Axions Accelerating the Universe

through $\mathcal{PT}$ Symmetry

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Joint work with N. E. Mavromatos (PRD 110, 045004 (2024))

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Last third is in preparation

ERG 2026 · 2–5 September 2026 · University of Sussex

Outline

- **Cosmological tensions**
- Response: a top-down approach, based on anomaly cancellation in string theory
- Non-renormalisable effective theory: the FRG approach
- Singularity of the beta function of the effective theory
- Truncation artefacts
- Possible generalisation of the model which can also address truncation issues

Cosmological tensions

ΛCDM — the standard model of cosmology

Λ cosmological constant

CDM cold dark matter + ordinary matterot

SUCCESSFUL (predicts CMB, abundance of light elements) — **BUT tensions**

H_0

Hubble constant: CMB vs distance ladders

a 5.3σ effect 67-68 km/s/Mpc, 73-74 km/s/Mpc

S_8 (DESI 2024-2025)

Clumpiness of space-time compared with predictions from CMB data 2.2*σ effect*

Whatever drives the late time expansion changes with time

Response to the tension

Not dark energy — repulsive gravity. Go to a top-down model.

STRING MODEL

e.g. heterotic $E_8 \times E_8$ (internal dimensions gauge degrees of freedom), or type-II orientifolds with intersecting branes stacks

FIELD CONTENT

axion field b, metric $g_{\mu\nu}$, gauge fields

The axion is a necessary introduction, to remove gauge anomalies in 10 dimensions

↓ **Green–Schwarz mechanism**

Compactify to 4 dimensions: $B_{\mu\nu}$ is the Kalb–Ramond field is a 2 form B

Anomaly cancellation and the coupling

Naively Field strength H=dB, $H_{\mu\nu\rho} = \varepsilon_{\mu\nu\rho\sigma}\, \partial_\sigma b$ (dualisation); b is a pseudoscalar axion

Anomaly cancellation A_{str} $b\, R\, \tilde{R}$ (gravitational) , $b\, F\, \tilde{F}$ (U(1) gauge field)

The axion coupling (decay constant): $A_{str} \sim (\alpha'/\kappa) \cdot \sqrt{(2/3)} \cdot 1/96 \equiv \bar{g}$

where $\alpha' \sim M_s^{-2}$, $\kappa \sim M_{pl}^{-1}$ — *the axion coupling is locked to gravity: makes KR axion* ***special***

String theory potentially allows N > 1 U(1) fields through e.g. reduction from $E_8 x E_8$ to standard model group; assume gauge field has not acquired mass

The model

$$S = \int d^4x \sqrt{g}\ [\quad (Z\_b/2)(\partial b)^2 + (m^2/2)\, b^2$$

$$+ \Sigma_{J=1..N} \left((Z\_F/4)\, F_J{}^{\mu\nu} F_{J\,\mu\nu} + (i\, A_{str}/4)\, b\, F_J{}^{\mu\nu} \tilde{F}_{J\,\mu\nu} \right)$$

$$+ \quad R / 2\kappa^2 \quad]$$

g is the determinant of the metric. The factor of i is forced by the Wick rotation to Euclidean

A^I *represent N gauge fields* $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu,\ \tilde{F}^{I\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F^i_{\rho\sigma}$

b couples to all the U(1) from Green-Schwarz mechanism

The model

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$+ \; \Sigma_{J=1..N} \; (\; (Z\_F/4)\, F_{J\mu\nu} F_{J\,\mu\nu} \; + \; (i\, A_{str}/4)\, b\, F_{J\mu\nu} \tilde{F}_{J\,\mu\nu} \;)$

$+ \; R / 2\kappa^2 \quad]$

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b couples to all the U(1) from Green-Schwarz mechanism

Is there a single Z_F?

$$-\frac{1}{4}\sum_{I=1}^{N} Z_{A^I}\, F^I_{\mu\nu} F^{I\mu\nu}$$

With S_N a consistent truncation: if all the Green-Schwarz couplings are equal, symmetry preserved under the flow

Can show:
Important for large N behaviour:

$$\eta_b \sim N g^2.$$

$$\eta_F \sim g^2.$$

Non-renormalisable → the FRG approach

A generalisation of axion electrodynamics (Eichhorn, Gies & Roscher, PRD 86, 125014 (2012))

$[A_{str}] = -1$ → non-renormalisable. FRG approach, axion decay constant locked to gravity.

(C. Wetterich, Phys. Lett. B 301, 90 (1993))

$$\text{LPA}': \quad g^2 \equiv k^2 \bar{g}^2 / (Z_b Z_F^2)$$

$$\partial_t g^2 = (2 + 2\eta_F + \eta_b) \, g^2$$

$$\eta_X \equiv -\partial_t \log Z_X , \quad t = \log k$$

g is not to be confused with the metric determinant.

