

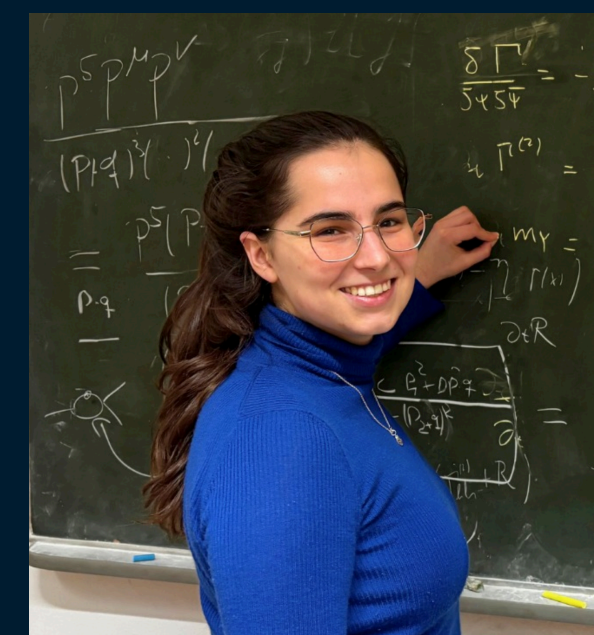
Quantum critical Dirac semimetals and finite-temperature effects

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ERG 2026
1st of September 2026



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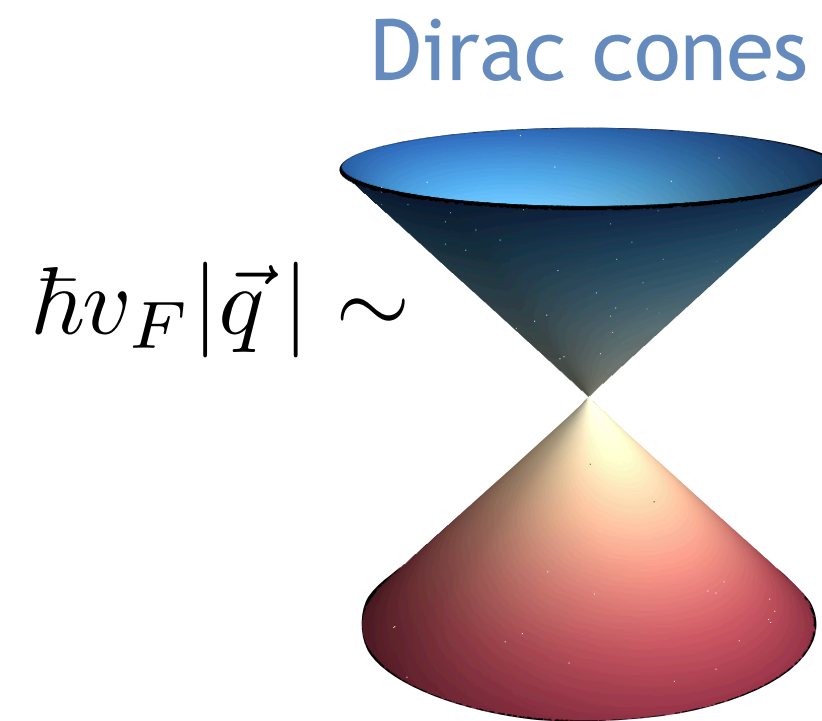


Michael Scherer
(RUB)

Dirac materials

Condensed matter systems with low-energy fermionic excitations which can be effectively described by the Dirac equation → Dirac fermions

$$(i\hbar v_F \gamma^\mu \partial_\mu - \cancel{m} v_F^2) \psi = 0$$



Limit of vanishing Dirac mass $m \rightarrow 0$

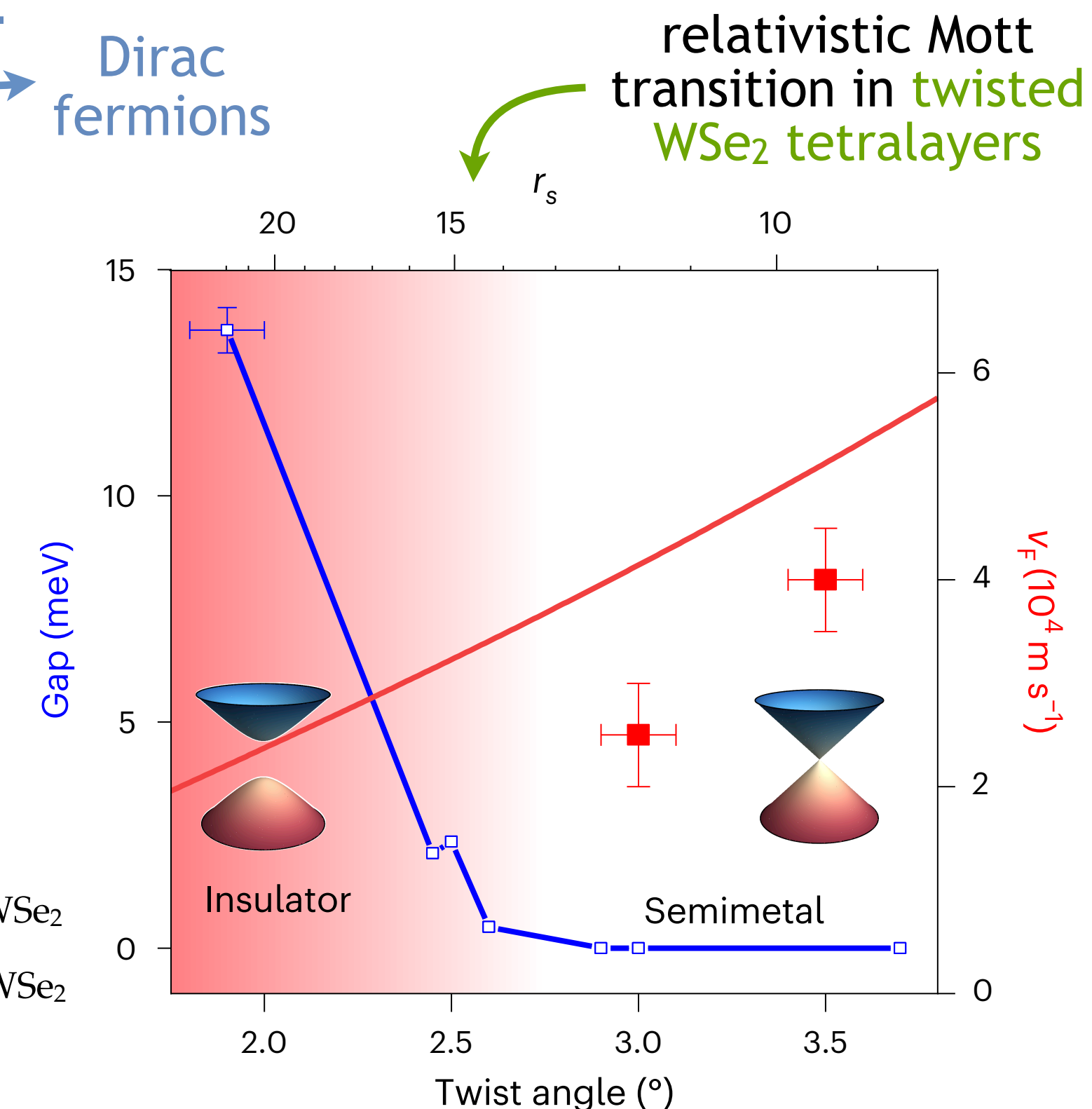
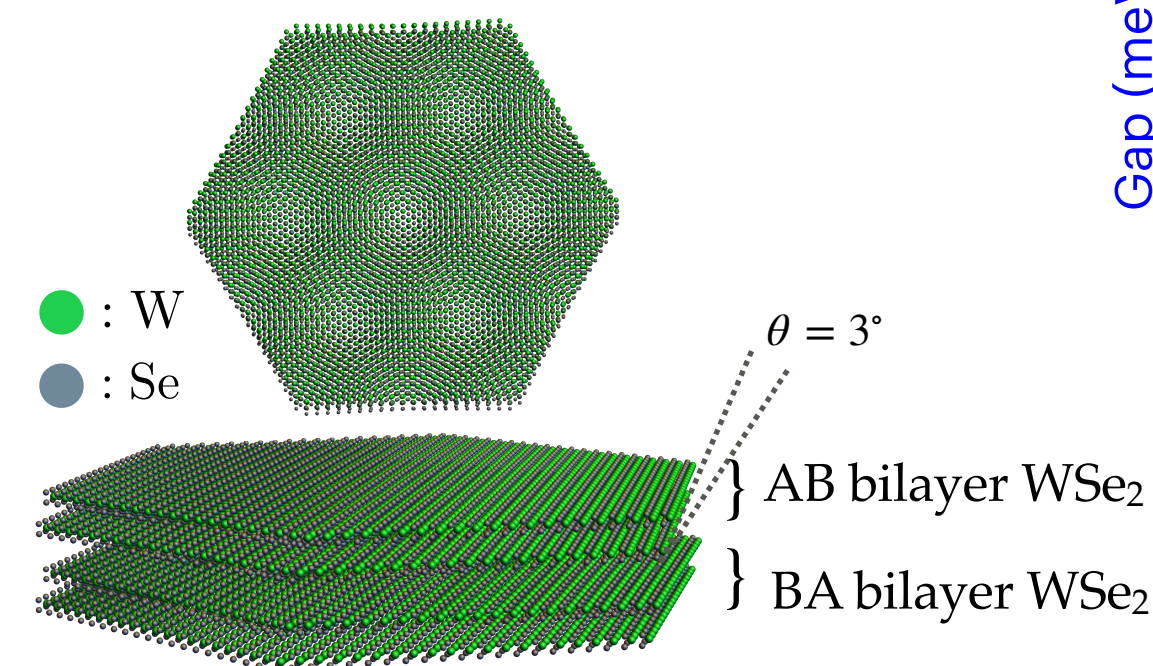
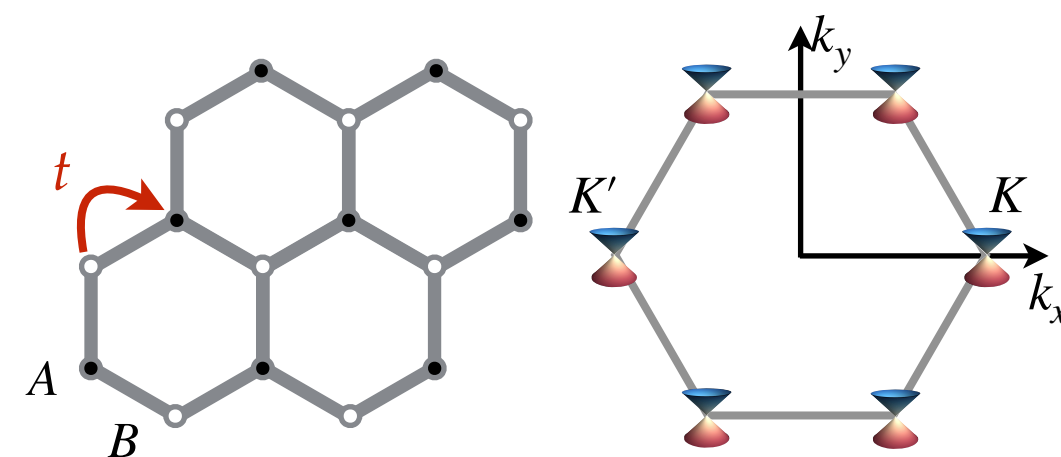
linear dispersion relation

← massless relativistic particles

Graphene

Tight-binding model

$$H_{\text{tb}} = -t \sum_{\langle ij \rangle} (a_i^\dagger b_j + b_j^\dagger a_i)$$



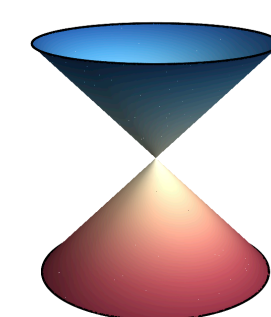
Ma et al., Nat. Mat. 24, 1935-1941 (2025)

Strong repulsive interaction → transition into ordered state → quantum phase transition

