https://fqcd-collaboration.github.io/

Precondensation and inhomogeneous instabilities in high-density QCD

Franz R. Sattler

University Bielefeld

2512.20510 [hep-ph] with Fabian Rennecke and Jan M. Pawlowski
2602.11265 [hep-ph] with Álvaro Pastor-Gutiérrez and Jan M. Pawlowski

UNIVERSITÄT BIELEFELD

Fakultät für Physik

ERG2026

University of Sussex

September 3rd, 2026

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Talk by Álvaro Pastor-Gutiérrez on wednesday

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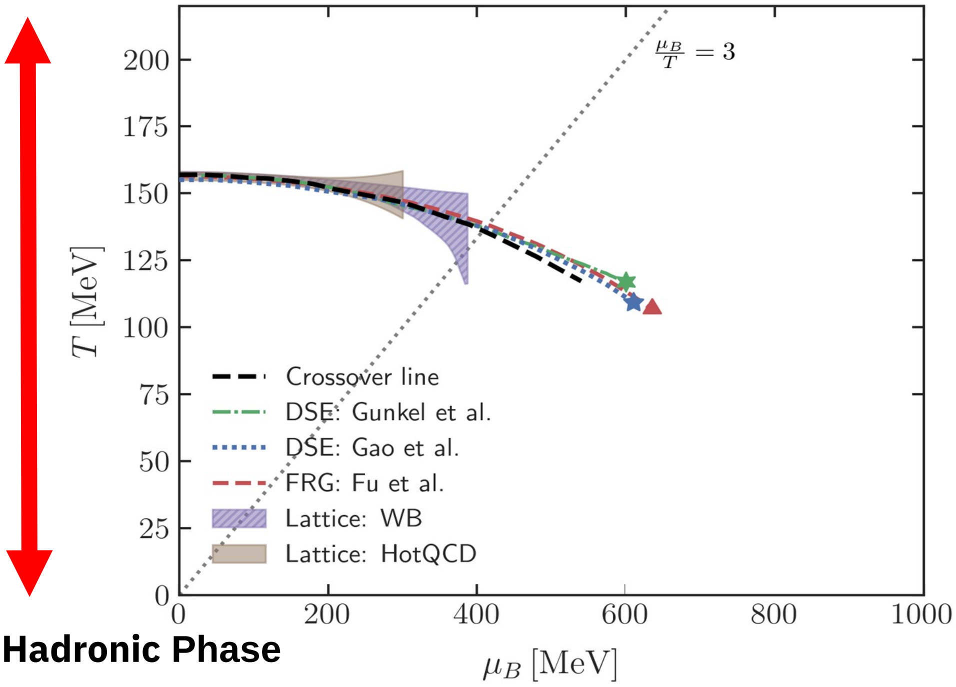
The QCD phase diagram

Quark-Gluon Plasma



Hadronic Phase

High densities:
Pawlowski, Rennecke, Sattler, [Eur. Phys. J. Spec. Top. (2026)]
Fu, Pawlowski, Rennecke [Phys. Rev. D 101 (2020), 054032]
Gao, Pawlowski [Phys.Lett.B820(2021) 136584]
Gunkel, Fischer [Phys.Rev.D 104 (2021) 5, 054022]

Low densities:
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Bazavov et al. (HotQCD) [Phys.Lett.B 795 (2019) 15-21]

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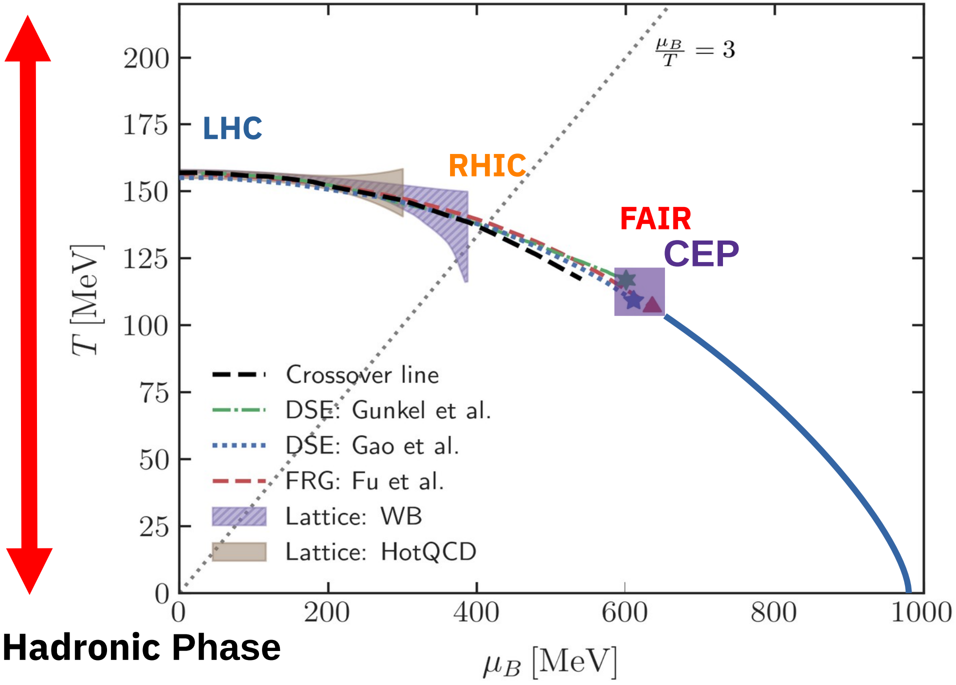

Hadronic Phase

Strongly correlated non-perturbative physics:

Dynamical Chiral Symmetry Breaking

Critical Endpoint (CEP) at $(T, \mu_B) \approx (110 \pm 10, 630 \pm 30)\mathrm{MeV}$

Direct access to phase structure using the **functional Renormalization Group**

Non-perturbative, first-principles, **direct access to finite μ**.

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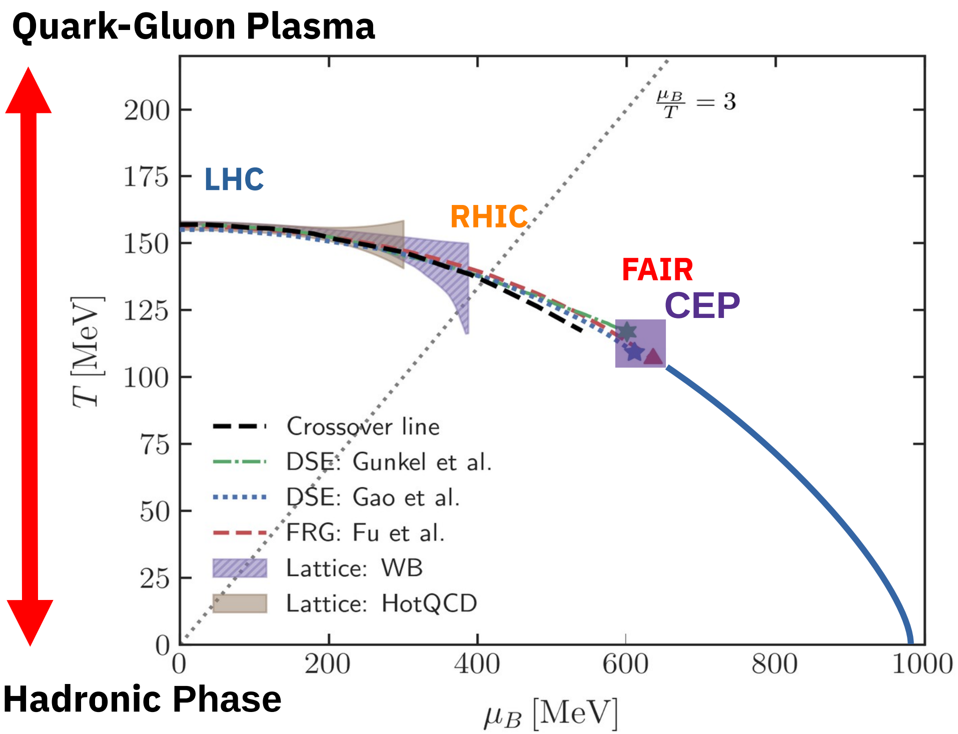
High-density QCD: some relevant channels and physics



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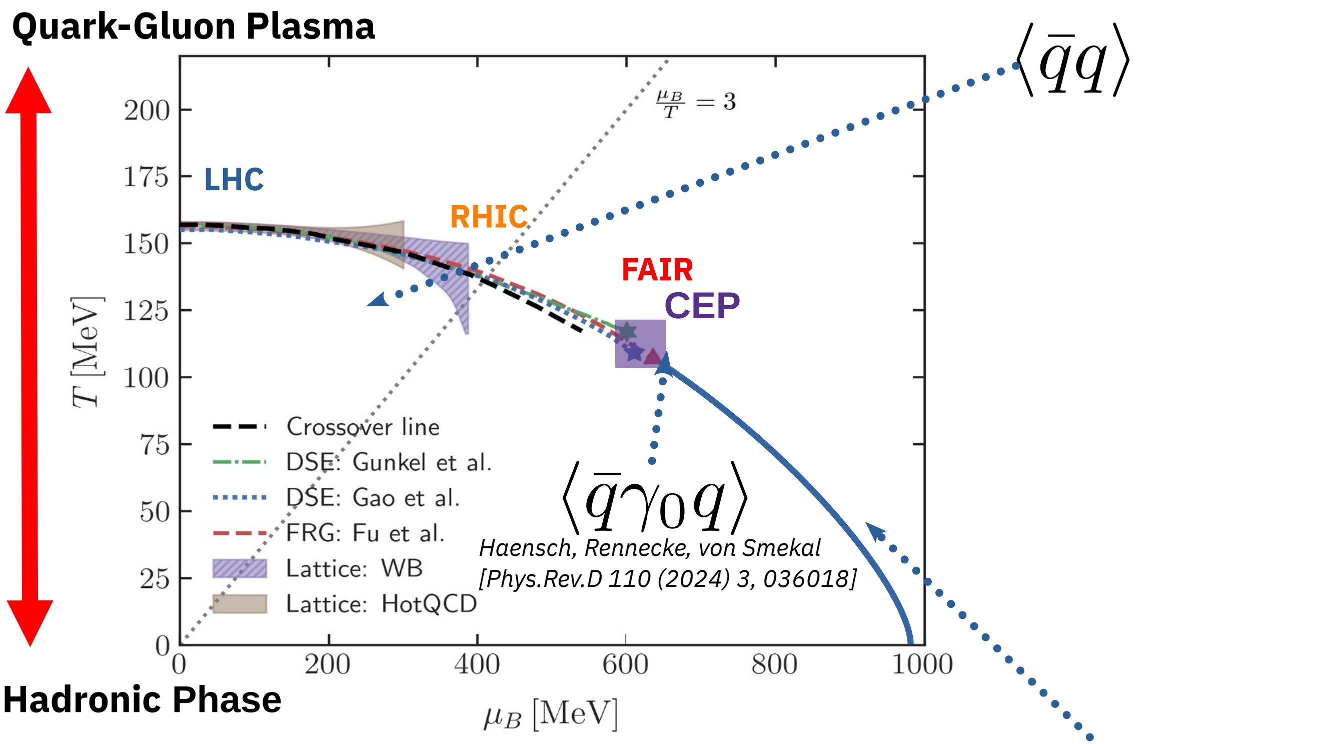

talk by Ugo Mire on wednesday

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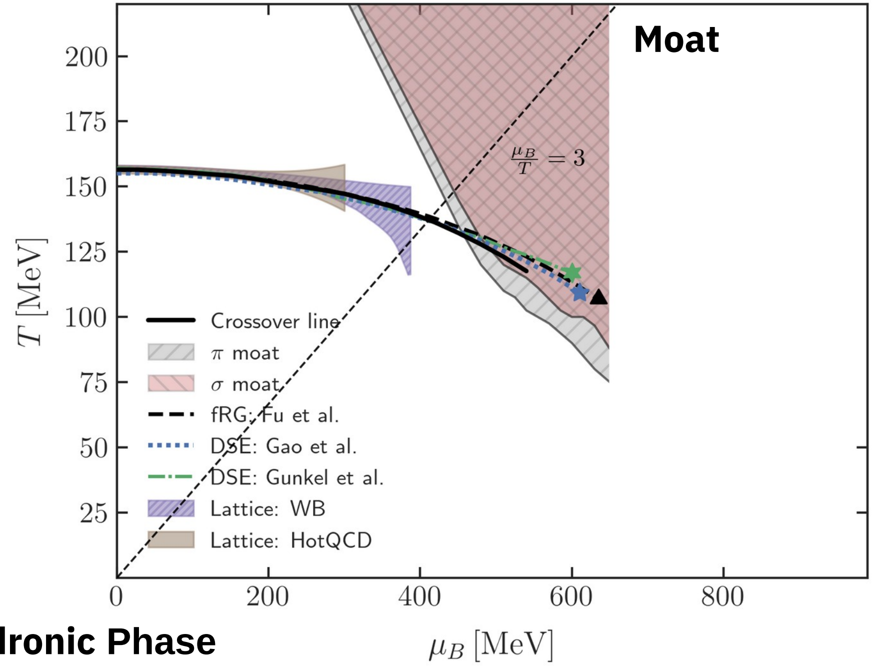
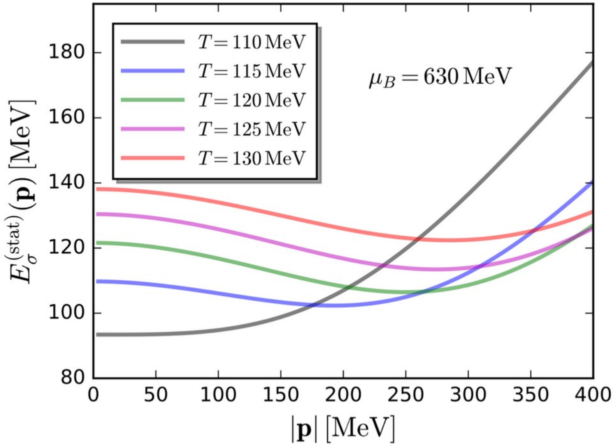


