RUHR
UNIVERSITÄT
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Exploring Thermal Order in Quantum Field Theories

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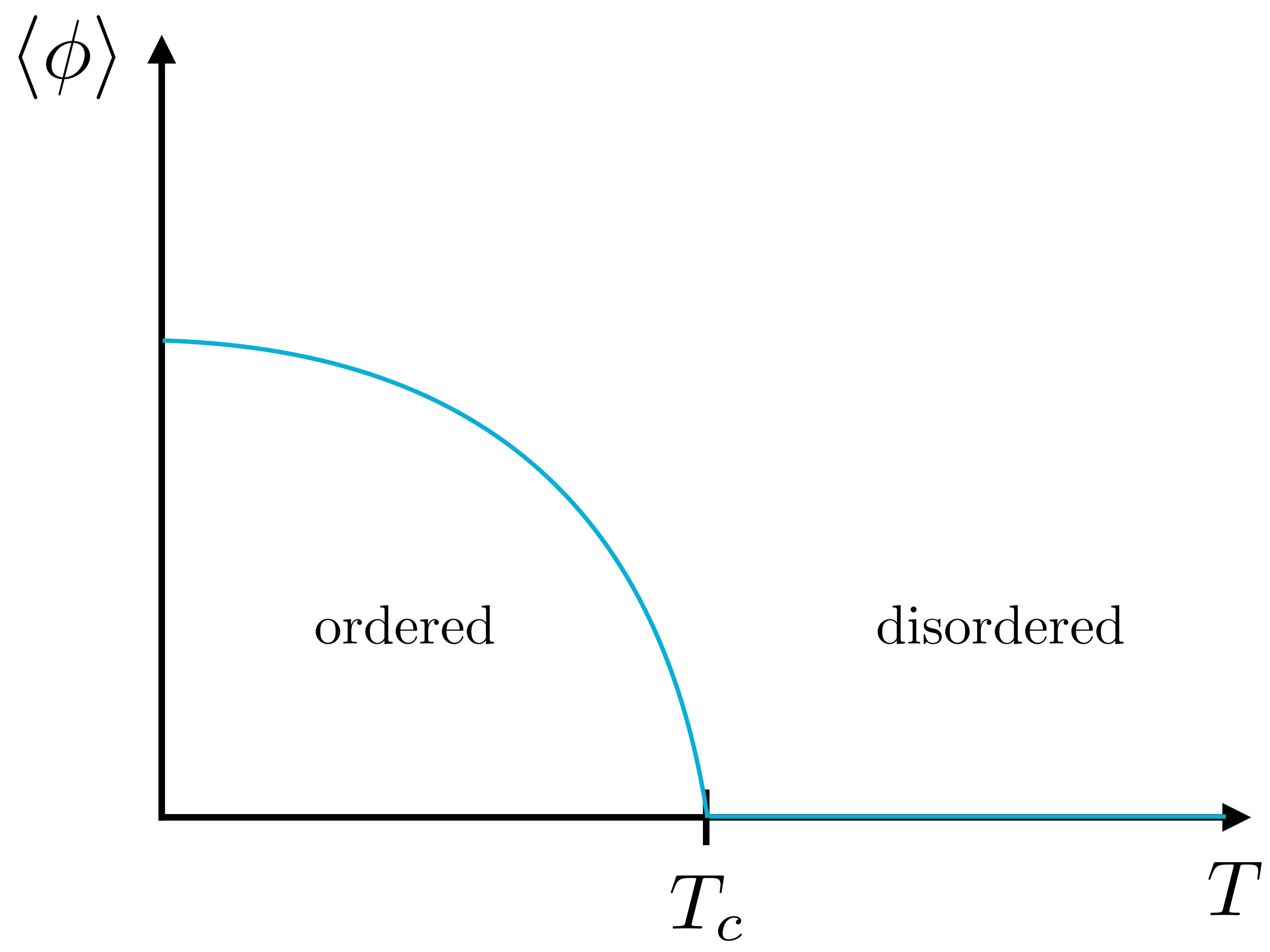

Many examples: magnetism, crystallization, superconductivity,…

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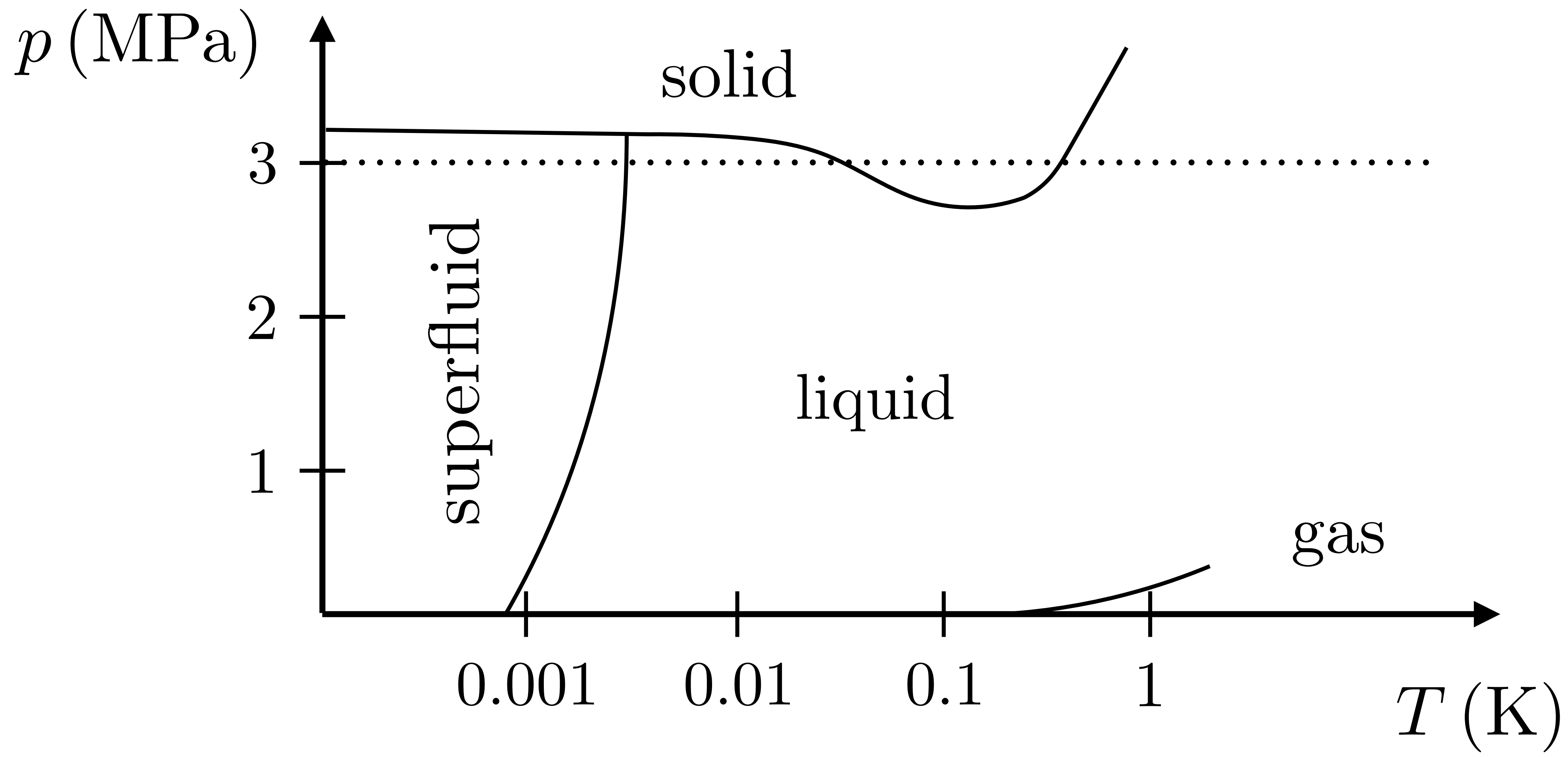
“Pomeranchuk” effect in ^{3}He

For $T \lesssim 0.3\,\mathrm{K}$:

$S_{\mathrm{liquid}} < S_{\mathrm{solid}}$ due to **disordered** nucleon spin

➔ additional fluctuating degree of freedom leads to **inverted transition**



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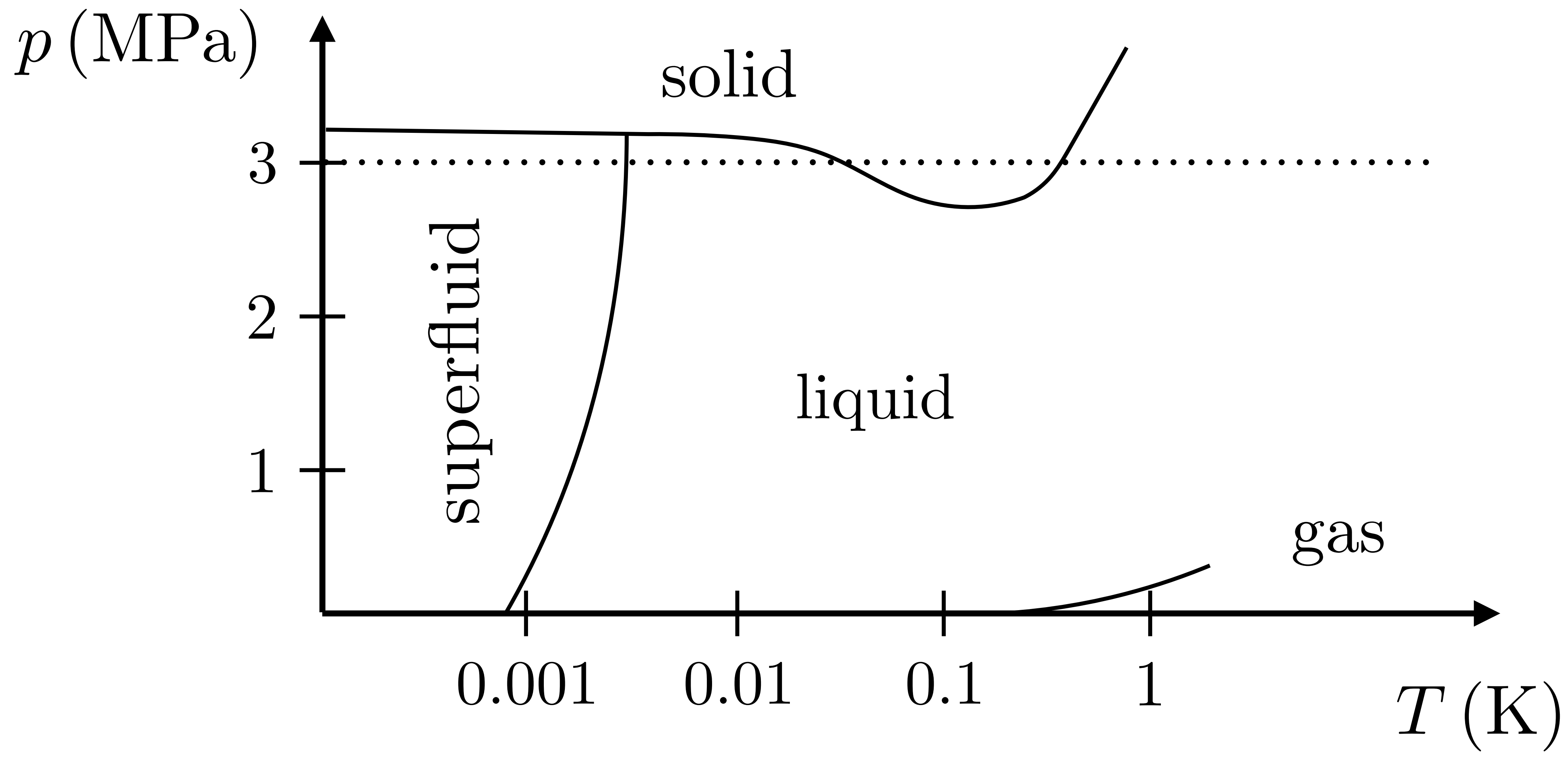