Chiral symmetry breaking

Relativistic Mott transition

Insufficient interaction strength



Dirac semimetal

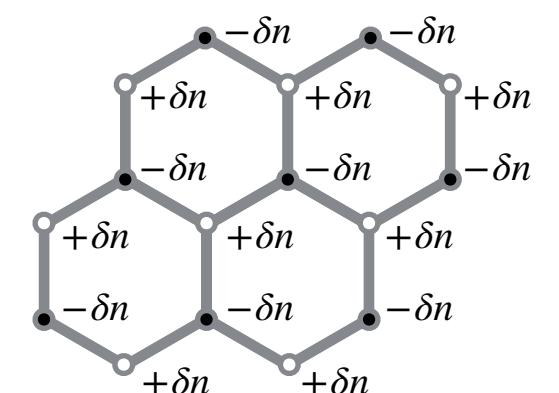
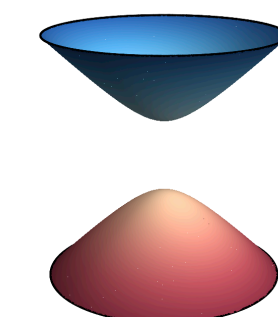
graphene

twisted WSe₂ tetralayer

QCP

int. strength

Ordered phase



Quantum critical Dirac materials

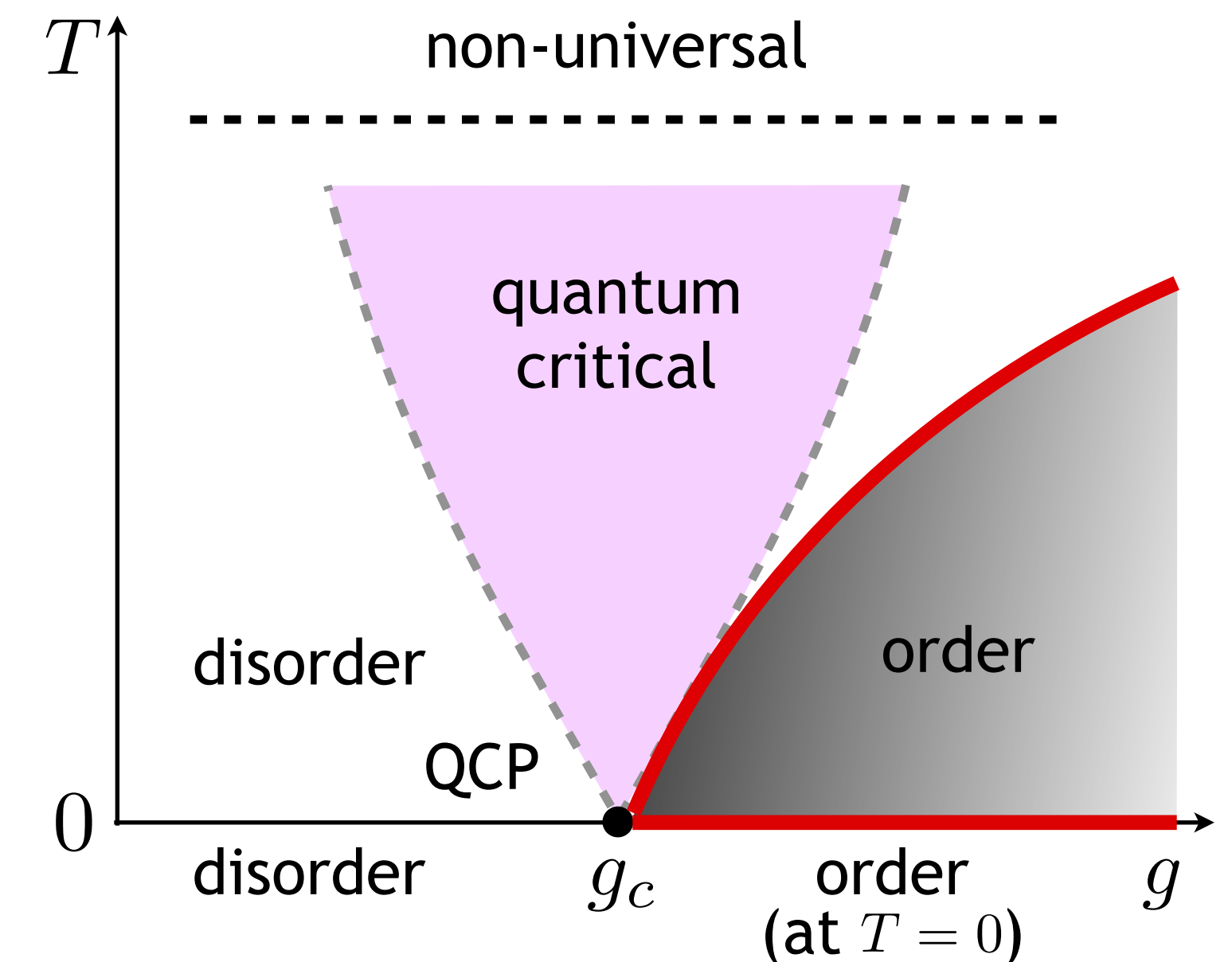
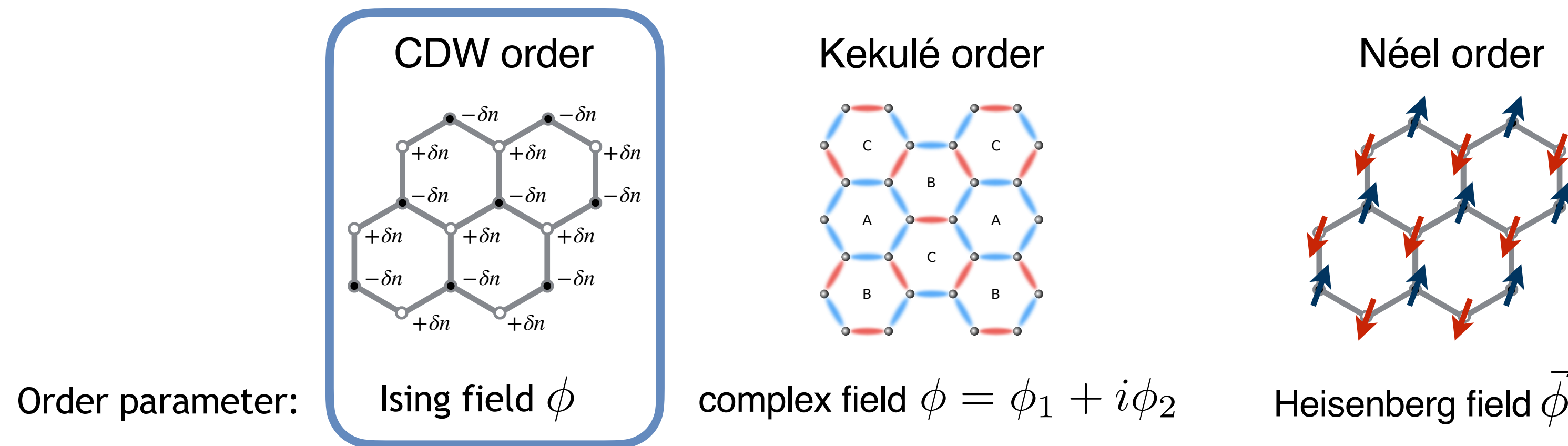
Quantum Phase Transitions (QPT) at $T = 0$ by variation of a non-thermal parameter.

Quantum Critical Point (QCP) influences behavior of electronic systems over wide range of phase diagram, even at finite temperature.

QPT are theoretically predicted and experimentally shown in Dirac materials.

R. Boyack, H. Yerzhakov, J. Maciejko, *EPJ ST* 230, 979-992 (2021)

Typical examples of insulating ordered phases:



Interacting Dirac materials

Effective theory for interacting Dirac materials
(Chiral Ising model)

$$S = \int d^3x \left[\bar{\psi} \gamma^\mu \partial_\mu \psi + \underbrace{\frac{g}{2} (\bar{\psi} \psi)^2}_{\text{Quartic interaction}} \right],$$

Dirac fields

👤 D.D. Scherer, J. Braun, H. Gies, *JPA MT* **46**, 285002 (2013)

👤 L. Janssen, I. Herbut, *PRB* **89**, 205403 (2014)

Interacting Dirac materials

Effective theory for interacting Dirac materials $\rightarrow$ Classical action of the Gross–Neveu–Yukawa model.
(Chiral Ising model)

Diagram illustrating the action S for the Hubbard-Stratonovich bosonic field ϕ . The action is given by:

$$S = \int d^3x \left[\bar{\psi} \gamma^\mu \partial_\mu \psi + h \phi \bar{\psi} \psi - \frac{1}{2} \phi \partial_\mu^2 \phi + \underbrace{\phi (m_\phi^2 + \lambda \phi^2) \phi}_{U(\phi)} \right],$$

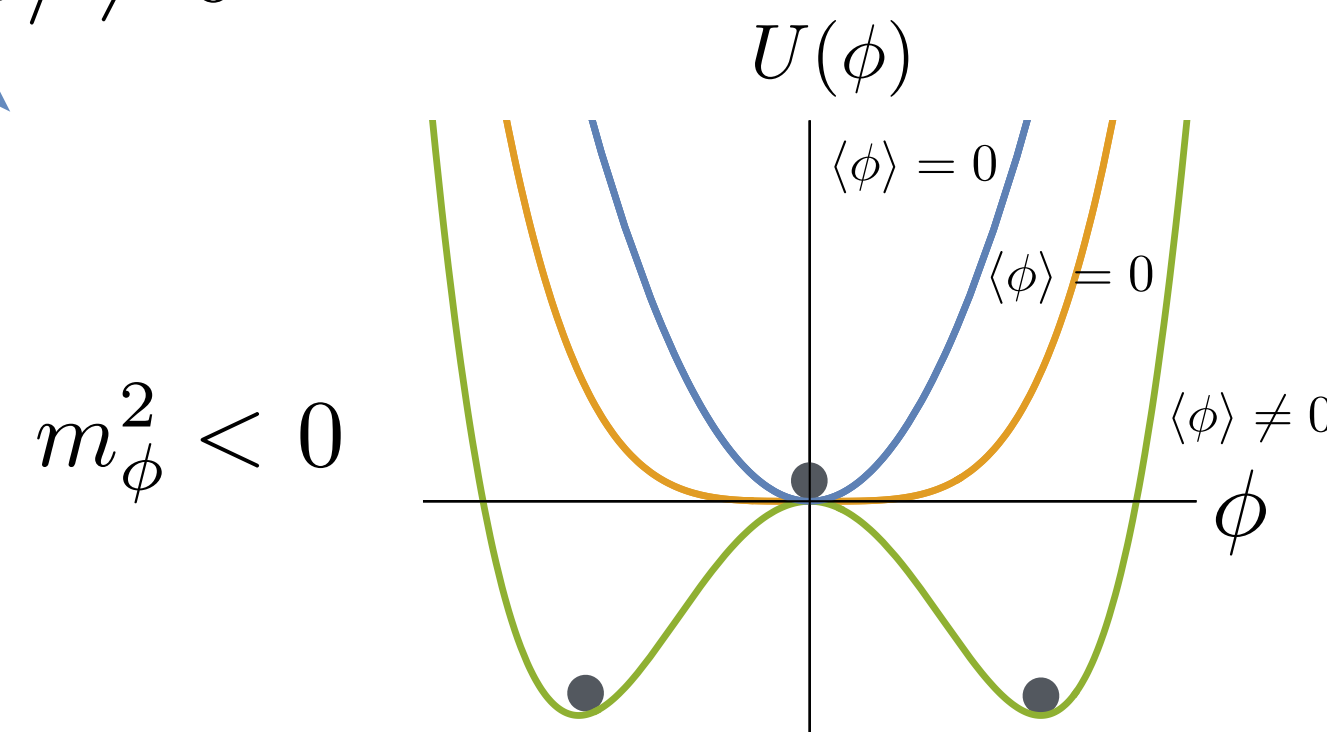
Annotations:

- Dirac fields**: Points to $\bar{\psi} \gamma^\mu \partial_\mu \psi$.
- Yukawa coupling**: Points to $h \phi \bar{\psi} \psi$.
- Hubbard-Stratonovich bosonic field**: Points to ϕ in the Yukawa coupling term, with the relation $\phi \sim \bar{\psi} \psi$ indicated below.
- bosonic mass**: Points to m_ϕ^2 in the potential term.
- coupling**: Points to $\lambda \phi^2$ in the potential term.
- Potential**: Points to the entire potential term $U(\phi) = \phi (m_\phi^2 + \lambda \phi^2) \phi$.

Fermions acquire mass gap $\Delta \sim h\langle\phi\rangle \neq 0$

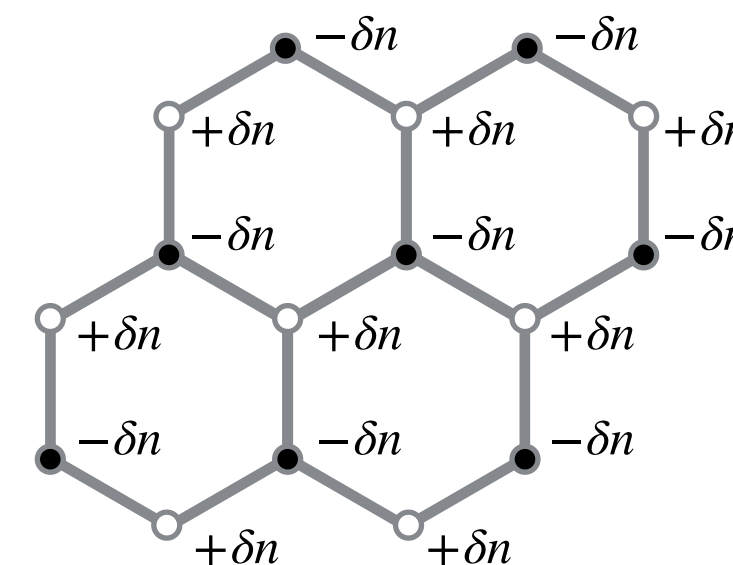
order parameter for
chiral $\mathbb{Z}_2$ symmetry
breaking

Shape of $U(\phi)$
affects the value
of the VEV of ϕ



Example. Graphene

Breaking of sublattice exchange symmetry



To investigate
macroscopic behavior...

Functional Renormalization Group

 D.D. Scherer, J. Braun, H. Gies, *JPA MT* **46**, 285002 (2013)

📌 L. Janssen, I. Herbut, *PRB* **89**, 205403 (2014)

Interacting Dirac materials

Effective theory for interacting Dirac materials → Classical action of the Gross–Neveu–Yukawa model.

(Chiral Ising model)

At microscopic scale, in general, different v_B and v_F , i.e., $v_B \neq v_F$.

Possible emergence of relativistic symmetry at zero temperature close to the QCP.

