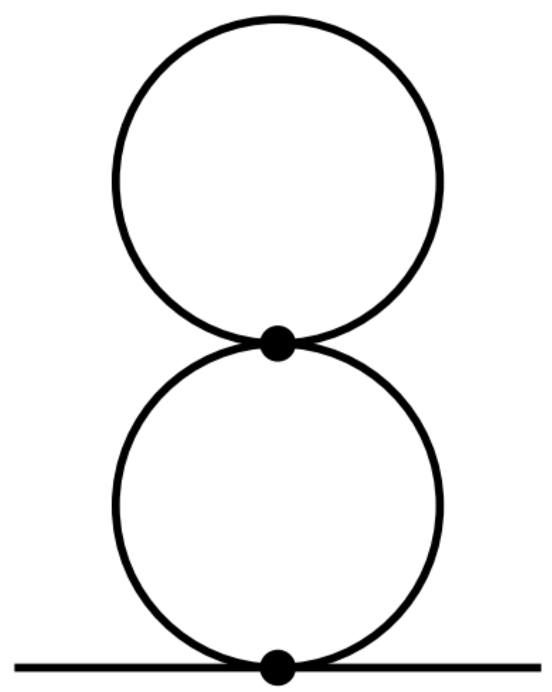
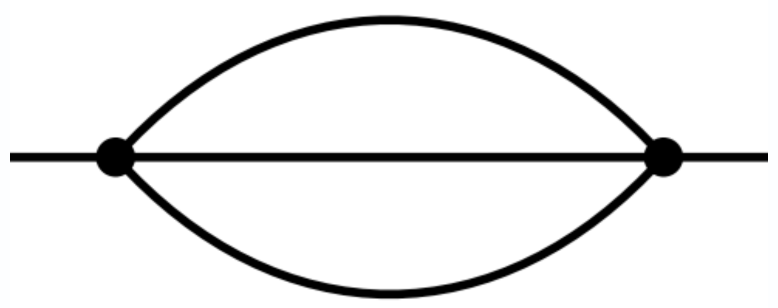


$$= -\frac{g^2}{4} T_1 T_2 h(\theta,\bar{\theta})$$

Only static sector

Sunset



$$- g^2 B(\omega,\boldsymbol{p})h(\theta,\bar{\theta})$$

$$-g^2 A(\omega,\boldsymbol{p})\left(1 - \frac{i\omega}{2} f(\theta,\bar{\theta})\right)$$

RG flow: statics & overdamped

Static sector = GL theory

$$\Gamma^{(2)}_{\text{stat}} = \int d\bar{\theta}_1\, d\theta_1 \lim_{\omega,\boldsymbol{p}\to 0} \Gamma^{(2)}$$

$$\Gamma^{(4)}_{\text{stat}} = \int \prod_{i=2}^{4} d\bar{\theta}_i\, d\theta_i \lim_{p_1,p_2,p_3\to 0} \Gamma^{(4)}$$

Zinn-Justin, Quantum Field Theory and Critical Phenomena

Overdamped dynamics = Model A

$$\Gamma^{(2)}_{\text{sun}} = -\frac{g^2}{6} \underbrace{X\Sigma_X}_{A(0,\mathbf{0})} \boxed{\left(1 - \frac{\mathrm{i}\omega}{2} f(\theta,\bar{\theta})\right)}$$

$$\downarrow$$

$$\boxed{z_{\text{od,WF}} = 2 + \frac{6\log(4/3) - 1}{54}\,\epsilon^2 + \mathcal{O}(\epsilon^3)}$$

reproduces the known two-loop Model A exponent

Hohenberg, Halperin, *Rev. Mod. Phys.* **49**, 435 (1977)