At sufficiently high temperatures, **symmetry** is **restored**… is this **always** the case?

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Single-scalar theory

Consider a **single scalar field** close to **four dimensions** with potential

$$V(\phi) = -\frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4!}\phi^4\,, \quad m_0^2, \lambda > 0$$

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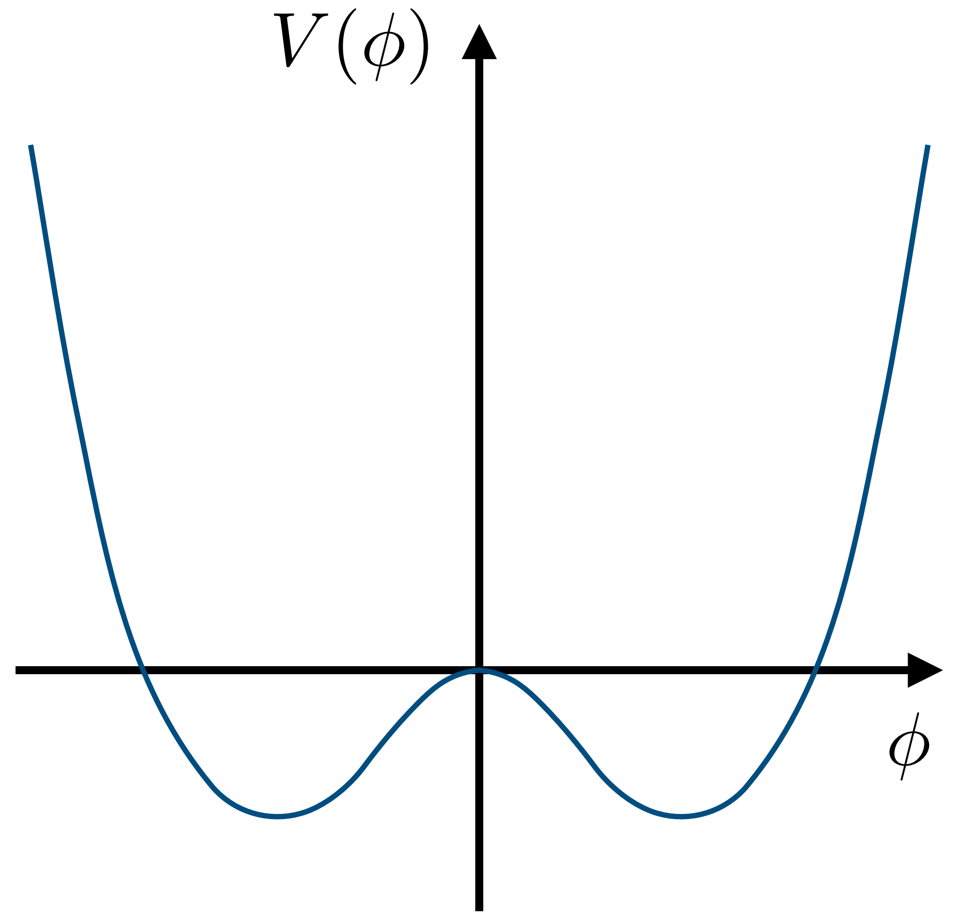

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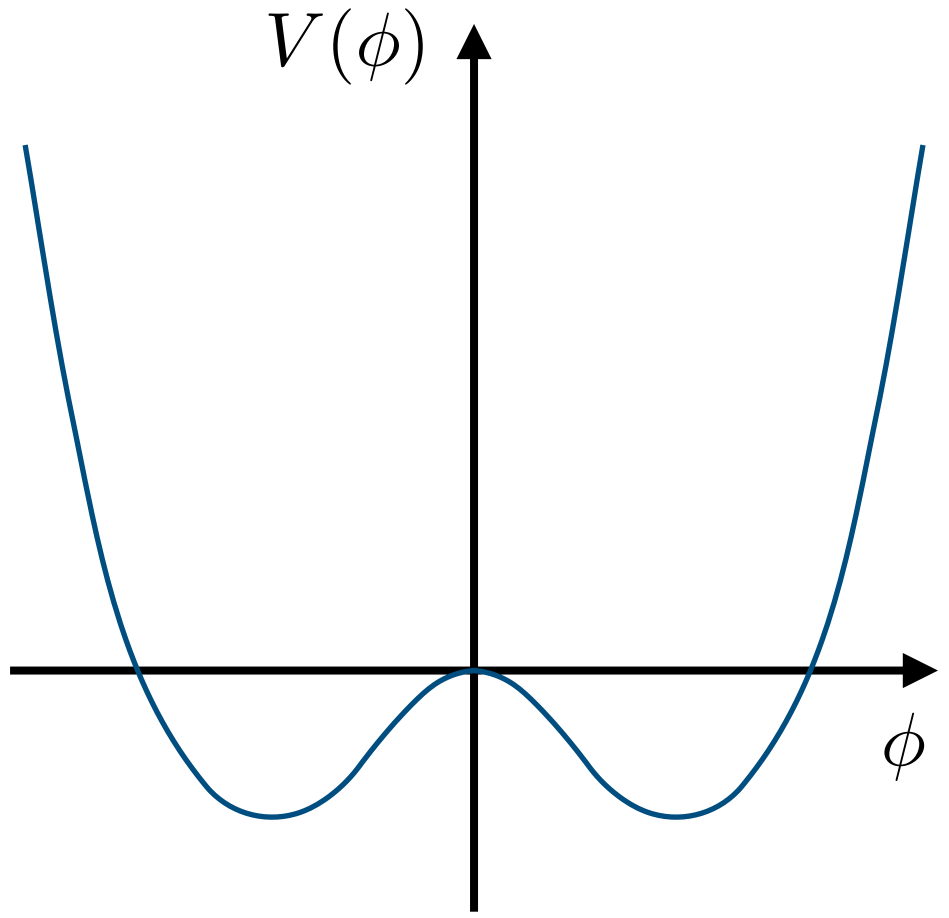

Effective potential at **finite temperature** at **leading order** in λ

$$V_{\text{eff}}(\phi) = \frac{1}{2}m(T)^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \mathcal{O}(\lambda^2)\,, \qquad m(T)^2 = \lambda\left(-\frac{\langle\phi\rangle_0^2}{6} + cT^2\right)\,, \qquad c > 0$$

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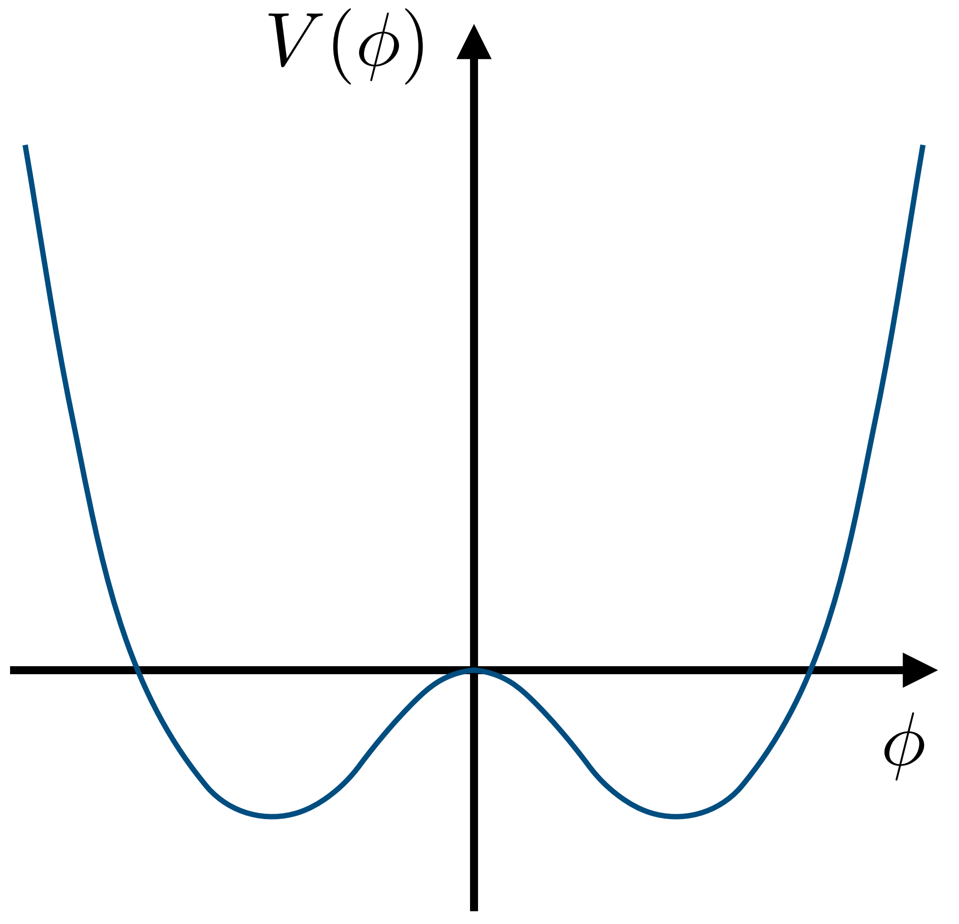

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➔ for sufficiently **high temperatures**, $m(T)^2 > 0$, and symmetry is **restored**.

