Dynamics in quantum random-field systems: activation or localization?

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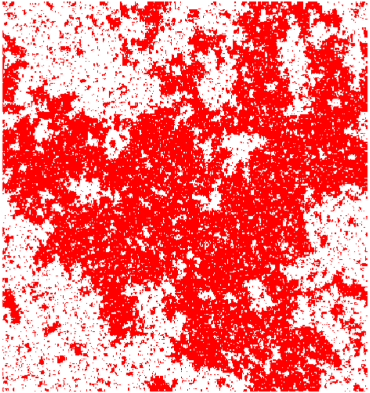
What is critical slowing down?

- Close to a critical point, correlated regions grow.
- A region of size ξ must rearrange collectively.
- The relaxation time is controlled by a single scale. Halperin, Hohenberg and Ma, PRL 29, 1548 (1972)

$$\tau \sim \xi^z$$



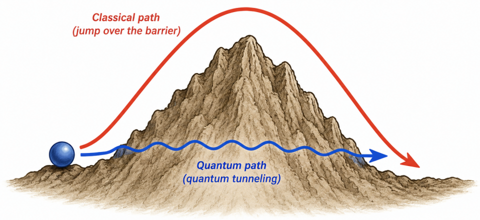
What changes in a rough landscape?

- Disorder creates many metastable configurations.
- Relaxation is no longer motion inside one smooth minimum.
- Large regions must cross or tunnel through barriers. Fisher: PRL 56, 416 (1986)

$$B(\xi) \sim \xi^{\Psi}, \qquad \tau \sim e^{B(\xi)}$$

$$\boxed{\tau \sim e^{\xi^{\Psi}}}$$

activated dynamics



Why the quantum RFIM?

A minimal model where random fields, exchange, and tunneling compete.

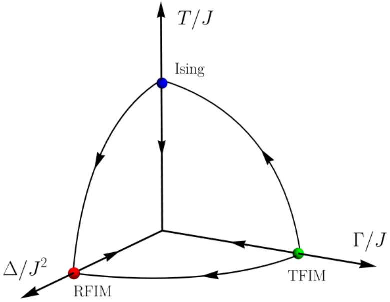
$$\hat{H} = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x - \sum_i h_i \sigma_i^z .$$

J: ferromagnetic ordering Γ: quantum tunneling h_i: quenched longitudinal random field

Aharony, Gefen, Shapir (1982)

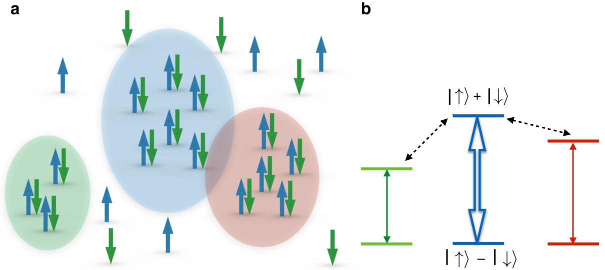
Boyanovski, Cardy (1983)

Senthil (1998)

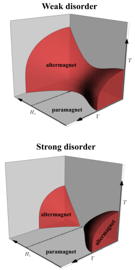


Experimental platforms

- **$Fe_xZn_{1-x}F_2$**: randomly diluted uniaxial antiferromagnet in a uniform applied field (classical RFIM). (Nash etal. PRB 43 1272 (1990))
- **$LiHo_xY_{1-x}F_4$**: dilution and transverse field generate effective longitudinal random fields. (Tabei etal. PRL 97 237203 (2006))
- **Mn_{12}-acetate**: transverse field plus distribution of easy-axis tilts realizes an effective RFIM. (Wen etal. PRB 82, 014406 (2010))
- **Inhomogeneous altermagnets**: magnetic field plus random strain can generate a tunable random field. (Chakraborty etal. PRB 112 035146 (2025))



(a) $LiHo_xY_{1-x}F_4$: clusters and two-level excitations. (Silevich etal. Nat. Comm. 10 4001 (2019))



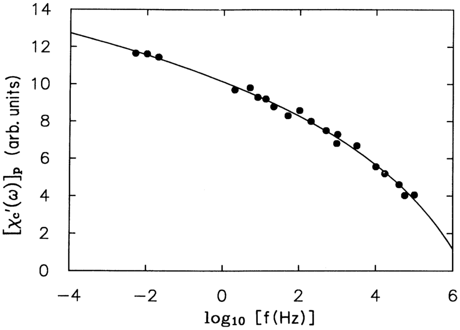
(b) Field-tuned random-field physics in altermagnets. (Chakraborty eta PRB 112 035146 (2025))