Hadronic Phase



Fu, Pawlowski, Pisarski, Rennecke, Wen, Yin [Phys.Rev.D 111 (2025) 9, 094026]

Moat Regime

Nunney Castle, England

e.g. bilayer graphene, see *McCann, Koshino, Reports on Progress in physics 76.5 (2013): 056503*

$$\langle \bar{q}q \rangle(\boldsymbol{x}) \sim \mathrm{const}$$

(homogeneous condensation)

$$\langle (\bar{q}q)(\bar{q}q) \rangle(0, \boldsymbol{x}) \sim \sin(\boldsymbol{q}\cdot\boldsymbol{x})\, e^{-x^2/m_\phi^2}$$

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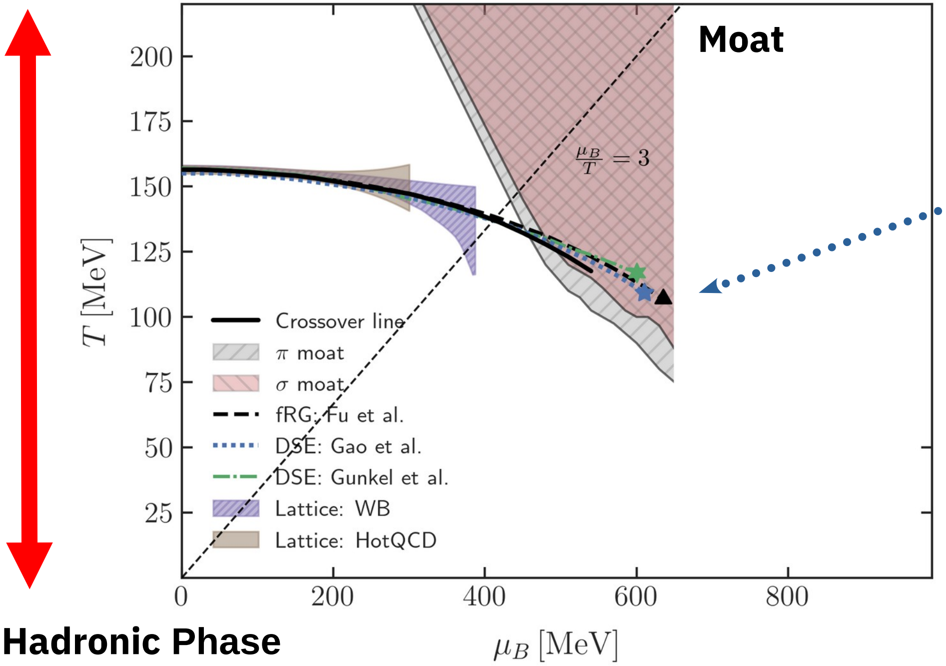
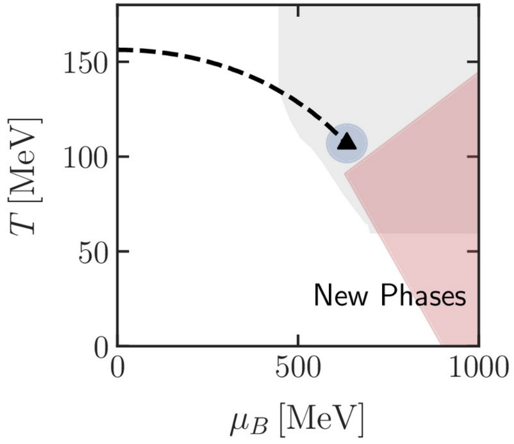


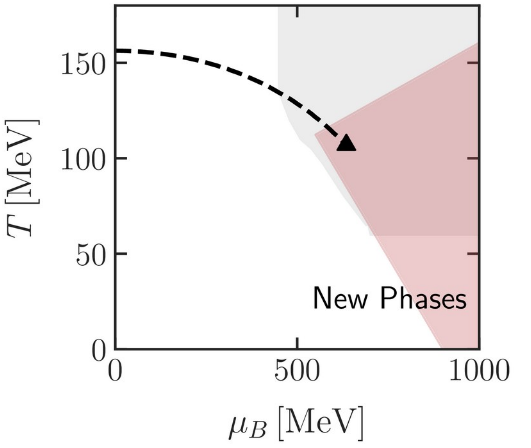
Hadronic Phase

Around/At the CEP:

Spatial modulations,
Inhomogeneous phases

e.g.
Buballa
[Prog.Part.Nucl.Phys. 81 (2015)]
Pisarski, Skokov, Tsvelik
[arXiv:2202.01036 (2022)]
...





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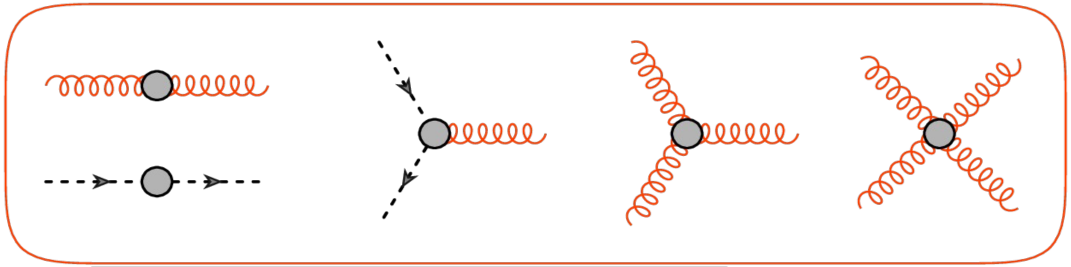
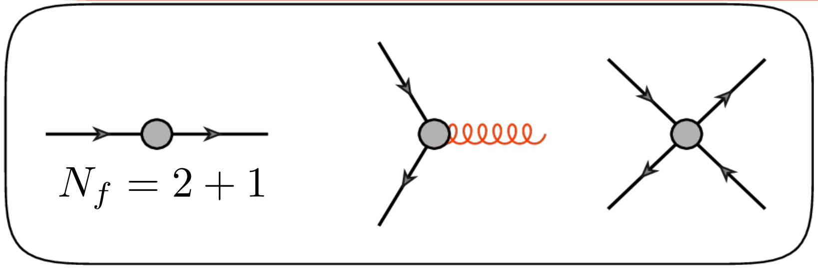
Bazavov et al. (HotQCD) [Phys.Lett.B 795 (2019) 15-21]