Propagating exponent

Propagating structure

$$\Gamma^{(2)}_{\text{dyn}} = -g^2 A(\omega, \boldsymbol{p}) \left(1 - \frac{\mathrm{i}\omega}{2} f(\theta, \bar{\theta})\right) - g^2 B(\omega, \boldsymbol{p}) h(\theta, \bar{\theta}) + \left(-\frac{\omega^2}{c^2} + \boldsymbol{p}^2 + m^2\right) \boxed{h(\theta, \bar{\theta})}$$

Propagating exponent

Propagating structure

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$$B(\omega,\mathbf{0}) = \int_{t,\boldsymbol{p}_1\boldsymbol{p}_2} e^{i\omega t}\,\frac{\cos(c\omega_{p_1}|t|)}{\omega_{p_1}^2}\,\frac{\cos(c\omega_{p_2}|t|)}{\omega_{p_2}^2}\,\frac{c\sin(c\omega_{-p_1-p_2}|t|)}{2\omega_{-p_1-p_2}}$$

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Computed using Hankel representation

Propagating exponent

Propagating structure

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Computed using Hankel representation

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Propagating structure

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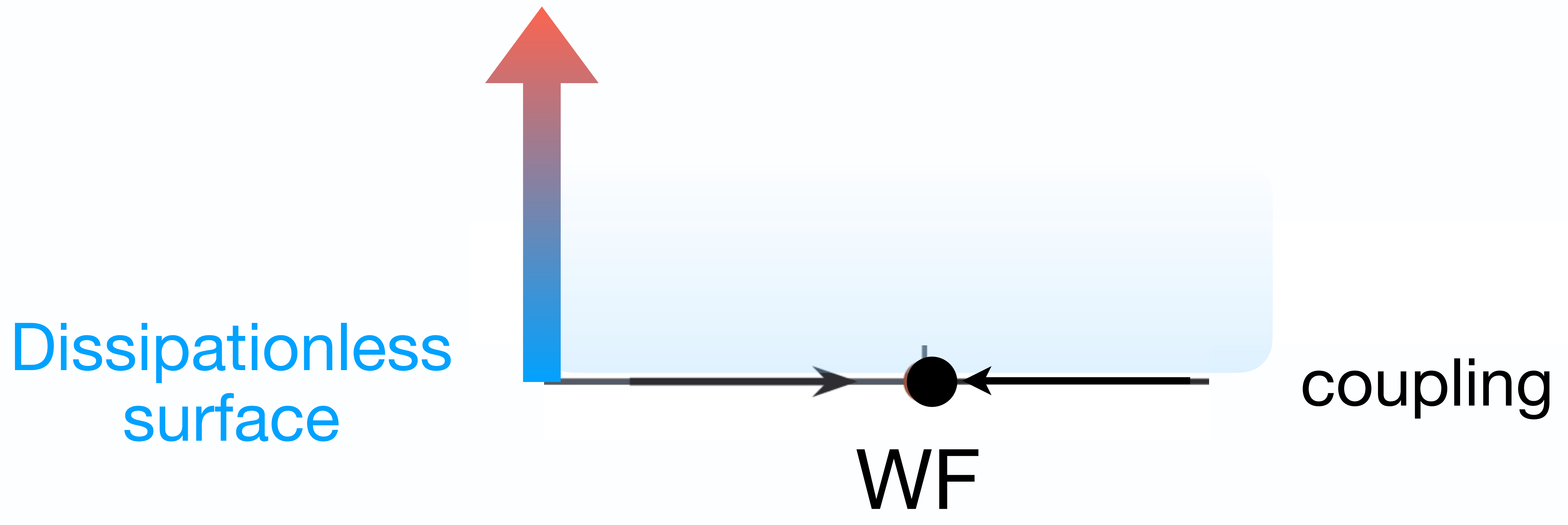
Computed using Hankel representation

$$z_{\text{prop,WF}} = 1-\frac{\epsilon^2}{9}\left(\frac{17}{12}-2\log 2\right)+\mathcal{O}(\epsilon^3)$$
$$\simeq 1-0.00337\,\epsilon^2$$



Does coarse-graining generate dissipation?