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Consider **two coupled scalar fields** with $\mathrm{O}(N) \times \mathbb{Z}_2$ symmetry and scalar potential

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Thermal mass at leading order in the quartics

$$m_\chi^2(T) = m_{\chi,0}^2 + cT^2 [3\lambda_\chi + N\lambda_{\phi\chi}]$$

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Problem: perturbation theory **breaks down** at **high temperatures**

➔ Is there a **well-defined** theory in **three dimensions** that shows this phenomenon?

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Towards a theory with persistent SSB

Chai et al., *Phys. Rev. D 102, 065014 (2020)*

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If theory has non-trivial fixed-point at zero temperature: **UV completion** at **fixed-point**

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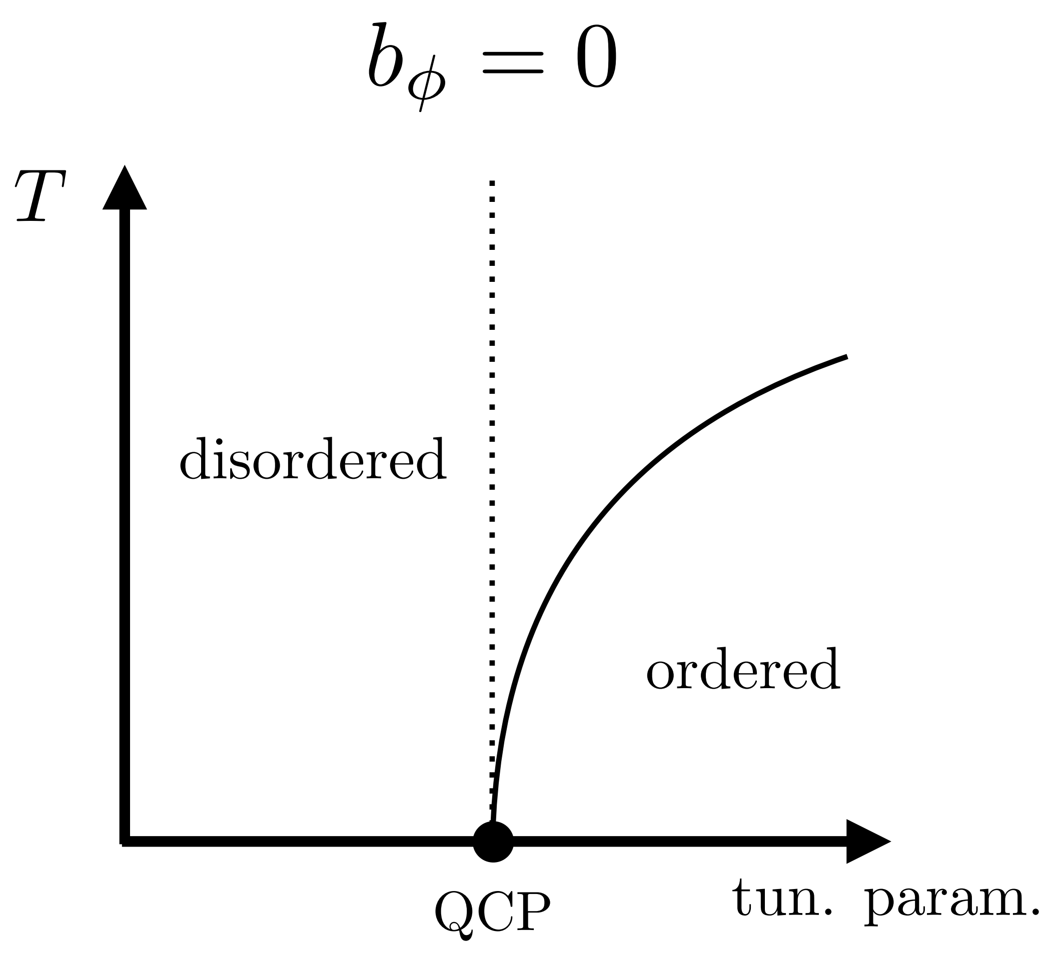

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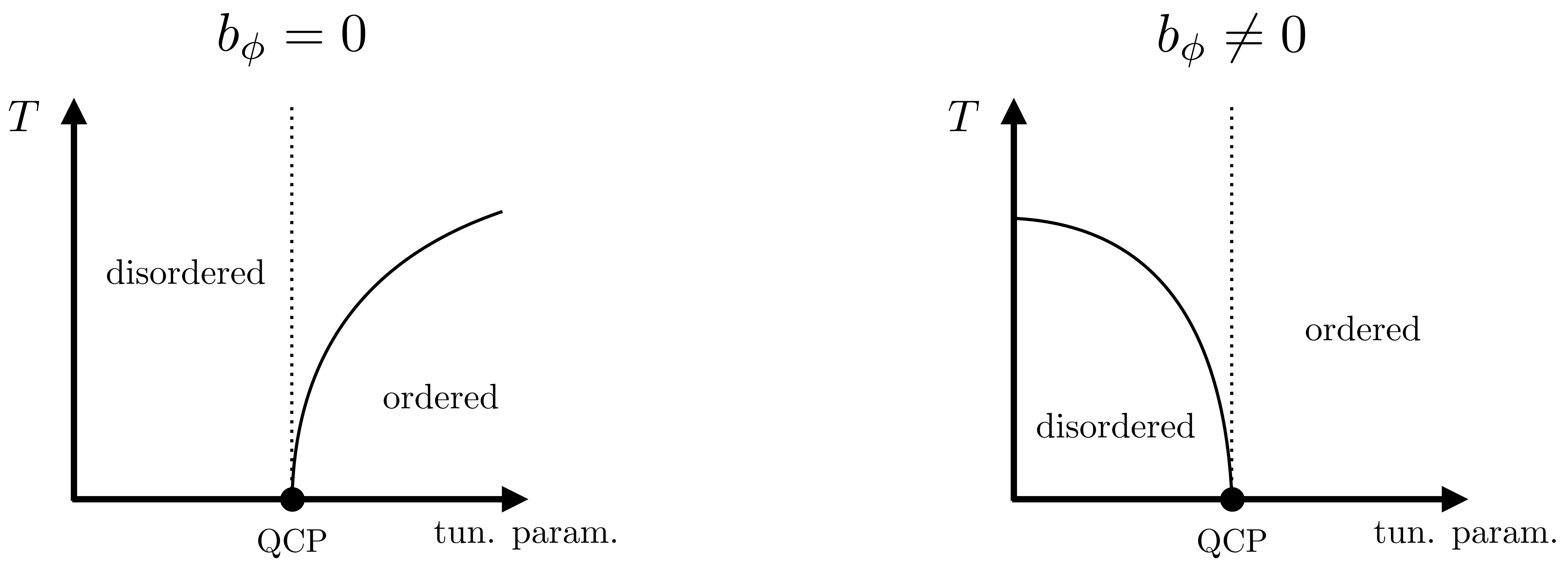

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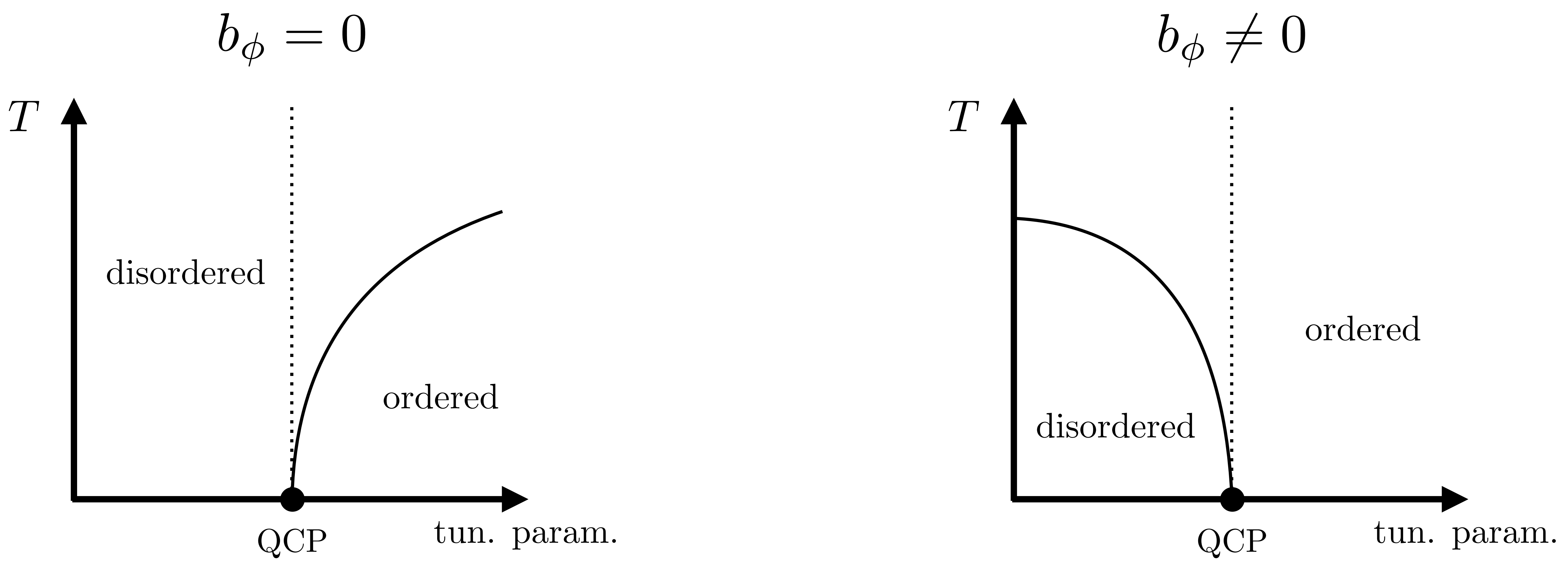

No-go theorems restrict the space of candidate theories!