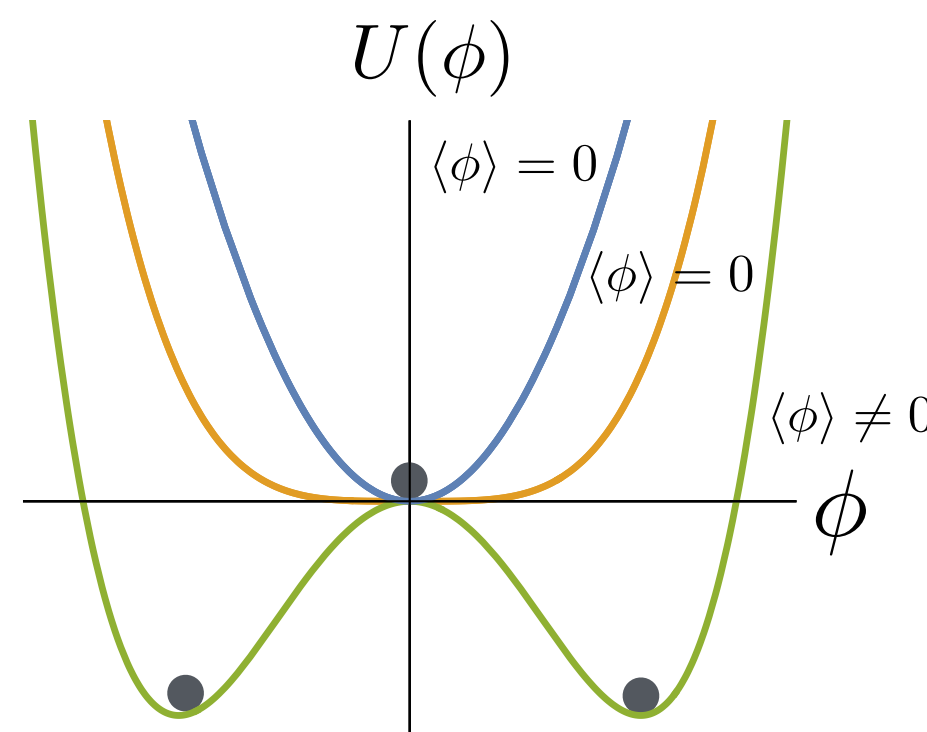
$$S = \int d^3x \left[\bar{\psi} \left(\gamma_0 \partial_\tau + v_F \vec{\gamma} \cdot \vec{\partial} \right) \psi + h\phi \bar{\psi} \psi + \frac{1}{2} \phi \left(-\partial_\tau^2 - v_B^2 \vec{\partial}^2 \right) \phi + \phi (m_\phi^2 + \lambda \phi^2) \phi \right]$$

Fermions acquire mass gap $\Delta \sim h\langle\phi\rangle \neq 0$

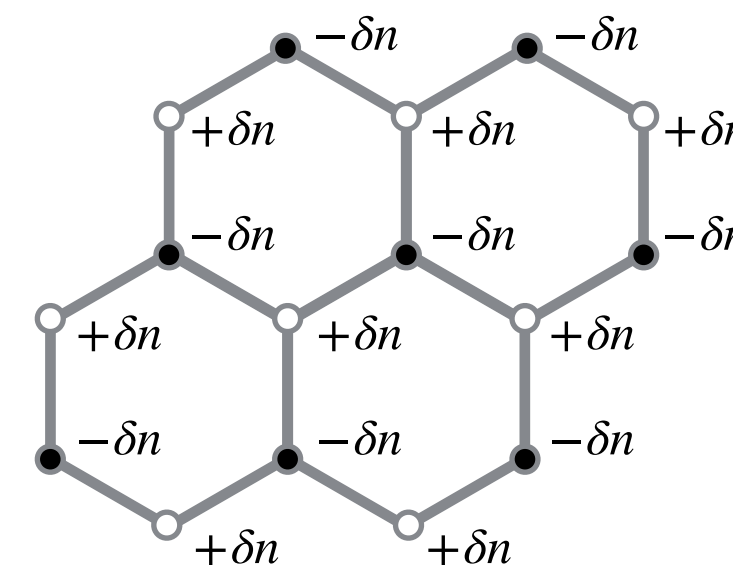
order parameter for
chiral $\mathbb{Z}_2$ symmetry
breaking

$$m_\phi^2 < 0$$

Shape of $U(\phi)$
affects the value
of the VEV of ϕ



Example. Graphene
Breaking of sublattice
exchange symmetry



To investigate
macroscopic behavior...

Functional
Renormalization
Group

☞ D.D. Scherer, J. Braun, H. Gies, *JPA MT* **46**, 285002 (2013)

☞ L. Janssen, I. Herbut, *PRB* **89**, 205403 (2014)

Functional Renormalization Group

The k evolution of Γ_k is determined by the [Wetterich equation](#)

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr} \left\{ \left[\Gamma_k^{(2)}[\Phi] + R_k \right]^{-1} (\partial_t R_k) \right\}, \quad \rightarrow \quad \boxed{\begin{array}{l} \beta \text{ functions} \\ \beta(g_i) = \partial_t g_i \\ \beta(g_i^*) = 0 \end{array}} \quad \leftarrow \quad \begin{array}{l} \text{Critical points correspond} \\ \text{to fixed points of the} \\ \text{flow equation} \end{array}$$

where R_k is an IR regulator and the RG time is $t = \ln(k/\Lambda)$.

Introduce [finite temperature](#)

$$q_0 \rightarrow \omega_n = 2\pi c_n T, \quad \int \frac{d^d q}{(2\pi)^d} \rightarrow T \sum_{n \in \mathbb{Z}} \int \frac{d^{d-1} q}{(2\pi)^{d-1}} \quad \text{with } \tau = 2\pi T/k$$

[Matsubara frequencies](#) with $c_n^F = n + 1/2, \quad c_n^B = n$

Ansatz [effective average action](#)

$$\Gamma_k = \int_0^{1/T} d\tau \int d^{d-1} x \left[Z_{\psi,k} \bar{\psi} D_{\psi,k} \psi - \frac{1}{2} Z_{\phi,k} \phi D_{\phi,k} \phi + h_k \phi \bar{\psi} \psi + U_k(\rho) \right] \quad \text{with} \quad \begin{array}{l} D_{\phi,k} = \partial_0^2 + v_{B,k}^2 \vec{\partial}^2, \\ D_{\psi,k} = \gamma_0 \partial_0 + v_{F,k} \vec{\gamma} \cdot \vec{\partial}. \end{array}$$

Critical exponents

Chiral Ising model in $d = 2 + 1$ at $T = 0$

Critical exponents $N_f = 2$

Very good
theoretical
agreement!

Chiral Ising	$1/\nu$	η_ϕ	η_ψ
<i>this work</i> , FRG (LPA'4)	1.00	0.76	0.032
FRG (NLO) [72]	0.994(2)	0.7765	0.0276
ϵ -exp w/ DREG3 [86]	0.993(27)	0.704(15)	0.043(12)
conformal bootstrap [43]	0.998(12)	0.7329(27)	0.04238(11)
QMC [46]	1.07(12)	0.72(6)	0.04(2)

👤 B. Knorr, *PRB* **94**, 245102 (2016)

👤 B. Ihrig, L. N. Mihaila, M. M. Scherer *PRB* **98**, 125109 (2018)

👤 R. S. Erramilli et al., *JHEP* 02(2023)036

👤 T-T. Wang, Z.Y. Meng, *PRB* **108**, L121112 (2023)

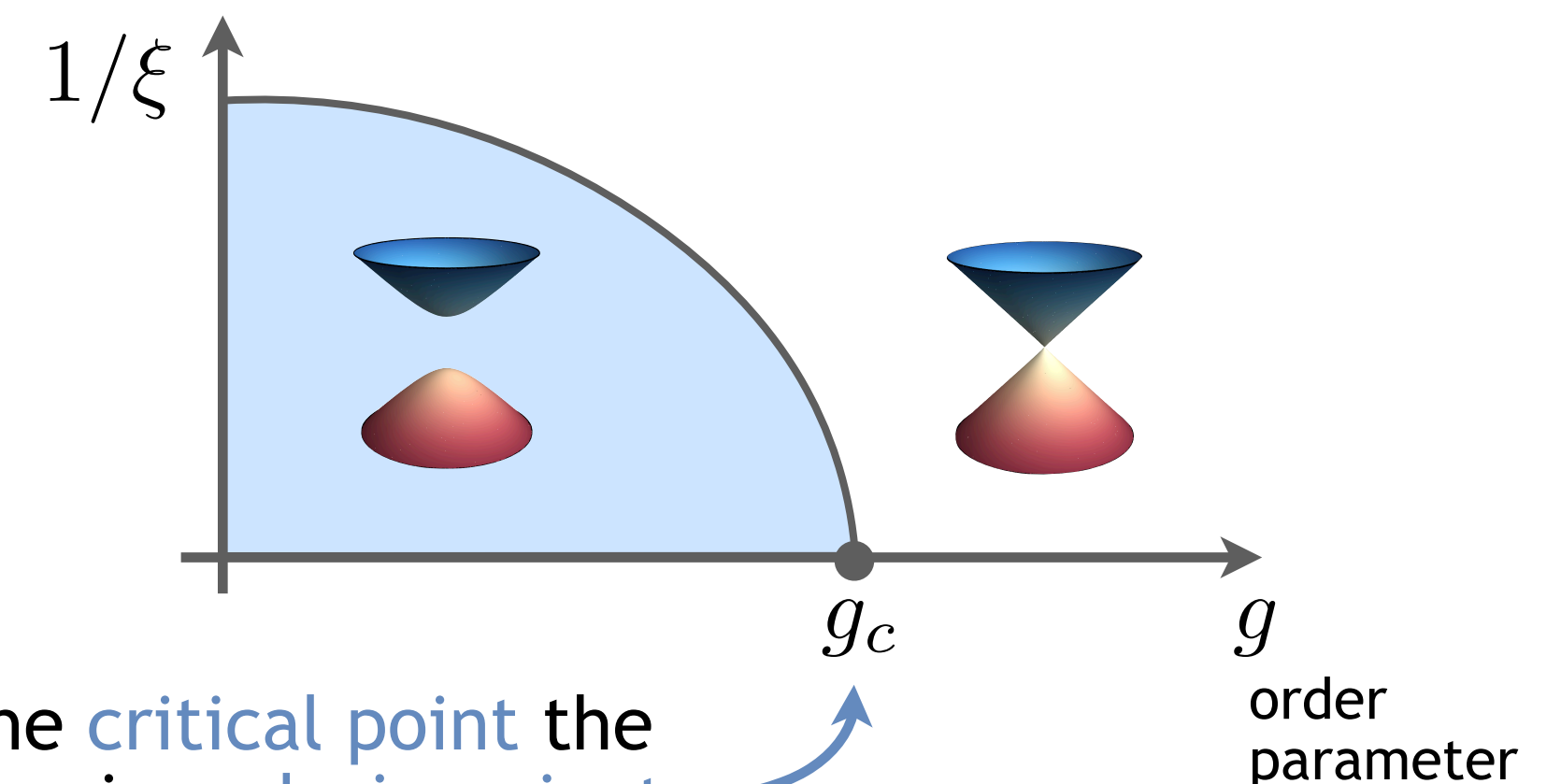
Correlation length diverges at criticality

$$\xi \propto |g - g_c|^{-\nu}$$

Correlation function at criticality

$$G_\psi(x) \propto \frac{1}{|x|^{d-1+\eta_\psi}} \quad G_\phi(x) \propto \frac{1}{|x|^{d-2+\eta_\phi}}$$

👤 MTS, L. Classen, M. M. Scherer, *PRB* **112**, 115133 (2025)



At the **critical point** the
system is **scale-invariant**
and $\xi \rightarrow \infty$.

Finite temperature phase diagram

Characteristic length scale (SYM)

$$\xi = \frac{1}{m_\phi} \propto T^{-1/a}$$



$$T \gg T_{\text{crit}}$$

Quantum critical fan (QCF)