$$\Gamma^{(2)}_{\text{dyn}} = -g^2 A(\omega,\boldsymbol{p})\boxed{\left(1-\frac{\mathrm{i}\omega}{2}f(\theta,\bar\theta)\right)} - g^2 B(\omega,\boldsymbol{p})h(\theta,\bar\theta) + \left(-\frac{\omega^2}{c^2}+\boldsymbol{p}^2+m^2\right)h(\theta,\bar\theta)$$

$$A(\omega,\boldsymbol{p}) = \int_{t,\boldsymbol{p}_1\boldsymbol{p}_2} e^{i\omega t}\,\frac{\cos(c\omega_{p_1}|t|)}{\omega_{p_1}^2}\,\frac{\cos(c\omega_{p_2}|t|)}{\omega_{p_2}^2}\,\frac{\cos(c\omega_{p-p_1-p_2}|t|)}{\omega_{p-p_1-p_2}^2}$$

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Following the same steps as before

$$\xrightarrow{\;d\,\to\,4\;} \qquad A(\omega,\boldsymbol{q}) = \frac{1}{(4\pi)^3}\frac{|\omega|}{c^2}(6\log 2-4)$$

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Following the same steps as before

$$\xrightarrow{d\to 4} \qquad A(\omega,\boldsymbol{q}) = \frac{1}{(4\pi)^3}\frac{|\omega|}{c^2}(6\log 2-4)$$

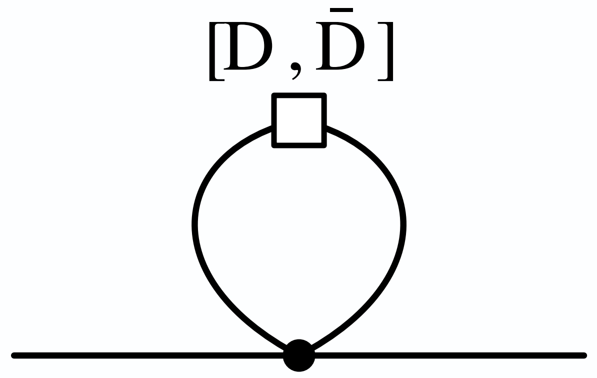
Cfr. **Dissipative kernel**

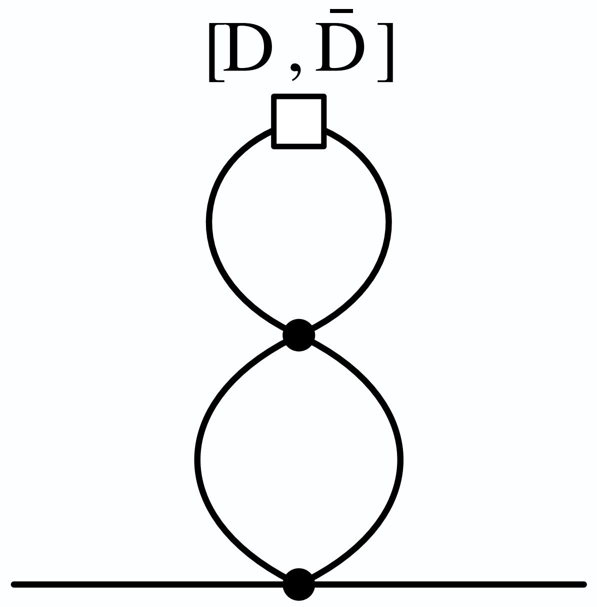
$$-2X\boxed{\left(1-\frac{\mathrm{i}\omega}{2}f(\theta,\bar{\theta})\right)} \longrightarrow$$