The experimental clue: very slow dynamics

- Hysteresis and slow relaxation in molecular magnets. (Nash etal. PRB 43 1272 (1990))
- Tunable relaxation in diluted dipolar Ising magnets. (Wen etal. PRB 82, 014406 (2010))
- Long-lived localized spin clusters in dilute $LiHo_xY_{1-x}F_4$. (Silevich etal. Nat. Comm. 10 4001 (2019))

Relaxation?

The theoretical question is not only static criticality, but the low-frequency clock



Activated critical dynamics in a classical RFIM experiment. (Nash etal. PRB 43 1272 (1990))

A theoretical clue: localization-like runaways

Dynamical RG flows in disordered quantum systems often run into localization-like singularities.

Quantum creep

A low-frequency approximation produces a finite-force "localization transition".

But this was identified as spurious: tunneling requires the dynamics at all frequencies.

Gorokhov–Fisher–Blatter, PRB 66, 214203 (2002)

Finite-T Bose glass

A more complete FRG truncation develops a finite-scale singularity.

One possible interpretation is MBL-like loss of thermalization, but the fate of the singularity remains open.

Daviet-Dupuis PRL 125, 235301 (2020)

Grison-Dupuis, PRB 113, 174208 (2026)

Question: what is the nature of dynamics in the QRFIM?

From the lattice model to the NPRG

Quantum RFIM

$$\hat{H} = -J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x - \sum_i h_i\sigma_i^z$$

$\xrightarrow{\text{Suzuki–Trotter}}$

Continuum field theory

$$S_B[\chi] = \int_{x,\tau}\left[\frac{1}{2}(\nabla\chi)^2 + \frac{1}{2}F_\Lambda(-\partial_\tau)\chi^2 + \frac{r}{2}\chi^2 + \frac{\lambda}{4!}\chi^4 - h(x)\chi\right]$$

$\xleftarrow{\text{integration from small to large scales}}$

NPRG

$$\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\partial_k R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Quenched disorder

$$h(x,\tau) = h(x)$$

The same disorder realization extends along the imaginary-time direction.

The multicopy formalism

Disorder-averaged effective action

$$\Gamma[\{\phi_a\}] = \sum_a \int_{x,\tau} \gamma_1[\phi_a(x,\tau)] - \frac{1}{2}\sum_{a,b}\int_{x,\tau,\tau'} \gamma_2[\phi_a(x,\tau),\phi_b(x,\tau')] + \frac{1}{3!}\sum_{a,b,c}\int_{x,\tau,\tau',\tau''} \gamma_3[\phi_a,\phi_b,\phi_c] + \cdots .$$

same disorder $h(x)$

copy a: $\phi_a(x,\tau)$ — γ_1: one copy

copy b: $\phi_b(x,\tau')$ — γ_2: two copies

copy c: $\phi_c(x,\tau'')$ — γ_3: three copies

τ, τ', τ'' are independent

- $\gamma_{1,k}$: average effective action.
- $\gamma_{n\geq 2,k}$: disorder cumulants.
- Quenched disorder makes higher cumulants multilocal in imaginary time.

Strategy at $T = 0$

1

Assume

the $T = 0$ fixed point is
the classical RFIM fixed point

2

Fluctuations around this
fixed point are **quantum**

$\Downarrow$

flow of $F_k(\omega)$

3

Check consistency

Are quantum corrections
irrelevant in the static sector?

yes $\Rightarrow$ assumption self-consistent

Static fixed point $\longrightarrow$ quantum dynamics $\longrightarrow$ consistency