$$T_{\text{QCF}} = A_{\pm} |g - g_c|^{z\nu}$$

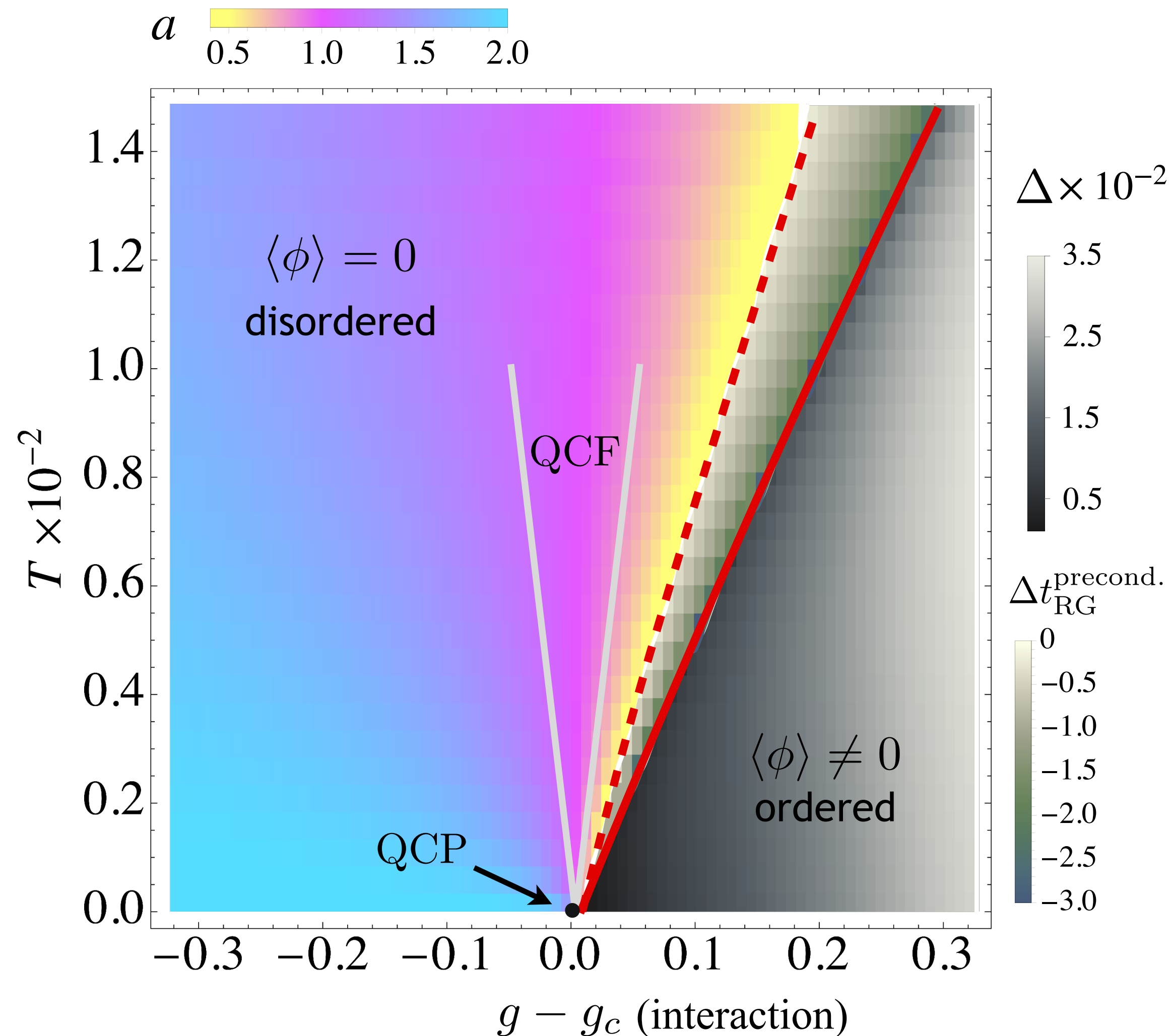


Dynamical critical exponent z ,

$$\xi \propto T^{-1/z}$$

with $a = z = 1$

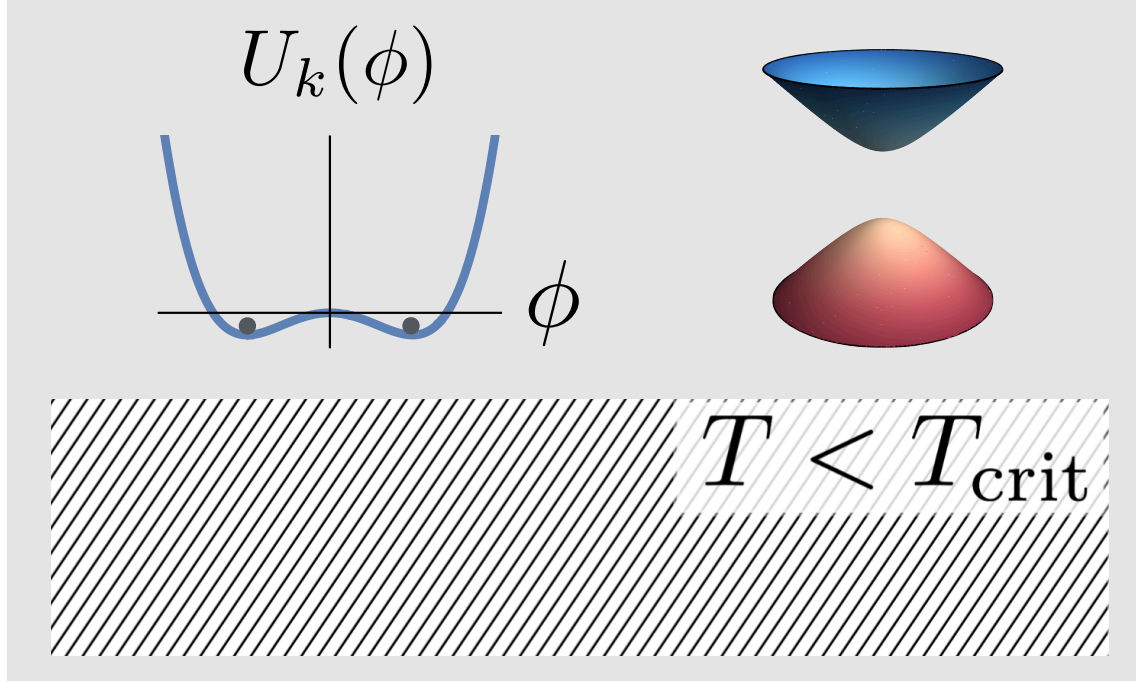
Chiral Ising model $N_{\text{Goldstone}} = 0$



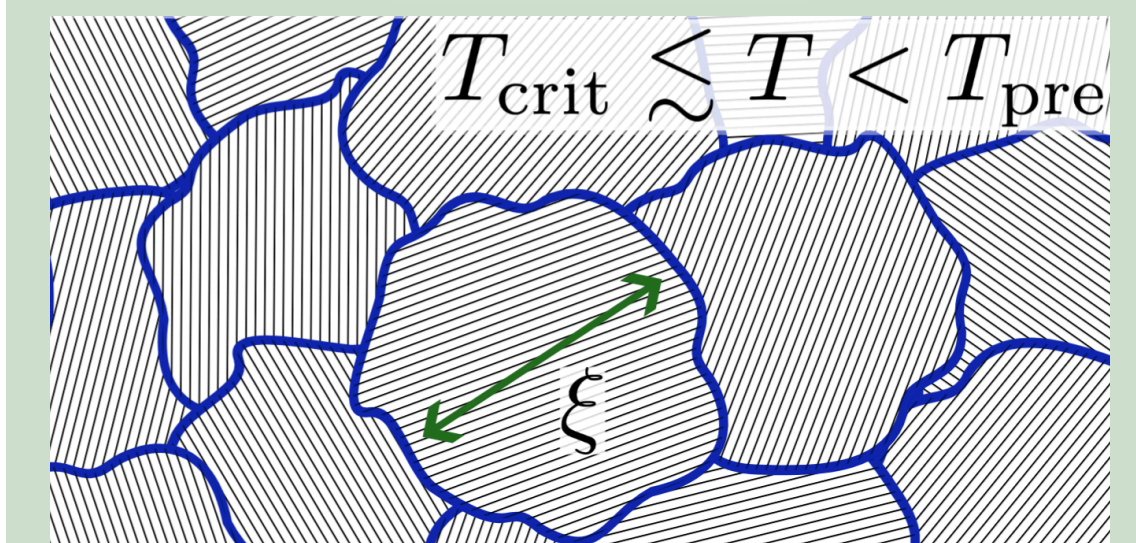
— Spontaneous chiral symmetry breaking

Fermion mass gap

$$\Delta \propto \langle \phi \rangle \text{ (SSB)}$$



Size of the precondensation regime $\Delta t_{\text{RG}}^{\text{precond.}}$ (SYM)



Finite temperature phase diagram

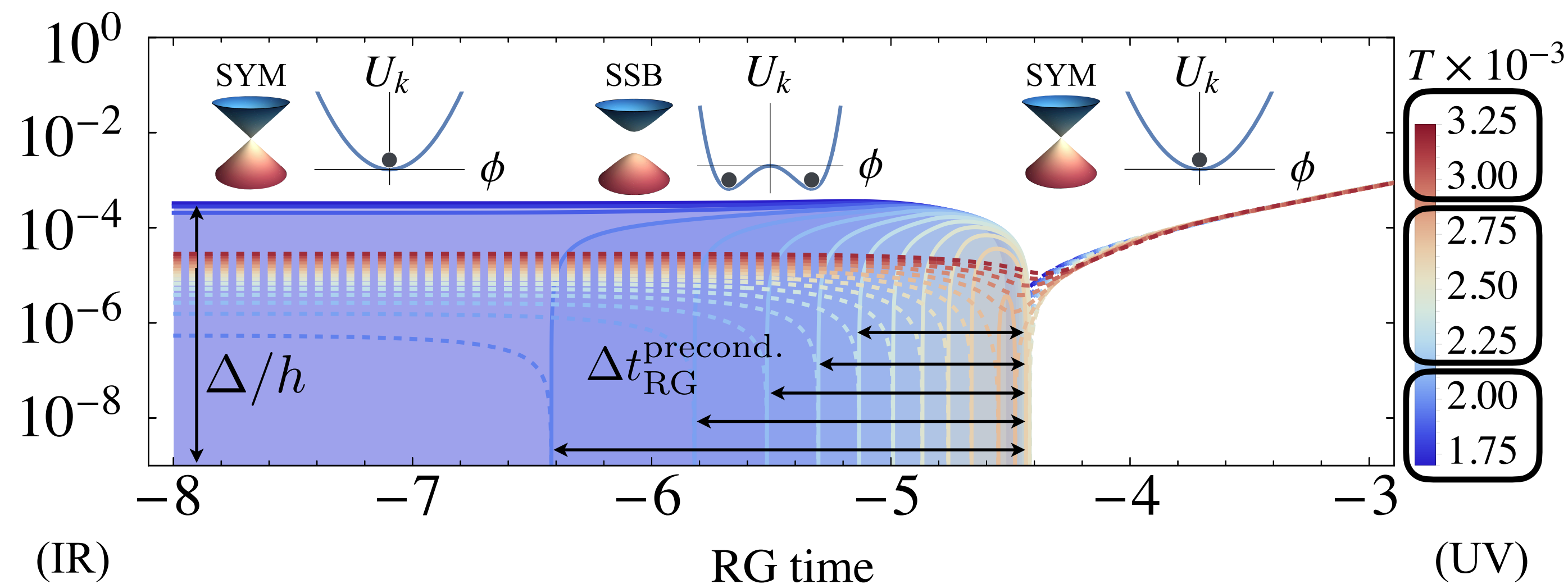
Chiral Ising model $N_{\text{Goldstone}} = 0$

RG flow of the effective potential:

(for a fixed coupling $g = g_c$)

----- m_ϕ^2 (SYM)

— $\langle \phi \rangle$ (SSB)