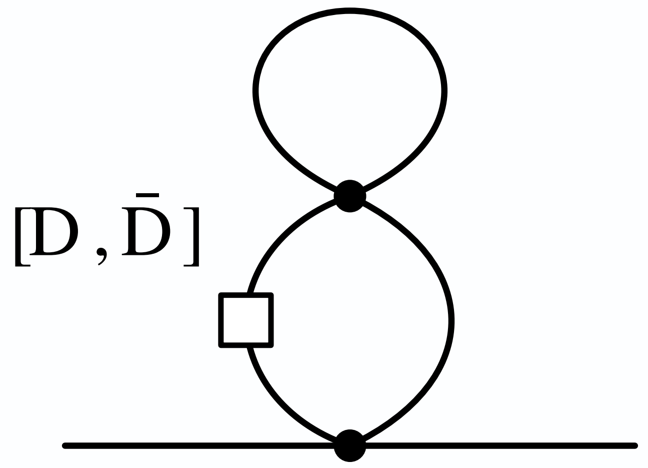
No ***local dissipative term generated***

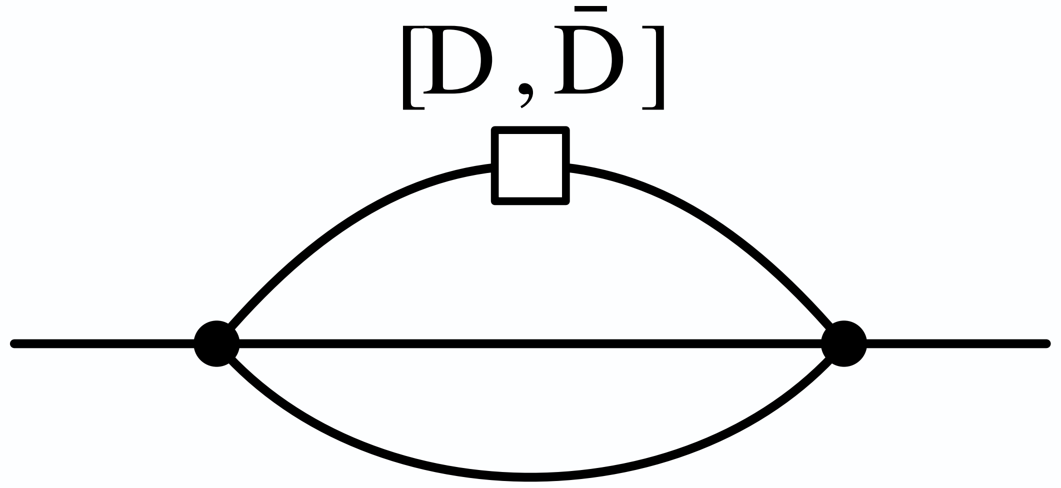
$$A(\omega=0,\boldsymbol{p}=\boldsymbol{0}) \longrightarrow$$ ***Invariant surface*** under RG