1) Start from the classical RFIM fixed point

Tarjus & Tissier 2004–2012

$$\gamma_{1,k}[\phi_a] = \frac{1}{2} \underbrace{\phi_a F_k(\partial_\tau)\phi_a}_{\text{"dynamics"}} + \underbrace{\frac{1}{2} Z_k(\phi_a)(\nabla\phi_a)^2 + U_k(\phi_a)}_{\text{classical RFIM}}$$

$$\gamma_{2,k}[\phi_a, \phi_b] = \underbrace{\phi_a\phi_b\, \Delta_k(\phi_a, \phi_b)}_{\text{renormalized disorder}}$$

$$\gamma_{p\geq 3,k} = 0.$$

Classical RFIM
fixed point

↓

$U_k,\ Z_k,\ \Delta_k$
known scaling

↓

$F_k(\omega)$
to be determined

Fixed-point scaling

$$\phi = k^{\frac{d-4+\bar\eta}{2}}\varphi, \qquad U_k = k^d u_*, \qquad Z_k = k^{-\eta} z_*, \qquad \Delta_k = k^{-(2\eta-\bar\eta)}\delta_*.$$

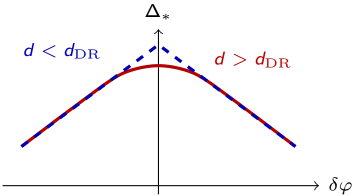
δ is crucial for the RFIM fixed point

δ encodes physics of metastability

Jumping between metastable states:

A) many small rearrangements of spins

B) one large rearrangement of spins



Nonperturbative FRG

$d > d_{\mathrm{DR}}$: δ_* analytic

$\Rightarrow$ dimensional reduction Parisi & Sourlas, PRL **43**, 744 (1979)

$\Rightarrow$ A)

$d < d_{\mathrm{DR}}$: δ_* cuspy

$\Rightarrow$ dimensional reduction breaks $\Rightarrow$ B)

Tarjus & Tissier, 2004–2012
Tarjus, Baczyk & Tissier, PRL (2013)

Below d_{DR}:

$$\delta_*(\varphi, \delta\varphi) = \delta_{*,0}(\varphi) + \delta_{*,1}(\varphi)|\delta\varphi| + \cdots$$

Cusp:

- DR is broken spontaneously
- $\eta \neq \bar{\eta}$ (3 independent critical exponents)

2) The cusp drives the dynamics

At the fixed point:

$$\delta_*(\phi, \delta\phi) \sim \delta_{*,0}(\phi) + \delta_{*,1}(\phi)|\delta\phi| + \cdots$$

fluctuationless fixed point
sharp cusp

↓

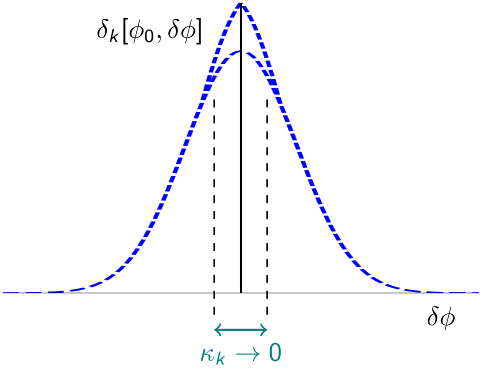
quantum fluctu-
ations at finite k
round the cusp

↓

boundary layer
$\kappa_k \quad \to \quad 0$

↓

large curvature
$\delta_k^{(0,2)}(0,0)$



Dangerously irrelevant quantum fluctuations

They disappear from the fixed point, but control how the cusp is approached.

2) The frequency sector feeds back on itself

Asymptotic flow

$$\partial_t F_k(\omega) \simeq -\frac{1}{4}\,\Delta_k^{(0,2)}(0,0) \int_q \Bigg[\frac{\partial_t R_{1,k}(q,\omega)}{\left(R_{1,k}(q,\omega) + q^2 Z_k + U_k'' + F_k(\omega)\right)^2} - \frac{\partial_t R_{1,k}(q,0)}{\left(R_{1,k}(q,0) + q^2 Z_k + U_k''\right)^2} \Bigg].$$

How we solve the problem:

- F is a full function of ω
- $R_{1,k}(q,\omega) = \alpha_0 k^2 Z_0\, s_0\left(\frac{q^2}{k^2} + \frac{\omega^2}{\omega_0^2}\right) + \alpha_1 k^{\varkappa}\, s_1\left(\frac{q^2}{k^2}, \frac{\omega^2}{\omega_0^2}\right)$
- The frequency regulator must follow the running scale of $F_k(\omega) \sim R_k^{\mathrm{dyn}}(\omega) \propto k^{\varkappa}$.

How does the dynamical kernel flow?