Two coupled equations for the η's

$\eta\_b = A\,(2 - \eta\_F/4)$

$\eta\_F = A\,(4 - \eta\_b/4 - \eta\_F/4)$

$\Rightarrow \quad M\,(\eta\_b, \eta\_F)^T = S$

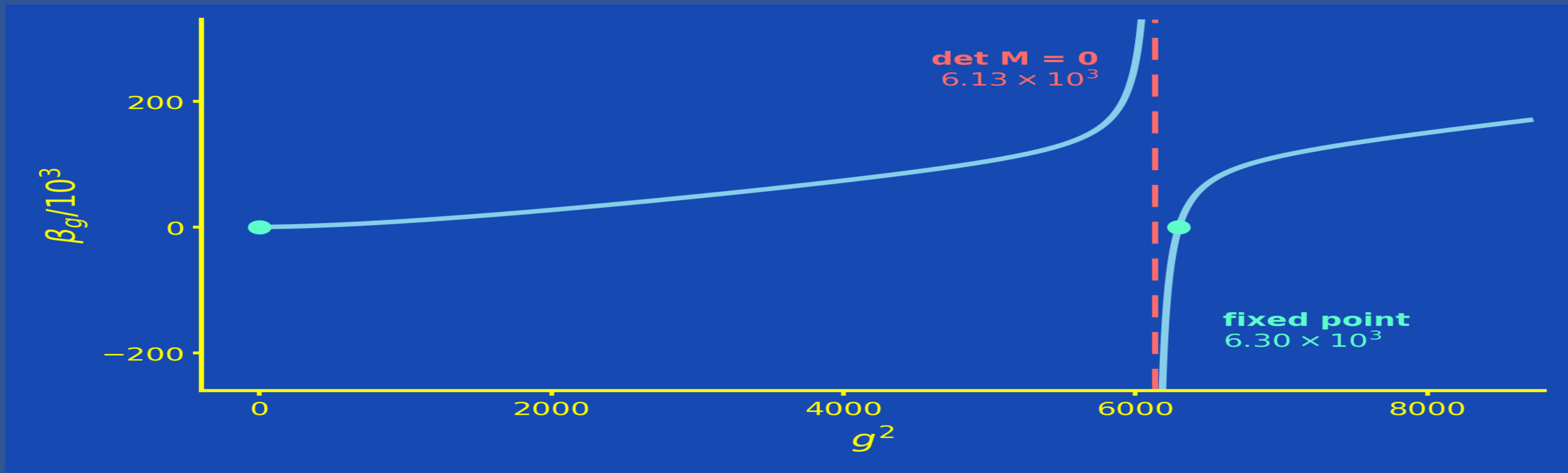
Denominator = det M

det M = 0 ⇔

$g^2 = 192\pi^2\,(1 \pm \sqrt{5})$

$$\partial_t g^2 = \frac{2\,g^2 \cdot (13\,g^4 - 8064\,\pi^2 g^2 - 147456\,\pi^4)}{(g^4 - 384\,\pi^2 g^2 - 147456\,\pi^4)}$$

Should you believe that pole?



$g^2_{sing} = 6.13{\times}10^3$;   numerator = 0 gives fixed points at 0 and $g^2_* = 6.30{\times}10^3$

- Coefficients are scheme dependent — change the regulator and the pole moves
- Near the pole η_b ≈ 88, η_F ≈ −45 — the derivative expansion is not good

Truncation artefacts

Near the pole

$\eta_b \approx 88, \quad \eta_F \approx -45$

The derivative expansion is not good.

Proximity

The fixed point is 2.7% away from the pole.

Suggests it may be spurious.

One way to check

go to higher orders in the derivative expansion — or find a limit in which the whole structure is under control.

We consider another way

A controlled limit

1 axion, N gauge fields: $N = 1 \rightarrow N \gg 1$

$$(i\,\bar{g}/4)\; b\; \Sigma_{I=1..N}\; F_I{}^{\mu\nu}\, \tilde{F}_{I\,\mu\nu} \qquad \lambda = N g^2 \text{ fixed}, \quad N \rightarrow \infty$$

Similar in spirit to the 't Hooft limit (G. 't Hooft, Nucl. Phys. B 72, 461 (1974)) although significant differences

Diagram counting

$\eta\_b = O(\lambda)$

axion self-energy — index summed

$\eta\_F = O(\lambda / N)$

photon self-energy — no sum

Integrating out the N gauge fields is exact power counting: $N\,(\sqrt{(\lambda/N)})^n = \lambda^{n/2} N^{1-n/2}$ for n axion legs

$n = 2$ is $O(1)$; quartic and higher axion self-interactions are suppressed by $1/N$, $1/N^2$, … see next slide

Power counting: V vertices, L loops

- Each vertex carries $g = \sqrt{\lambda/N}$.
- Each closed gauge-index loop carries a factor N — the index is summed. Call the number of these P ($P \le L$; axion loops carry no index).

