



Density functional theory of renormalization group in nuclear matter

Wei-jie Fu

Dalian University of Technology

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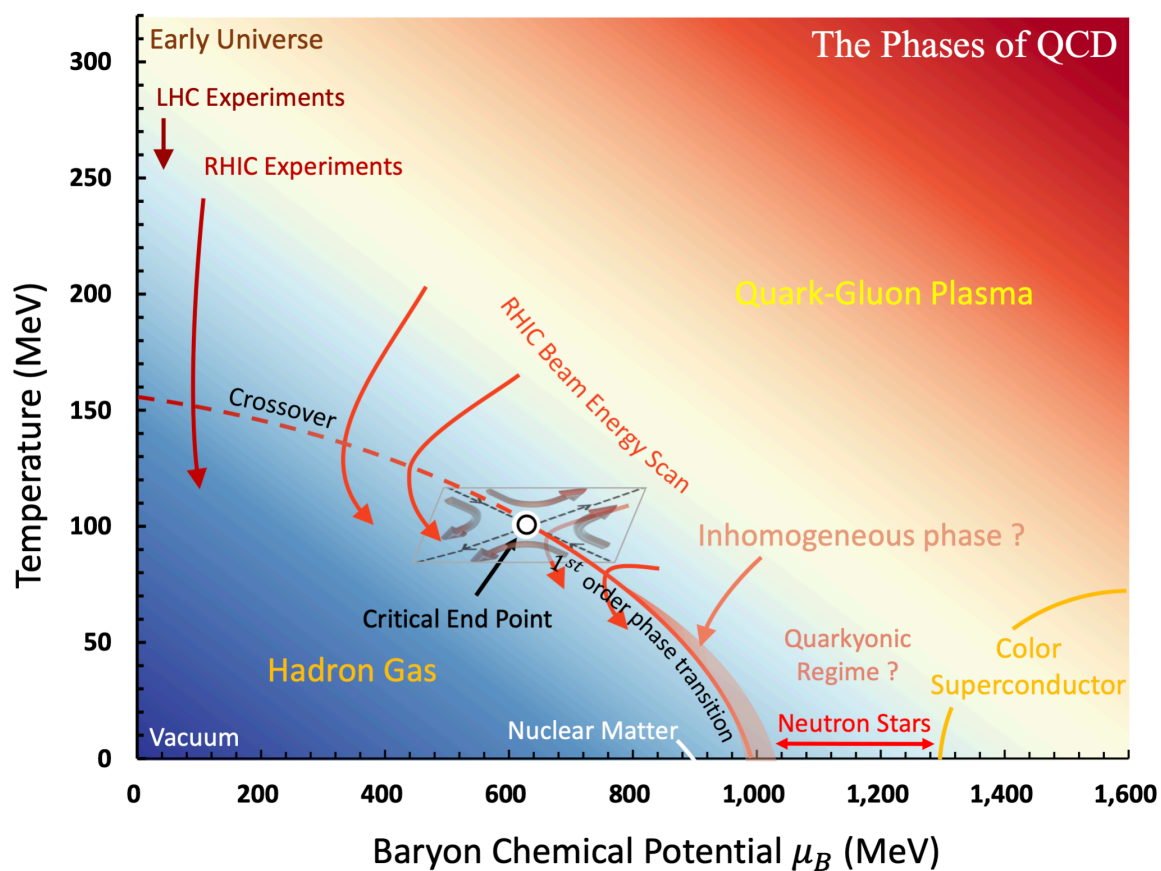
Based on:

Yong-rui Chen, WF, Yang-yang Tan, *PRC* 113 (2026) 045801, arXiv:2508.02575.

Yong-rui Chen, Sheng-nan Han, Yang-yang Tan, Jing Wu, Zhi-yi Zhao, WF, in preparation.

QCD phase diagram at high densities

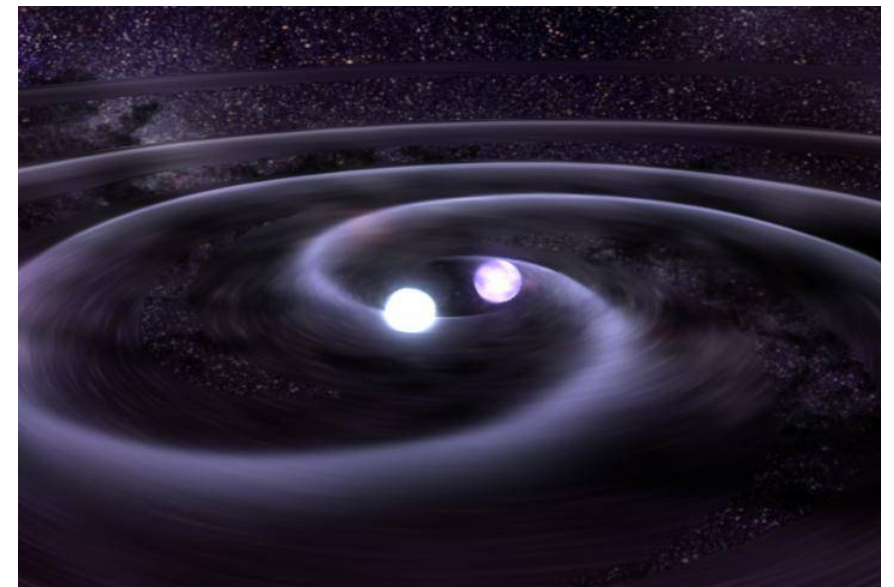
QCD phase diagram



Neutron Star



Gravitational waves

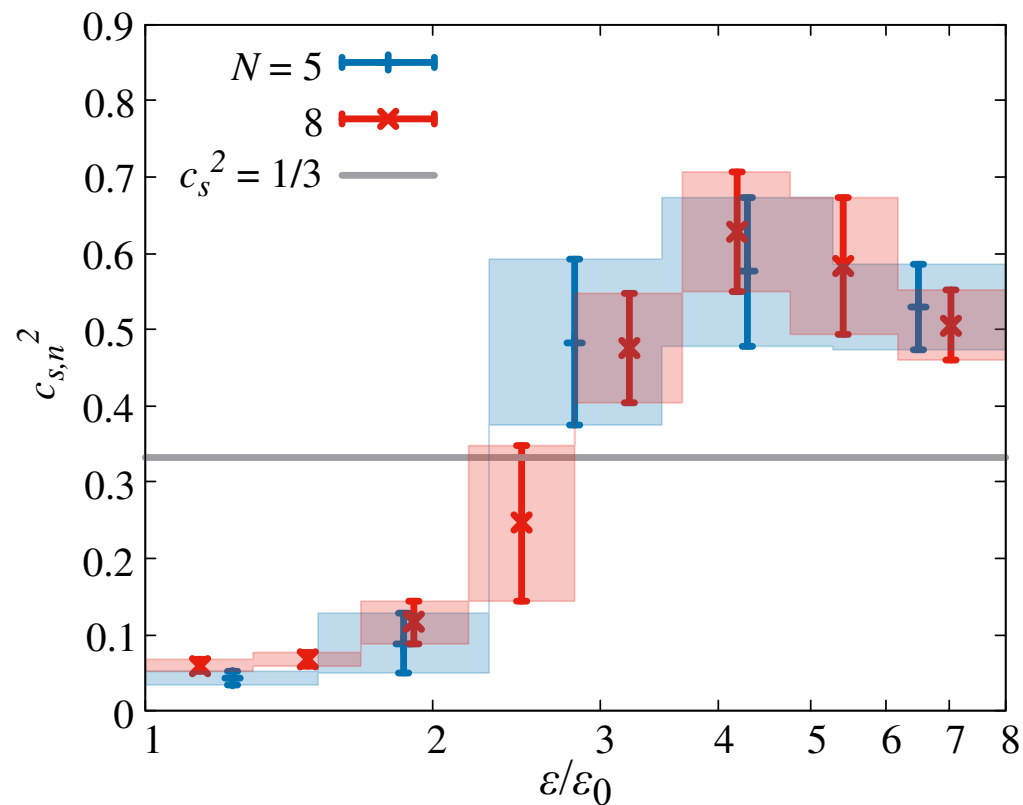


See also the talk by Fabian Rennecke on Wed

- Studies of QCD phase diagram at high densities play an important role in exploring the nature of strong interactions, compact stars, etc.