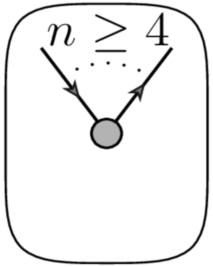
Truncation

Glue sector



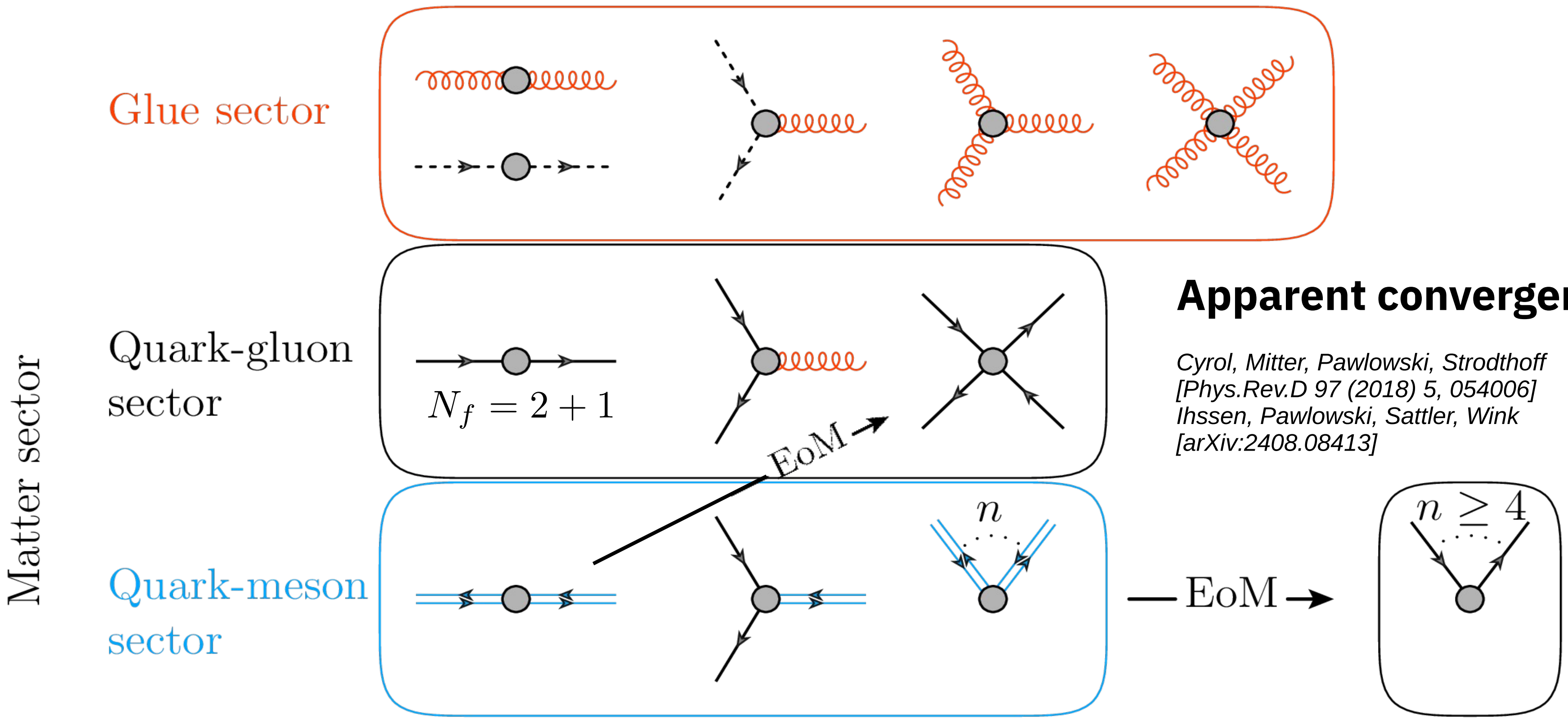
Quark-gluon sector





Matter sector

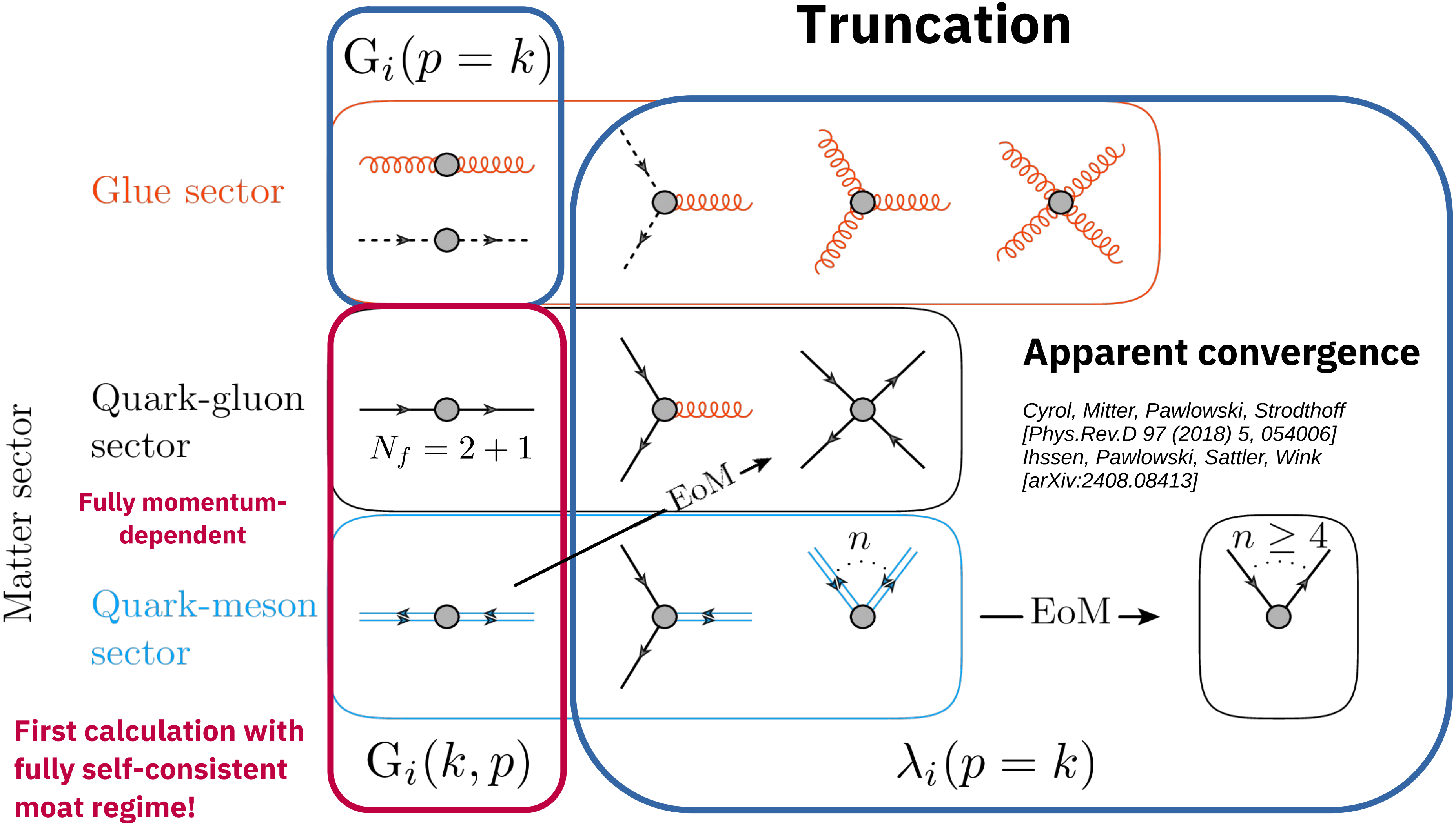
Truncation



Apparent convergence

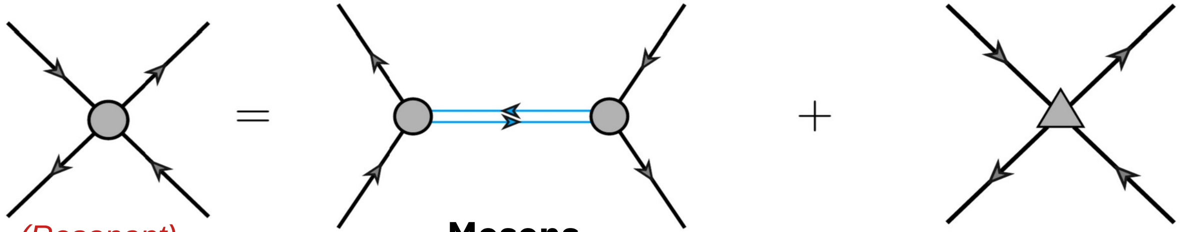
Cyrol, Mitter, Pawlowski, Strodthoff [Phys.Rev.D 97 (2018) 5, 054006]
Ihssen, Pawlowski, Sattler, Wink [arXiv:2408.08413]

Truncation



$G_i(p=k)$
Glue sector
Matter sector
Quark-gluon sector
$N_f = 2+1$
EoM
Quark-meson sector
n
EoM
$n \geq 4$
$G_i(k,p)$
$\lambda_i(p=k)$
Fully momentum-dependent
First calculation with fully self-consistent moat regime!
Apparent convergence
Cyrol, Mitter, Pawlowski, Strodthoff [Phys.Rev.D 97 (2018) 5, 054006]
Ihssen, Pawlowski, Sattler, Wink [arXiv:2408.08413]

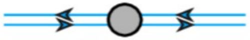
Dynamical hadronisation



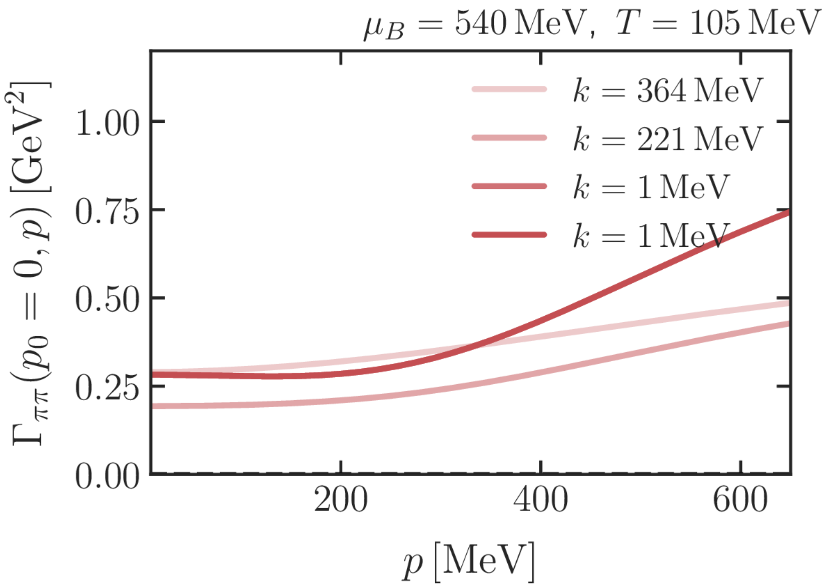
Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
Fu, Huang, Pawlowski, Tan, Zhou
[Phys.Rev.D 112 (2025) 5, 054047]

Gies, Wetterich
[Phys.Rev. D 65 (2002), 065001]

$N_f = 2 + 1$



$\mathrm{G}_i(k, p)$



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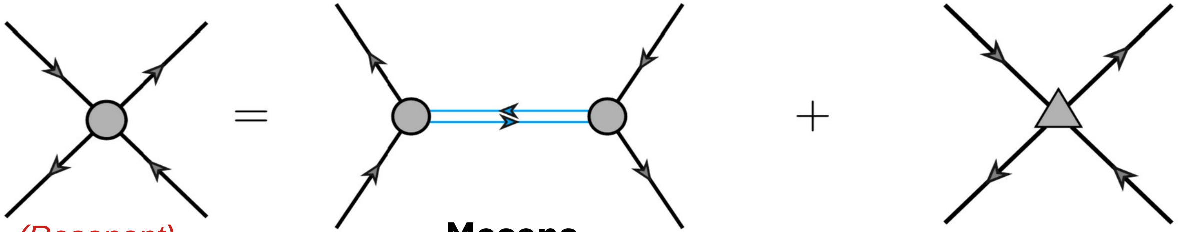

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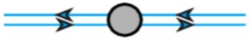
Full momentum dependence

$N_f = 2 + 1$



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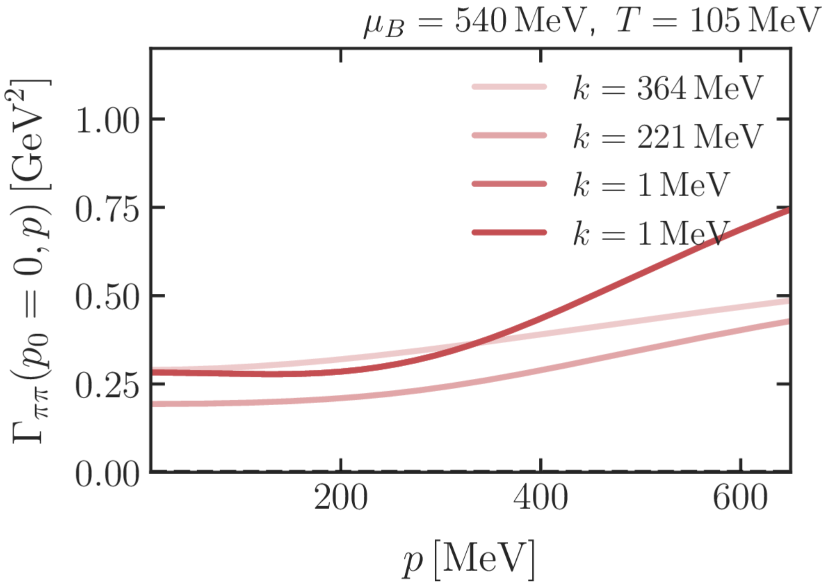

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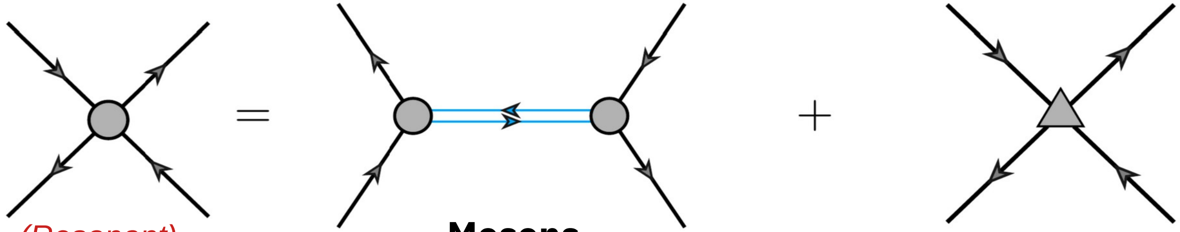

(Resonant) | **Mesons** | Remainder

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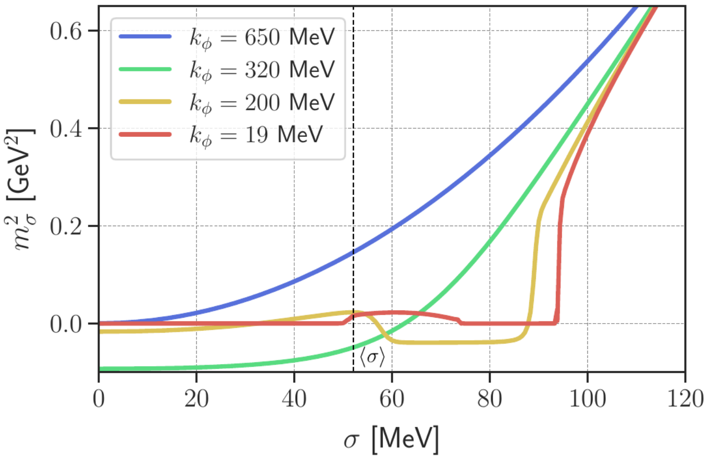
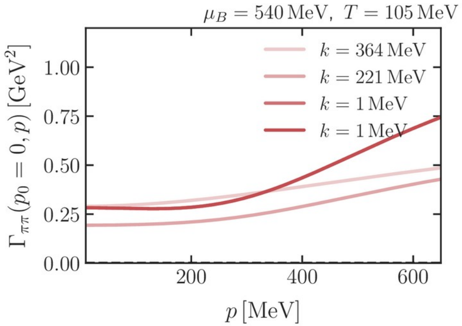


Ihssen, Pawlowski, Sattler, Wink
[Phys.Rev.D 111 (2025) 3, 036030]



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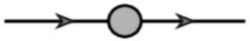

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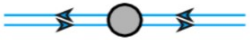
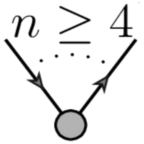


$$\mathrm{G}_i(k, p)$$

Full scattering potential



$$\longrightarrow \quad V\left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{2}\right)$$

All orders of n-meson scatterings

(+ quantitative access to chiral limit)
[Ihssen, Pawlowski, Sattler, Wink
Phys.Rev.D 113 (2026) 9, 094038]

TensorBases
Mathematica package

[J. Braun, J. Pawlowski, A. Geißel, **FRS**, N. Wink, Annals Phys. 484 (2026)]

- Automatically derived projectors
- Library of tensor bases, extendable by everyone

```
Needs["TensorBases`"]

In[2]:= TBGetProjector["transAqbq", 1, {p1, mu, a}, {p2, d2, A2, F2}, {p3, d3, A3, F3}]
TBGetInnerProduct["transAqbq"][TBGetProjector, 1, TBGetBasisElement, 1] // FormTrace // Simplify
TBGetInnerProduct["transAqbq"][TBGetProjector, 1, TBGetBasisElement, 8] // FormTrace // Simplify
```

Out[2]= $\frac{1}{6\,Nf - 6\,Nc^2\,Nf}\, i\, \text{deltaFundFlav}[F2, F3] \times \text{gamma}[nu\$20834, d2, d3] \times \text{TCol}[a, A2, A3] \times \text{transProj}[p1, mu, nu\$20834]$

Out[3]= 1

Out[4]= 0

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DiFfRG framework

[**FRS**, J. Pawlowski, Comput.Phys.Commun. 327 (2026)]

- Vertex and/or derivative expansions of fRG systems
- Hydrodynamic methods for **full field dependences (FE/FV)**
- **Full momentum dependences**, numerical Matsubara sums
- Performance portable: (many) **GPU**s, CPUs accelerated

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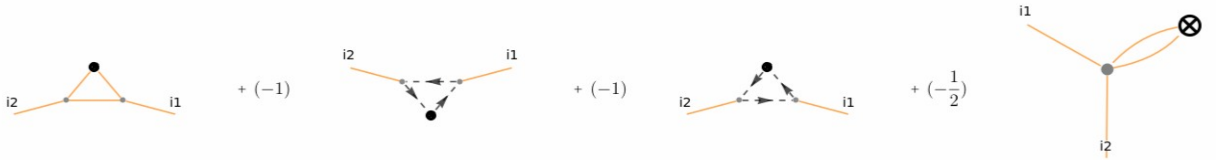
FunKit