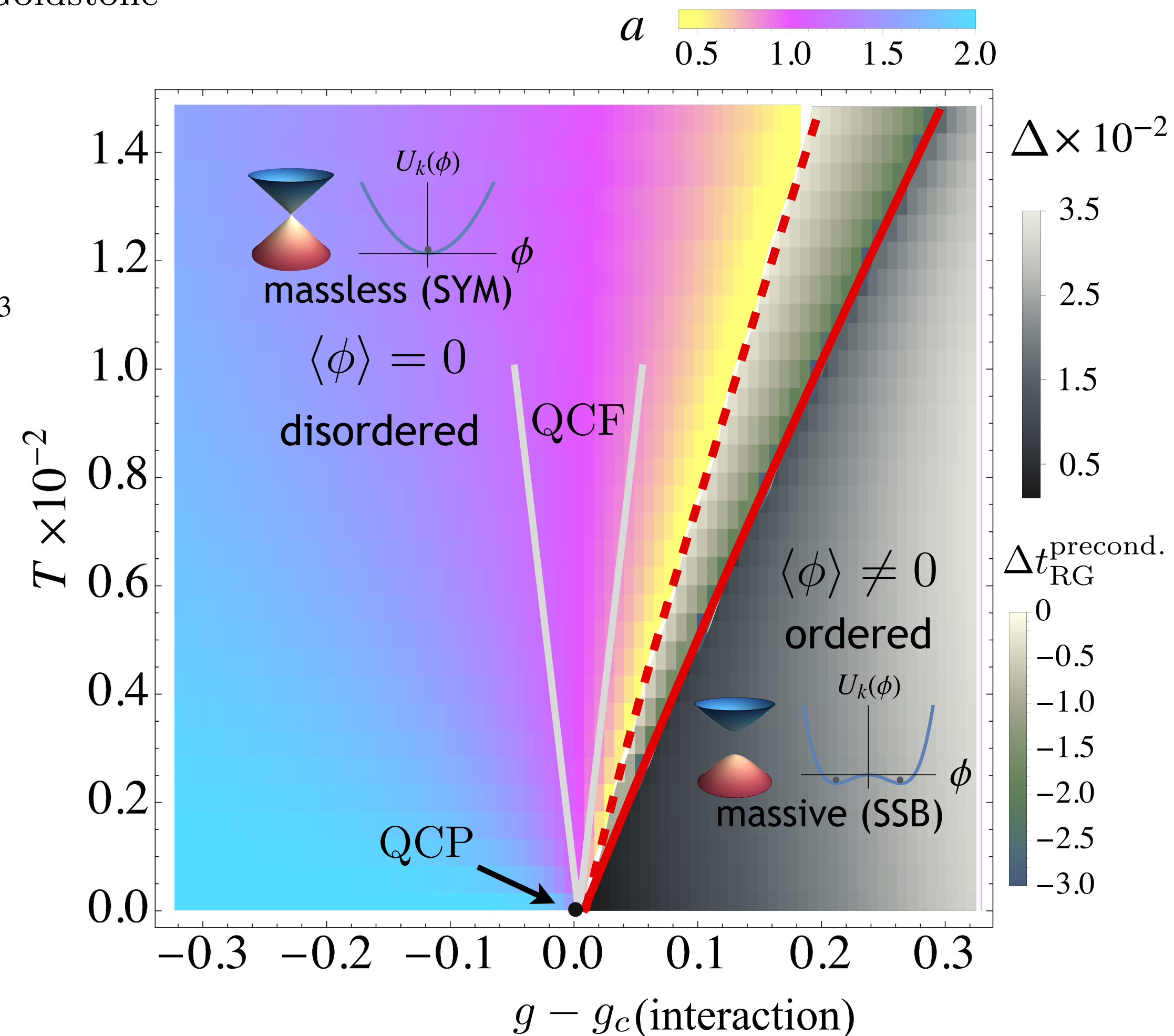


Fermion mass gap

$$\Delta \sim h \langle \phi \rangle \neq 0 \text{ (SSB)}$$

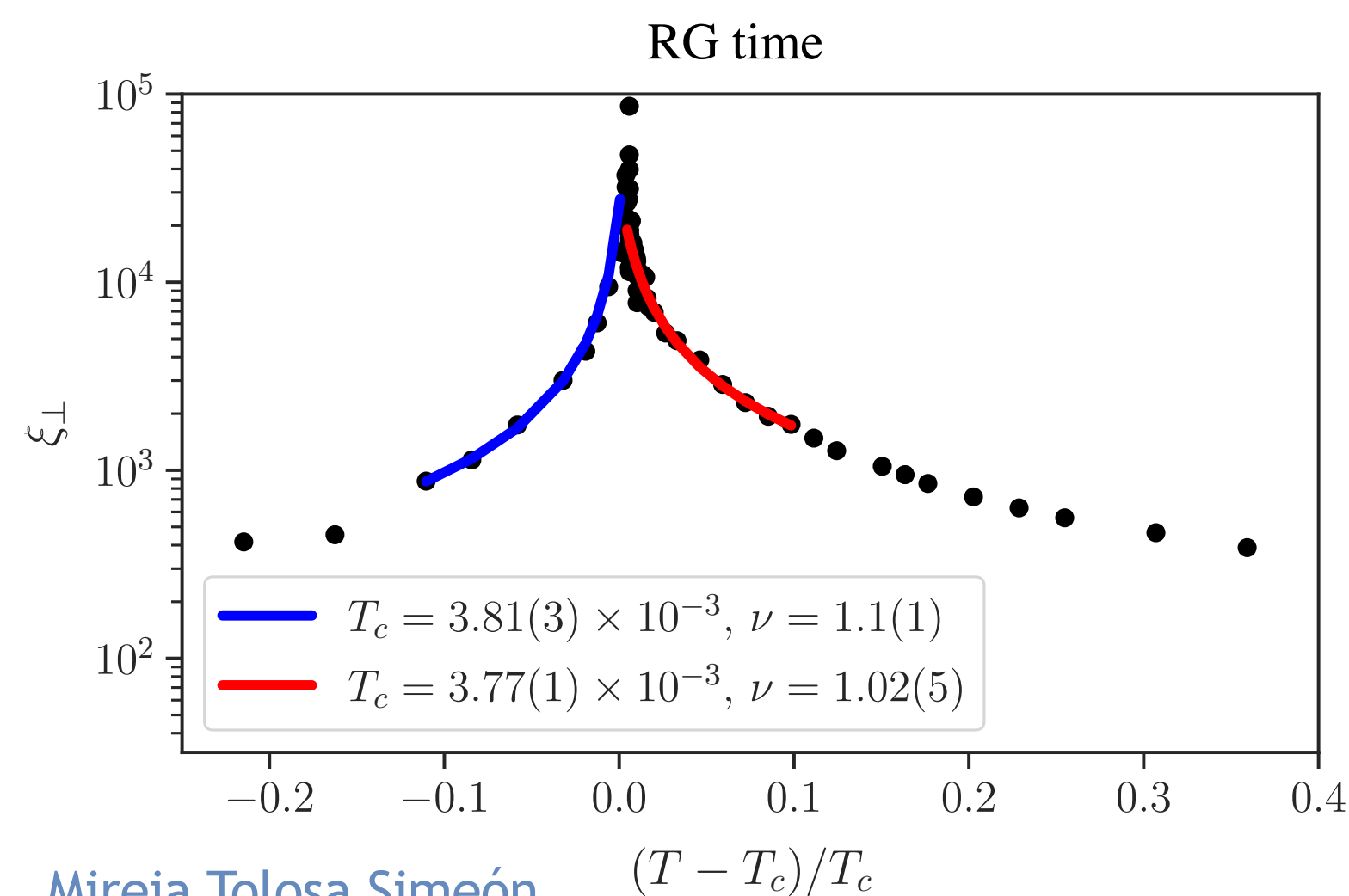
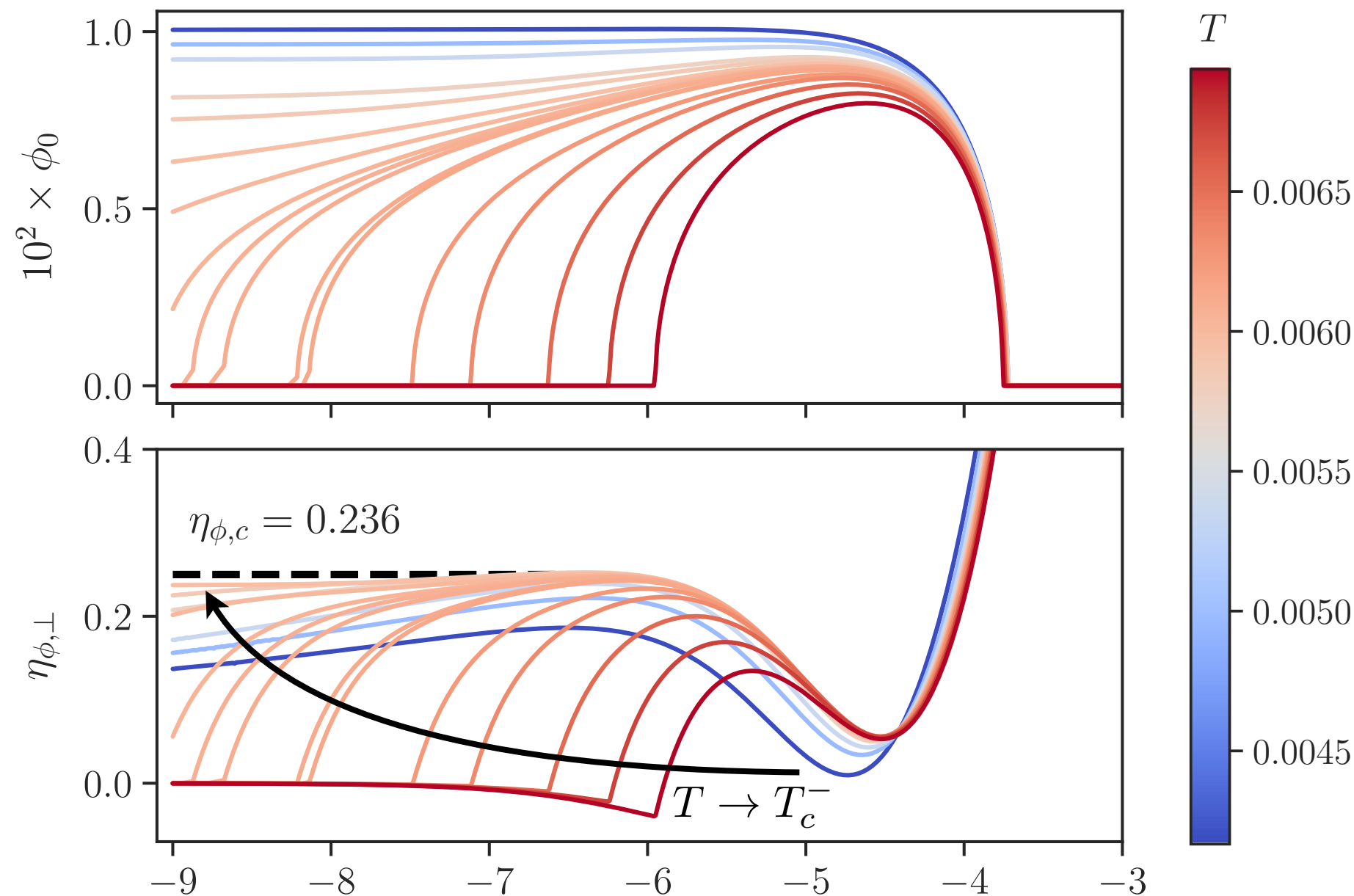
Correlation length

$$\xi = \frac{1}{m_\phi} \propto T^{-1/a} \text{ (SYM)}$$



Finite temperature phase diagram

Chiral Ising model $N_{\text{Goldstone}} = 0$



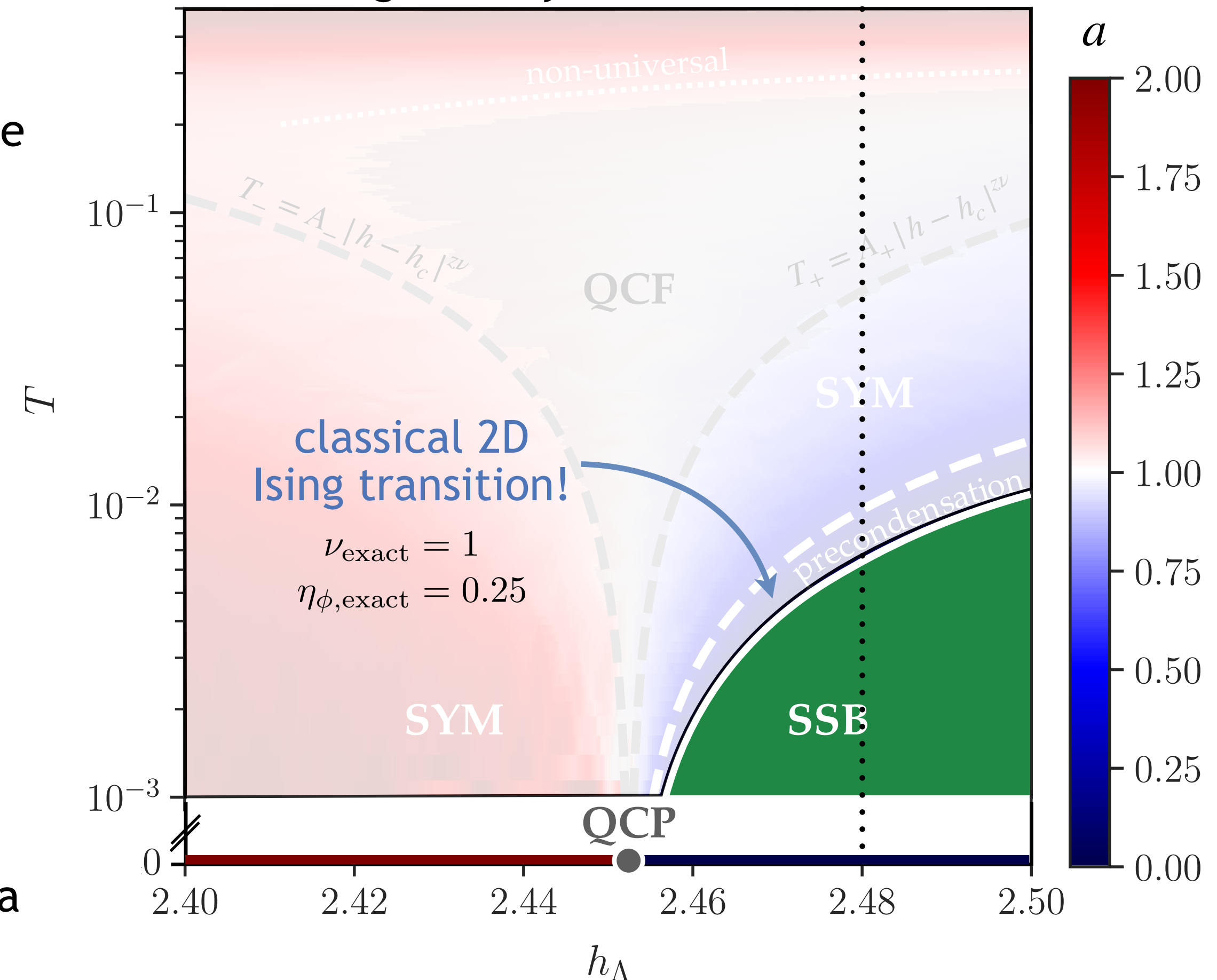
In the vicinity of the finite-temperature phase transition.

Finite-temperature phase transition

$$\xi_{\perp} \propto |T - T_c|^{\nu}$$

Correlation length develops a singularity as expected.

DiFfRG framework.
Using fluid dynamical methods.



F. R. Sattler, J. M. Pawłowski, *Computer Physics Communications* **327**, 110262 (2026)

F. Ihssen, B. Hawashin, *MTS*, M. M. Scherer, arXiv:2607.18396 (2026)

1st September 2026

Emergence of relativistic symmetry at $T = 0$

Chiral Ising model $N_{\text{Goldstone}} = 0$

$$\Gamma_k = \int_0^{1/T} d\tau \int d^{d-1}x \left[Z_{\psi,k} \bar{\psi} \left(\gamma_0 \partial_0 + v_{F,k} \vec{\gamma} \cdot \vec{\partial} \right) \psi - \frac{1}{2} Z_{\phi,k} \phi \left(\partial_0^2 + v_{B,k}^2 \vec{\partial}^2 \right) \phi + h_k \phi \bar{\psi} \psi + U_k(\rho) \right]$$

For $v_B/v_F = 1$, the microscopic action is fully **Lorentz symmetric**.

With a **covariant regulator**, the velocity flows are always in the Lorentz-symmetric subspace. $\rightarrow$ $v_B/v_F = 1$ is a **fixed point** of the flows of v_B and v_F .

Consider **linear perturbations** $\delta r = \frac{v_B^2}{v_F^2} - 1$ out of the subspace with relativistic symmetry.