EoS at high baryon densities

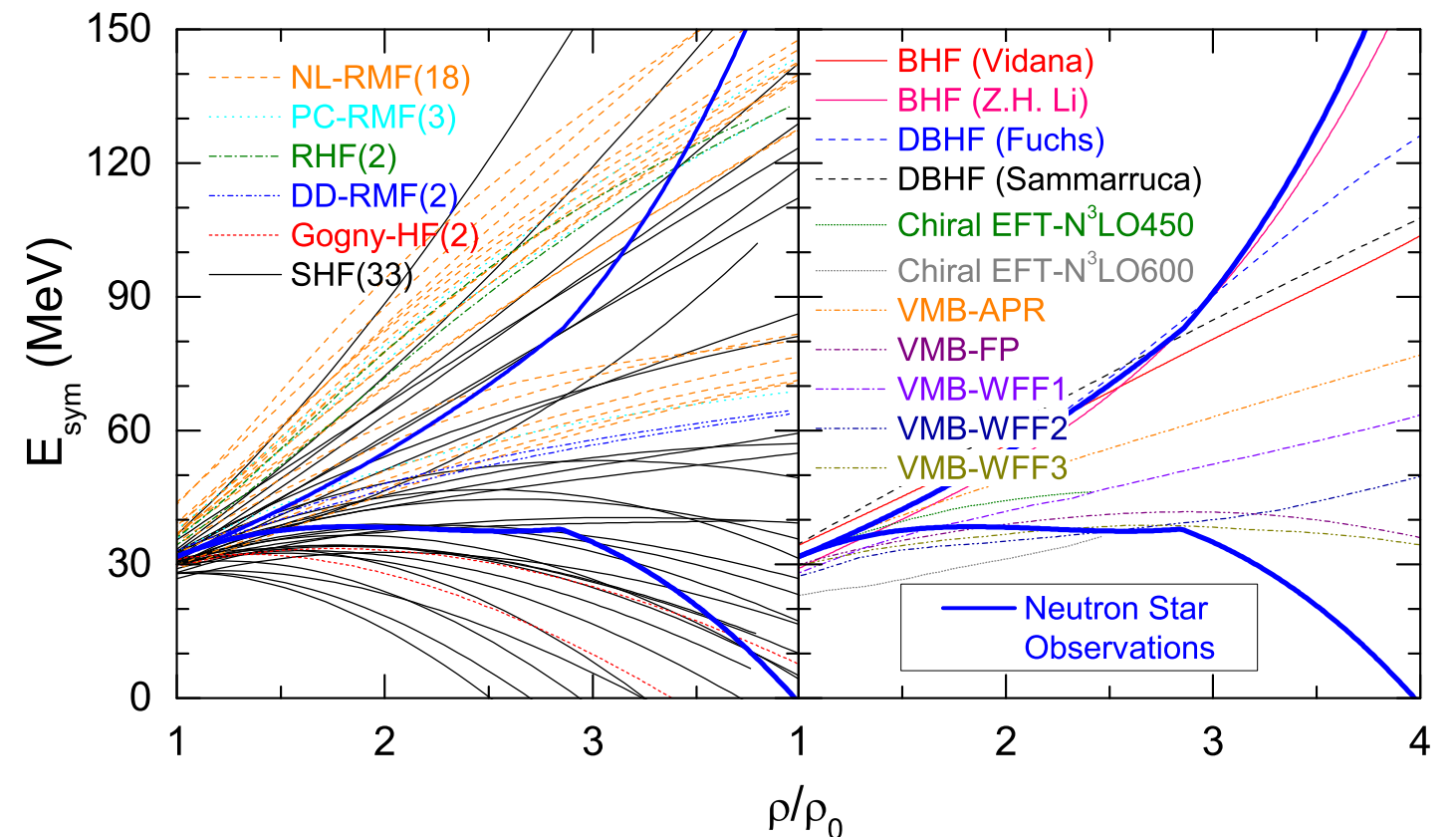
Speed of sound



Inferred from NS observations with machine learning

K. Fukushima, arXiv:2501.01907

Nuclear symmetry energy



Nai-Bo Zhang, Bao-An Li, *ApJ* 902 (2020) 38

- Both the speed of sound and symmetry energy of nuclear matter beyond the saturation density are not well determined.

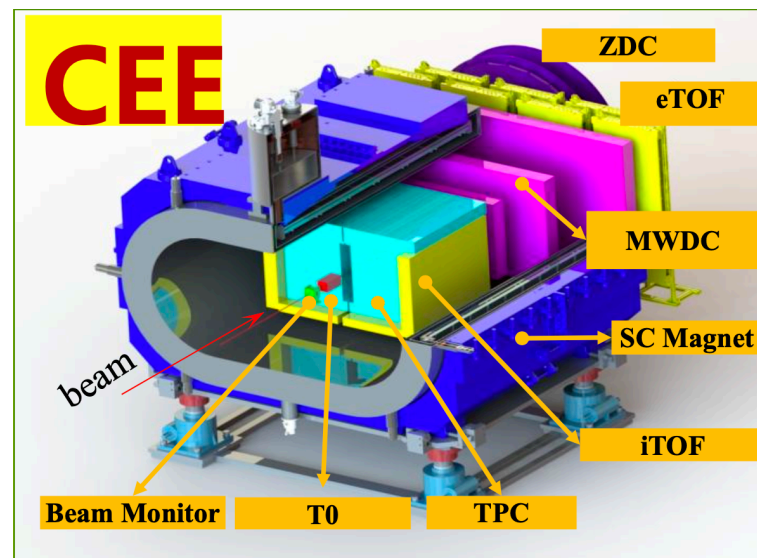
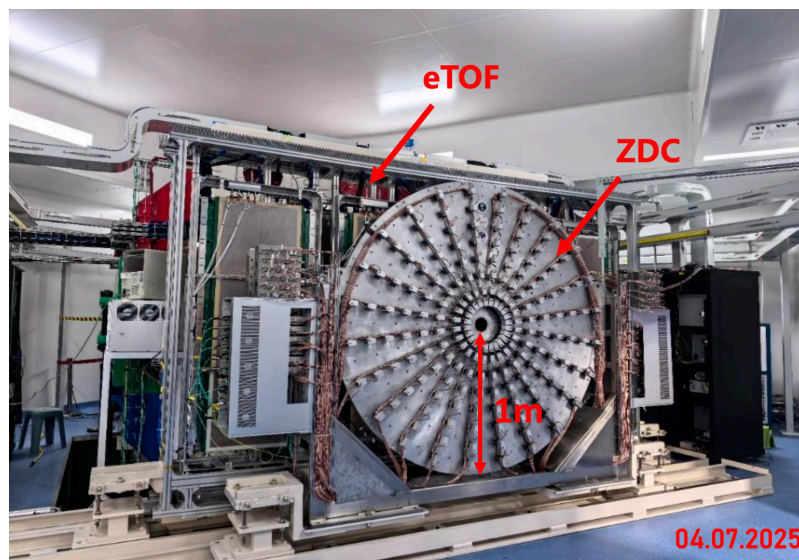
Facilities at high baryon densities

HIAF



High Intensity
heavy-ion
Accelerator
Facility (HIAF)
and CEE
detector in
China

CEE



- HIAF and CEE, facilities at high baryon densities in China, have already begun to operate from last year, which provide us with the opportunity to study the physics of strong interactions at high baryon densities.

Outline

- **Introduction**
- **Formalism of density-fRG**
- **Some numerical results**
- **Silver Blaze property and two-loop flows**
- **Summary and outlook**

Walecka model in fRG

- The effective action in the Euclidean spacetime

$$\Gamma_k[\Phi] = \int_x \left\{ \bar{\psi}_N \left[\gamma_\mu \partial_\mu + m_N - \gamma_0 \hat{\mu}_N - h_{\sigma,k} \sigma T^0 + i h_{\omega,k} \gamma_\mu \omega_\mu T^0 + i h_{\rho,k} \gamma_\mu \vec{\rho}_\mu \cdot \vec{T} \right] \psi_N \right. \\ \left. + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{\lambda_3}{3!} \sigma^3 + \frac{\lambda_4}{4!} \sigma^4 + \frac{1}{4} \omega_{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega_\mu + \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}_{\mu\nu} \right. \\ \left. + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}_\mu + V_k(\sigma, \omega, \rho) \right\},$$

where V_k is the effective potential arising from density fluctuations, and thus it is vanishing in the vacuum, i.e.,

$$V_{k \rightarrow \infty} = 0$$

In the density-fRG, the density fluctuations are taken into account through the evolution of the RG scale k . The vacuum case corresponds to $k \rightarrow \infty$, and in the other infrared limit $k \rightarrow 0$ the full density effects are encoded.

Yong-rui Chen, WF, Yang-yang Tan, *PRC* 113 (2026) 045801, arXiv:2508.02575

Flow of the effective potential

Flow equation of the effective action:

$$\partial_t \Gamma_k[\Phi] = - \text{[solid circle with cross]} + \frac{1}{2} \text{[dashed circle with cross]}$$

From the flow equation of the effective action above, one readily arrives at the flow of effective potential

$$\partial_t V_k(\sigma, \omega_0, \rho_{03}) = -\frac{2}{3\pi^2} k^4 \left[\Delta \mathcal{F}_{(1)}(\bar{m}_N^{*2}; T, \mu_p^*) + \Delta \mathcal{F}_{(1)}(\bar{m}_N^{*2}; T, \mu_n^*) \right],$$

with

$$\Delta \mathcal{F}_{(1)}(\bar{m}_f^2; T, \mu) = \mathcal{F}_{(1)}(\bar{m}_f^2; T, \mu) - \mathcal{F}_{(1)}(\bar{m}_f^2; 0, 0),$$

where the vacuum part in the threshold function is subtracted. The effective chemical potentials for the proton and neutron read

$$\begin{aligned} \mu_p^* &= \mu_p - \frac{1}{2} \left(h_{\omega,k} i\omega_0 + h_{\rho,k} i\rho_{03} \right), \\ \mu_n^* &= \mu_n - \frac{1}{2} \left(h_{\omega,k} i\omega_0 - h_{\rho,k} i\rho_{03} \right). \end{aligned}$$