$$\text{amplitude} \sim (\lambda/N)^{V/2}\, N^{P} = \lambda^{V/2}\, N^{P-V/2}$$

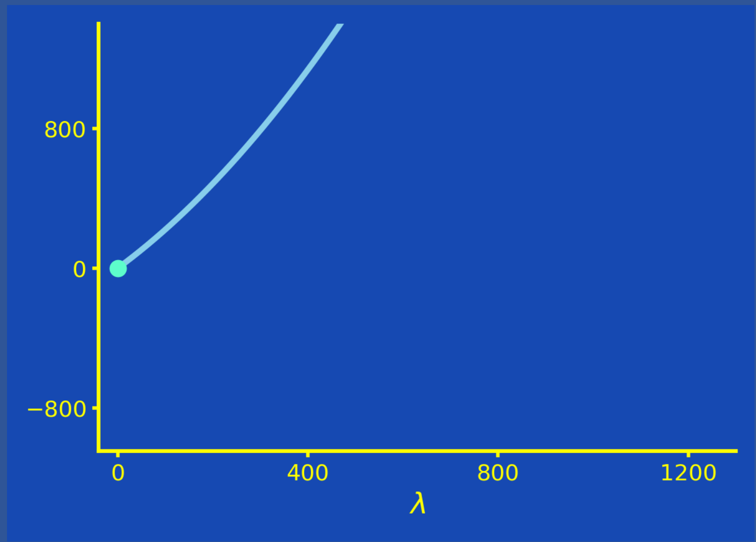
Every index loop needs at least two vertices, so $P \le V/2 \Rightarrow$ nothing exceeds O(1).
Equality $\Leftrightarrow$ every index loop is a two-vertex bubble $\Rightarrow$ the leading set is chains of bubbles.

diagram	V	P	order
axion self-energy	2	1	λ
photon self-energy	2	0	λ / N
n axion legs (Tr ln)	n	1	$\lambda^{n/2}\, N^{1-n/2}$
chain of n bubbles	2n	n	λ^{n}
vertex correction	4	1	λ^{2} / N

The pole is a 1/N effect

det M = 1 + O(1/N)
No pole.

Further analysis gives (Mavromatos & Sarkar, in preparation)



Hermitian branch

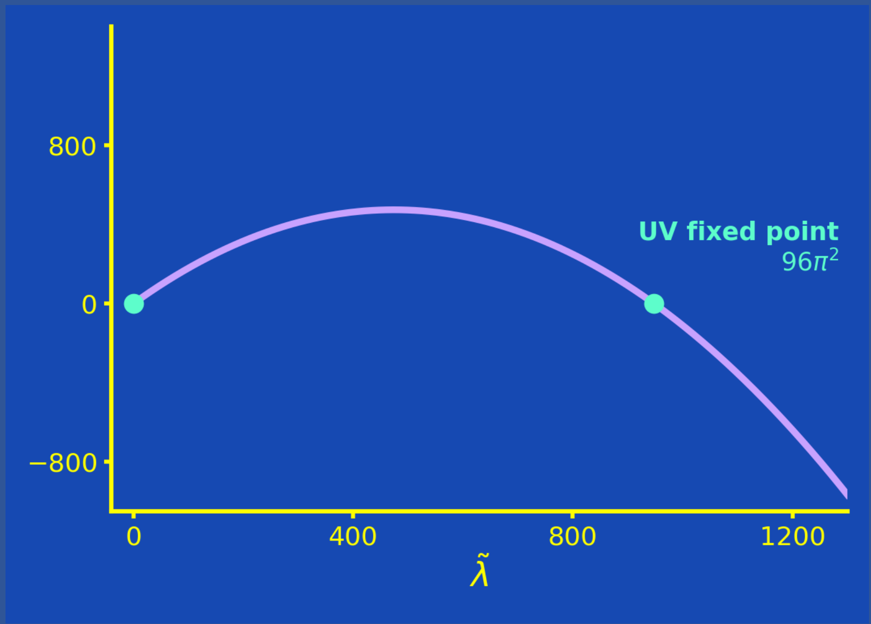
β_λ = 2Cλ + Dλ²/48π²

a Riccati equation — soluble in closed form. The only zero is at λ < 0, so it is unreachable: every Hermitian trajectory runs away.

Single Z_F: without S_N, the splitting is $1/N$ suppressed.

So the single-Z_F picture is recovered at large N without invoking any permutation symmetry at all

The $\mathcal{PT}$ branch



$\mathcal{PT}$: $\lambda \to -\tilde{\lambda} \Rightarrow G \to -G$

UV fixed point at $\tilde{\lambda}_* = 96\pi^2$

η_a∗ = −2, so b F F̃ is exactly marginal; one relevant direction survives once shift symmetry is imposed.

Caveats

$N \gg 1$ is not a consequence of string theory.

Regard this as a model with control — a laboratory, not a claim about the world.

Gravity effect:

In the large-N limit η_F does not vanish outright: $\eta_F = -c_v g_N + O(1/N)$.

c_v is the standard Einstein–Hilbert coefficient for a minimally coupled vector

gravity couples universally, so the surviving
piece is common to all N factors.

Gravity is enslaved to the axion coupling

Unsolved what happens to the gravitational coupling with negative sign in FRG.