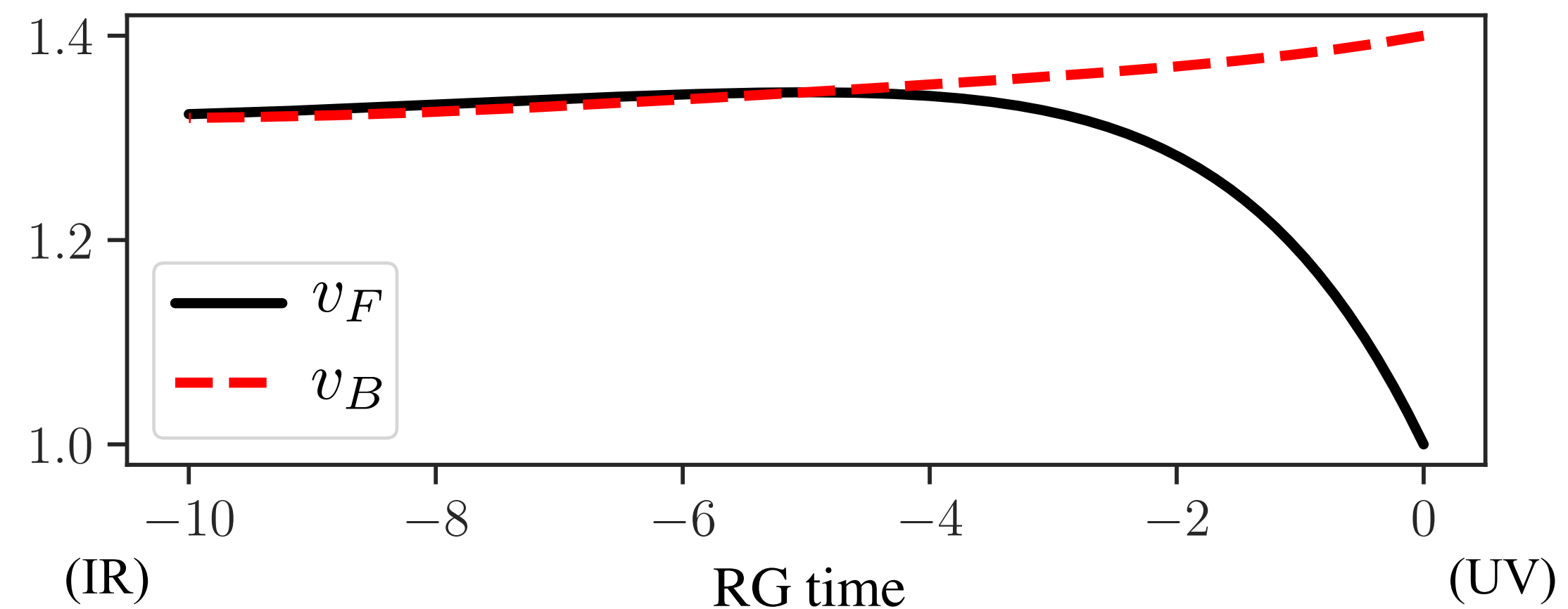
For the **covariant Litim regulator**, we find

$$\partial_t \left(\frac{v_B^2}{v_F^2} \right) = C \left(\frac{v_B^2}{v_F^2} - 1 \right) + \mathcal{O}(\delta r^2)$$

with $C > 0$ at the QCP and on both sides of the transition.

Small perturbations out of this subspace are **irrelevant** close to the QCP.

Relativistic symmetry emerges in the IR



B. Roy, V. Juričić, I. F. Herbut, *JHEP* **04**, 018 (2016)

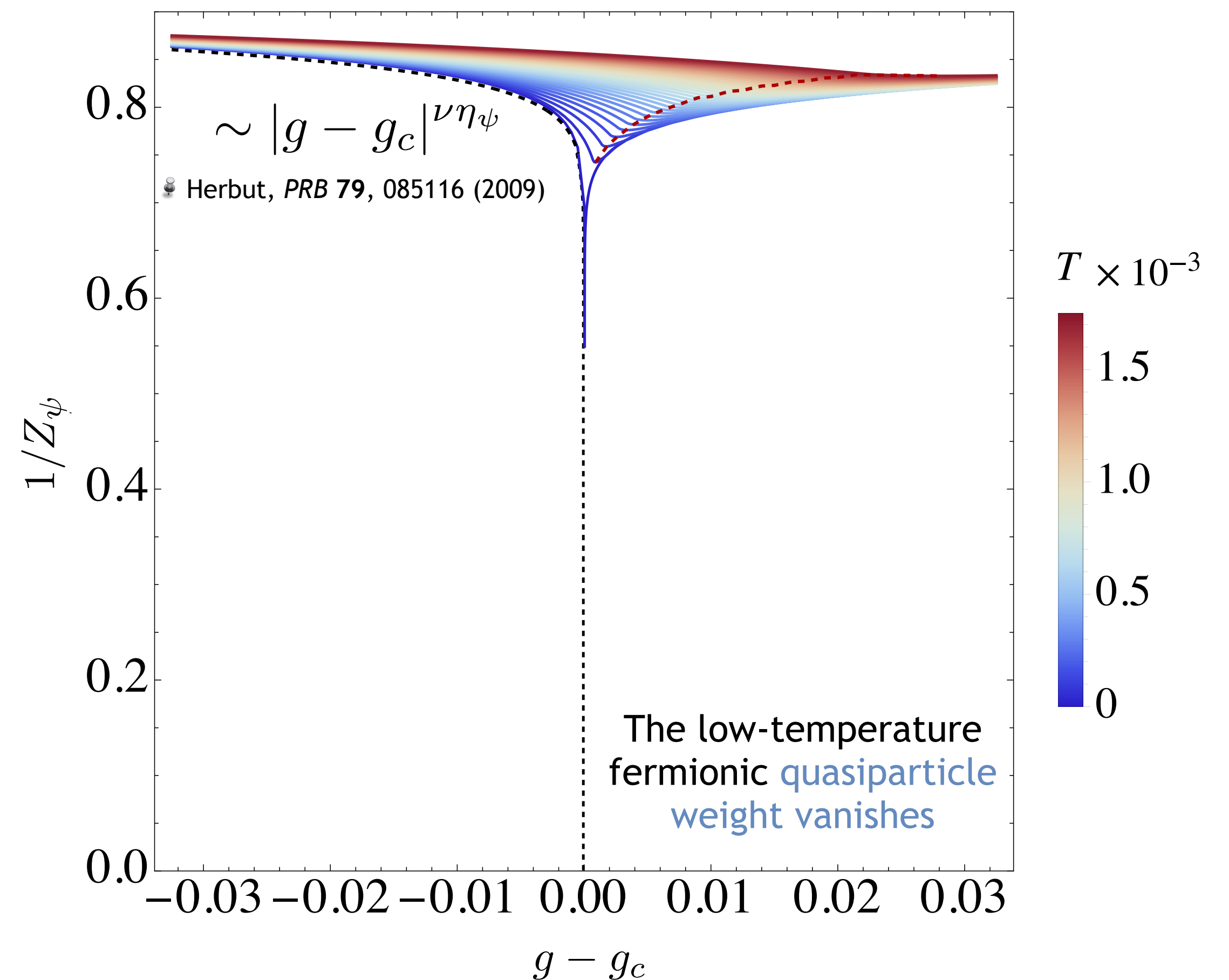
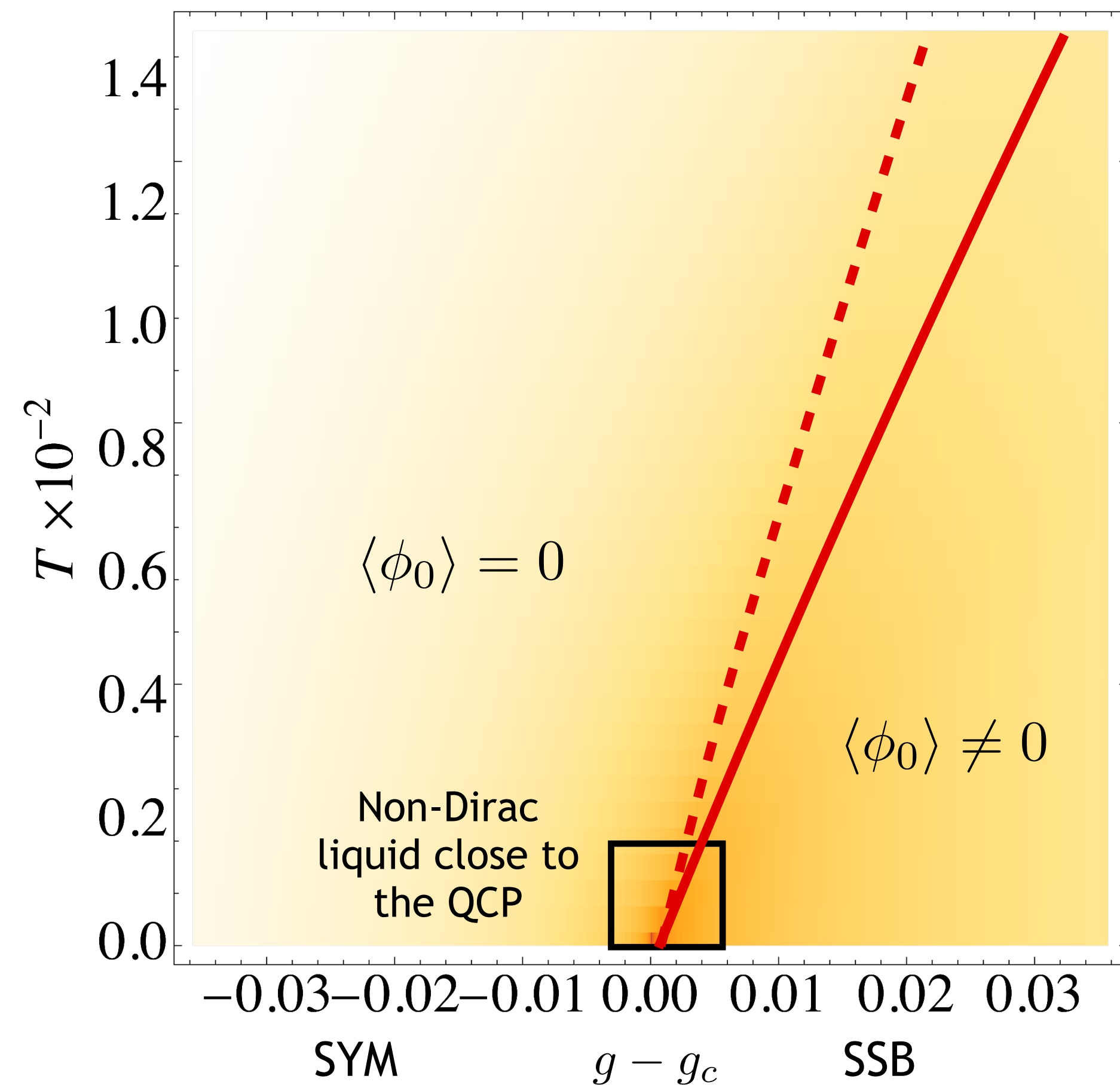
Z. H. Liu, L. Janssen, arXiv:2603.22259v2 (2026)

F. Ihssen, B. Hawashin, *MTS*, M. M. Scherer, arXiv:2607.18396 (2026)

Quasiparticle weight

Chiral Ising model $N_{\text{Goldstone}} = 0$

Full fermionic propagator: $G_\psi(q) = \frac{1}{Z_\psi} \frac{\not{q} + m}{q^2 - m^2 + i\epsilon}$ with $Z_\psi = 1 - \left. \frac{d\Sigma}{d\not{q}} \right|_{\not{q}=m}$ ← Quasiparticle weight $\sim 1/Z_\psi$



Finite temperature phase diagram

Chiral XY and Heisenberg models $N_{\text{Goldstone}} > 0$

$$S = \int d^3x \left[\bar{\psi} (I_{N_f} \otimes \gamma^\mu) \partial_\mu \psi - \frac{1}{2} \phi_a \partial_\mu^2 \phi_a + h \phi_a \bar{\psi} M_a \psi + \frac{1}{2} m_\phi^2 \phi_a \phi_a + \frac{1}{8} \lambda (\phi_a \phi_a)^2 \right]$$