Dissipationless surface: instability

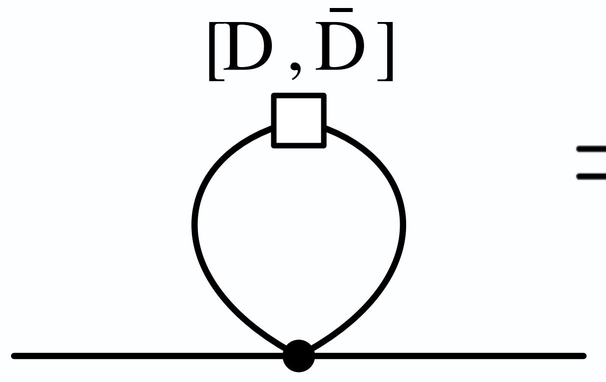




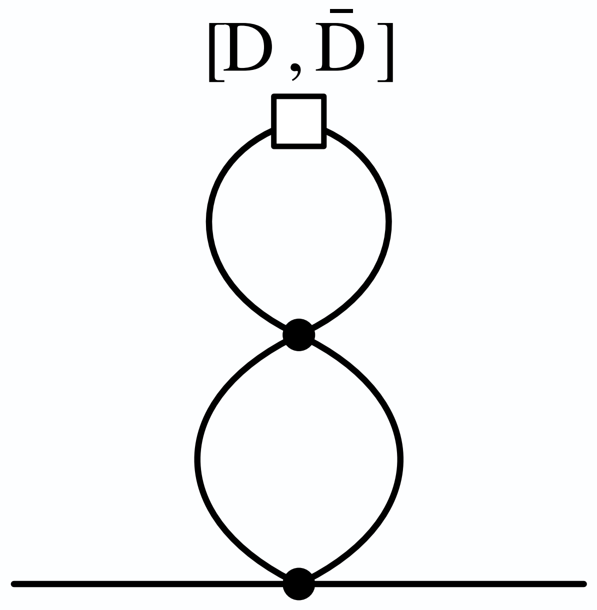




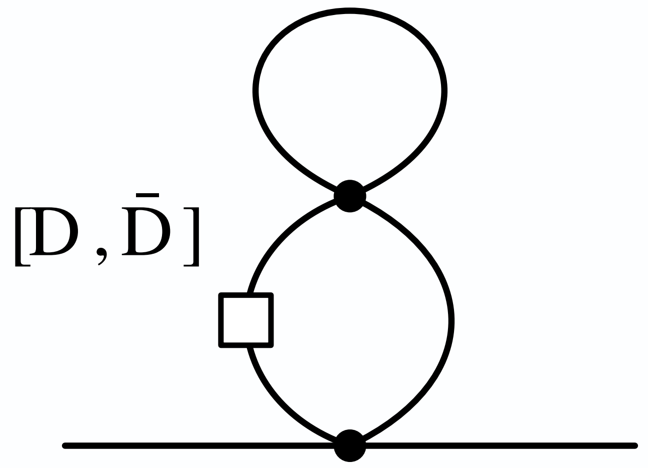
Dissipationless surface: instability



$= 0$



$= 0$