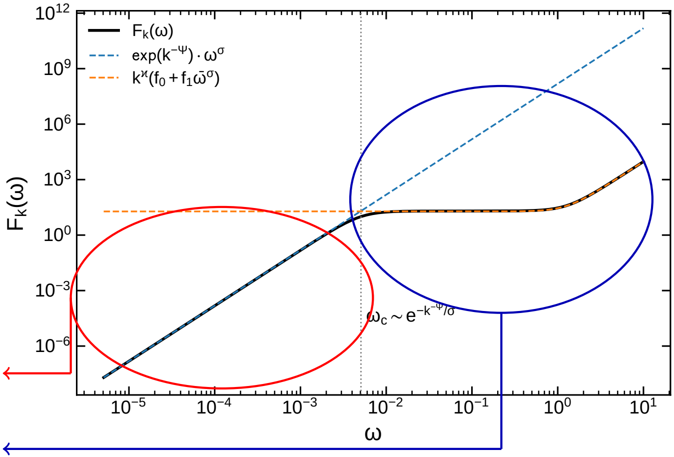
Key message

The flow of $F_k(\omega)$ is not uniform in frequency.

- at UV $F_\Lambda \sim \omega^\sigma$
- $\omega_c(k) \sim e^{-k^{\frac{\Psi}{\sigma}}}$

$$\boxed{\omega \lesssim \omega_c(k) \;\Rightarrow\; F \sim e^{k^{-\Psi}} \omega^\sigma}$$

$$\boxed{\omega \gg \omega_c(k) \;\Rightarrow\; F \sim k^{\varkappa}}$$



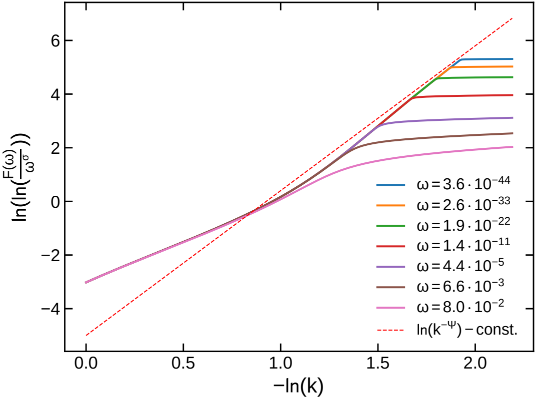
One function, two scaling regimes.

The activation exponent Ψ $(\omega \lesssim \omega_c(k))$

- $F \sim e^{k^{-\Psi}} \omega^{\sigma}$

$$\Psi = \begin{cases} \sigma\Theta + 2 - \eta, & T = 0, d < d_{DR}, \\ \sigma(\Theta - 2\lambda) + 2 - \eta, & T = 0, d \geq d_{DR}, \\ \Theta, & T > 0, d < d_{DR}, \\ \Theta - 2\lambda, & T > 0, d \geq d_{DR}, \end{cases}$$

d	$\Theta(= \Psi_{T>0})$	$\Psi_{T=0,\sigma=1}$	$\Psi_{T=0,\sigma=2}$
3	1.45	2.89	4.35
4	1.81	3.58	5.39
5	1.997	3.952	5.949



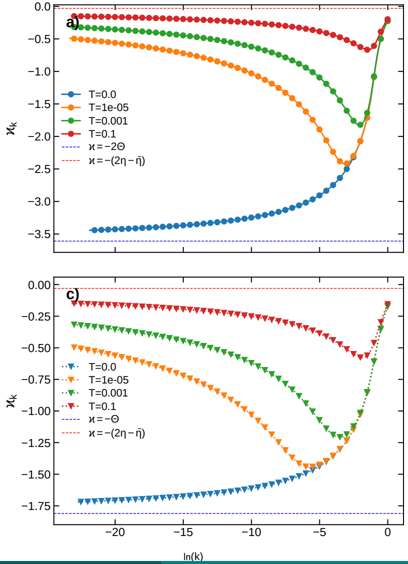
Controlled flow, not a runaway

The scaling exponent $\varkappa$ ($\omega \gg \omega_c(k)$)

- $F \sim k^{\varkappa}$
- F remembers the UV shape

$$\varkappa = \begin{cases} -\sigma\Theta, & T = 0, d < d_{DR}, \\ -\sigma(\Theta - 2\lambda), & T = 0, d \geq d_{DR}, \\ -(2\eta - \bar{\eta}), & T > 0, d < d_{DR}, \\ -\eta + 2\lambda, & T > 0, d \geq d_{DR} \end{cases}$$