The effective mass of nucleons

$$m_N^* = m_N - \frac{1}{2} h_{\sigma,k} \sigma.$$

Thermodynamic potential

It is straightforward to integrate out the flow equation and obtain the effective potential

$$V_{k=0}(\sigma, \omega_0, \rho_{03}) = -\frac{1}{3\pi^2} \int_0^\infty dk \frac{k^4}{E_k} \left[n_F(\bar{m}_N^{*2}; T, \mu_p^*) + n_F(\bar{m}_N^{*2}; T, -\mu_p^*) \right. \\ \left. + n_F(\bar{m}_N^{*2}; T, \mu_n^*) + n_F(\bar{m}_N^{*2}; T, -\mu_n^*) \right],$$

with

$$E_k = \sqrt{k^2 + m_N^{*2}},$$

and the fermionic distribution function

$$n_F(\bar{m}_f^2; T, \mu) = \frac{1}{\exp\left\{ \frac{1}{T} \left[k(1 + \bar{m}_f^2)^{1/2} - \mu \right] \right\} + 1}.$$

Then, one arrives at the thermodynamic potential density,

$$\Omega = V_{k=0} + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{\lambda_3}{3!} \sigma^3 + \frac{\lambda_4}{4!} \sigma^4 - \frac{1}{2} m_\omega^2 (\mathrm{i}\omega_0)^2 - \frac{1}{2} m_\rho^2 (\mathrm{i}\rho_{03})^2.$$

- Note that if the k -dependence of the Yukawa couplings is neglected, that is, the Yukawa couplings do not depend on the density, the effective potential would return to the result obtained in the mean field approximation.

Flows of Yukawa couplings

Flow equation of the Yukawa coupling:

$$\partial_t \left(\text{triangle diagram with red dashed line } \sigma \right) = \tilde{\partial}_t \left(\text{triangle diagram with red dashed line } \sigma + \text{triangle diagram with blue dashed line } \sigma + \text{triangle diagram with green dashed line } \sigma + \text{triangle diagram with red dashed line } \sigma \right)$$

Yong-rui Chen, WF, Yang-yang Tan, *PRC* 113 (2026) 045801, arXiv:2508.02575

$$\partial_t \left(\text{triangle diagram with blue dashed line } \omega \right) = \tilde{\partial}_t \left(\text{triangle diagram with red dashed line } \omega + \text{triangle diagram with blue dashed line } \omega + \text{triangle diagram with green dashed line } \omega \right)$$

$$\partial_t \left(\text{triangle diagram with green dashed line } \rho \right) = \tilde{\partial}_t \left(\text{triangle diagram with red dashed line } \rho + \text{triangle diagram with blue dashed line } \rho + \text{triangle diagram with green dashed line } \rho \right)$$

Silver Blaze property:

In order to recover the Silver Blaze property, we modify the threshold functions in the flows of Yukawa couplings as such,

The flow of the Yukawa coupling is obtained from the projections as follows,

$$\partial_t h_{\sigma,k} = -\frac{1}{2} \text{Re} \left[\text{Tr} \left(T^0 \partial_t \Gamma_{\bar{\psi}\psi\sigma}^{(3)}(p_0) \right) \right],$$

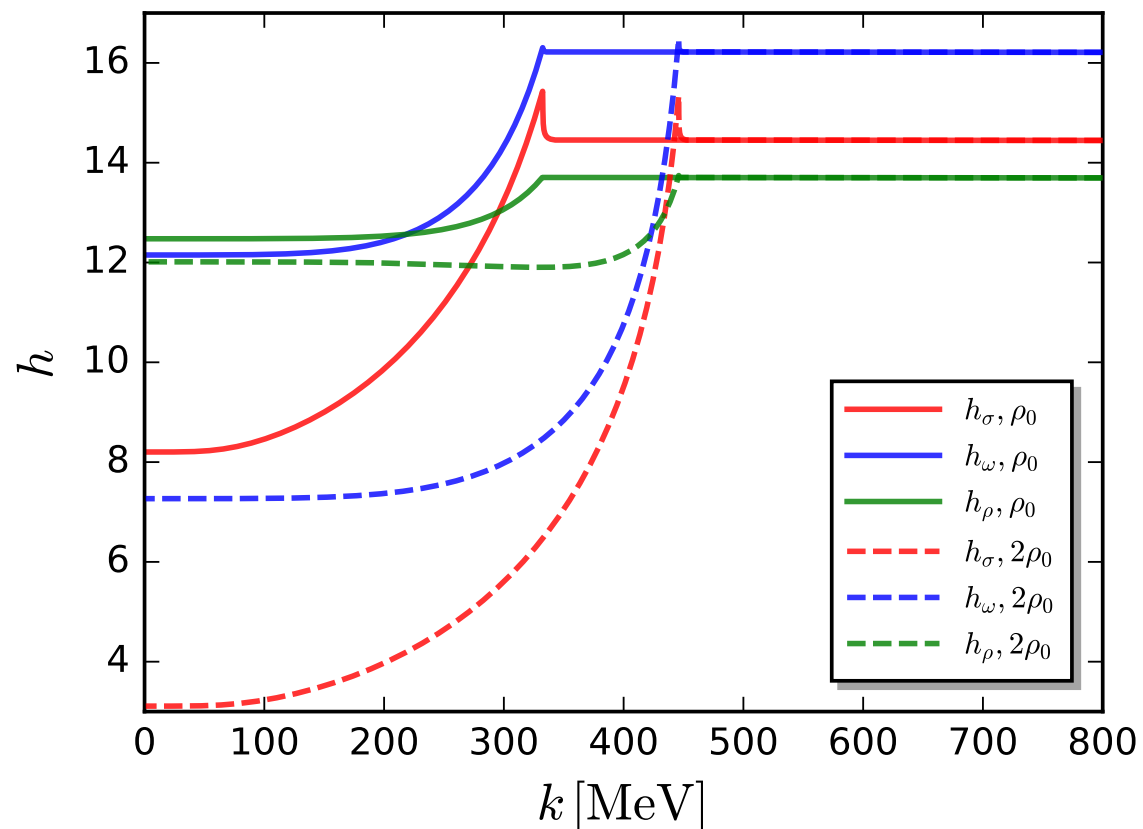
$$\partial_t h_{\omega,k} = -\frac{1}{6} \text{Re} \left[\text{Tr} \left(T^0 i \vec{\gamma}_\mu \partial_t (\Gamma_{\bar{\psi}\psi\omega}^{(3)})_\mu(p_0) \right) \right],$$

$$\partial_t h_{\rho,k} = -\frac{1}{6} \text{Re} \left[\text{Tr} \left(T^3 i \vec{\gamma}_\mu \partial_t (\Gamma_{\bar{\psi}\psi\rho}^{(3)})_\mu^{i=3}(p_0) \right) \right],$$

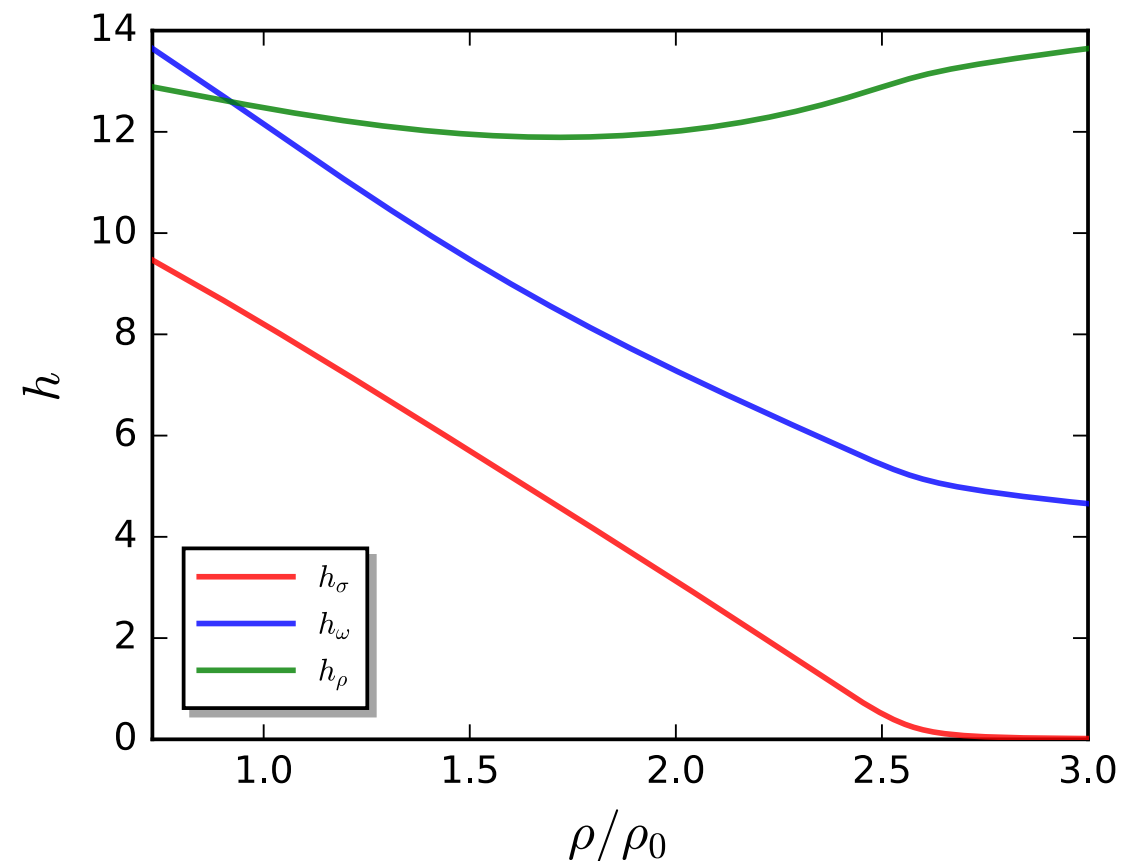
$$\begin{aligned} & \Delta \mathcal{F} \mathcal{B}_{(n_f, n_b)}(\bar{m}_f^2, \bar{m}_b^2; T, \mu, p_0) \\ &= \left[\mathcal{F} \mathcal{B}_{(n_f, n_b)}(\bar{m}_f^2, \bar{m}_b^2; T, \mu, p_0) \right. \\ & \quad \left. - \mathcal{F} \mathcal{B}_{(n_f, n_b)}(\bar{m}_f^2, \bar{m}_b^2; 0, 0, 0) \right] \left[n_F(\bar{m}_f^2; T, \mu) \right. \\ & \quad \left. + n_F(\bar{m}_f^2; T, -\mu) \right], \end{aligned}$$