[**FRS**, 2605.28935]

```
In[16]:= fRGAA = FTakeDerivatives[WetterichEquation, {A[i1], A[i2]}] // FTruncate // FPlot // FRoute // FPrint;
```



$$\begin{aligned}
&\int_{l_1} G_{A^aA^b}(-l_1,l_1)\,\Gamma_{A^mA^cA^b}(p_1,l_1-p_1,-l_1)\,G_{A^eA^c}(l_1-p_1,p_1-l_1)\,\Gamma_{A^nA^gA^e}(-p_1,l_1,p_1-l_1)\,G_{A^iA^g}(l_1,-l_1)\,\partial_t R_{A^iA^a}(-l_1,l_1)\\
&+\int_{l_1^{(\mathrm{f})}}\Big(-G_{c^a\bar{c}^b}(l_1^{(\mathrm{f})},-l_1^{(\mathrm{f})})\,\Gamma_{A^m\bar{c}^cc^a}(p_1,l_1^{(\mathrm{f})}-p_1,-l_1^{(\mathrm{f})})\,G_{c^e\bar{c}^c}(l_1^{(\mathrm{f})}-p_1,p_1-l_1^{(\mathrm{f})})\,\Gamma_{A^n\bar{c}^gc^e}(-p_1,l_1^{(\mathrm{f})},p_1-l_1^{(\mathrm{f})})\,G_{c^i\bar{c}^g}(l_1^{(\mathrm{f})},-l_1^{(\mathrm{f})})\,\partial_t R_{\bar{c}^bc^i}(l_1^{(\mathrm{f})},-l_1^{(\mathrm{f})})\Big)\\
&+\int_{l_1^{(\mathrm{f})}}\Big(-G_{c^a\bar{c}^b}(-l_1^{(\mathrm{f})},l_1^{(\mathrm{f})})\,\Gamma_{A^m\bar{c}^bc^d}(p_1,-l_1^{(\mathrm{f})},l_1^{(\mathrm{f})}-p_1)\,G_{c^d\bar{c}^f}(p_1-l_1^{(\mathrm{f})},l_1^{(\mathrm{f})}-p_1)\,\Gamma_{A^n\bar{c}^fc^h}(-p_1,p_1-l_1^{(\mathrm{f})},l_1^{(\mathrm{f})})\,G_{c^h\bar{c}^j}(-l_1^{(\mathrm{f})},l_1^{(\mathrm{f})})\,\partial_t R_{\bar{c}^jc^a}(-l_1^{(\mathrm{f})},l_1^{(\mathrm{f})})\Big)\\
&+\int_{l_1}\Big(-\frac{1}{2}G_{A^aA^b}(l_1,-l_1)\,\Gamma_{A^iA^jA^cA^b}(-p_1,p_1,-l_1,l_1)\,G_{A^eA^c}(-l_1,l_1)\,\partial_t R_{A^eA^a}(l_1,-l_1)\Big)
\end{aligned}$$

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Mathematica package

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```

Open Source, see github.com/satfra

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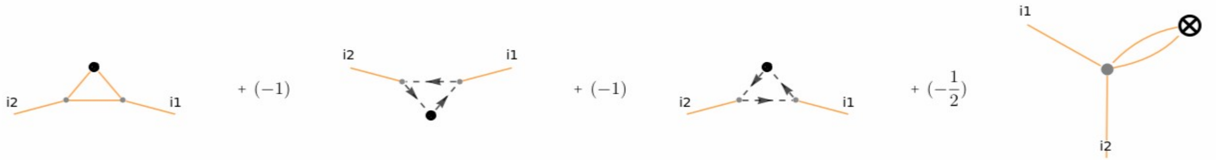
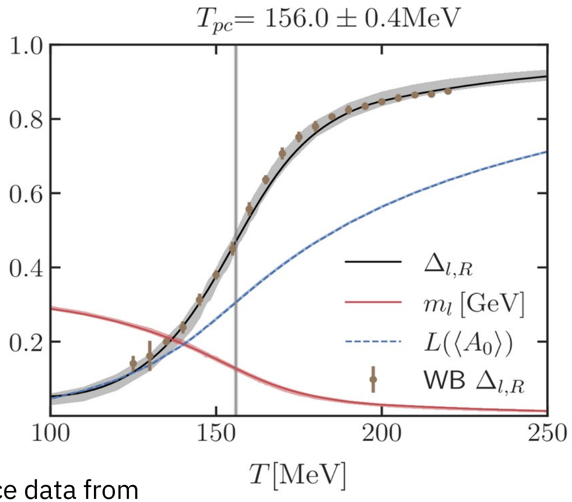


$$\int_{l_1} G_{A^aA^b}(-l_1,l_1)\,\Gamma_{A^mA^cA^b}(p_1,l_1-p_1,-l_1)\,G_{A^eA^c}(l_1-p_1,p_1-l_1)\,\Gamma_{A^nA^gA^e}(-p_1,l_1,p_1-l_1)\,G_{A^iA^g}(l_1,-l_1)\,\partial_t R_{A^iA^a}(-l_1,l_1)$$
$$+\int_{l_1^{(f)}}\Big(-G_{c^a\bar c^b}(l_1^{(f)},-l_1^{(f)})\,\Gamma_{A^m\bar c^c c^a}(p_1,l_1^{(f)}-p_1,-l_1^{(f)})\,G_{c^e\bar c^c}(l_1^{(f)}-p_1,p_1-l_1^{(f)})\,\Gamma_{A^n\bar c^g c^e}(-p_1,l_1^{(f)},p_1-l_1^{(f)})\,G_{c^i\bar c^g}(l_1^{(f)},-l_1^{(f)})\,\partial_t R_{\bar c^b c^i}(l_1^{(f)},-l_1^{(f)})\Big)$$
$$+\int_{l_1^{(f)}}\Big(-G_{c^a\bar c^b}(-l_1^{(f)},l_1^{(f)})\,\Gamma_{A^m\bar c^b c^d}(p_1,-l_1^{(f)},l_1^{(f)}-p_1)\,G_{c^d\bar c^f}(p_1-l_1^{(f)},l_1^{(f)}-p_1)\,\Gamma_{A^n\bar c^f c^h}(-p_1,p_1-l_1^{(f)},l_1^{(f)})\,G_{c^h\bar c^j}(-l_1^{(f)},l_1^{(f)})\,\partial_t R_{\bar c^j c^a}(-l_1^{(f)},l_1^{(f)})\Big)$$
$$+\int_{l_1}\Big(-\frac{1}{2}G_{A^aA^b}(l_1,-l_1)\,\Gamma_{A^iA^jA^cA^b}(-p_1,p_1,-l_1,l_1)\,G_{A^eA^c}(-l_1,l_1)\,\partial_t R_{A^eA^a}(l_1,-l_1)\Big)$$

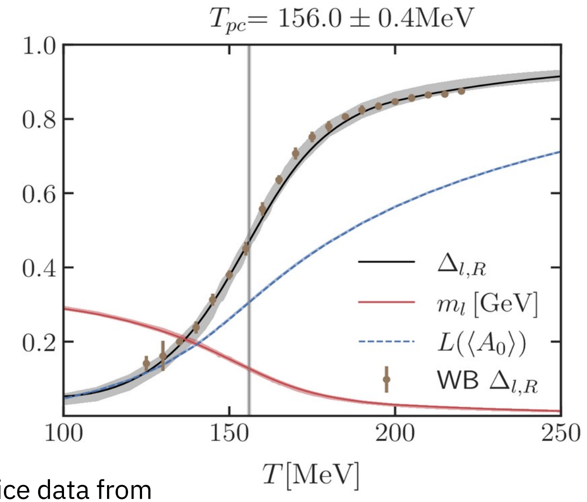


Lattice data from
Borsanyi et al (Wuppertal-Budapest), JHEP 09, 073 (2010)

$$T_{pc}^{\mathrm{lat}} = 155 - 158\mathrm{MeV}$$

We have control over
- Gauge consistency
- UV-limit

Partial systematic error estimate from
- Regulator variation
- LEGO®



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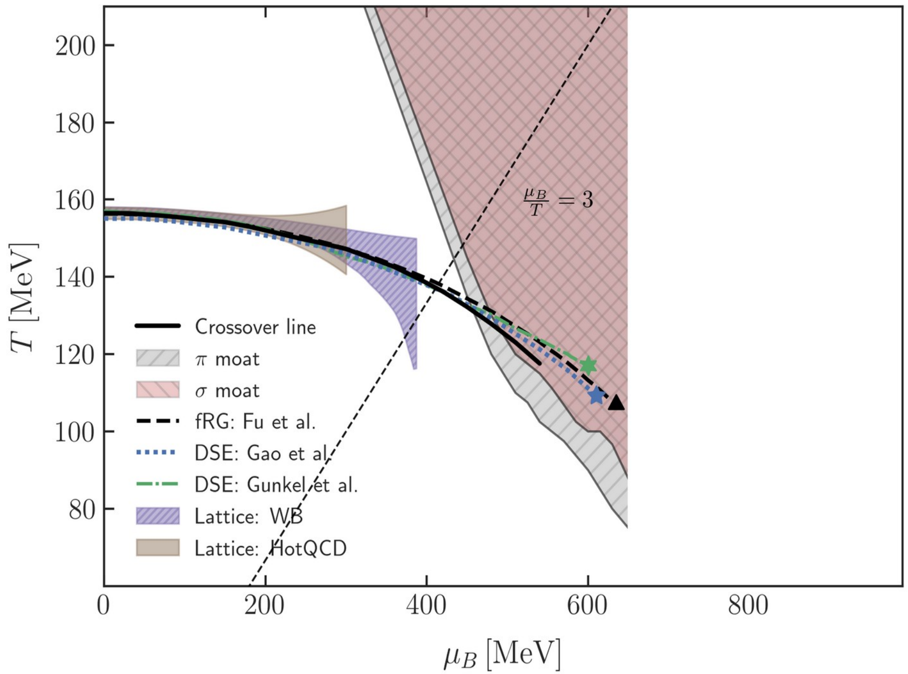
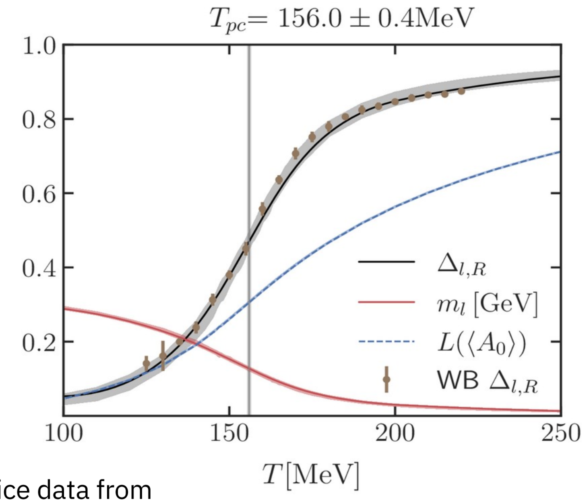


$$T = T_{pc}\left(1 - \kappa\left(\frac{\mu_B}{T_{pc}}\right)^2 + \lambda_2\left(\frac{\mu_B}{T_{pc}}\right)^4 + \lambda_3\left(\frac{\mu_B}{T_{pc}}\right)^6 + \dots\right)$$

	this work	lattice [52]	lattice [364]	lattice [54]	fRG [42]	DSE [57]
κ	0.0160(21)	0.015(4)	0.0144(26)	0.0153(18)	0.0142(2)	0.0147(5)

[52] A. Bazavov et al, Phys. Lett. B795 (2019)
[364] Bonati et al, Phys. Rev. D98.5 (2018) 054510
[54] Borsanyi, et al, Phys. Rev. Lett. 125.5 (2020) 052001
[42] Fu, Jan M. Pawlowski, and Fabian Rennecke Phys. Rev. D 101.5 (2020), 054032
[57] Gao, Pawlowski Phys. Lett. B 820 (Oct. 2021), 136584