chiral XY $M_a = I_2 \otimes \sigma_a \otimes \sigma_2$ with $a = \{1, 2\}$

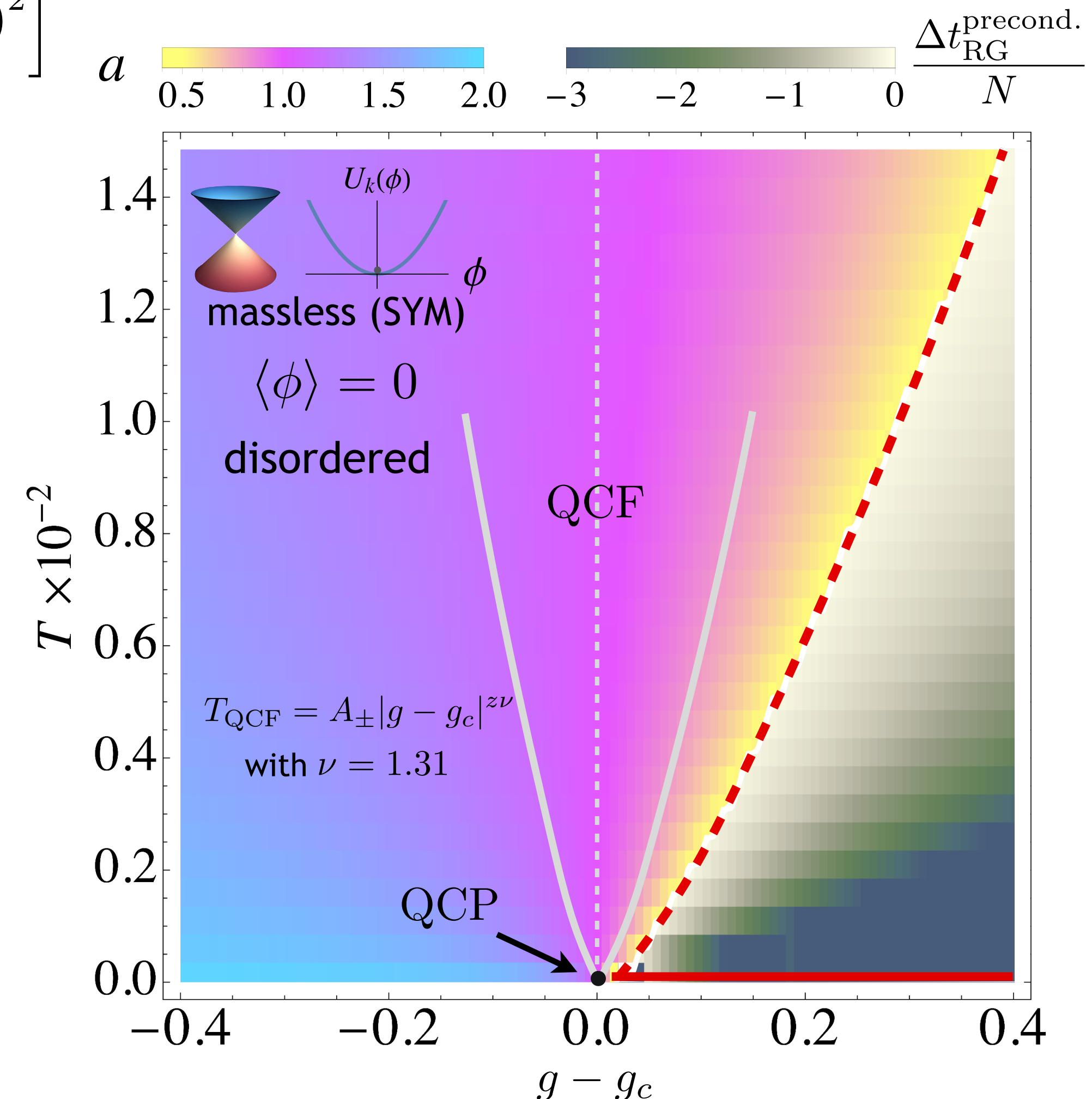
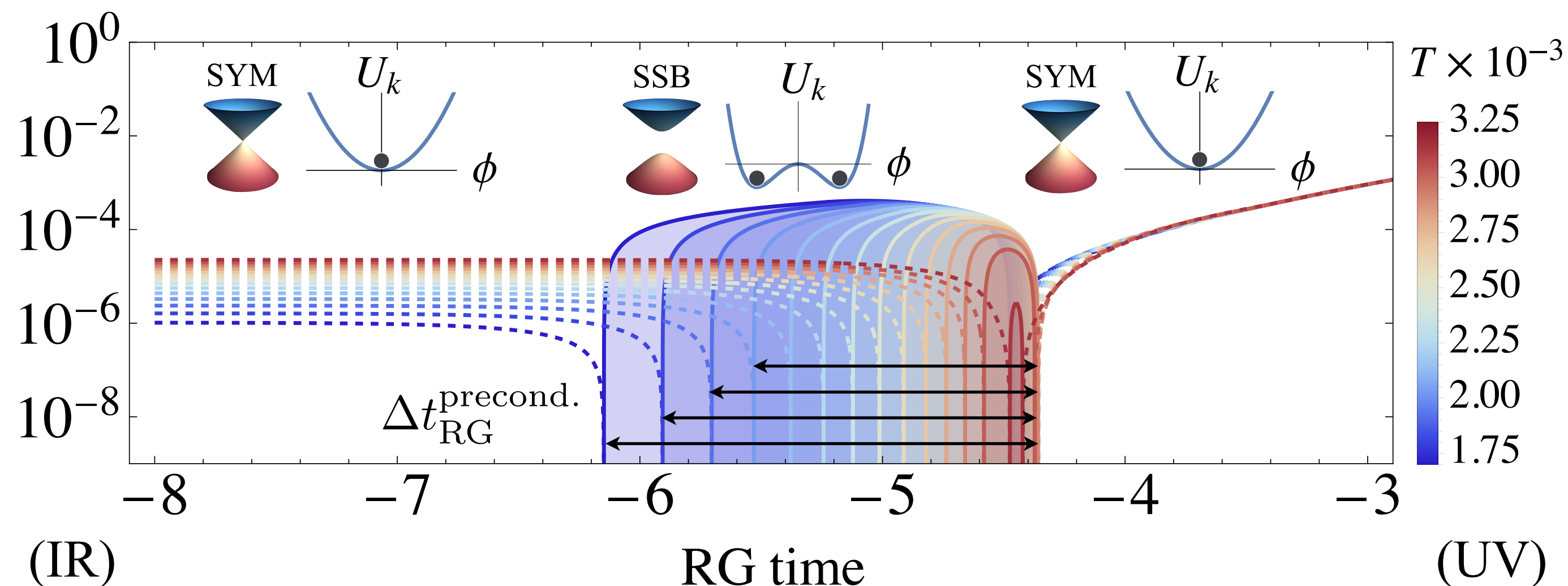
chiral Heisenberg $M_a = \sigma_a \otimes I_4$ with $a = \{1, 2, 3\}$

Continuous $O(N)$ SSB at finite T in $d = 2 + 1$ at intermediate RG scale

FORBIDDEN
by Mermin-Wagner theorem

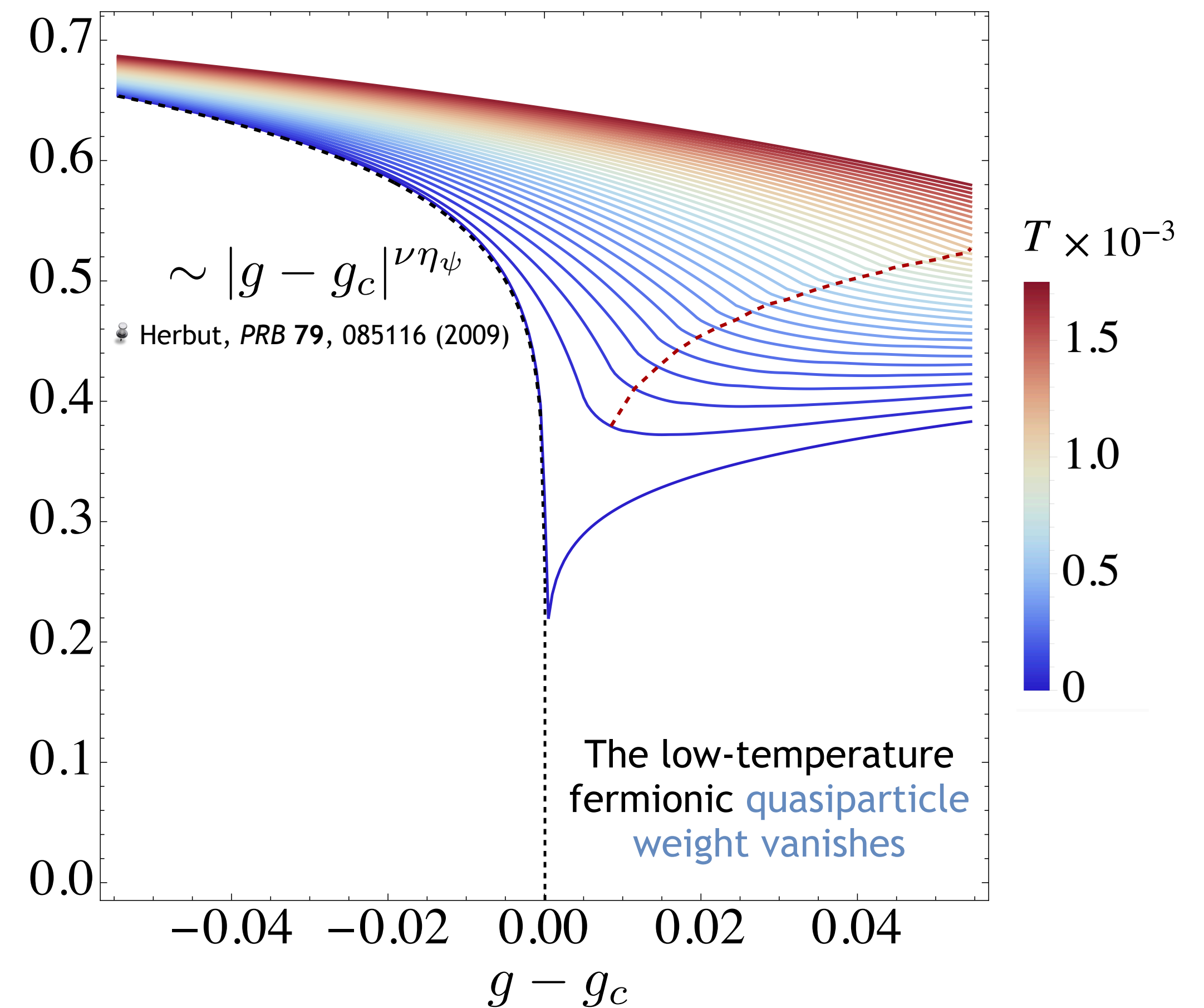
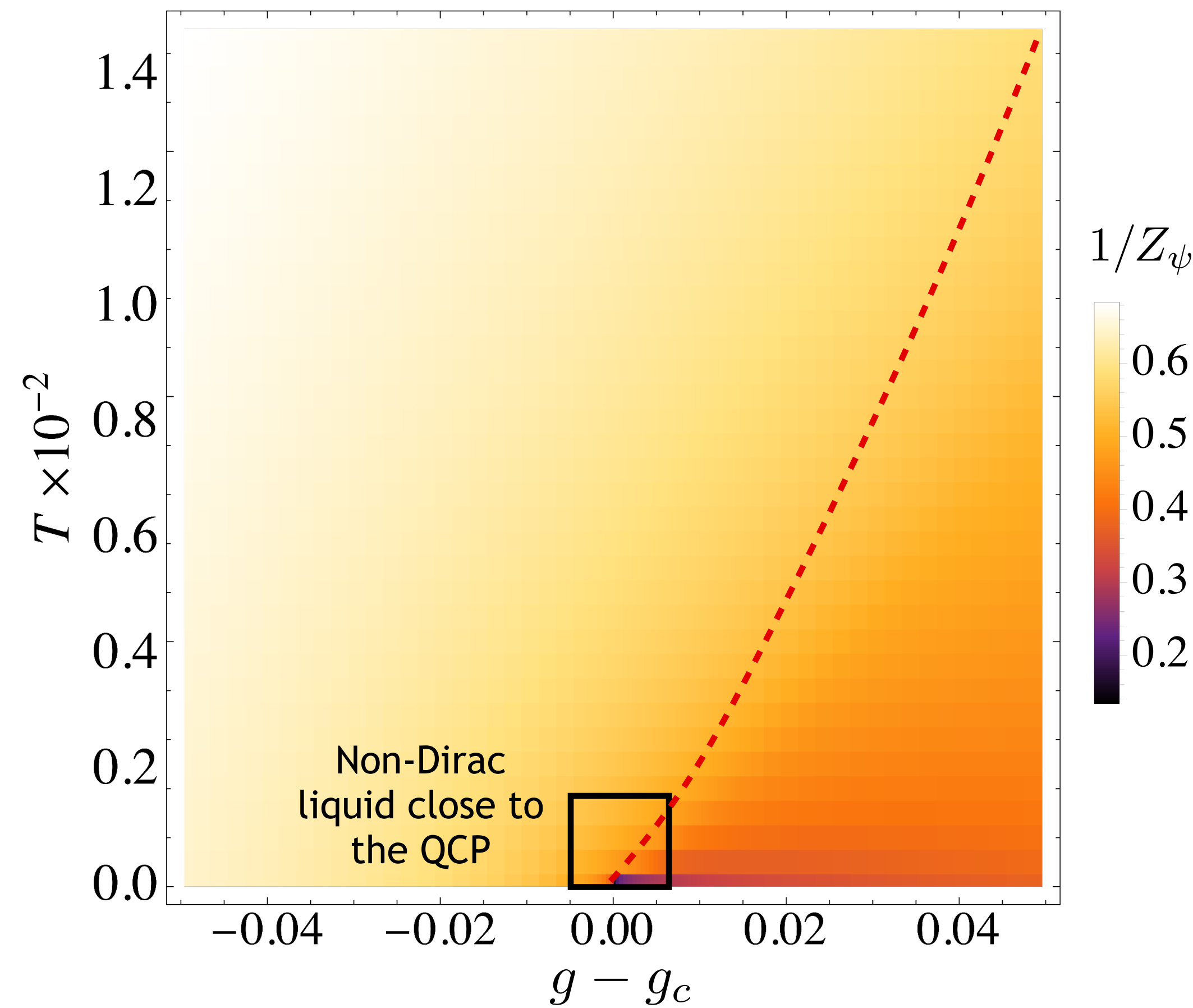
Goldstone modes melt down
condensation as $t \rightarrow t_{IR}$

NO condensation at finite T
only **precondensation**



Quasiparticle weight

Chiral XY and Heisenberg models $N_{\text{Goldstone}} > 0$

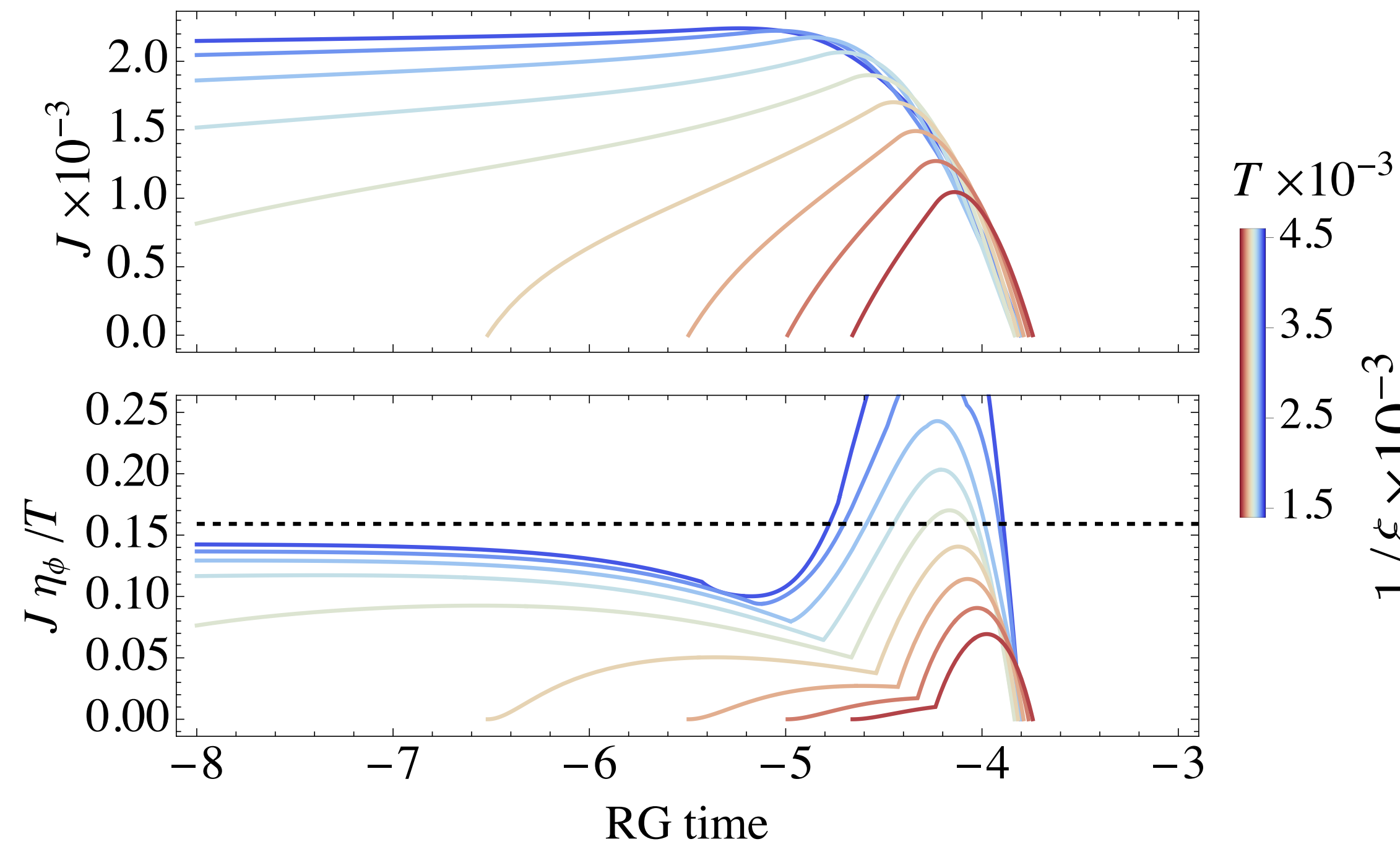


Signatures of BKT transition

Chiral XY model $N_{\text{Goldstone}} = 1$

Phase stiffness J finite $\rightarrow$ very long RG flow.

