Probability distribution of order parameter in $O(N)$ models

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Objective

We want to find:

$$P(\mathbf{s}) \propto \int \mathcal{D}\mathbf{S}\ \delta(\mathbf{s} - \frac{1}{L^d}\sum_x \mathbf{S}_x) e^{-H[\mathbf{S}]}$$

FRG method

Introducing effective probability action:

$$\Gamma_{M,k}[\phi] + \ln P_J(s) = \sum_x J(x)\phi(x) - \frac{M^2}{2}(s - \frac{1}{L^d}\sum_x \phi(x))^2 - \sum_{x,y} \frac{1}{2}\phi(x) R_k(x-y)\phi(y)$$

Obtaining flow equation:

$$\partial_k \Gamma_{M,k}[\phi] = \frac{1}{2}\sum_{x,y} \partial_k R_{M,k}(x,y) \left(\Gamma^{(2)}_{M,k} + R_{M,k}\right)^{-1}(y,x)$$

Real effective probability distribution is given by: $\Gamma_k[\phi] = \lim_{M\to\infty} \Gamma_{M,k}[\phi]$
LPA asnsatz:

$$\Gamma_{M,k}[\phi] = \int_x \frac{(\nabla\phi(x))^2}{2} + I_{M,k}\left[\frac{\phi(x)^2}{2}\right]$$

For $N\to\infty$ LPA approximation is exact and furthermore it is possible to find analytical solution for flow equations.

Analytical solution for $N\to\infty$

In limit $N\to\infty$ flow for probability potential reduces to:

$$\partial_k I_k[\rho] = \frac{N}{2L^d}\sum_{q\neq 0} \frac{\partial_k R_k(q^2)}{q^2 + R_k(q^2) + I'_k[\rho]}$$

Solution to probability potential can be given in form:

$$L^d I[\rho]/N = \int_0^{\frac{L^{d-2}}{N}\rho} \mathrm{d}\alpha\ F^{-1}[\alpha + \Delta]$$

where Δ is parameter dependent on distance from criticality (at criticality it is equal to 0), while function F is given by:

$$F[\alpha] := -\frac{1}{2}\int_0^\infty \mathrm{d}u\ \left(e^{-u\alpha}\theta(u)^d - u^{d/2}\right)$$

MC simulation and comparison with FRG

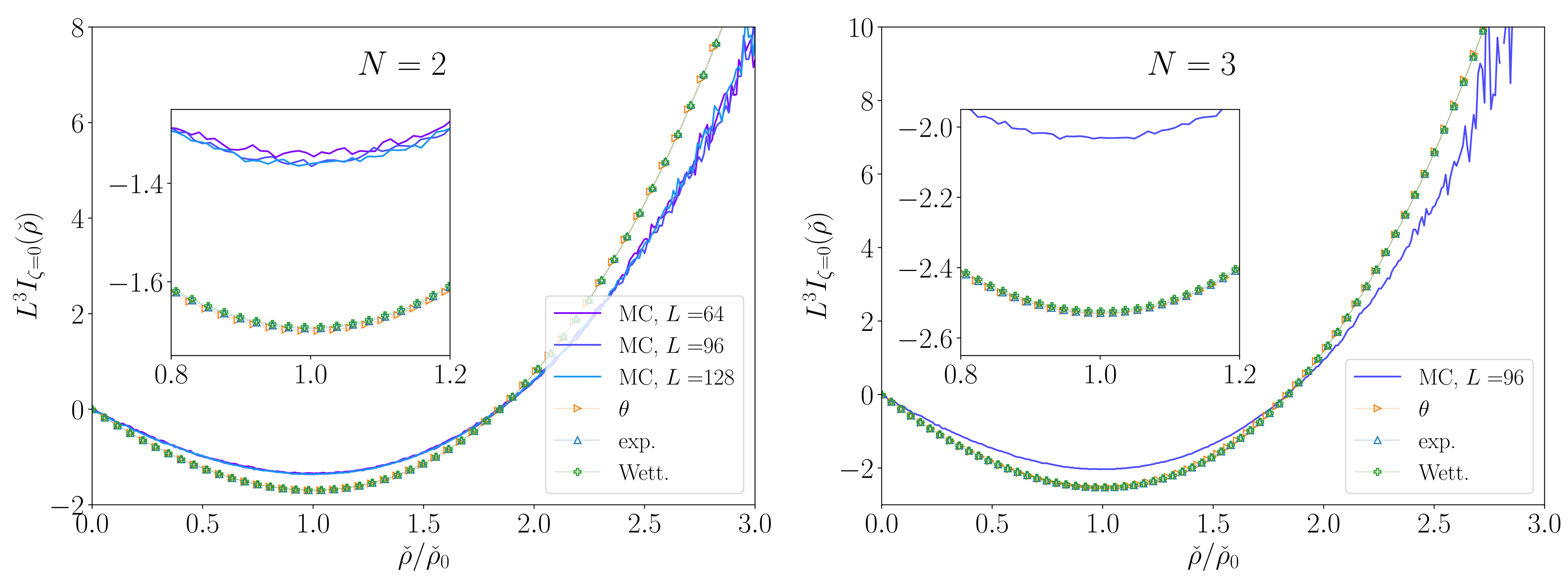

Figure 1: Comparison of MC data for different sample sizes and LPA approximation for different regulators, at criticality