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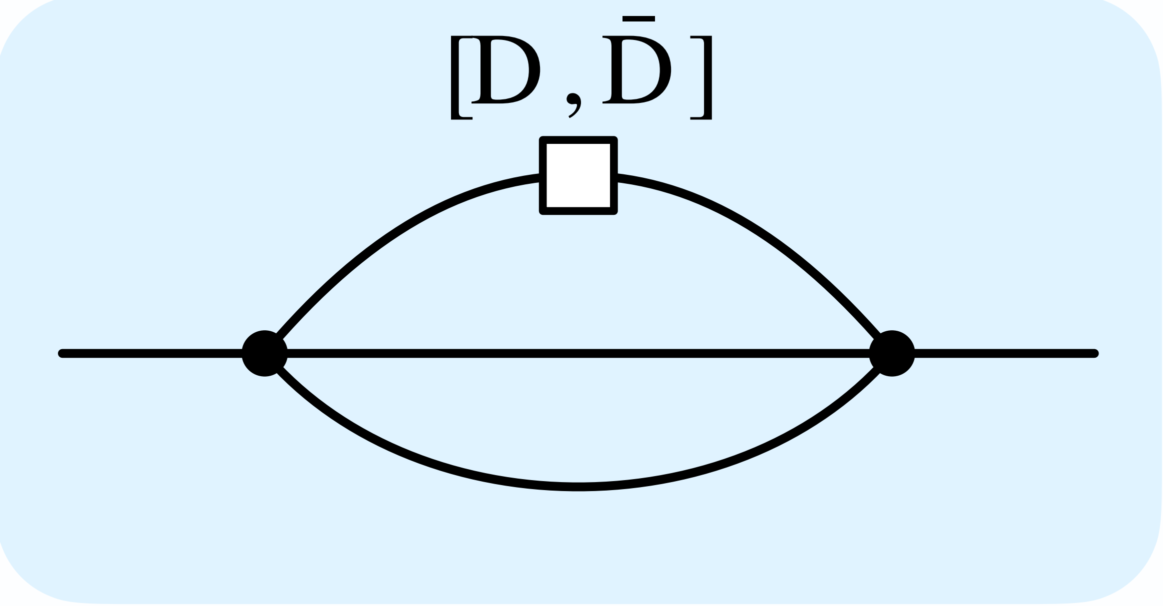
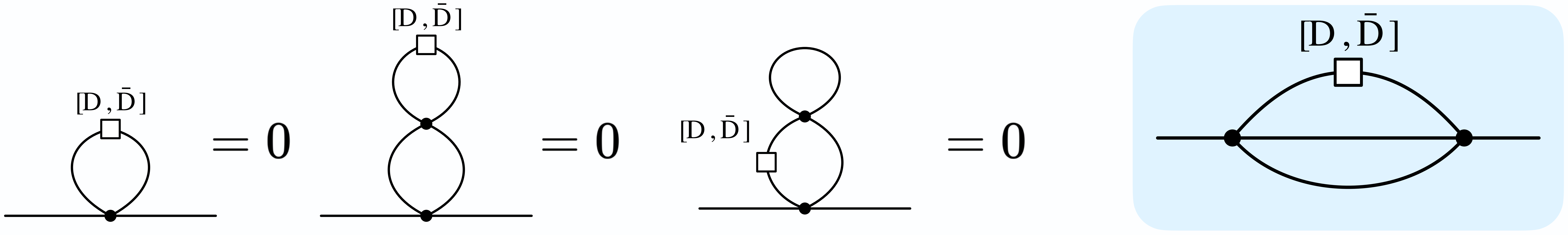
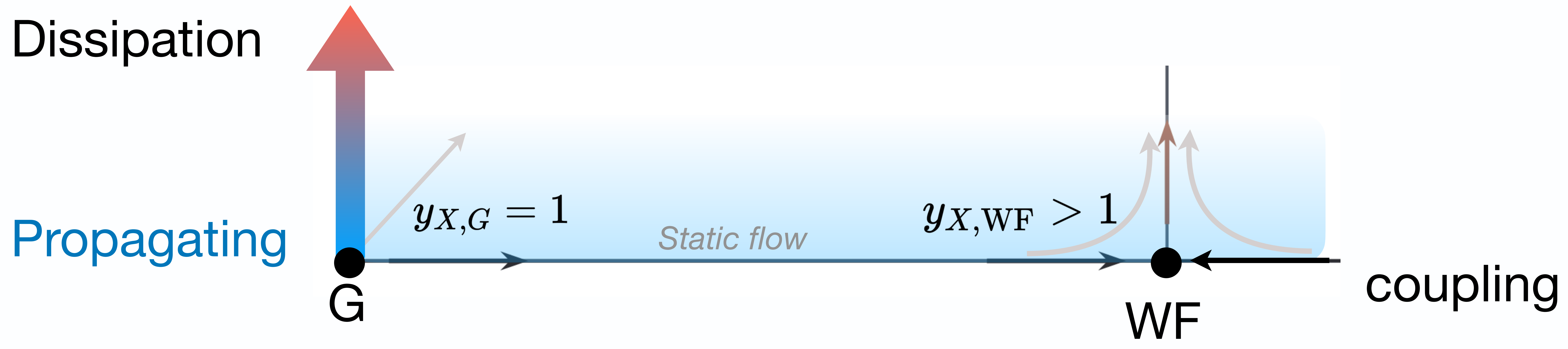
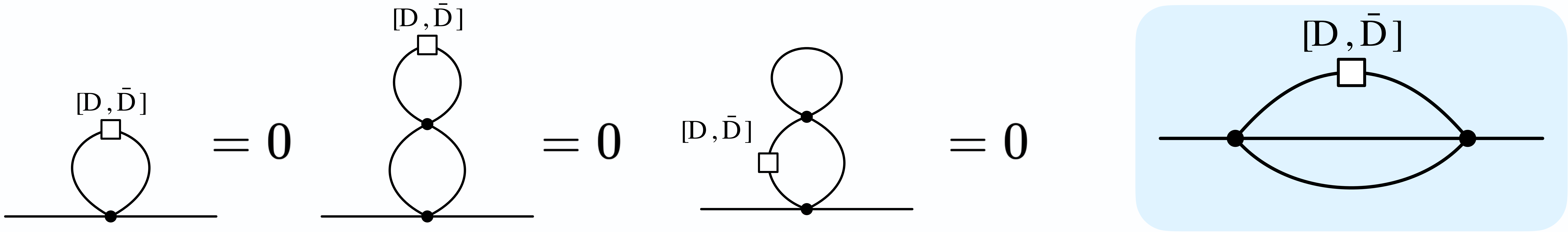