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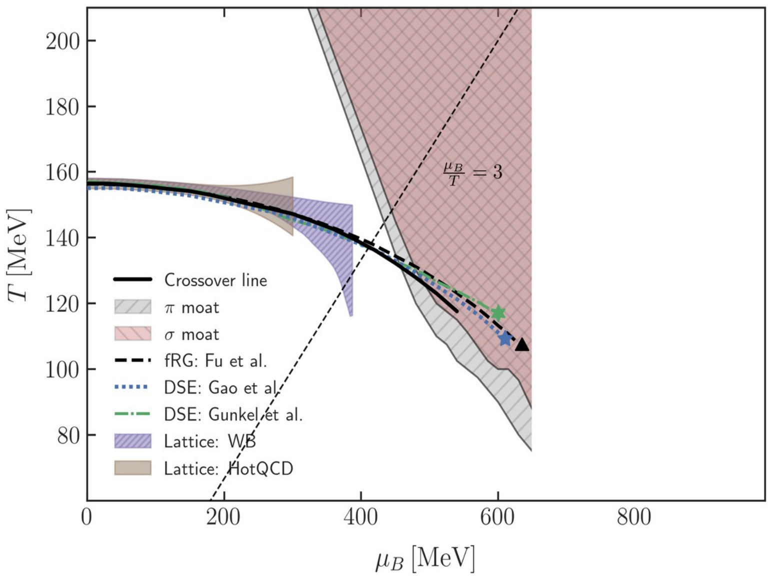
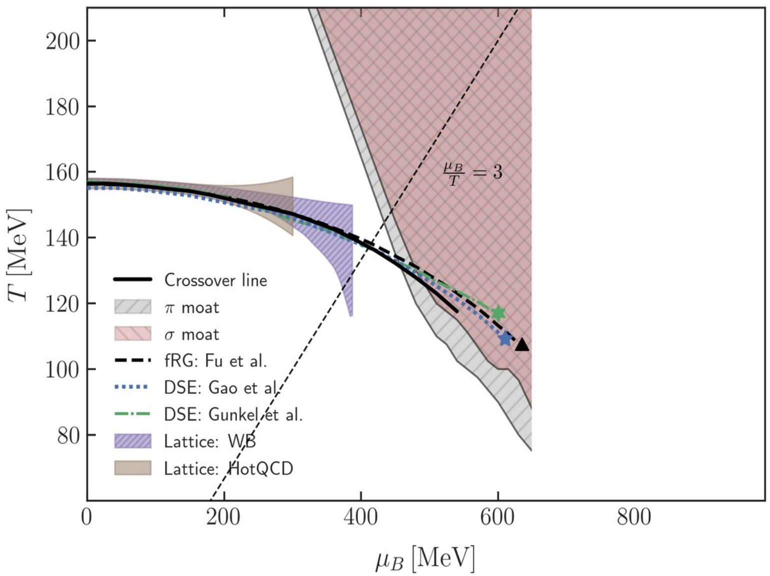
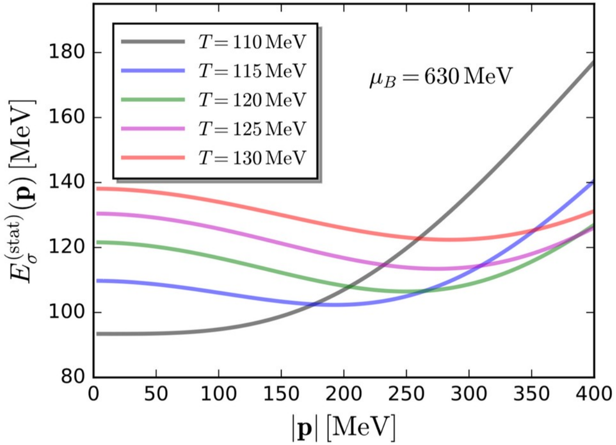


$\frac{\mu_B}{T} = 3$
Crossover line
π moat
σ moat
fRG: Fu et al.
DSE: Gao et al.
DSE: Gunkel et al.
Lattice: WB
Lattice: HotQCD
T [MeV]
μ_B [MeV]

Moat Regime





Fu, Pawlowski, Pisarski, Rennecke, Wen, Yin [Phys.Rev.D 111 (2025) 9, 094026]

Nunney Castle, England

e.g. bilayer graphene, see *McCann, Koshino, Reports on Progress in physics 76.5 (2013): 056503*

$$\langle \bar{q}q \rangle(\boldsymbol{x}) \sim \mathrm{const}$$

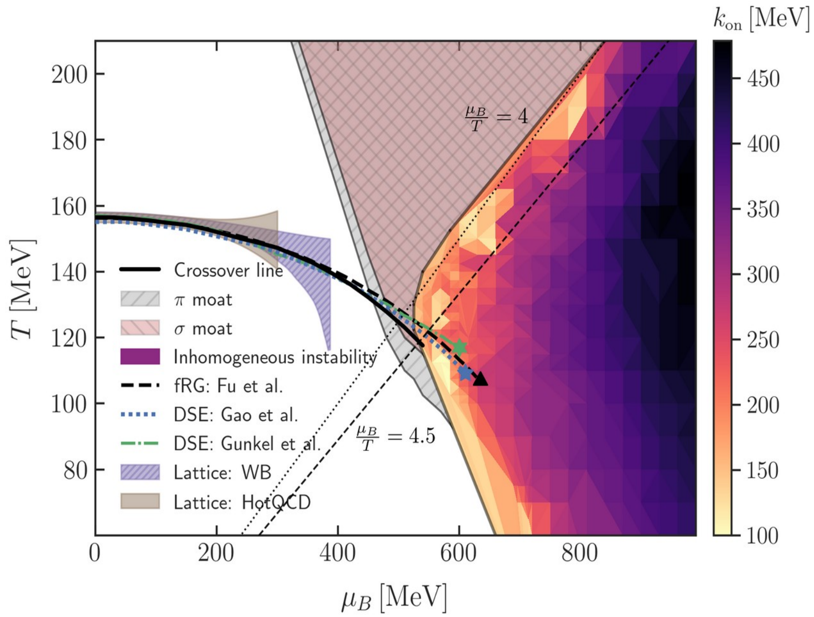
(homogeneous condensation)

$$\langle (\bar{q}q)(\bar{q}q) \rangle(0, \boldsymbol{x}) \sim \sin(\boldsymbol{q} \cdot \boldsymbol{x})\, e^{-x^2/m_\phi^2}$$

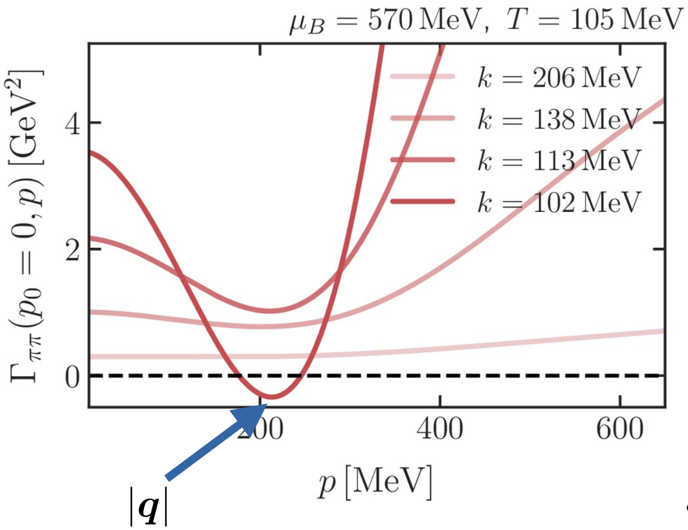
Deeper moat

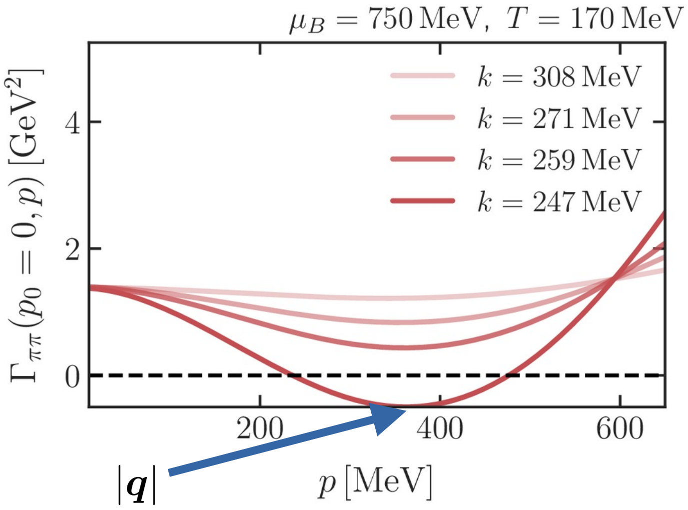
Stable?

Moat Regime



First calculation with fully self-consistent moat regime!





Nunney Castle, England

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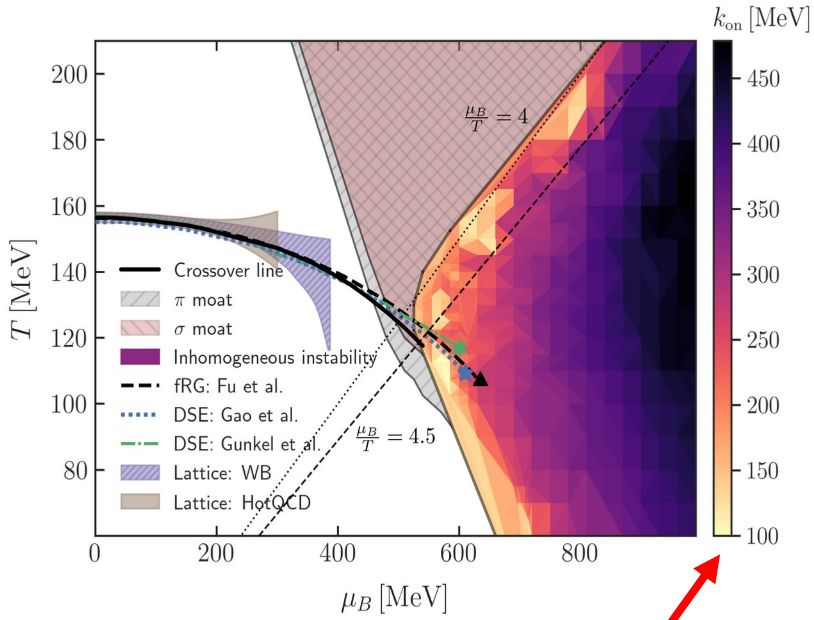
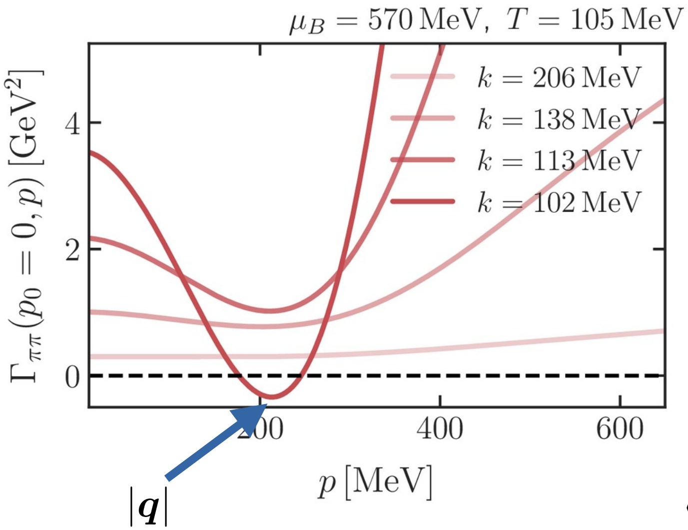
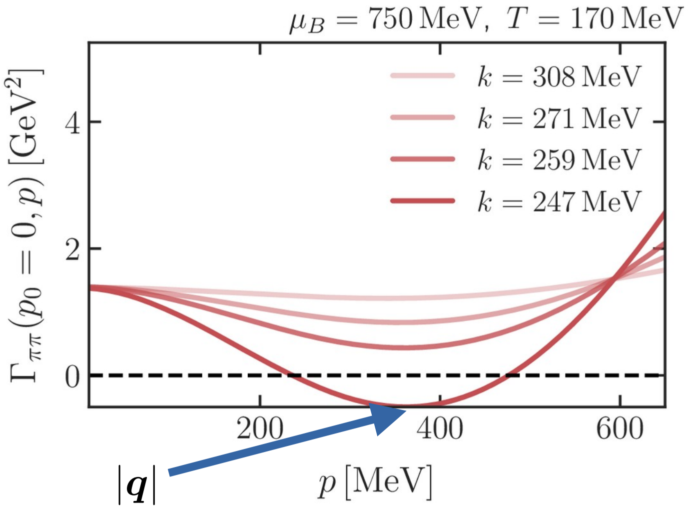
Inhomogeneous Instability

$$\langle \bar{q}q\rangle(\boldsymbol{x}) \sim \sin(\boldsymbol{q}\cdot\boldsymbol{x}) \quad (???)$$







Moat Regime

Nunney Castle, England

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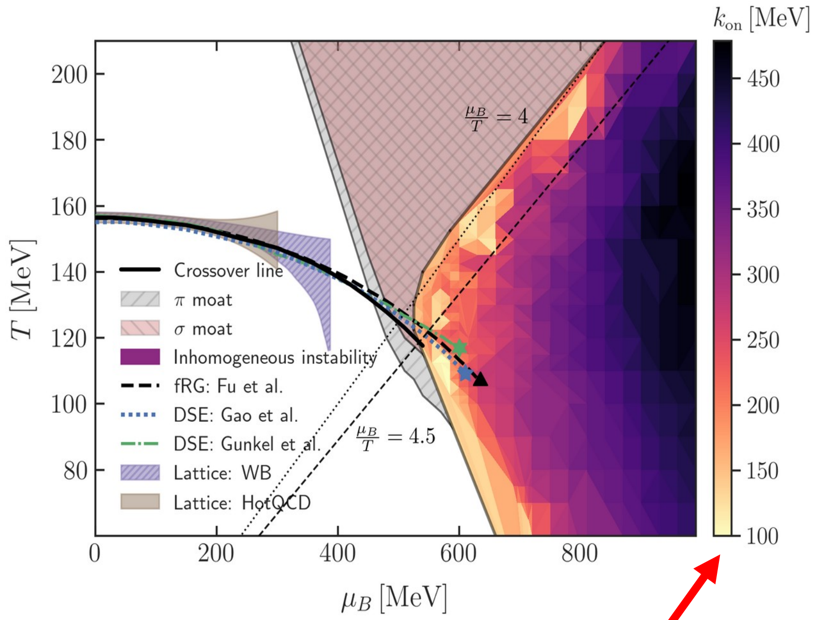
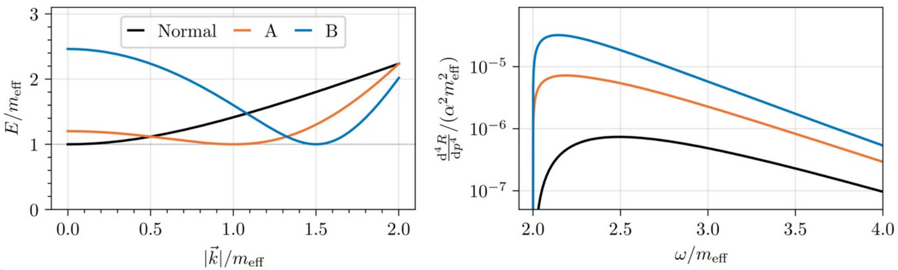
Breakdown of expansion around homogeneous ground-state

→ *Expand around inhomogeneous ground state* with *Fu, Huang, **Kockler**, Pawlowski, Rennecke, Yin*



Dilepton production

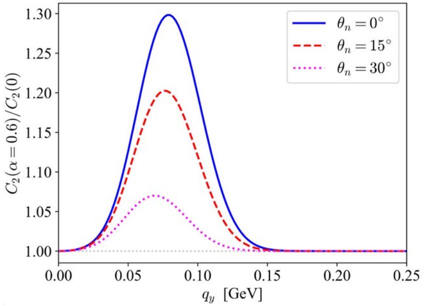


Nussinov, Ogilvie, Pannullo, Pisarski, Rennecke
Phys.Rev.Lett. 135 (2025) 10, 101904

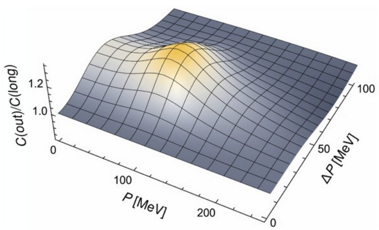
First calculation with fully self-consistent moat regime!