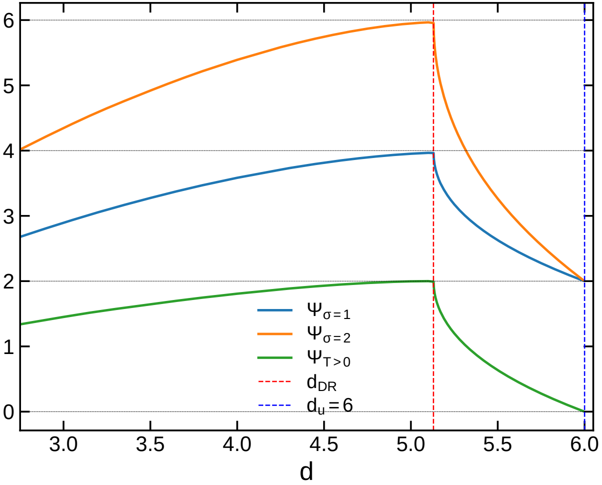
d	Θ	$\varkappa_{T>0}$	$\varkappa_{T=0,\sigma=1}$	$\varkappa_{T=0,\sigma=2}$
3	1.45	−0.009	−1.45	−2.90
4	1.81	−0.03	−1.81	−3.91
5	1.997	−0.004	−1.997	−3.993



3) Criterion of consistency

We show that $\Psi > 0$ implies:

- Δ is rounded in a boundary layer that shrinks to 0 as $k \to 0$
- Simplified equation for F_k is asymptotically correct



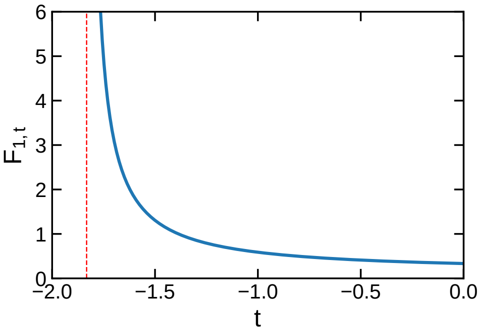
How to get localization mathematically?

Scenario A)

- Derivative expansion of the dynamic kernel
- Even with the special regulation the divergence appears
- $F_{1,k} = \left(p_1 \left(\bar{F}_1 - \frac{k^{-p_2}}{p_2} \right) \right)^{\frac{1}{-p_1}}, \qquad p_1, p_2 > 0$

Scenario B)

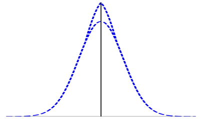
- Dynamic kernel as a full function of ω
- The special regulator is not used but a standard one $\propto k^{2-\eta}$
- self consistency $\Rightarrow \varkappa_{i+1} = -(4 - \bar{\eta}) + 2\varkappa_i - \frac{\varkappa_i}{\sigma}$ $\Rightarrow \varkappa \to -\infty$.



The mechanism

$\Psi > 0$

Cusp rounding



$\kappa_k \to 0$

Simplified flow of F

$\partial_t F \propto \Delta^{(02)}$

Additional frequency regulator

scaling like

$F \propto k^{\varkappa}$

Full frequency dependence

$F = F(\omega)$

Activated dynamics

- $\omega \lesssim \omega_c(k) \Rightarrow F \sim e^{k^{-\Psi}} \omega^{\sigma}$
- $\omega \gg \omega_c(k) \Rightarrow F \sim k^{\varkappa}$

Where could it appear?

- Pinned elastic manifolds and quantum creep.
- Bose glass and Mott glass problems?
- Possible warning for localization-like finite-scale singularities in FRG flows.

Conclusions

- The QRFIM static criticality is controlled by the classical RFIM zero-temperature fixed point.
- Quantum fluctuations are dangerously irrelevant but dynamically essential.
- They round the cuspy disorder cumulant in an anomalous boundary layer.
- The rounded cusp generates the flow of the full dynamic kernel $F_k(\omega)$.
- With an adapted frequency regulator, the flow gives activated dynamics rather than localization.
- Analytical predictions of the activation exponent Ψ and the dynamic kernel scaling $\varkappa$

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