Low temperature limit $J\eta_\phi/T \rightarrow 1/(2\pi)$.

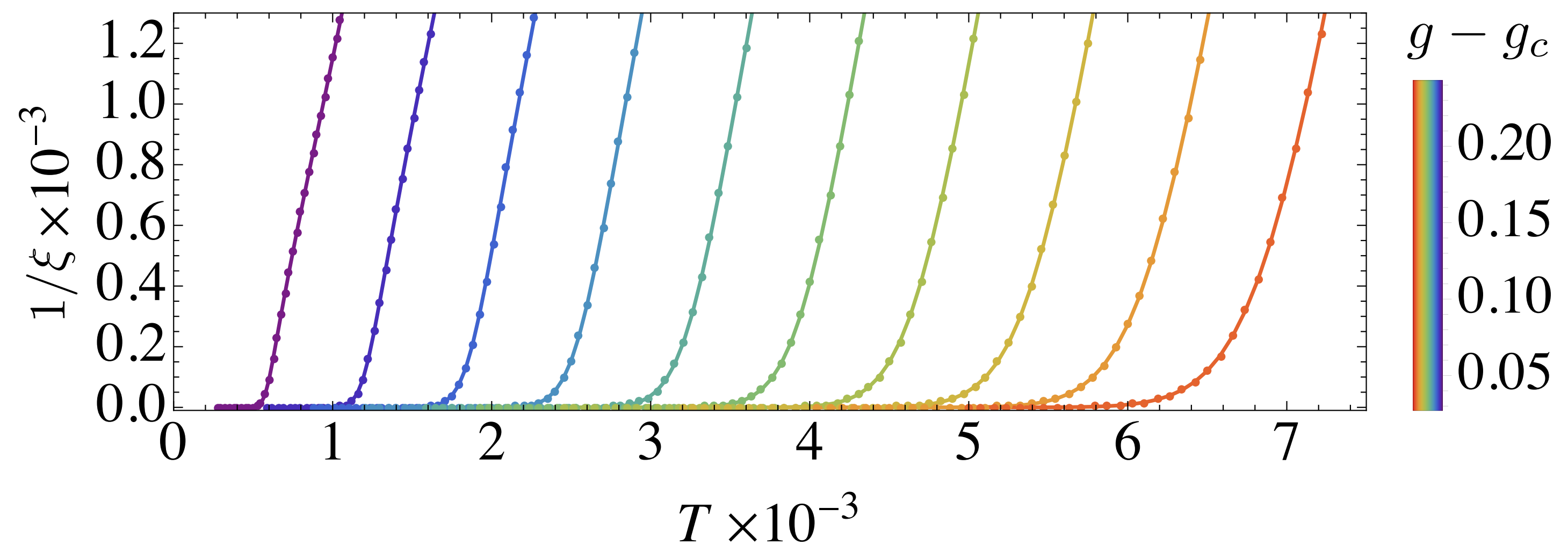


Phase fluctuations prevent LRO at finite T , but qLRO.

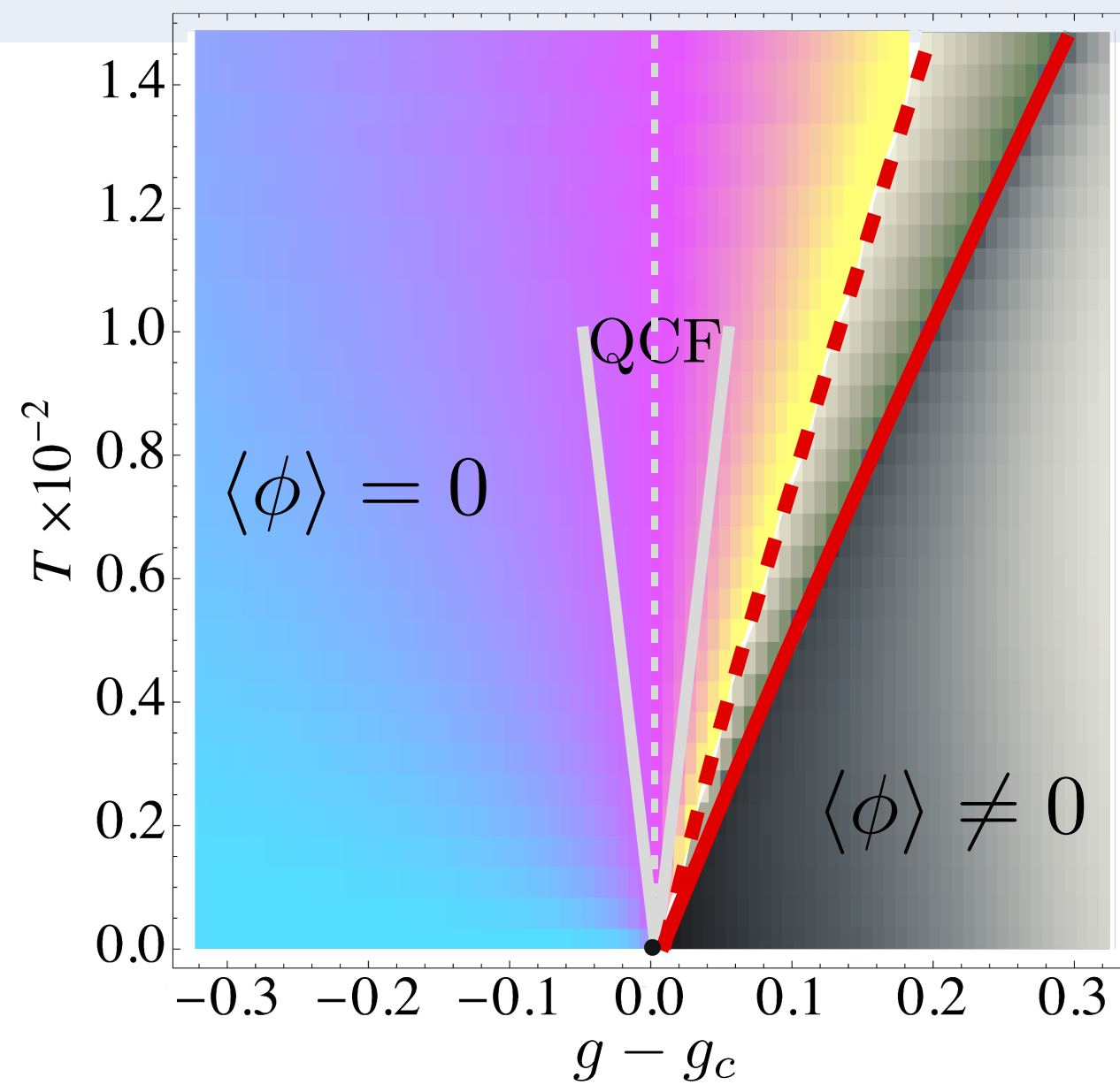
At the BKT phase ($T < T_{\text{BKT}}$), infinite correlation length ξ .

Above but close T_{BKT} , essential scaling of the correlation length

$$\xi \sim Ae^{c/\sqrt{T-T_{\text{BKT}}}}$$

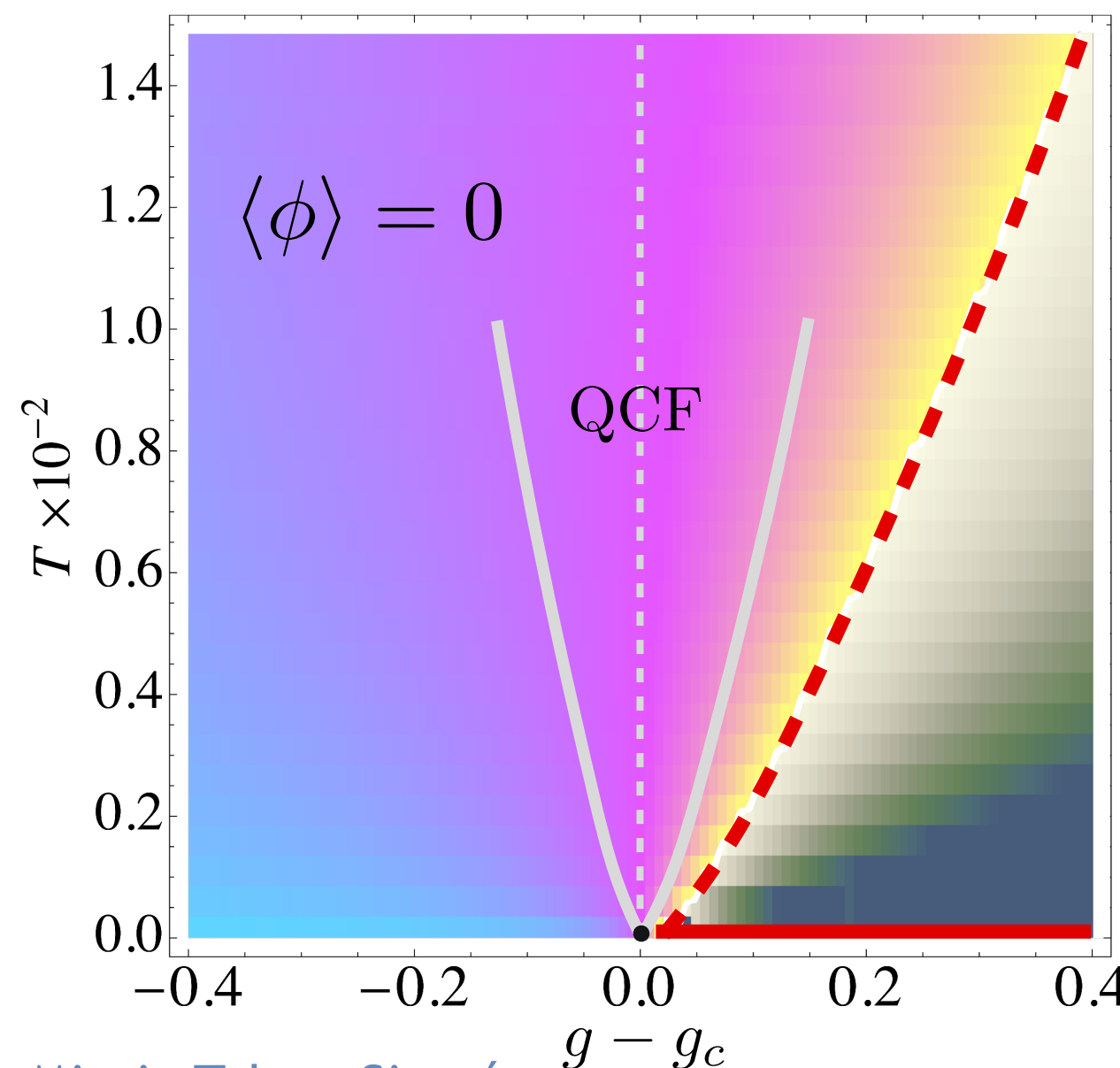


Summary



ALLOWED
by Mermin-Wagner
theorem
↓
discrete SSB
in $d = 2 + 1$ at finite T

CHIRAL ISING MODEL
 $N_{\text{Goldstone}} = 0$



**CHIRAL XY and
HEISENBERG MODELS**
 $N_{\text{Goldstone}} > 0$

continuous SSB
in $d = 2 + 1$ at finite T
↑
FORBIDDEN
by Mermin-Wagner
theorem

Unified approach describing **phase diagram** and **(quantum) critical behavior** for interacting Dirac materials.

Quantum critical behavior at $T = 0$ in terms of the **critical exponents**.

Quantum critical fan above QCP at finite T .

Precondensation region on intermediate scales.

In chiral Ising model, **condensation** at finite T .

Emergence of relativistic symmetry close to the QCP.

Classical phase transition at finite temperature.

In chiral XY and chiral Heisenberg models, **no condensation** at finite T .

Non-Dirac liquid close to QCP.

Signatures of BKT transition in chiral XY model.

Outlook

Thermodynamic properties such as specific heat.

Connection to microscopic (lattice) models.

**Thank you for
your attention!**