Fluctuation observables?

HBT interferometry



K Fukushima, Y. Hidaka, K Inoue, K Shigaki, Y Yamaguchi,
Phys. Rev. C 109, L051903 (2024)



Pisarski, Rennecke, Rischke
Phys.Rev.D 107 (2023) 11, 116011

Breakdown of expansion around homogeneous ground-state

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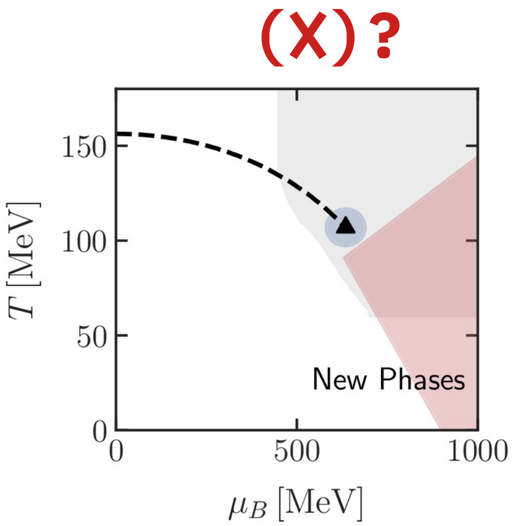
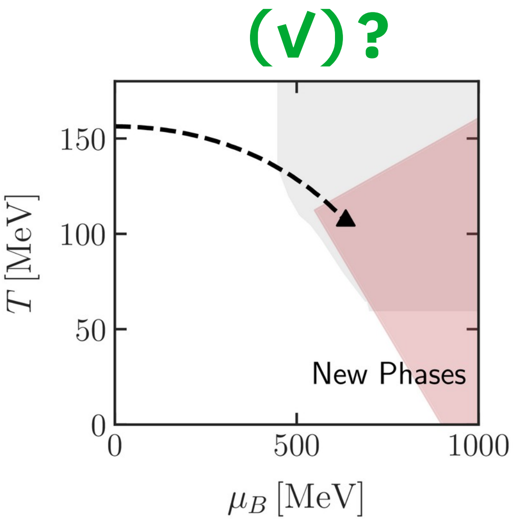
Existence of the CEP?

Existence of the CEP?





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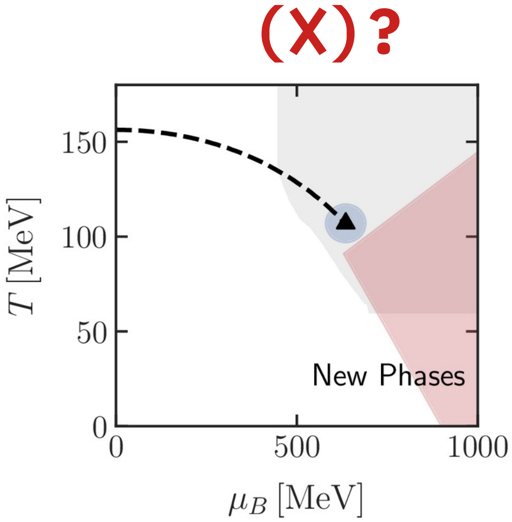

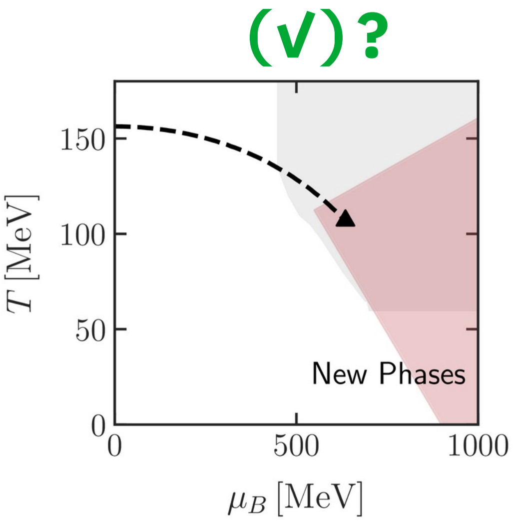

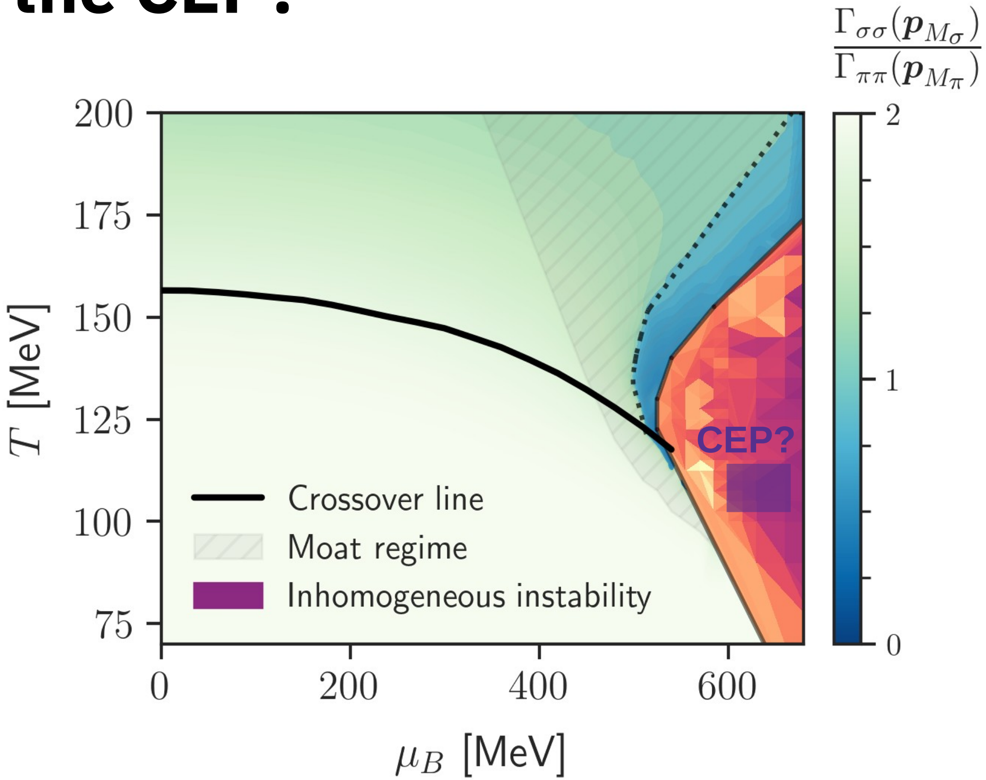

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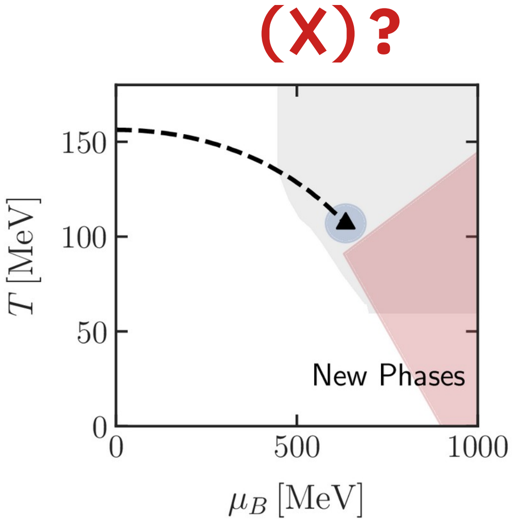

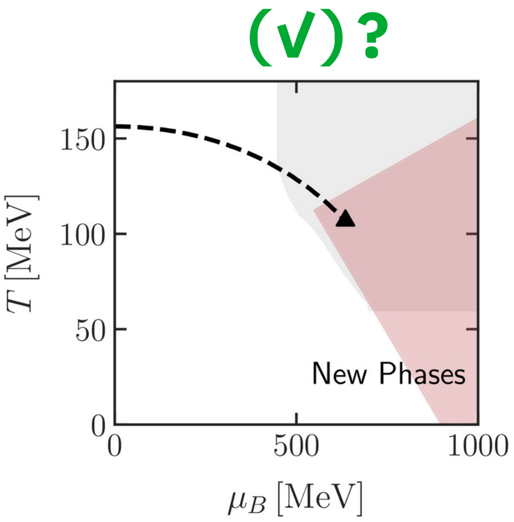

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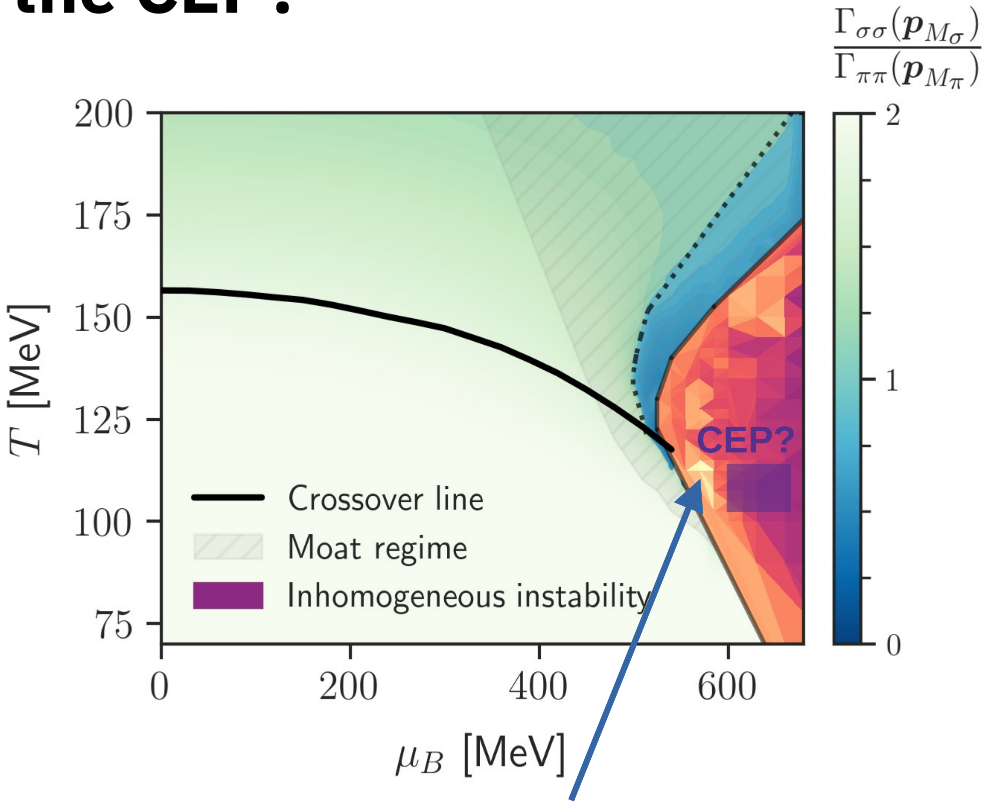

Inhomogeneous precondensation?

Pastor-Gutiérrez, Pawlowski, Sattler arXiv: 2602.11265 [hep-ph]

Talk by Álvaro Pastor-Gutiérrez on wednesday

Thermal precondensation in gauge-fermion theories

Pastor-Gutiérrez, Pawlowski, Sattler arXiv: 2602.11265 [hep-ph]

What is precondensation?