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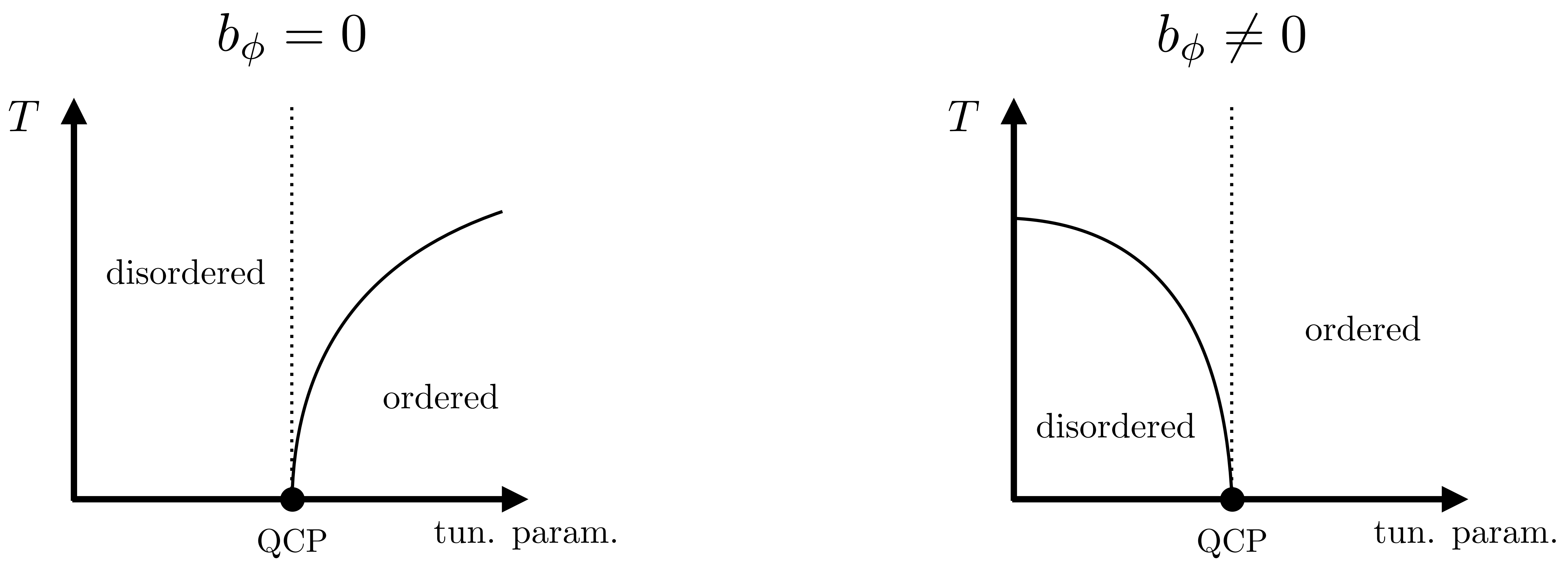

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Does a **non-trivial CFT** exist, that shows SSB at finite T?

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The biconical model

Model of two coupled scalar fields with $\mathrm{O}(N) \times \mathbb{Z}_2$ symmetry in $d = 2+1$

$$S[\phi, \chi] = \int d^d x \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{\lambda_\phi}{8}(\phi^2)^2 + \frac{\lambda_\chi}{8}\chi^4 + \frac{\lambda_{\phi\chi}}{4}\phi^2\chi^2 \right]$$

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Biconical fixed point (BFP): $\lambda_\phi^*, \lambda_\chi^* > 0,\ \lambda_{\phi\chi}^* < 0$

- studied with ϵ-expansion, FRG, large N

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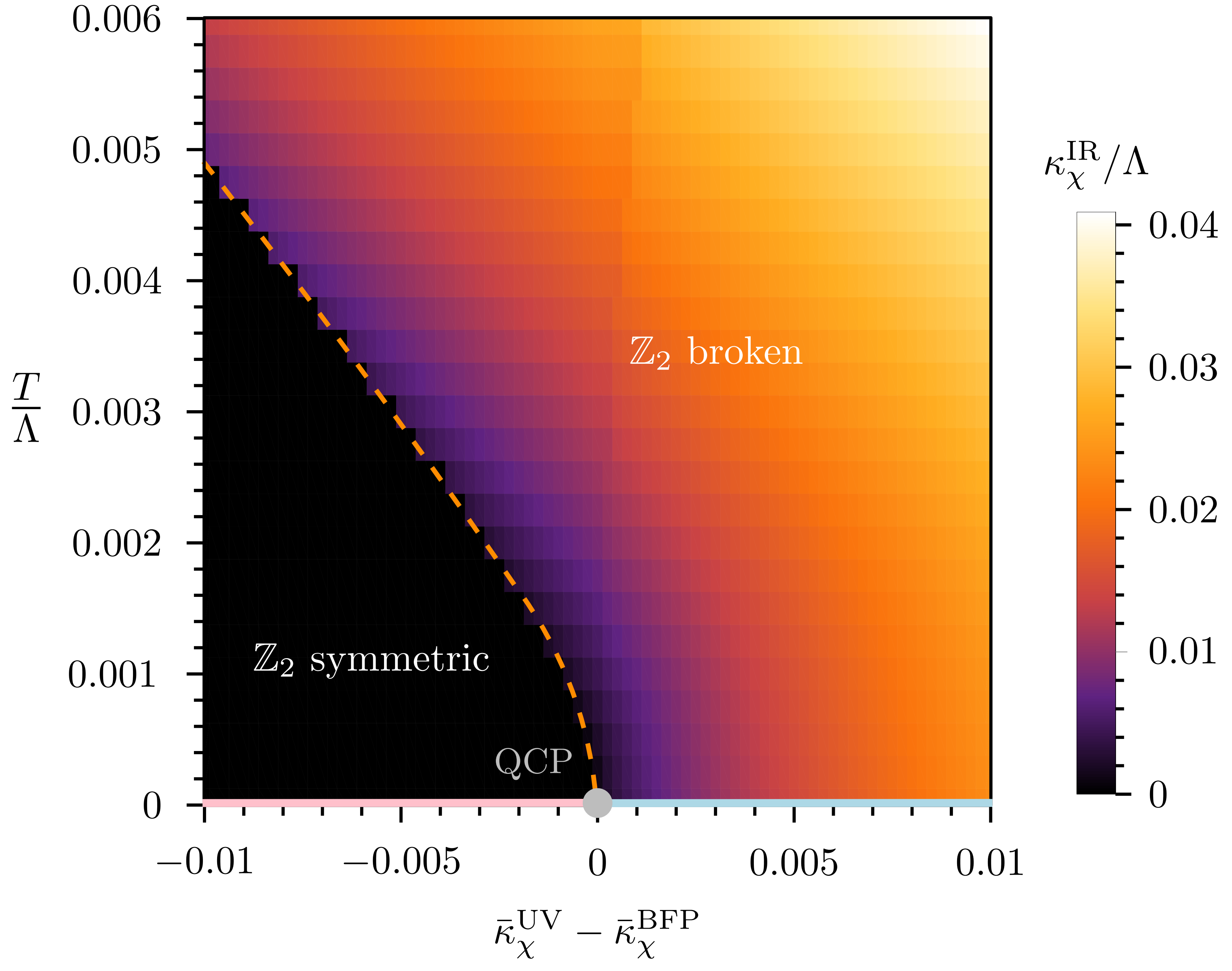

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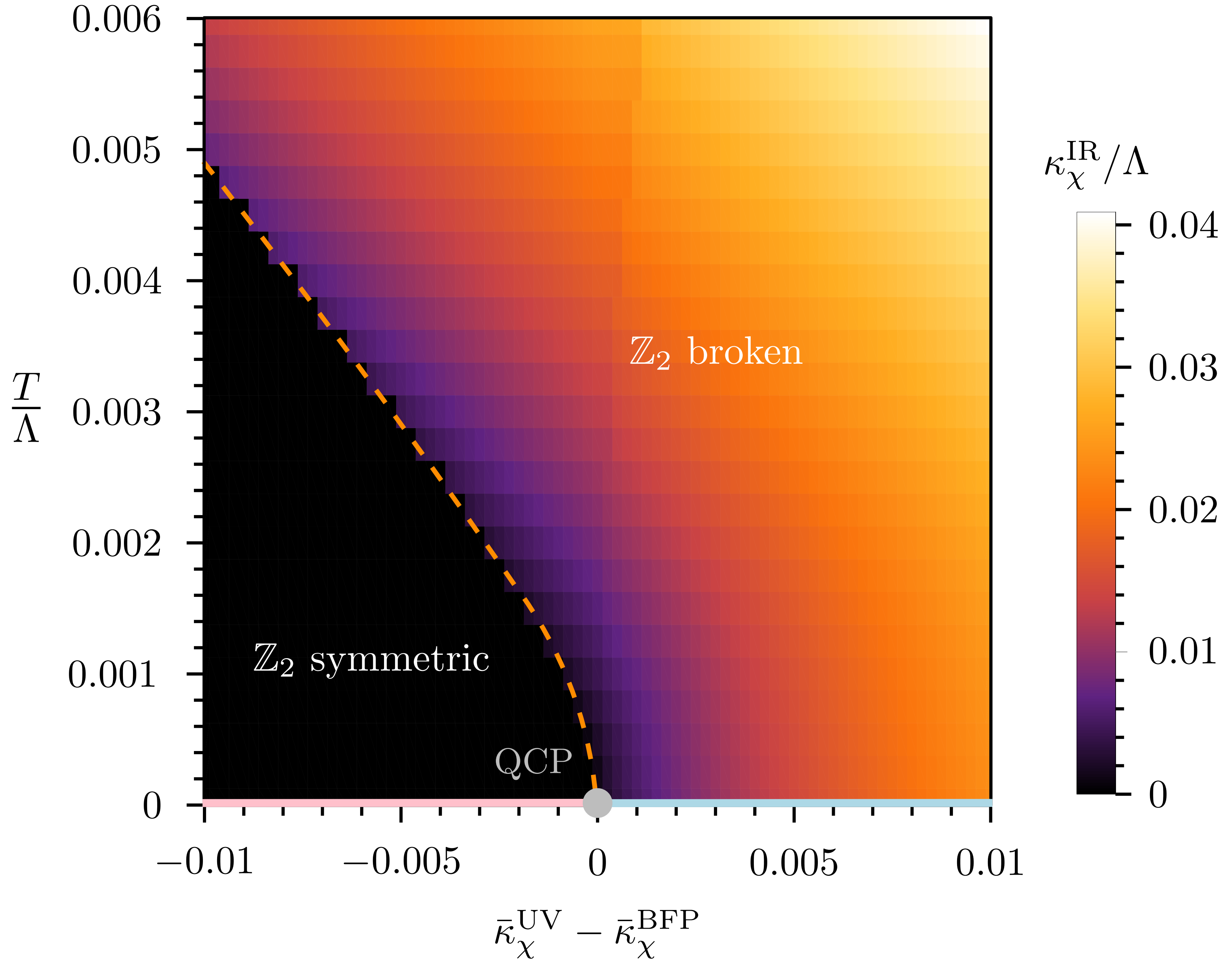
➜ Can we extend this theory to lower N_c ?