Density dependent Yukawa couplings

Yukawa vs RG scale:



Yukawa vs density:

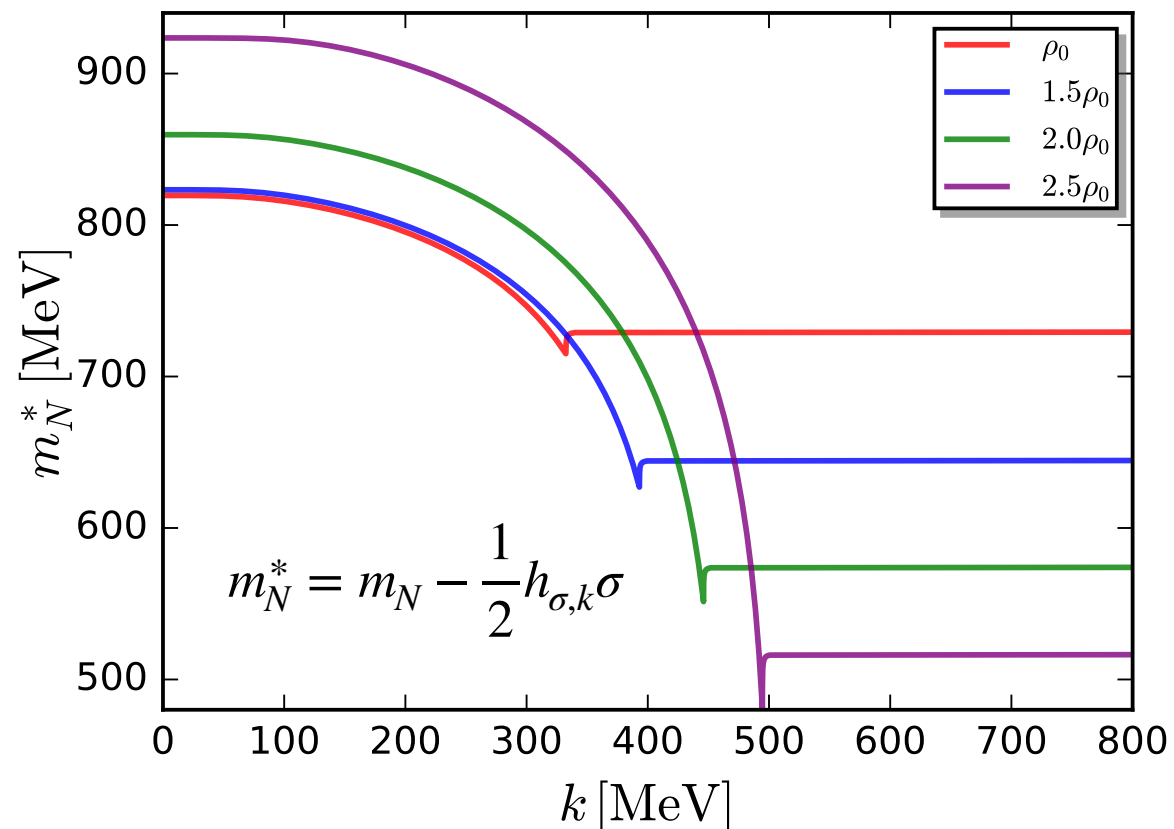


- Density fluctuations reduce h_σ , h_ω , and the dependence of h_ρ is mild.
- Both the attractive and repulsive interactions are **screened** by the **nuclear medium**.

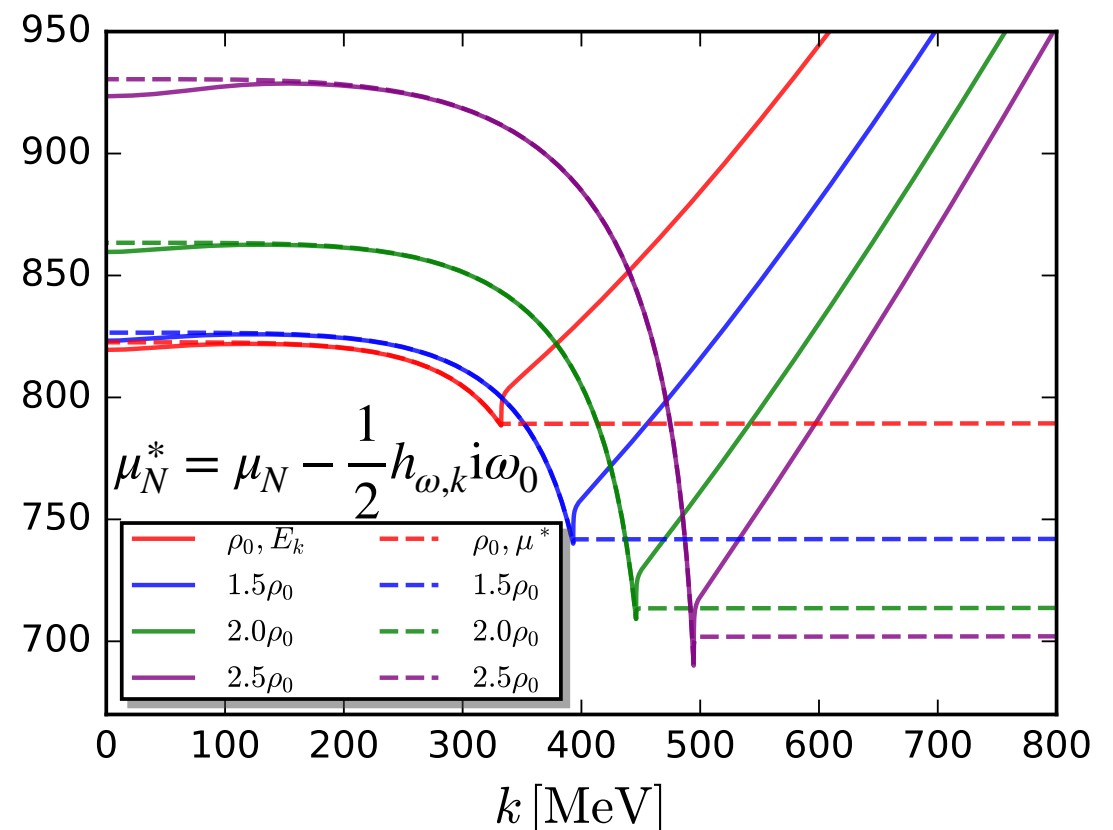
Yong-rui Chen, WF, Yang-yang Tan, *PRC* 113 (2026) 045801, arXiv:2508.02575

Running nucleon mass and chemical potentials

Effective nucleon mass:



Effective energy and chemical potential:

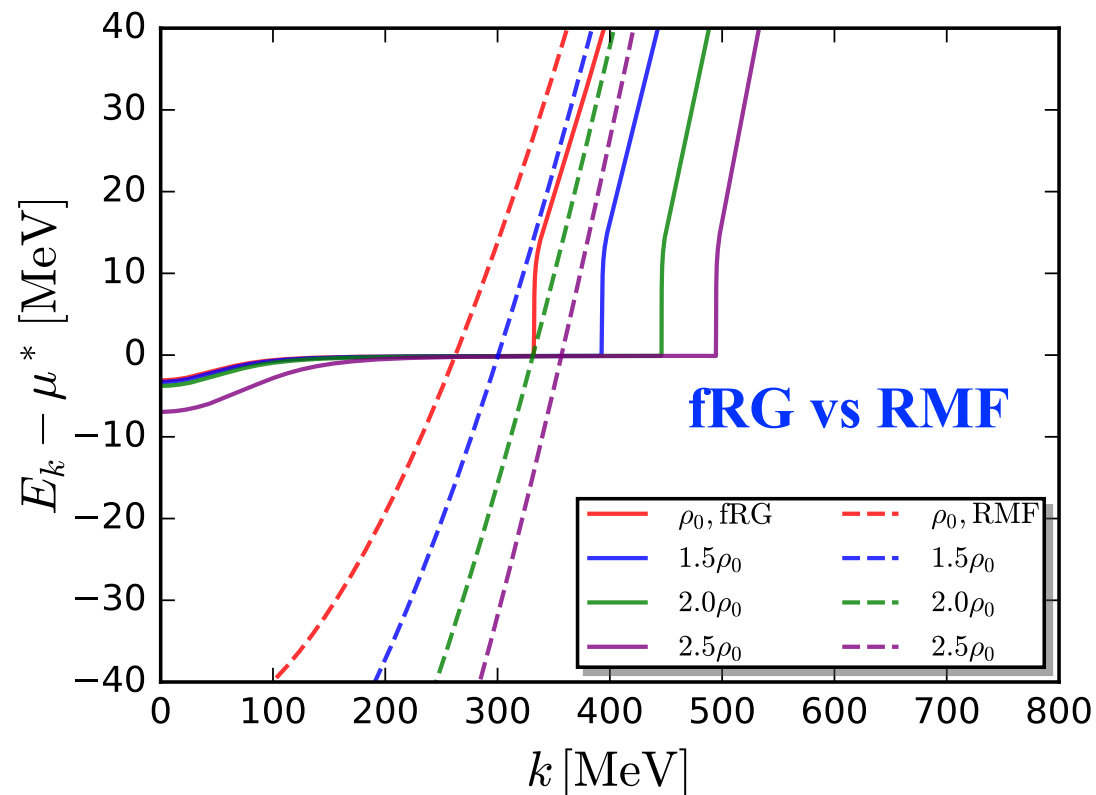


Yong-rui Chen, WF, Yang-yang Tan, *PRC* 113 (2026) 045801, arXiv:2508.02575

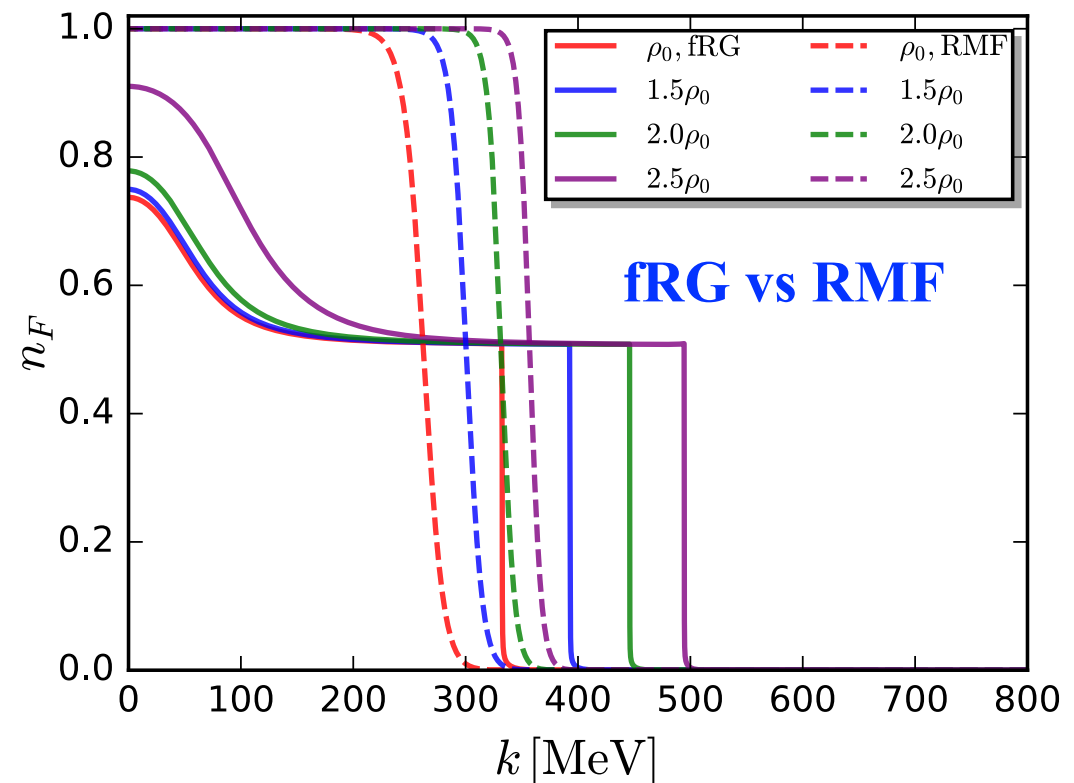
- This is different from the RMF case, where both the effective mass and chemical potential are constants at a fixed baryon density.
- Due to the **screening** effect, the nucleon mass **increases** with the decrease of the RG scale.

Fermi surface locking

Difference between the energy and the chemical potential:



Fermionic distribution function:

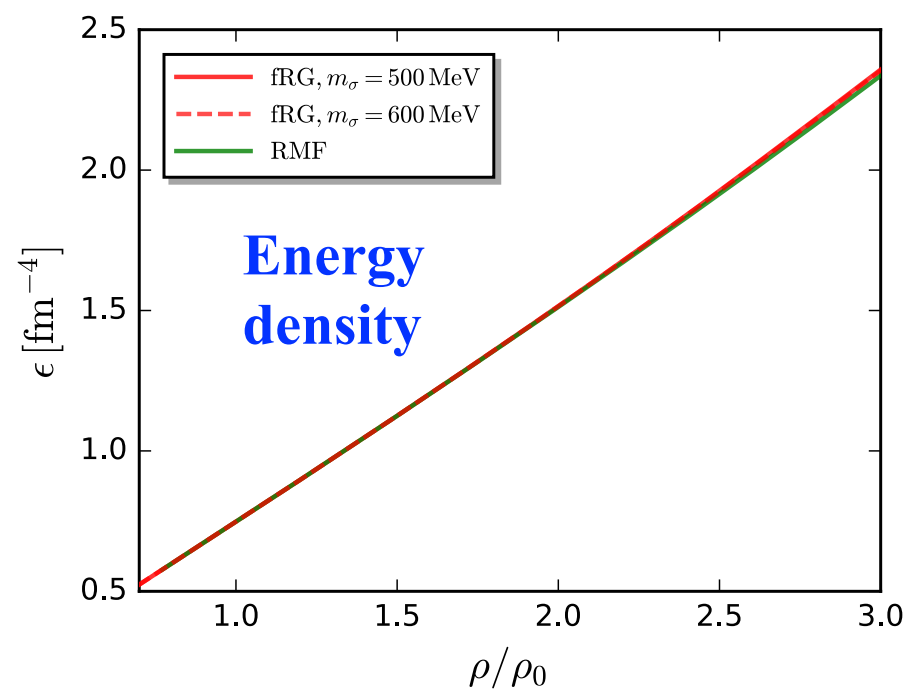
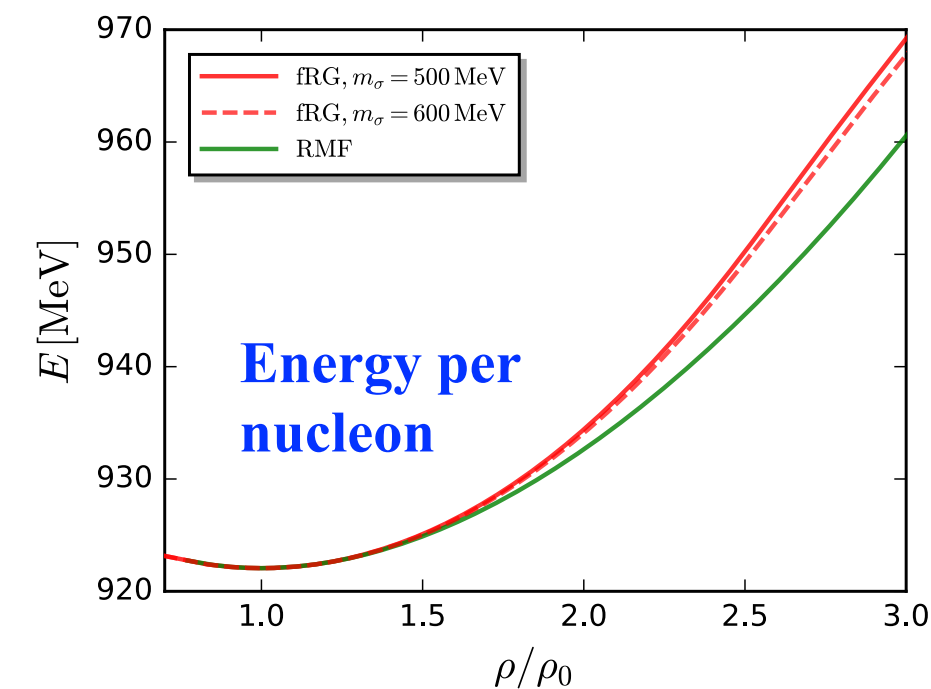
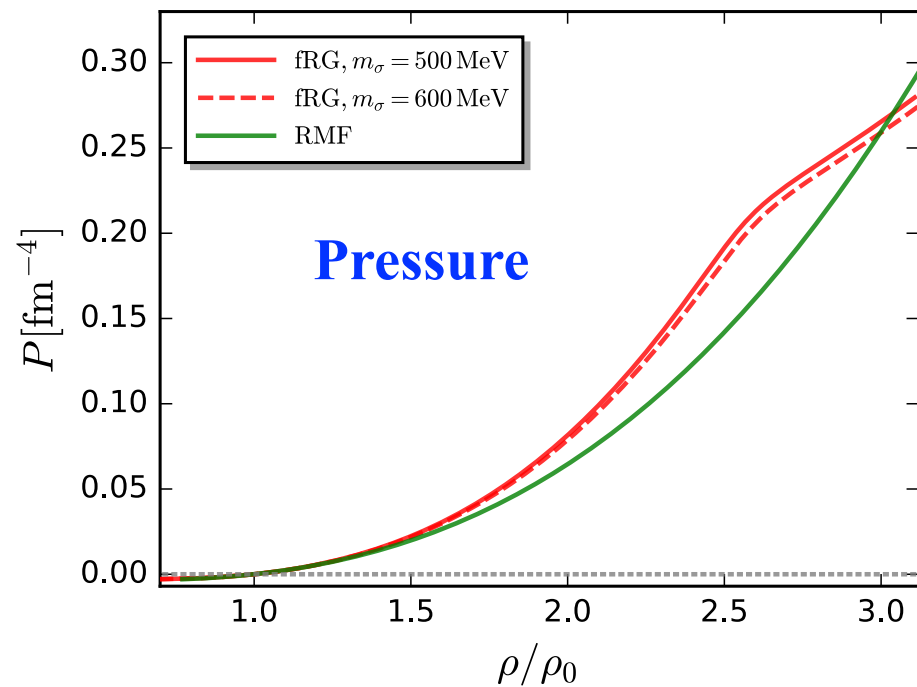
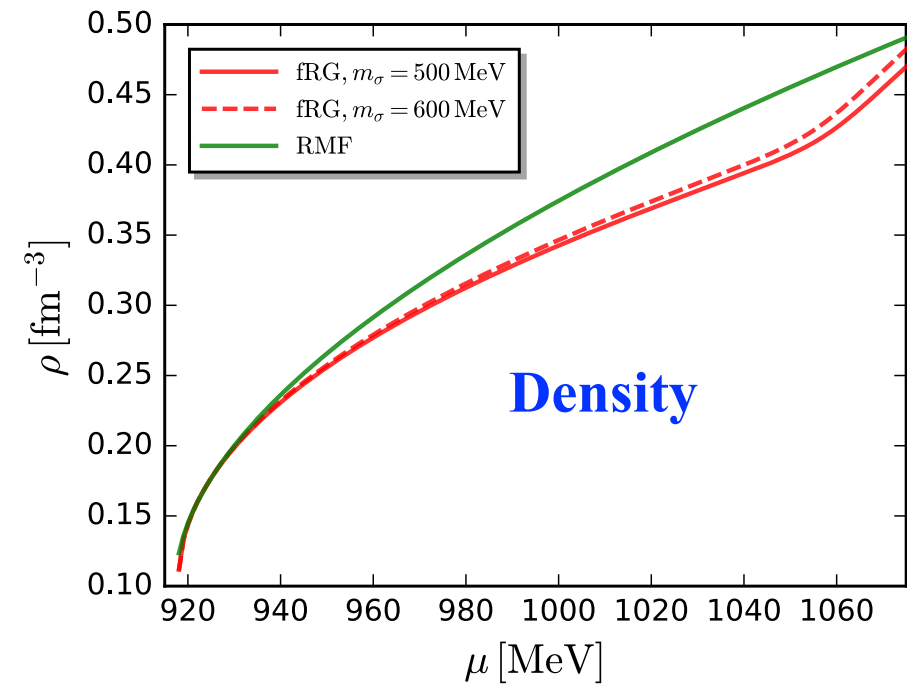