Condensate exists only on certain length-scales,
often close to a 2nd order PT

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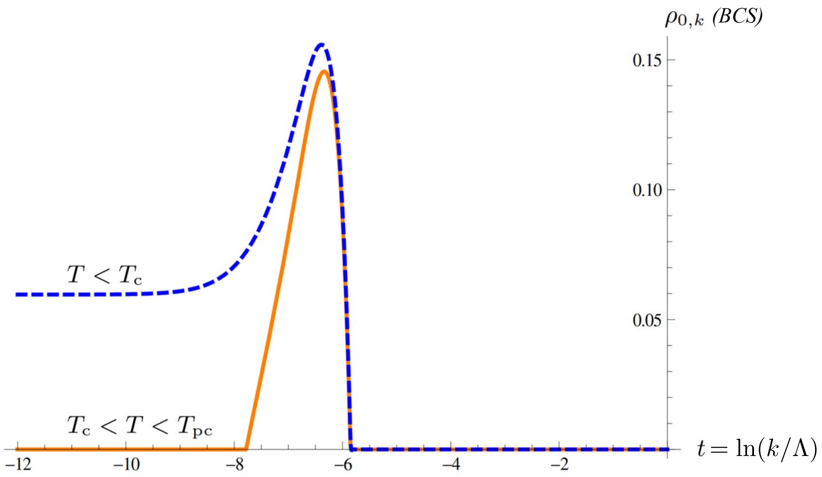

Previously discussed, e.g. in

- Cold atoms: BCE-BCS crossover *Boettcher, Pawlowski, Diehl [1204.4394]*

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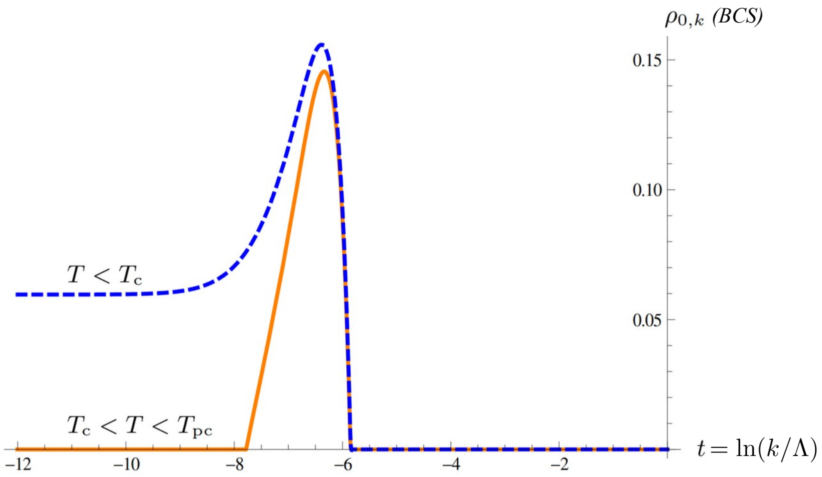

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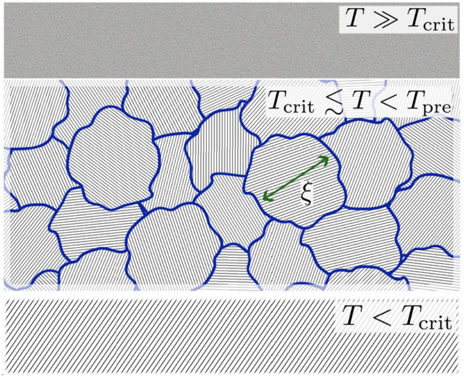

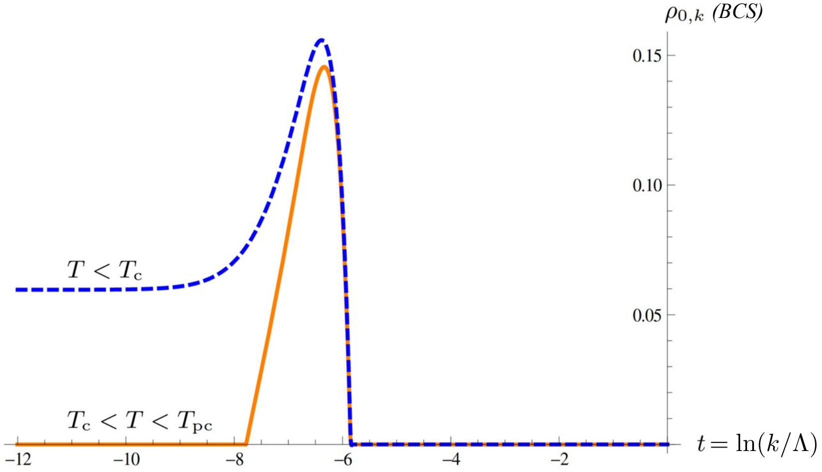

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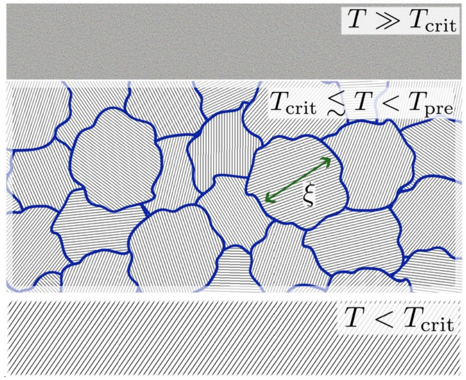

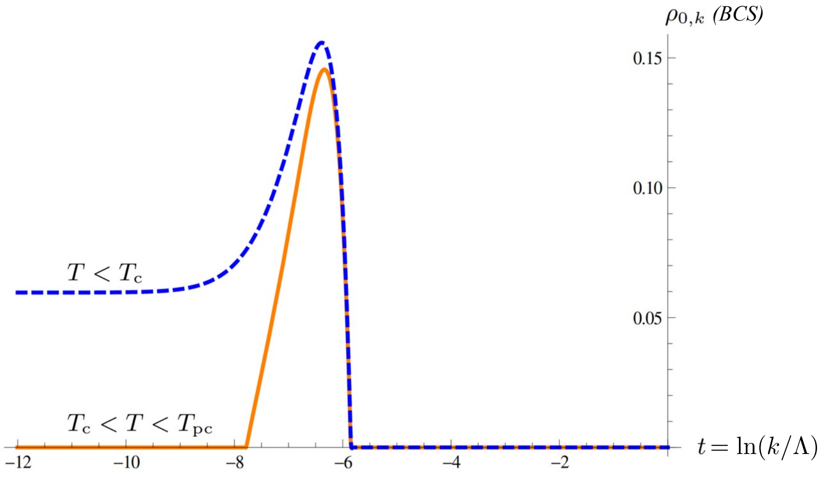

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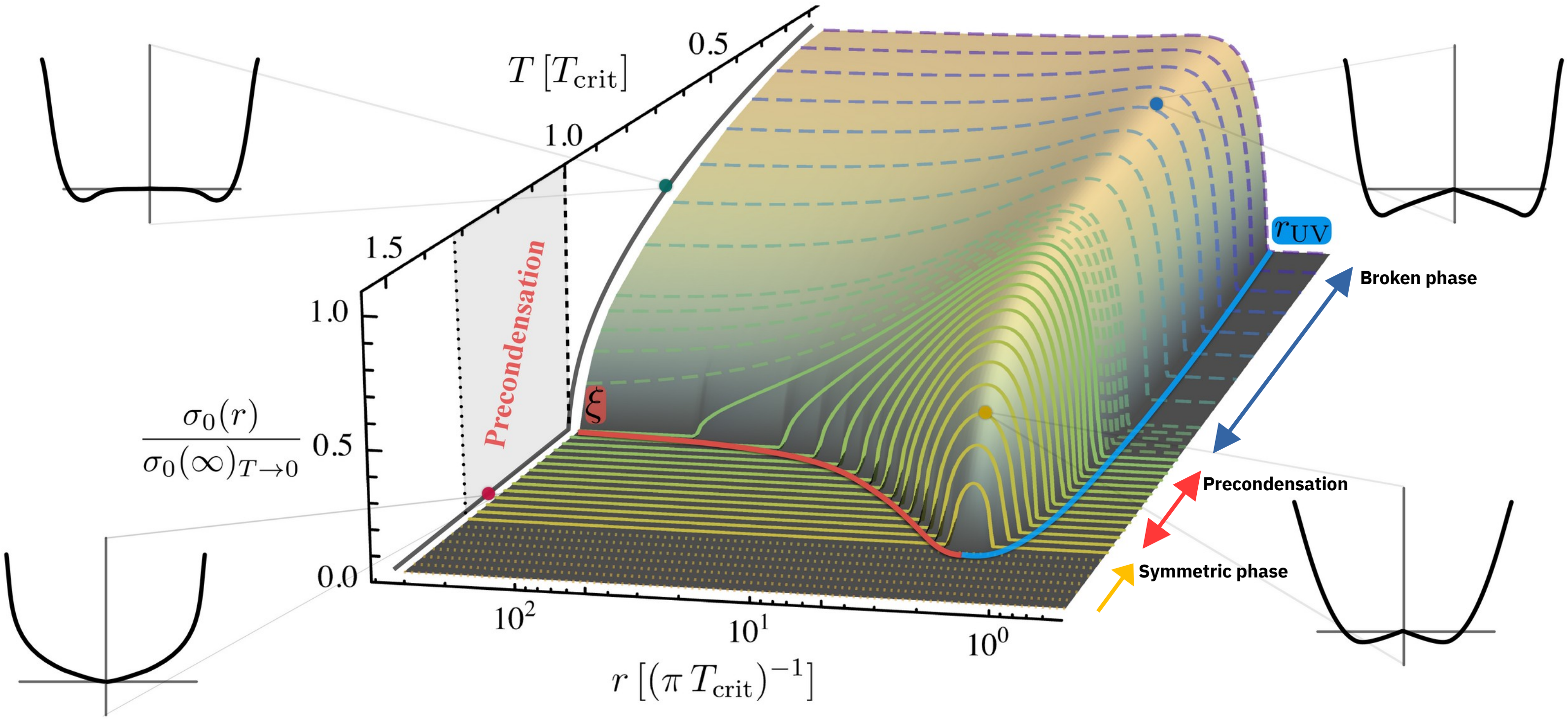

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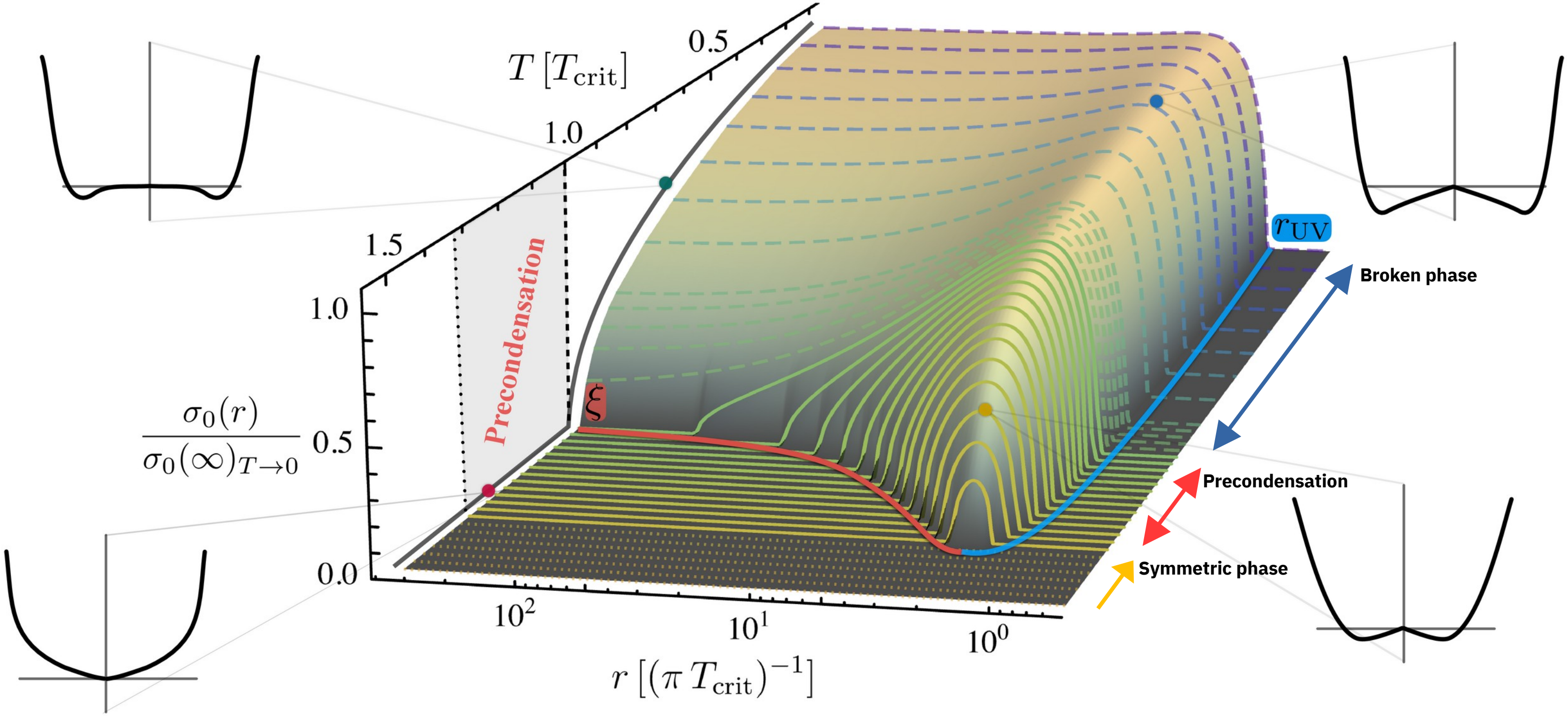