➜ Exclusive to internal symmetries?



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The biconical model with fermions

Add Dirac fermions and couple to $\mathbb{Z}_2$ sector through Yukawa interaction

$$S = S_{\mathrm{O}(N)\times\mathbb{Z}_2} + \int d^d x\, \bar{\psi}\partial\!\!\!/\psi + \int d^d x\, h\chi\bar{\psi}\psi$$

In $d = 4 - \epsilon$, $\eta_\chi = \mathcal{O}(\epsilon^2)$ ➜ h is **slightly relevant** deformation of BFP

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For $h \neq 0$, the action admits the **parity symmetry**

$$\mathcal{P}: \quad x_1 \to -x_1, \quad \chi \to -\chi, \quad \psi \to \gamma^1\psi, \quad \bar{\psi} \to -\bar{\psi}\gamma^1$$

and is spontaneously broken for $\langle\chi\rangle > 0$.

FRG for the biconical model with fermions

Study RG flow using **FRG within extended local potential approximation**

$$\Gamma_k[\Phi] = \int d^d x \left\{ \frac{Z_{\phi,k}}{2}(\partial\phi)^2 + \frac{Z_{\chi,k}}{2}(\partial\chi)^2 + U_k[\rho_\phi, \rho_\chi] + Z_{\psi,k}\bar{\psi}\partial\!\!\!/\psi + h_k \chi\bar{\psi}\psi \right\}.$$

with **truncated** effective potential

$$U(\rho_\phi, \rho_\chi) = \sum_{n+m\geq 1}^{n+m\leq n_{\max}} \frac{\lambda_{nm}}{n!m!}(\rho_\phi - \kappa_\phi)^n (\rho_\chi - \kappa_\chi)^m \,, \qquad (\partial_{\rho_i} U)(\kappa_\phi, \kappa_\chi) = 0$$

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System can be in **four phases:**

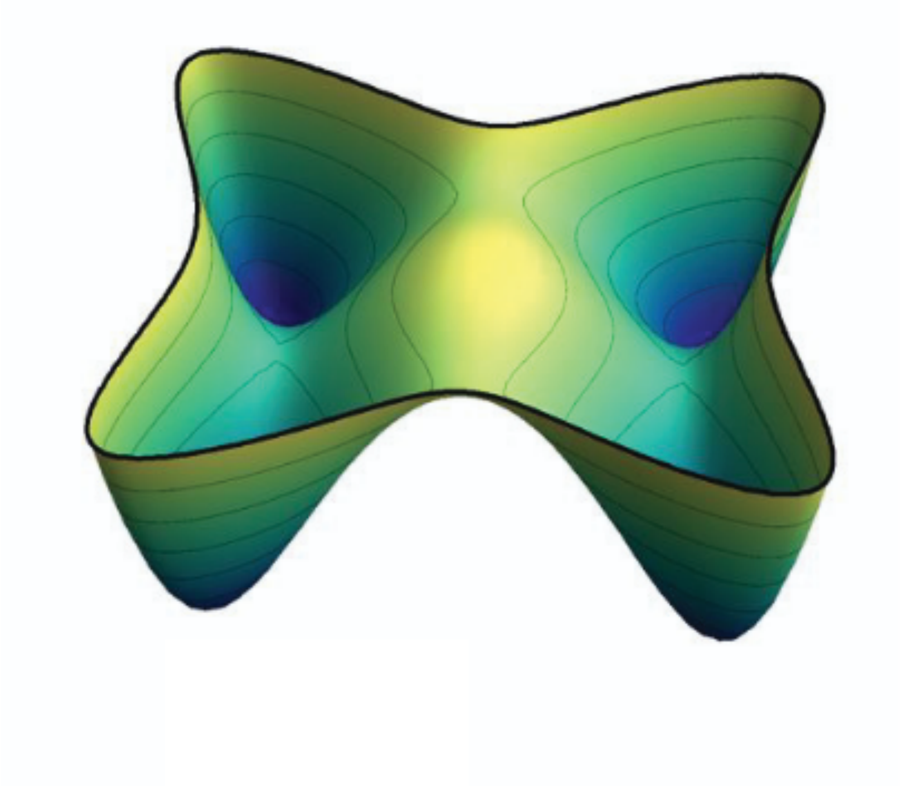
(1) SSB-SSB: $\mathrm{O}(N) \times \mathbb{Z}_2 \to \mathrm{O}(N-1)$

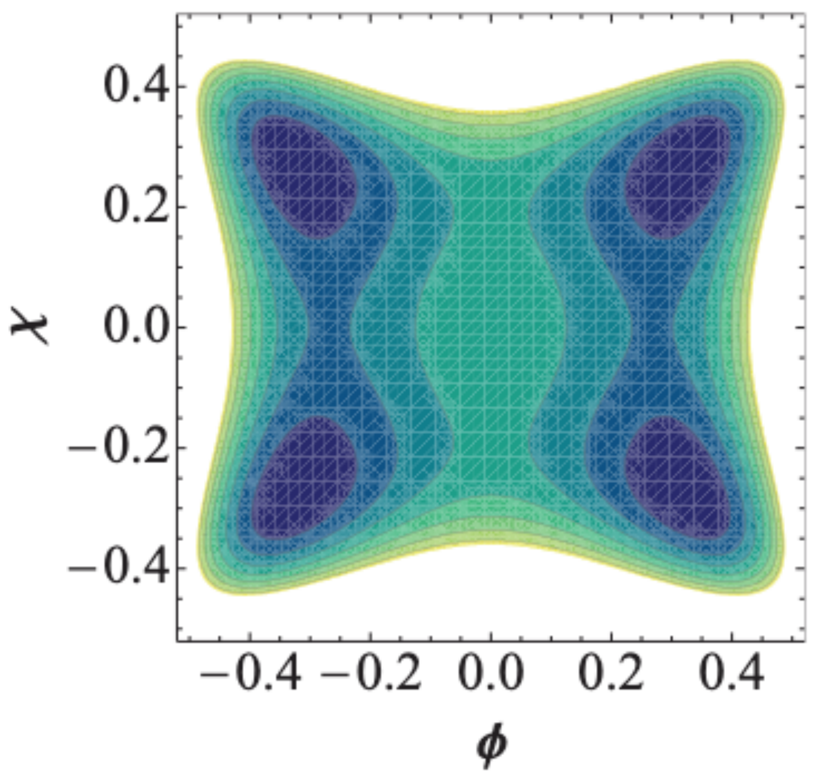
(2) SSB-SYM: $\mathrm{O}(N) \times \mathbb{Z}_2 \to \mathrm{O}(N-1) \times \mathbb{Z}_2$

(3) SYM-SSB: $\mathrm{O}(N) \times \mathbb{Z}_2 \to \mathrm{O}(N)$

(4) SYM-SYM: $\mathrm{O}(N) \times \mathbb{Z}_2$

SSB-SSB phase





Eichhorn et al., *Phys. Rev. E 88, 042141 (2013)*

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In $d = 4 - \epsilon$, theory is **weakly coupled.** Include only **canonically relevant** scalar couplings

$$U_k[\rho_\phi, \rho_\chi] = m_\phi^2\rho_\phi + m_\chi^2\rho_\chi + \frac{\lambda_\phi}{2}\rho_\phi^2 + \frac{\lambda_\chi}{2}\rho_\chi^2 + \lambda_{\phi\chi}\rho_\phi\rho_\chi$$

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At the fixed point, $m_i^2 = \mathcal{O}(\lambda) = \mathcal{O}(\epsilon)$, and at leading order in the couplings,

$$\begin{aligned}
\beta_{\lambda_\phi} &= -\epsilon\lambda_\phi + (N+8)\lambda_\phi^2 + \lambda_{\phi\chi}^2, \\
\beta_{\lambda_{\phi\chi}} &= -\epsilon\lambda_{\phi\chi} + 4N_f\lambda_{\phi\chi}h^2 + 3\lambda_\chi\lambda_{\phi\chi} + \lambda_{\phi\chi}^2 + (N+2)\lambda_\phi\lambda_{\phi\chi}, \\
\beta_{\lambda_\chi} &= -\epsilon\lambda_\chi + \left(9\lambda_\chi^2 + N\lambda_{\phi\chi}^2 + 8N_f\lambda_\chi h^2 - 9N_f h^4\right), \\
\beta_{h^2} &= -\epsilon h^2 + (4N_f + 6)h^4.
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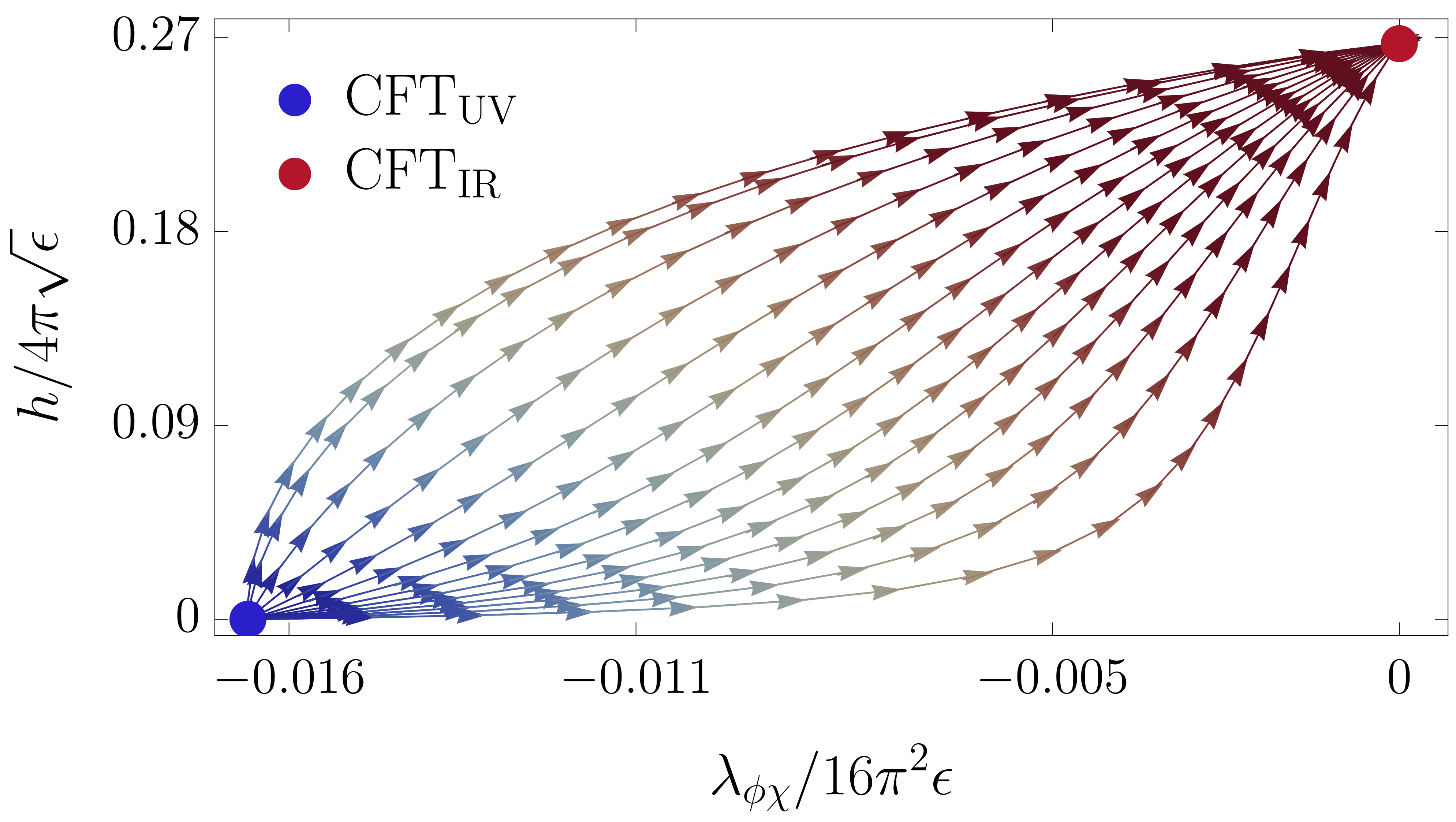