Yong-rui Chen, WF, Yang-yang Tan, *PRC* 113 (2026) 045801, [arXiv:2508.02575](https://arxiv.org/abs/2508.02575)

- There is a quite large region that the running energy and chemical potential are always close to each other in fRG, quite different from the conventional RMF. We call it Fermi surface locking.
- The Fermi surface locking is the consequence of the **balance** between the **attractive** and **repulsive running** Yukawa couplings.

Equation of state

fRG vs RMF:

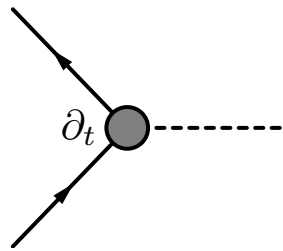


- A stiffer EoS in the regime of $\rho_0 \lesssim \rho \lesssim 2.5\rho_0$, then a softer EoS when $\rho \gtrsim 2.5\rho_0$.

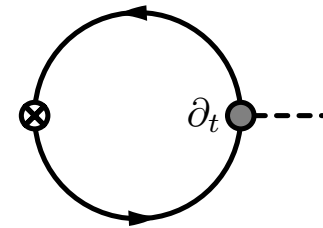
Yong-rui Chen, WF, Yang-yang Tan,
PRC 113 (2026) 045801,
 arXiv:2508.02575

Silver Blaze property

Breaking of the Silver Blaze property arises from the truncation of frequency for the external fermionic line



If the fermionic frequencies are summed with a loop, e.g.,



The Silver Blaze property can be restored.

Flow of Yukawa vertices is changed:

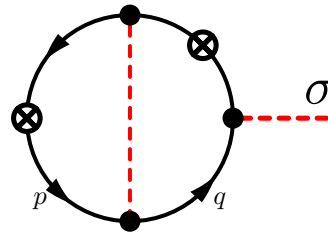
$$\partial_t \left(\text{triangle diagram} \right) = \tilde{\partial}_t \left(\text{triangle diagram with red dashed line} + \text{triangle diagram with blue dashed line} + \text{triangle diagram with green dashed line} + \text{triangle diagram with red dashed line} \right)$$



$$\text{loop diagram} = \text{loop diagram with red dashed line} + \text{loop diagram with blue dashed line} + \text{loop diagram with green dashed line} + \text{loop diagram with red dashed line} + \dots$$

Two-loop flows

Summation of the external frequencies of fermionic line would necessarily result in two-loop flows, e.g.,



A similar case was also found in WF, [J. M. Pawłowski, F. Rennecke, B.-J. Schaefer, PRD 94 \(2016\) 116020, arXiv:1608.04302](#). The two-loop flow would give rise to not only the usual fermionic distribution function

$$n_F(E_{\text{nucleon}} - \mu),$$

but also new kinds of

$$n_F(E_{\text{nucleon}} - E_{\text{meson}} - \mu),$$

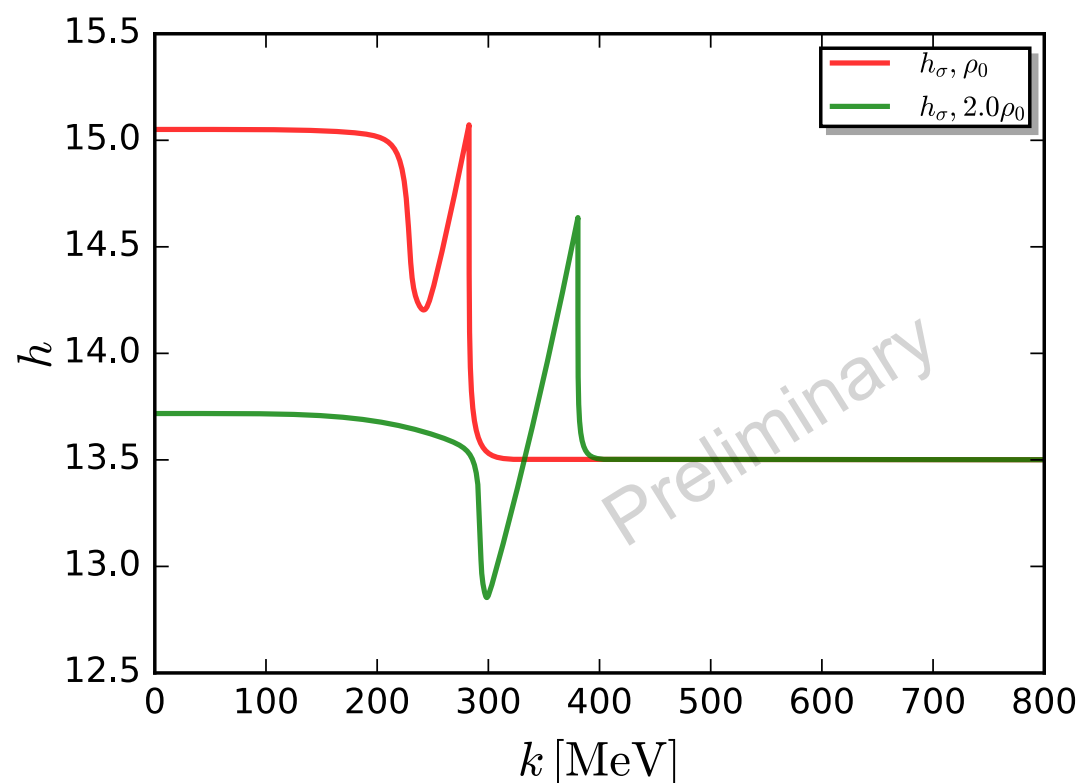
$$n_F(E_{\text{nucleon}} + E_{\text{meson}} - \mu),$$