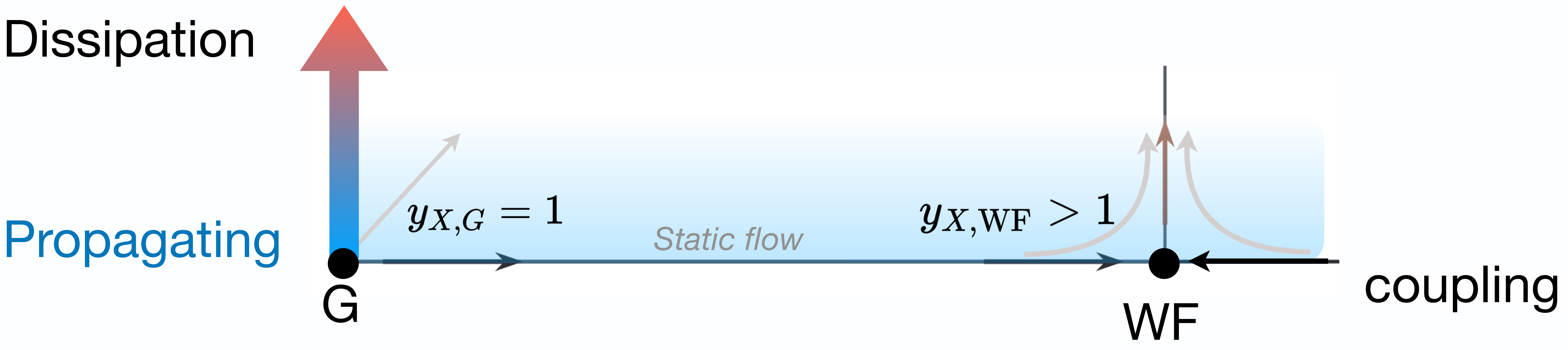
Dissipationless surface: instability





Dissipationless surface: instability

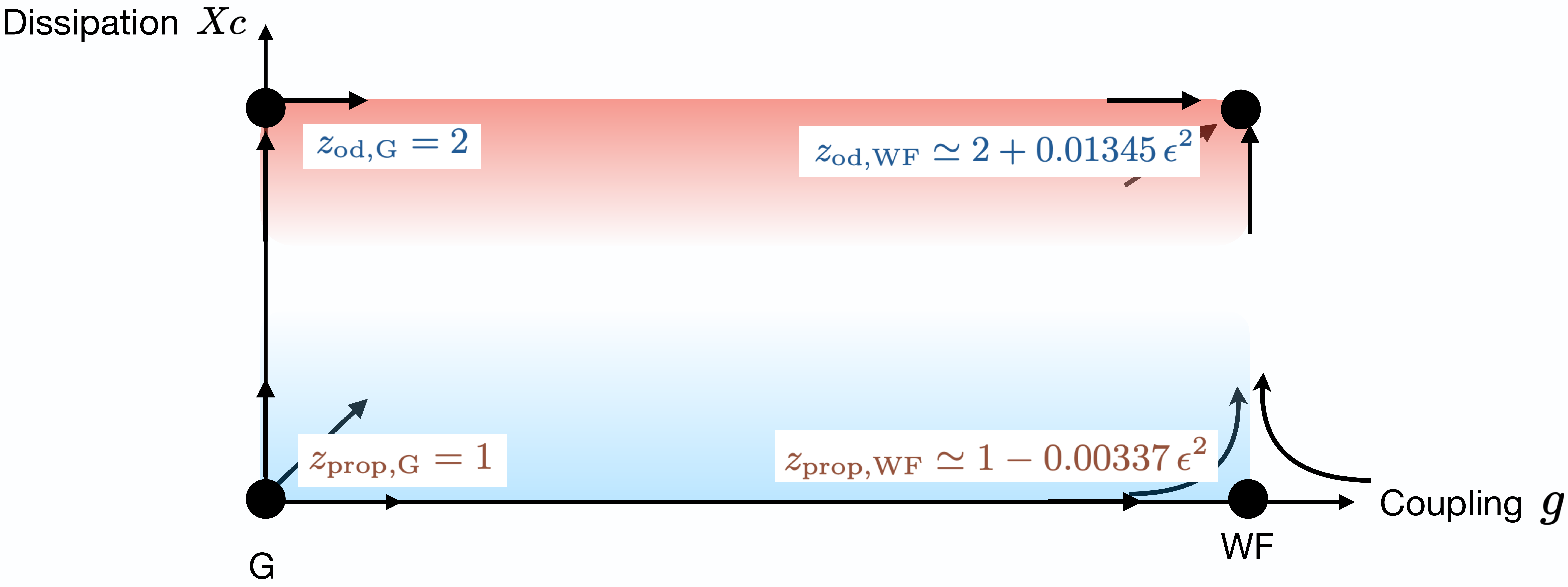




repulsive in both directions

saddle

Summary



Outlook:

- Critical dynamics of **Model G, ..**
- Nonperturbative RG crossover: functional RG

Eduardo Grossi

Thank you
for your attention!

Backup slides

Effective action

Generating functional:

$$Z[\mathcal{J}] = \int \mathcal{D}\Psi \, \exp\left[-S[\Psi] + \int_Z \mathcal{J}(Z)\Psi(Z)\right]$$

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$$\Gamma[\Phi] = \int_Z \mathcal{J}(Z)\Phi(Z) - \log Z[\mathcal{J}] \qquad \Gamma^{(n)}[\Phi](Z_1, \ldots, Z_n) \equiv \frac{\delta^n \Gamma[\Phi]}{\delta\Phi(Z_1)\cdots\delta\Phi(Z_n)}$$