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$$N = 100, N_f = 2$$

SSB-SSB	κ_ϕ	κ_χ	λ_ϕ	λ_χ	$\lambda_{\phi\chi}$	h	θ_1	θ_2	θ_3	θ_h
$\mathrm{CFT}_{\mathrm{UV}}$	1.93	0.26	0.29	0.31	–0.29	0	1.99	1.01	0.95	0.50

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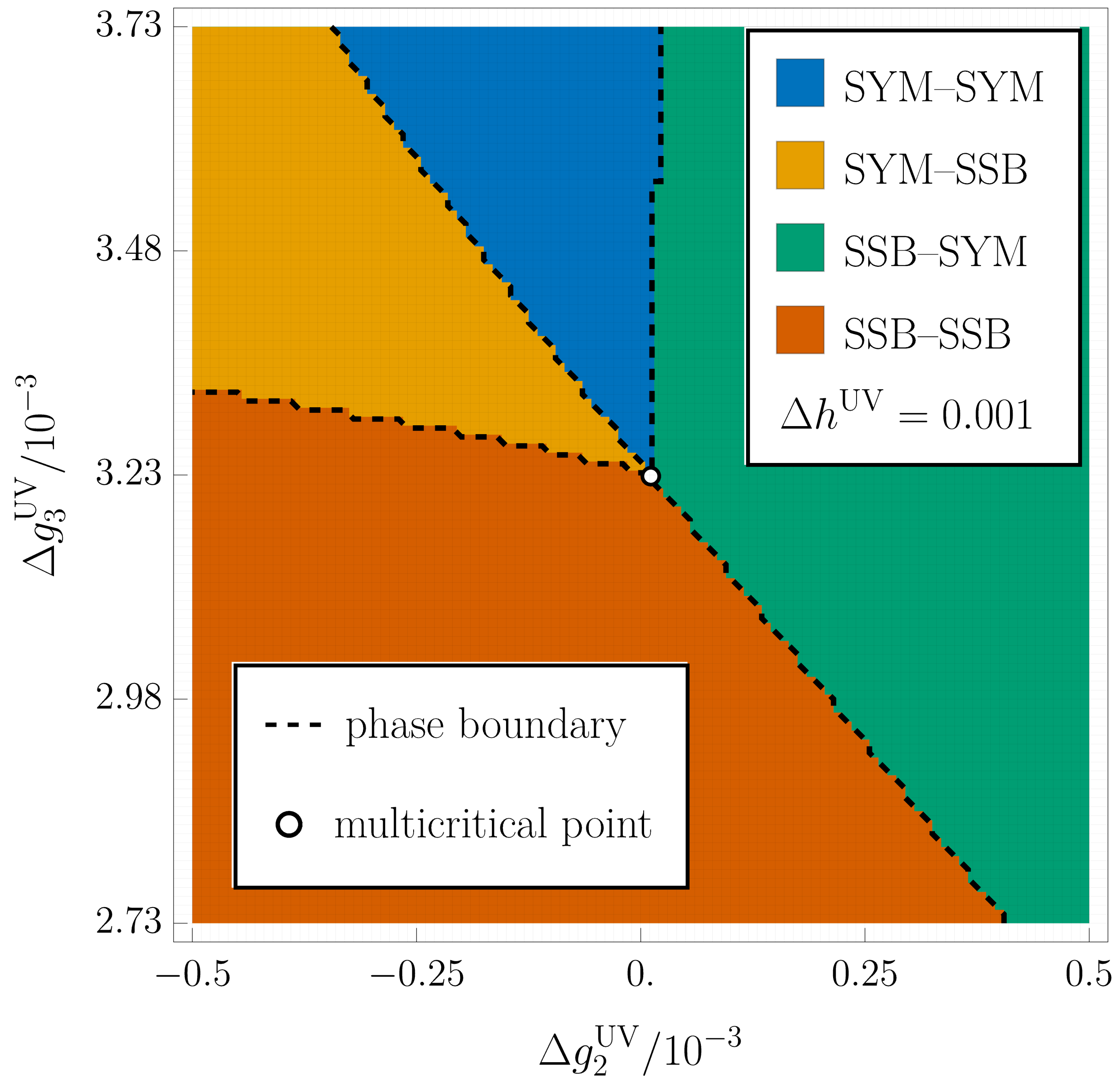

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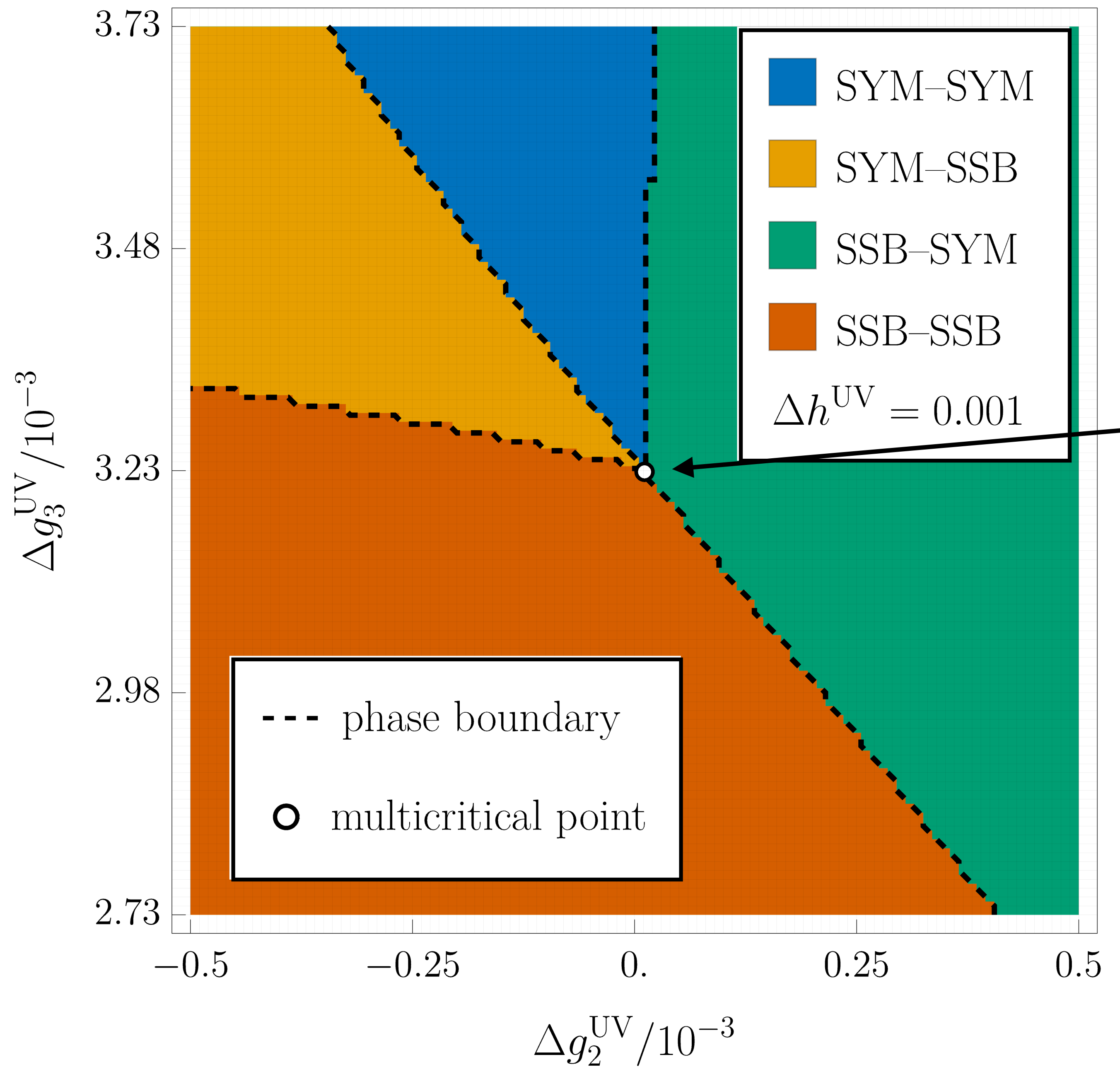