$$n_F(E_{\text{nucleon}} - E_{\text{meson}} + \mu), \text{ etc.}$$

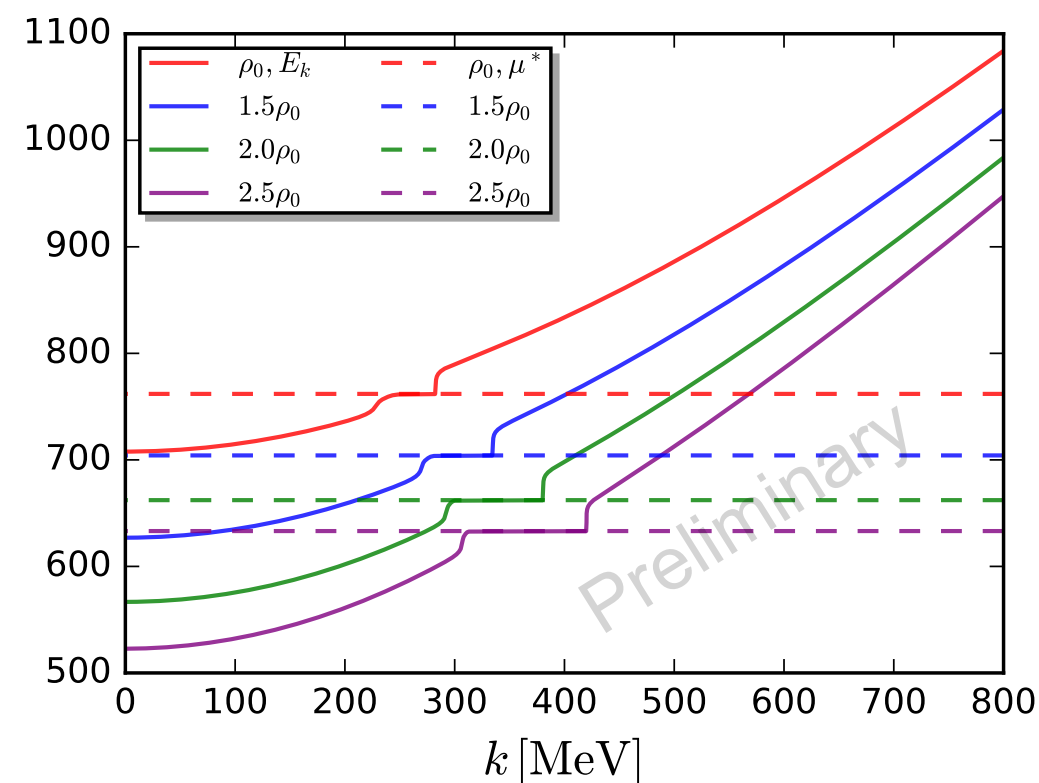
which are different from usual Fermi surface.

Some preliminary results of two-loop flows

h_σ vs RG scale:



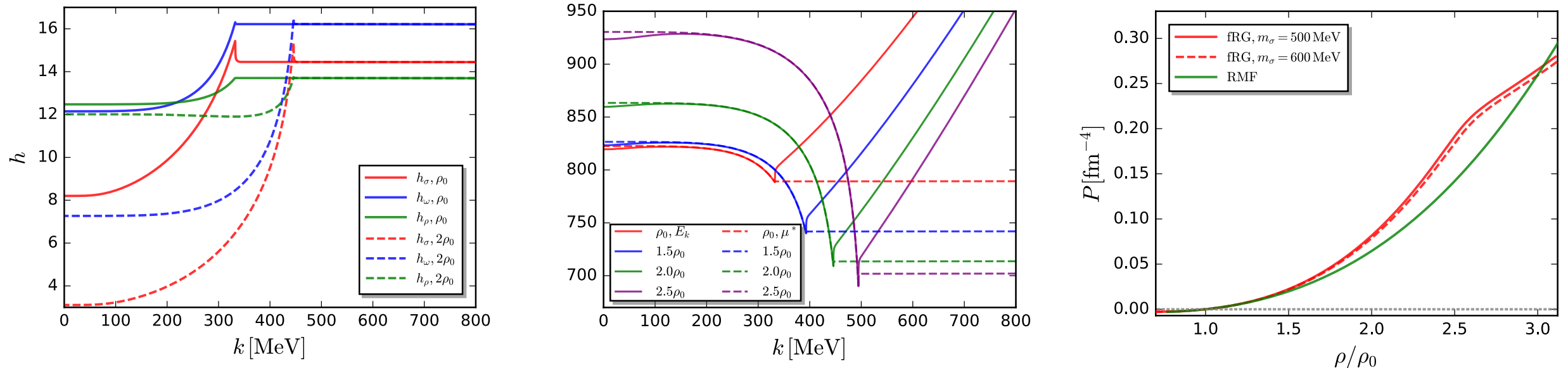
Effective energy and chemical potential:



- Note that only the flow of h_σ is implemented for the moment.
- Stay tuned for the **full** results of two-loop flows.

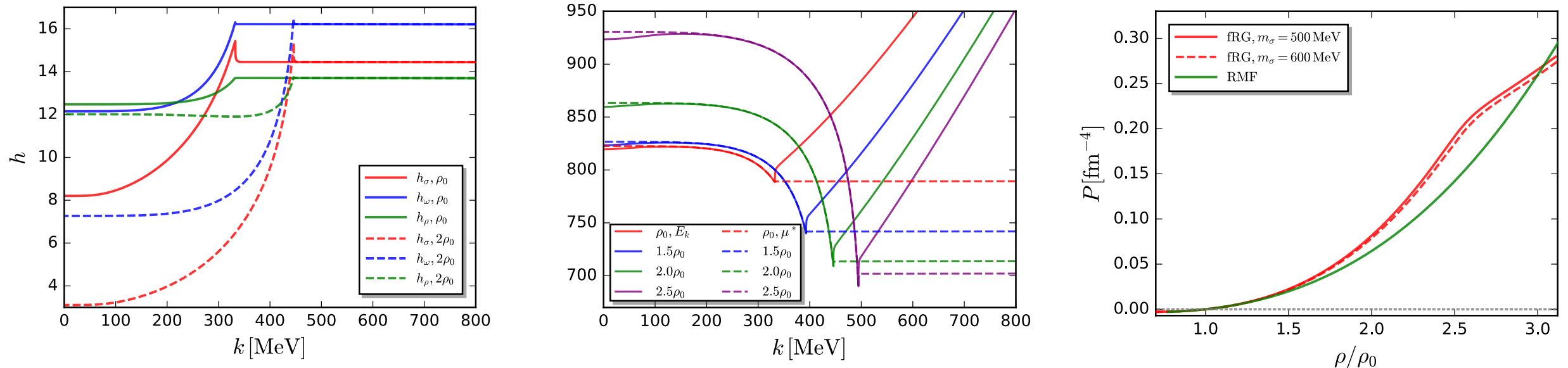
Yong-rui Chen, Sheng-nan Han, Yang-yang Tan, Jing Wu, Zhi-yi Zhao, WF, in preparation

Summary and outlook



- ★ The density functional renormalization group (density-fRG) is proposed to investigate the high density physics in the nuclear matter.
- ★ Both the attractive and repulsive nucleon meson interactions are screened by the high density medium, which results in a stiffer EoS in the regime of $\rho_0 \lesssim \rho \lesssim 2.5\rho_0$, then a softer EoS when $\rho \gtrsim 2.5\rho_0$.
- ★ Two-loop flows are proposed to recover the Silver Blaze property, which are very important for the computations at high densities.

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- ★ Two-loop flows are proposed to recover the Silver Blaze property, which are very important for the computations at high densities.

Thank you very much for your attentions!

Backup

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathcal{D}\hat{\Phi}) \exp \left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

flow of the Schwinger function:

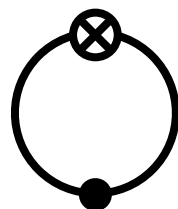
$$\partial_t W_k[J] = -\frac{1}{2} \text{STr} \left[(\partial_t R_k) G_k \right] - \frac{1}{2} \Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

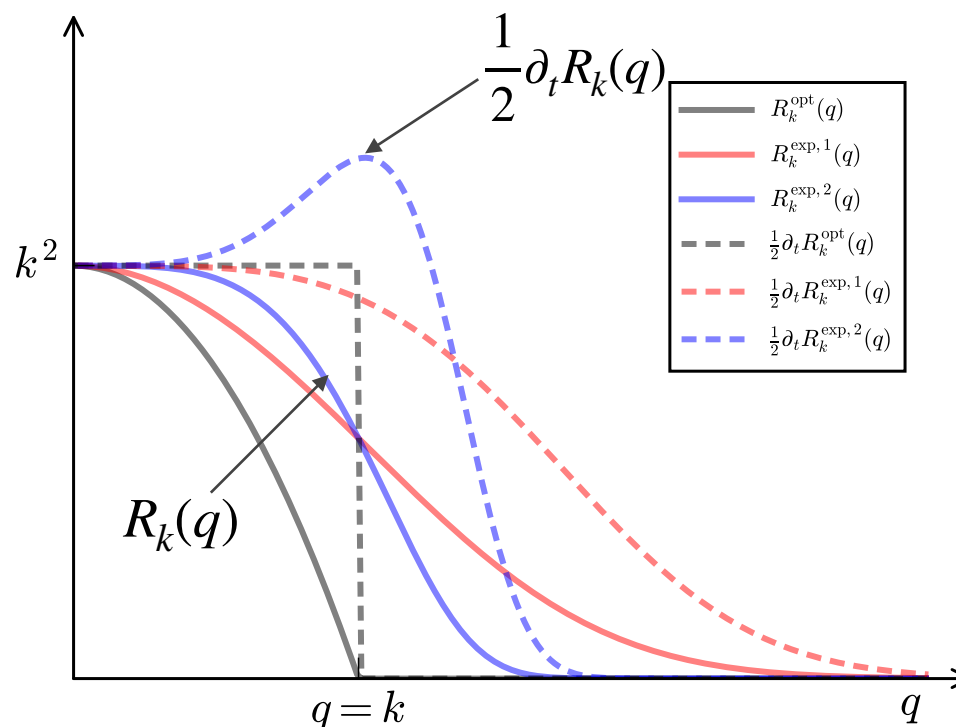
flow of the effective action:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr} \left[(\partial_t R_k) G_k \right] = \frac{1}{2}$$