Effective action

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- **Two-loop expansion**

$$\Gamma[\Phi] = S[\Phi] + \Gamma_{1\ell}[\Phi] + \Gamma_{2\ell}[\Phi] + \cdots$$

$$\Gamma_{1\ell}[\Phi] = \frac{1}{2}\ \mathrm{Tr}\log S^{(2)}[\Phi]$$

$$\Gamma_{2\ell}[\Phi] = \frac{1}{8}\int_{1,2,3,4} S^{(4)}(1,2,3,4)\,\Delta_\Phi(1,2)\,\Delta_\Phi(3,4) - \frac{1}{12}\int_{1,\ldots,6} S^{(3)}(1,2,3)\,S^{(3)}(4,5,6)\,\Delta_\Phi(1,4)\,\Delta_\Phi(2,5)\,\Delta_\Phi(3,6)$$

Hankel representation

$$\mathcal{H}_n(x) = \frac{\sqrt{\pi}}{2\pi i} \int_{\mathcal{C}_H} \mathrm{d}s \, s^{-n-\frac{1}{2}} \exp\left(s - \frac{x}{4s}\right)$$

$$\mathcal{H}_0(x) = \cos\sqrt{x}$$

$$\mathcal{H}_1(x) = 2\,\frac{\sin\sqrt{x}}{\sqrt{x}}$$

$$\dots$$