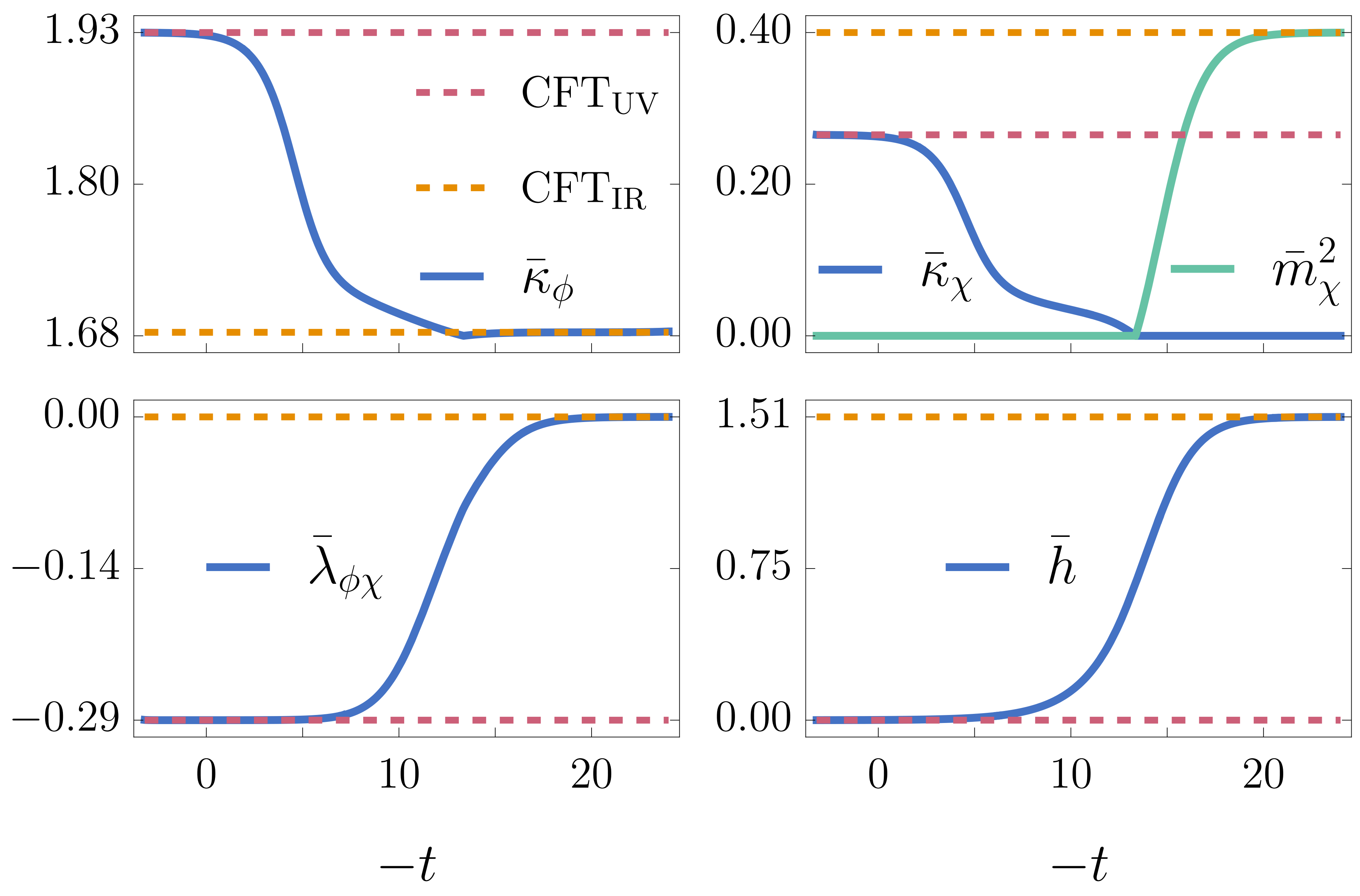

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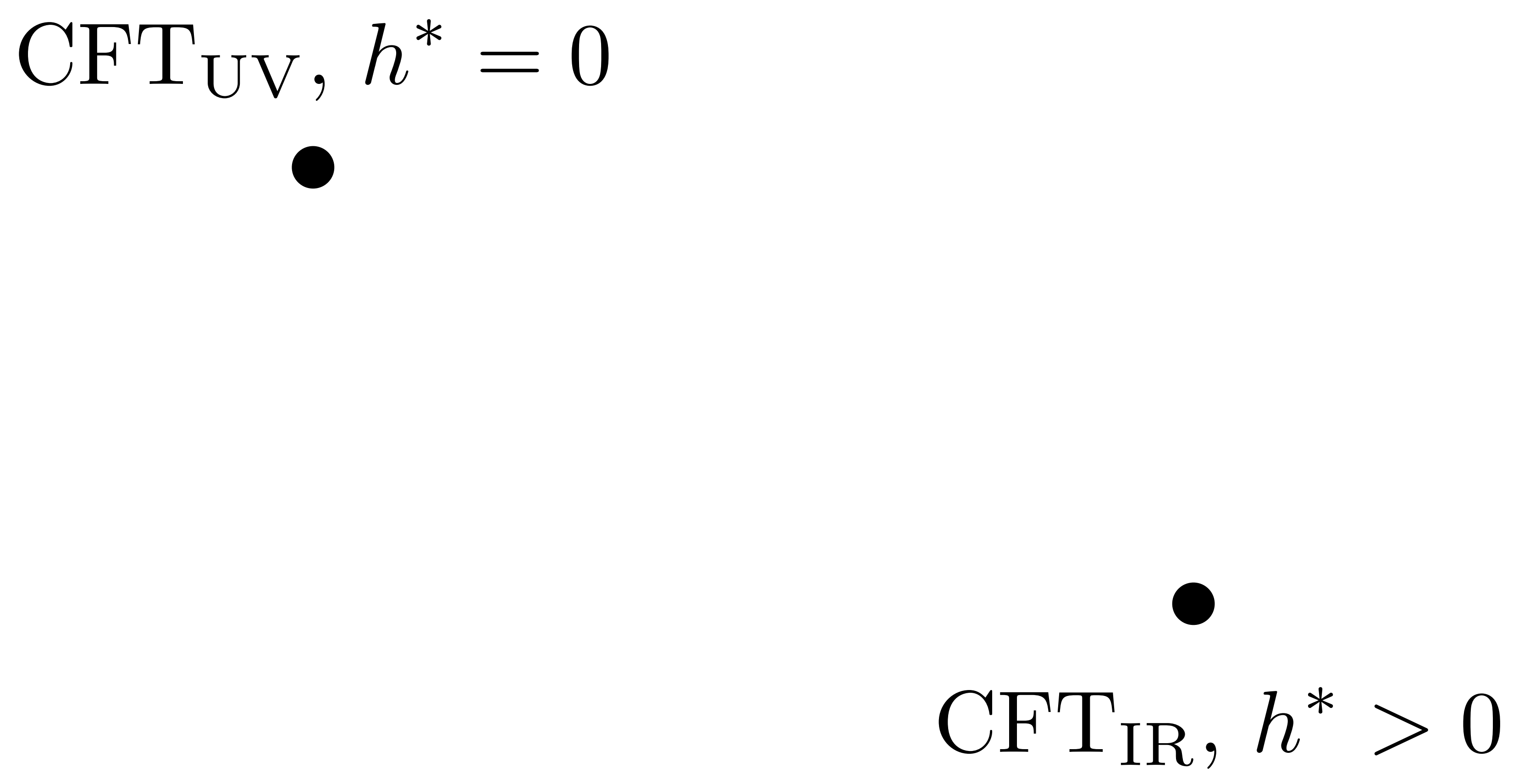

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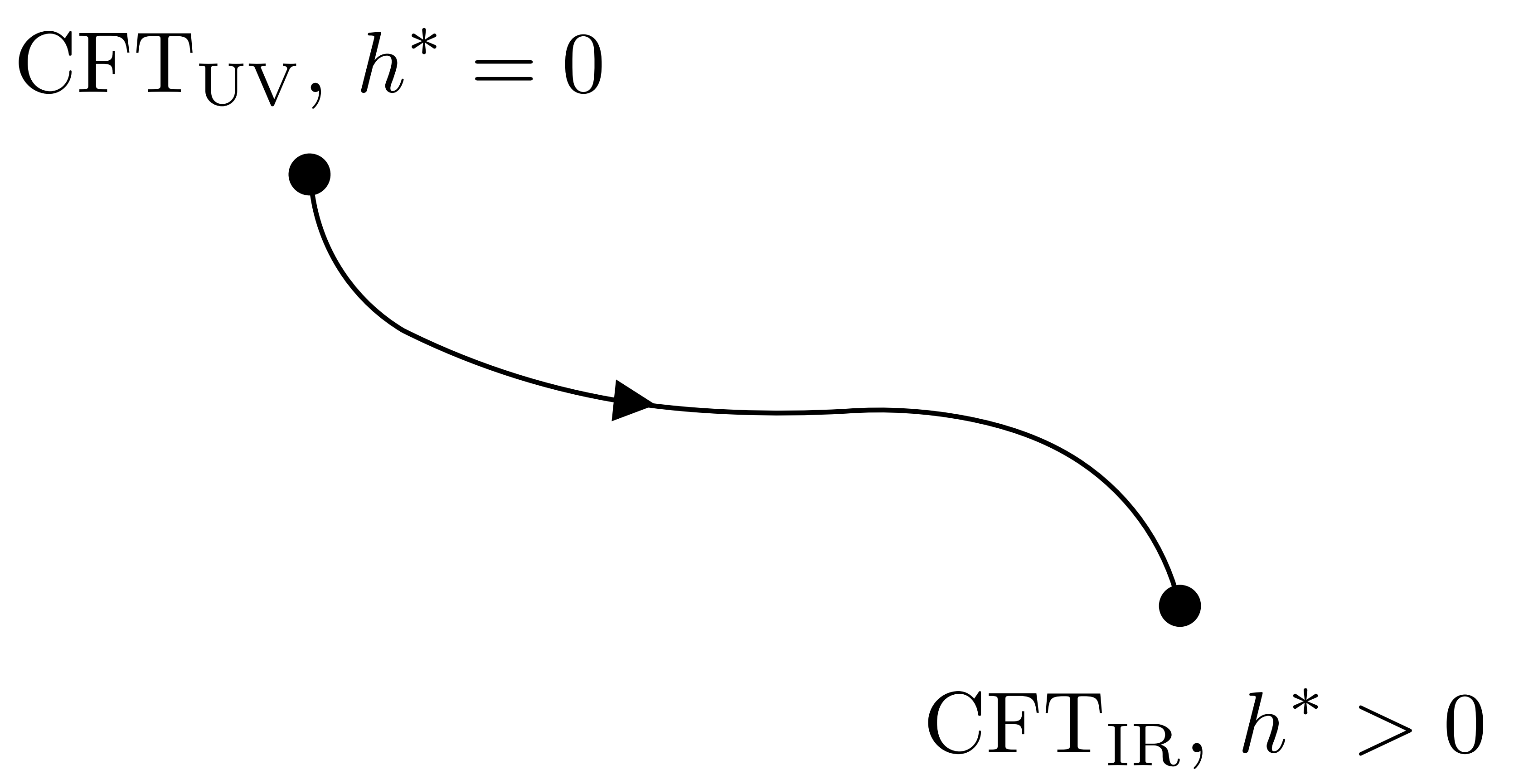