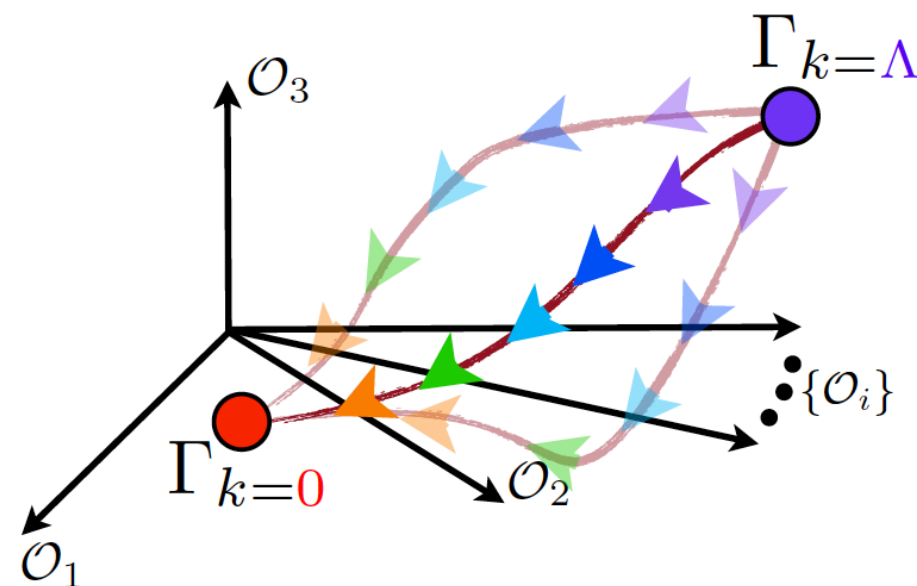


Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90



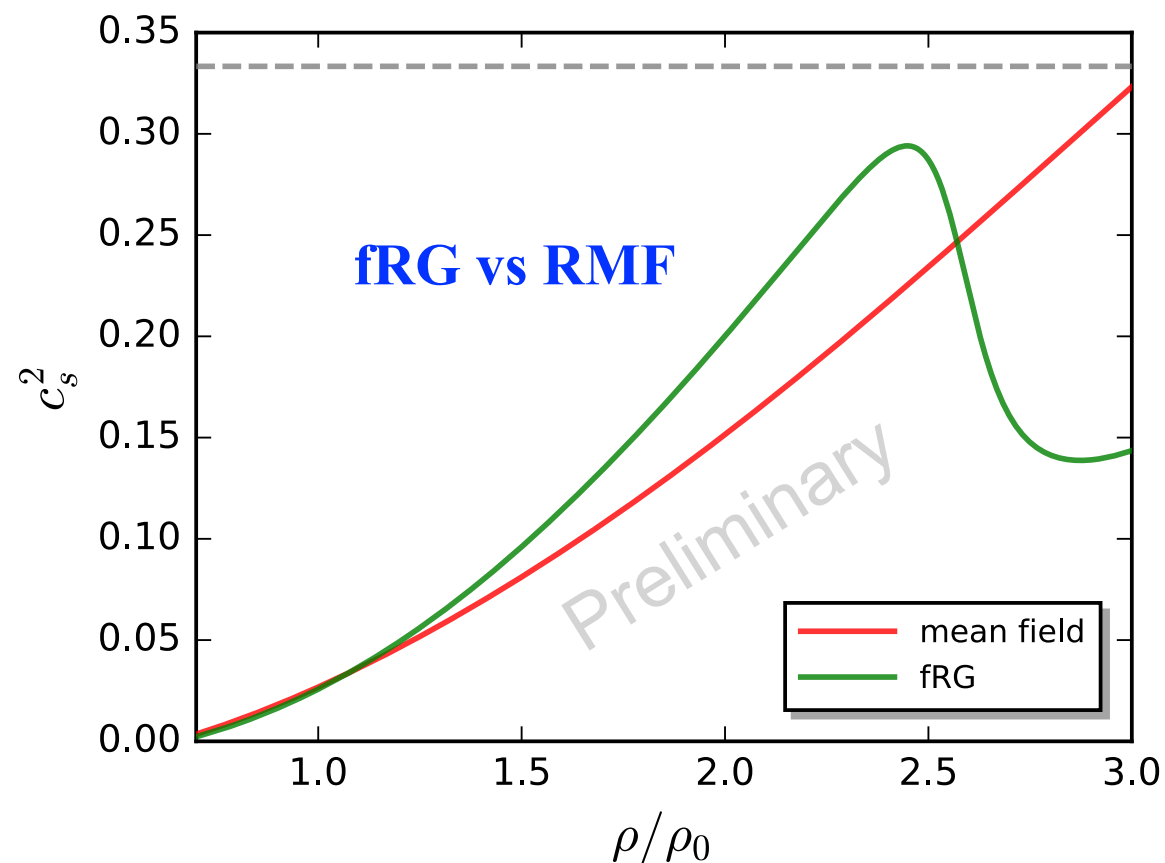
$$G_{k,ab} = \gamma^c_a \left(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi] \right)^{-1}_{cb},$$



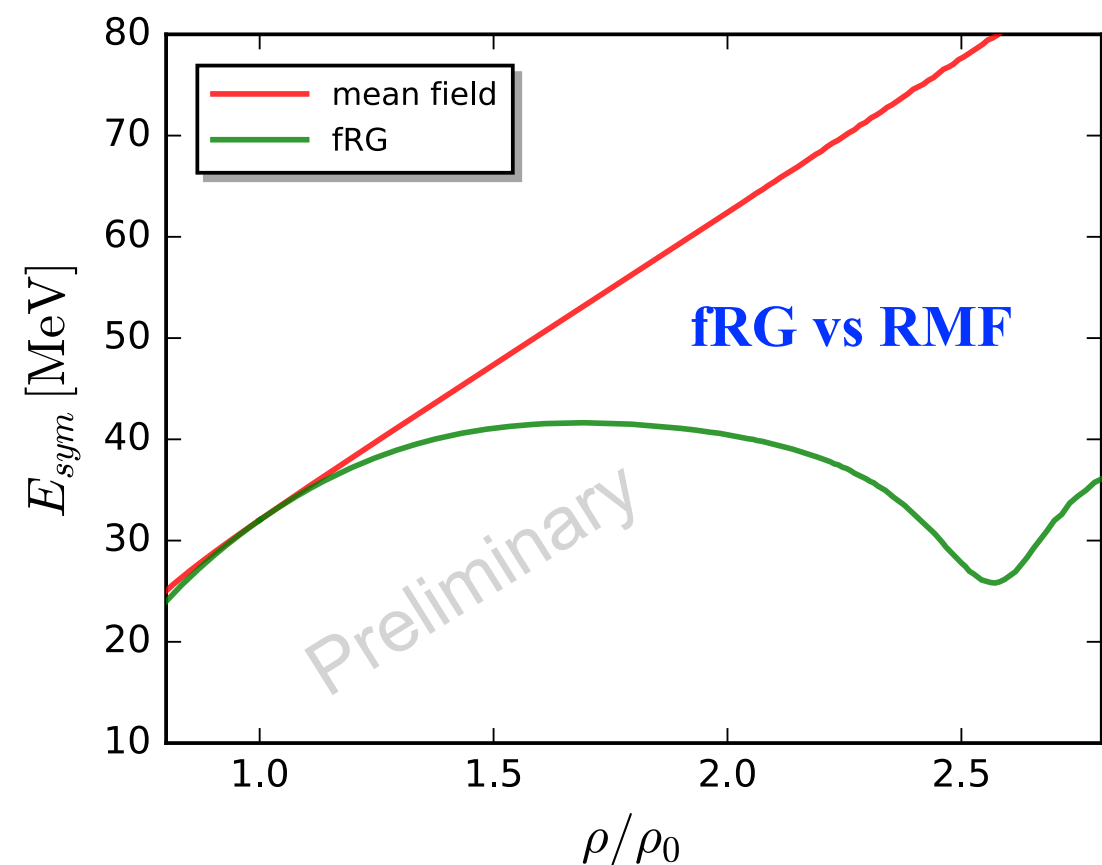
Review: WF, *CTP* 74 (2022) 097304,
arXiv: 2205.00468 [hep-ph]

Sound speed and symmetry energy

Speed of sound:



Symmetry energy:



Yong-rui Chen, Yang-yang Tan, WF, Xiaofeng Luo,
Lie-Wen Chen, in preparation.

- Speed of sound develops a **peak structure** around $\rho \sim 2.5\rho_0$.
- A **non-monotonic** dependence of the symmetry energy on the density is found.