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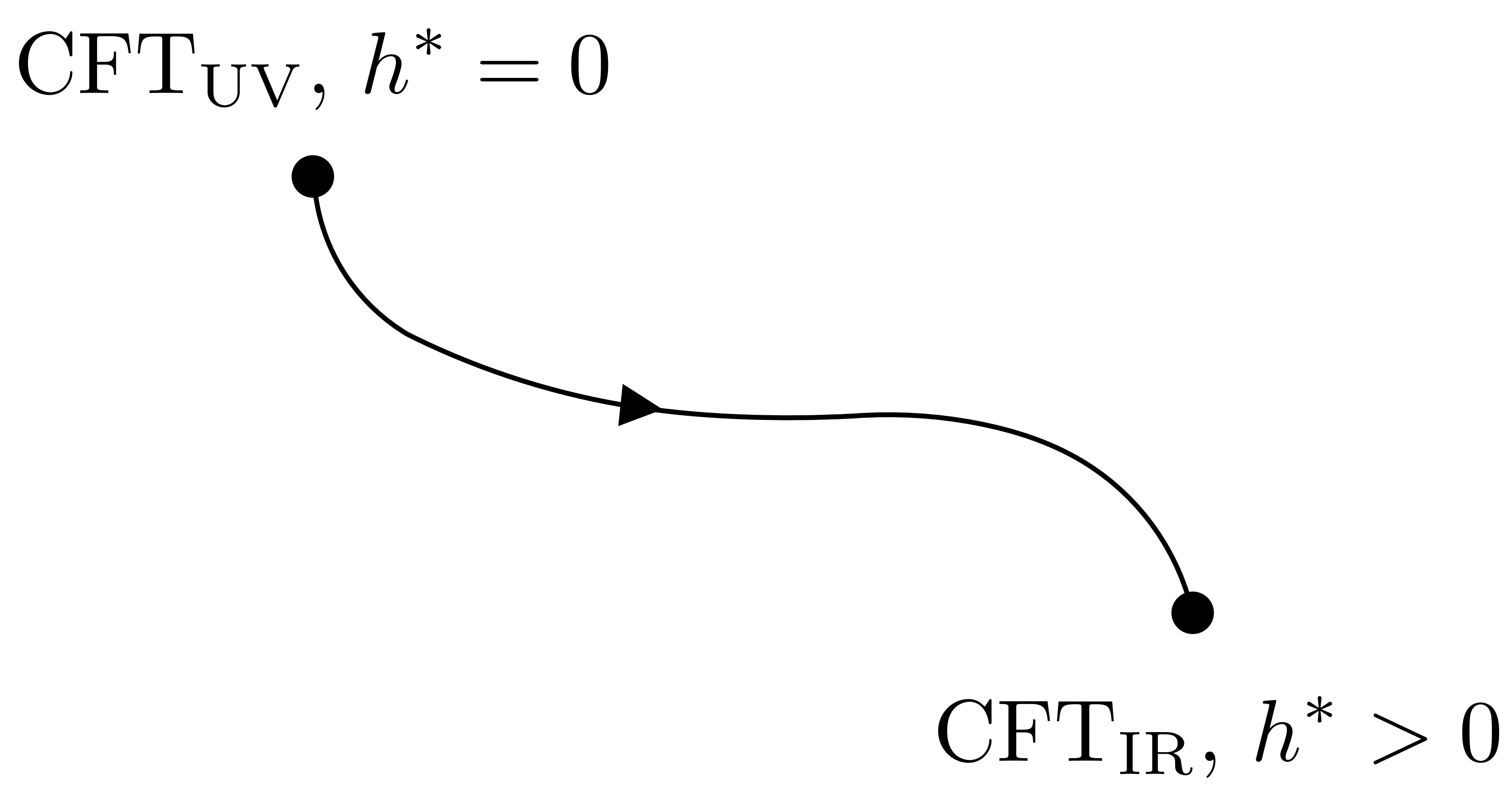

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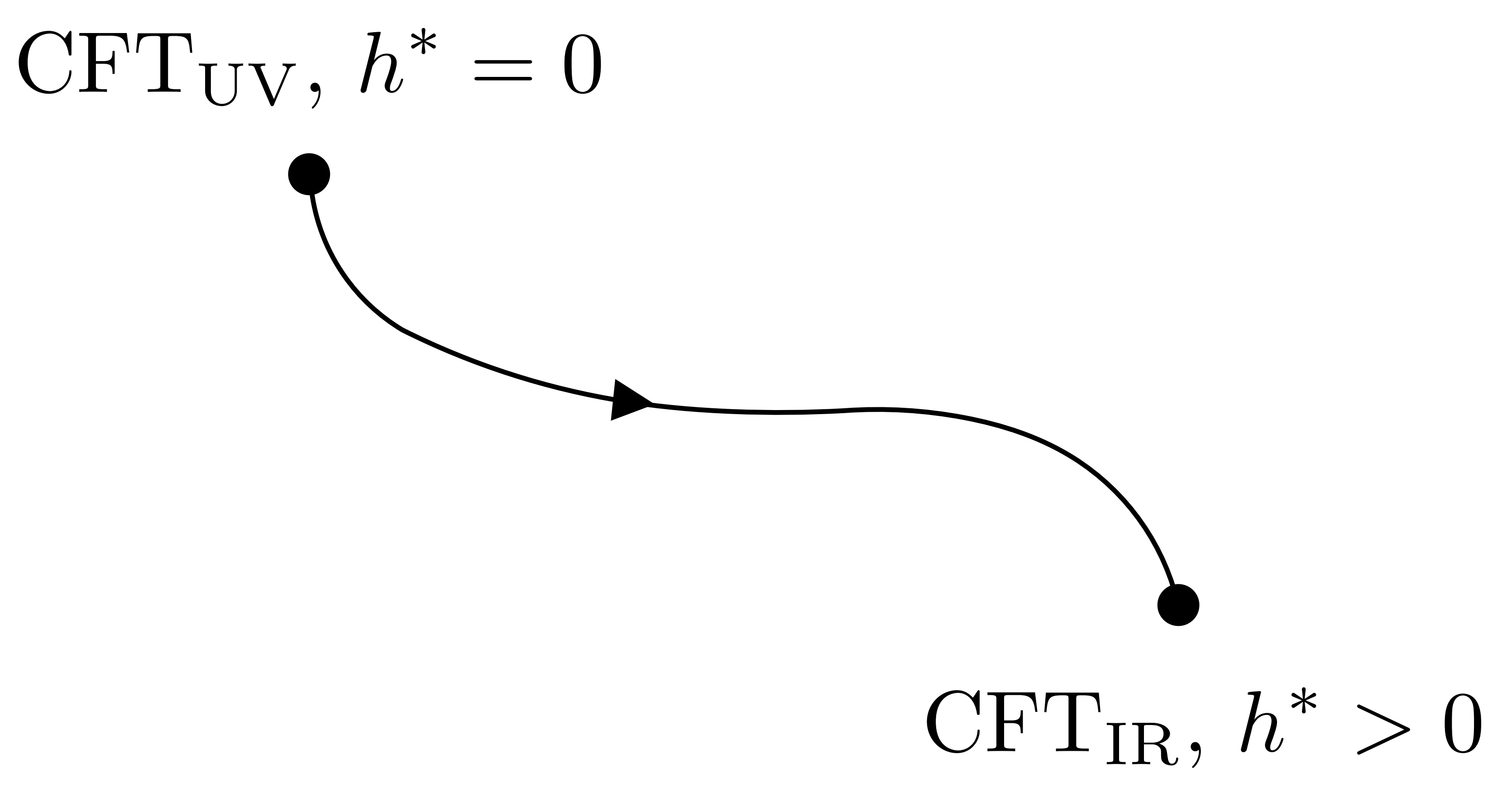

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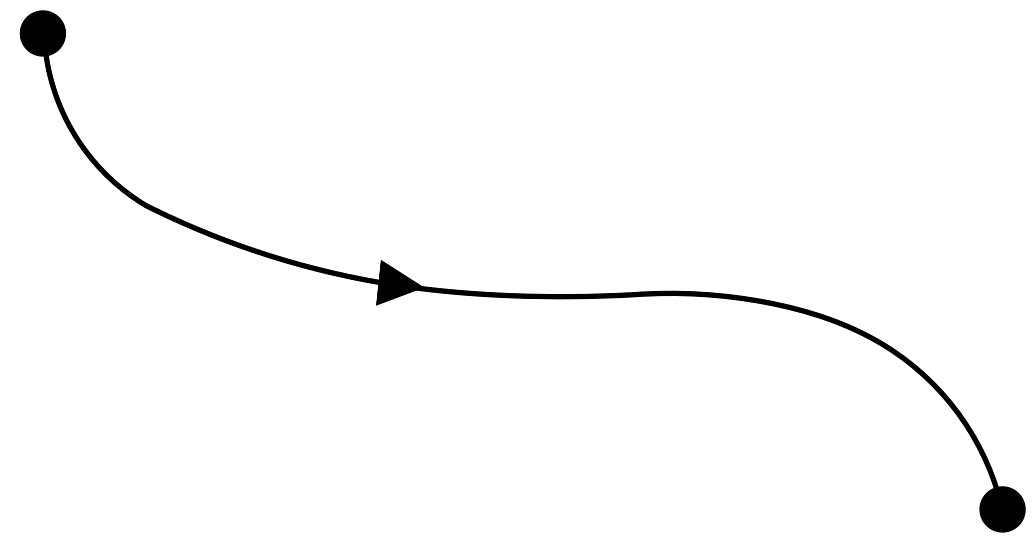

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➔ spacetime parity is **broken** for $T > T_c$

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Collaborators:

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