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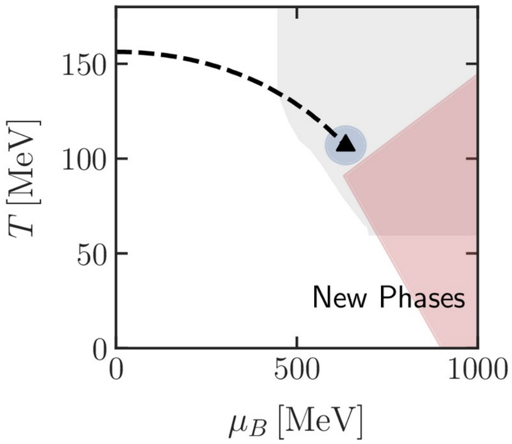
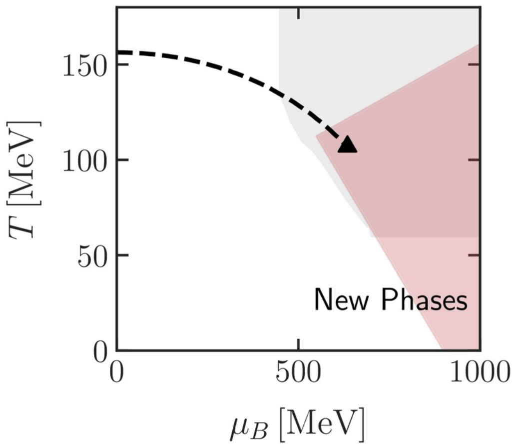
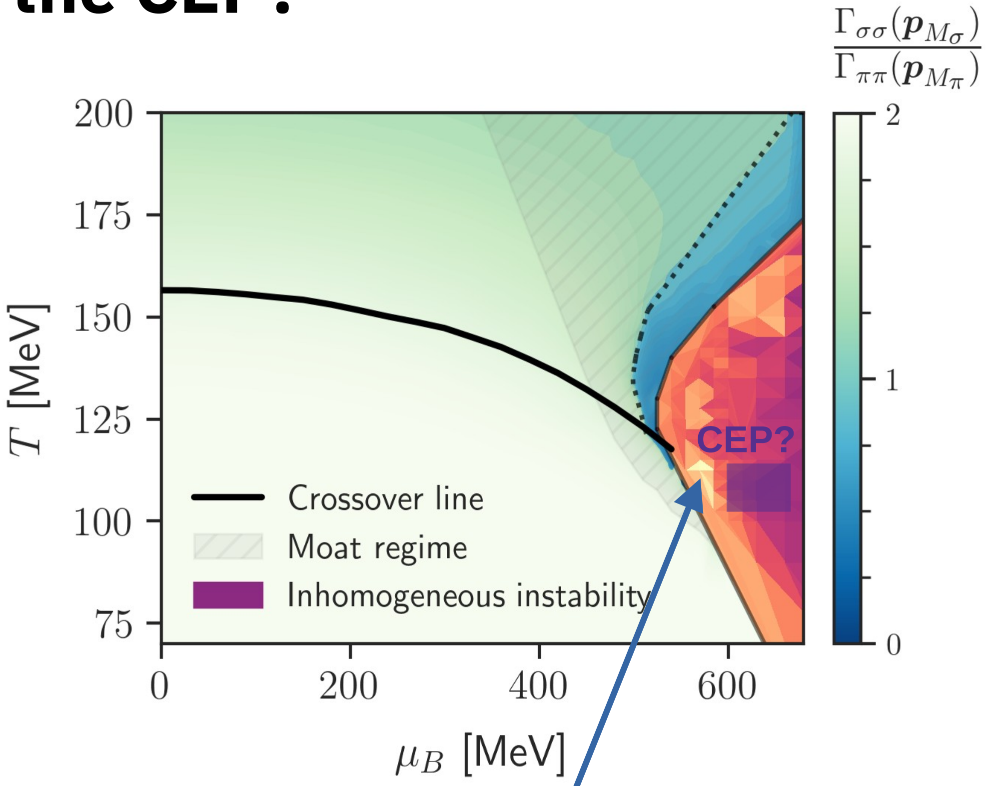
Existence of the CEP?

(X)?



(√)?





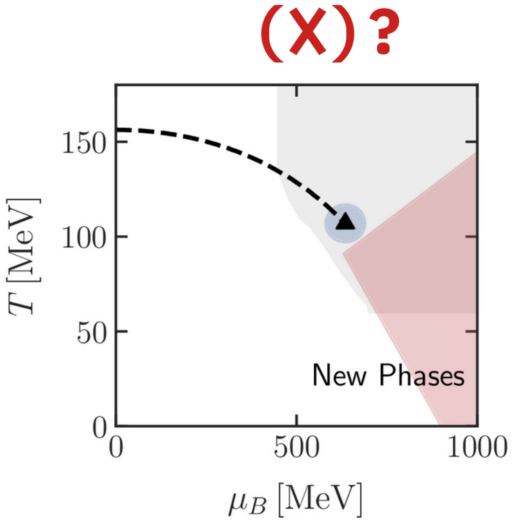
No CEP?

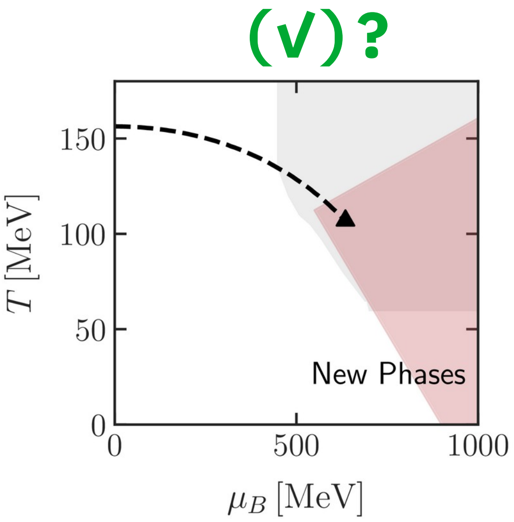
Inhomogeneous precondensation?

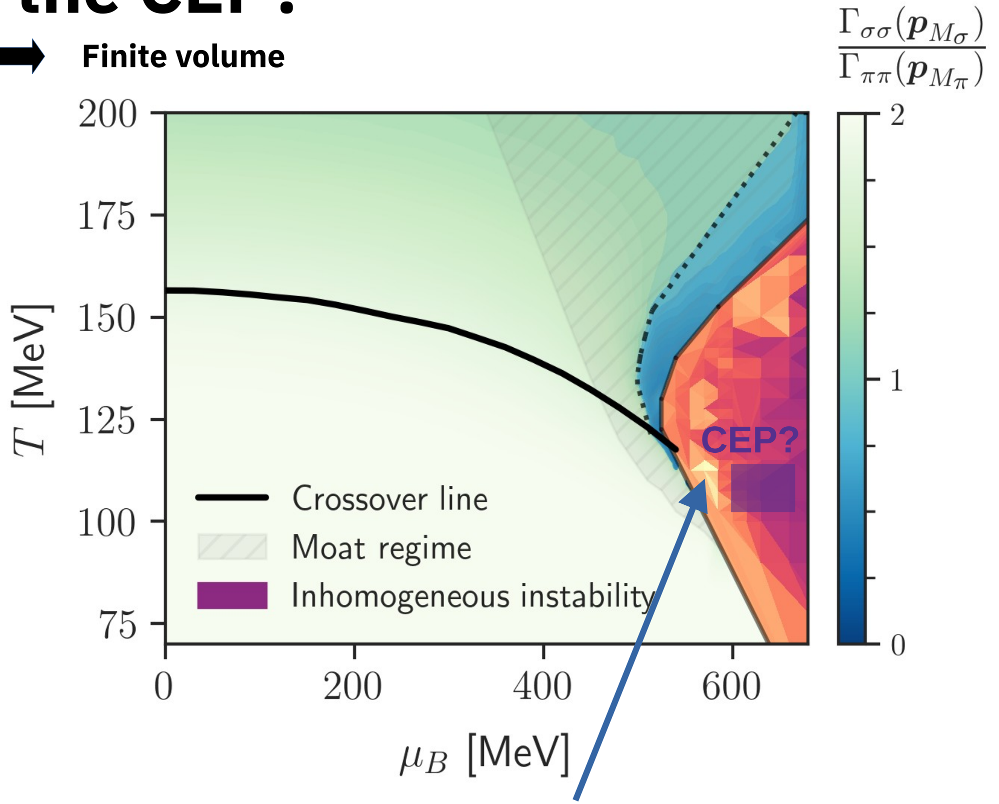
Pastor-Gutiérrez, Pawlowski, Sattler arXiv: 2602.11265 [hep-ph]

Existence of the CEP?

Thermodynamic limit ⟷ **Finite volume**







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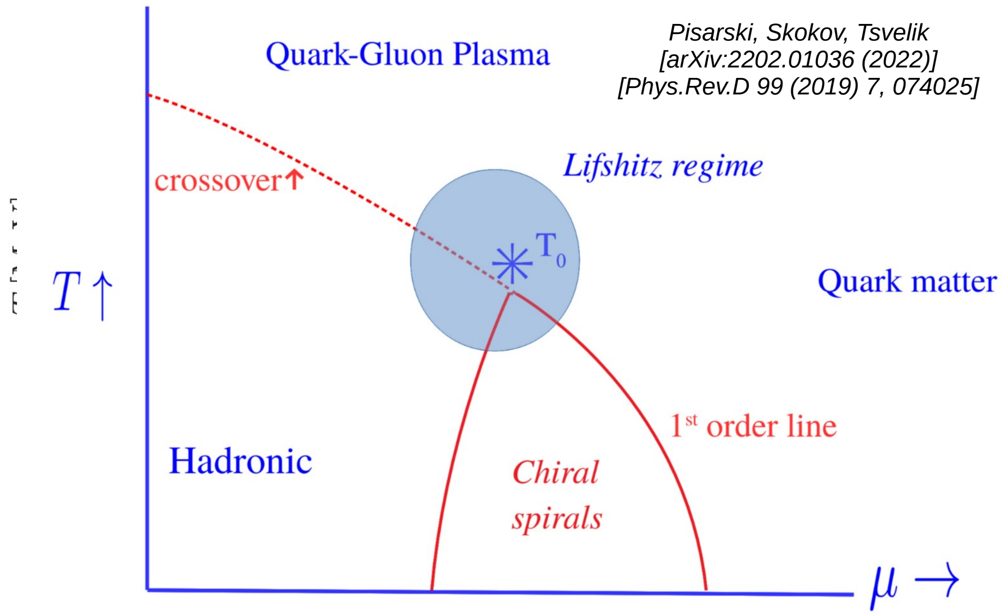
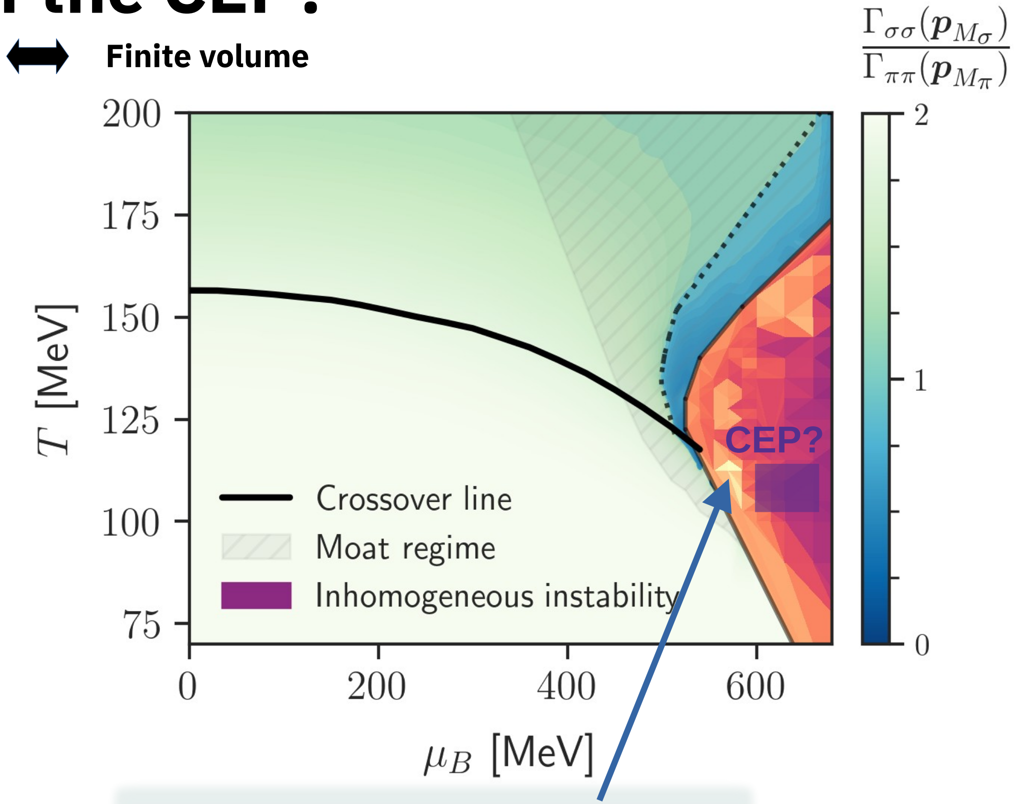
To Be Continued

Existence of the CEP?

Thermodynamic limit ⟷ **Finite volume**

Pisarski, Skokov, Tsvelik [arXiv:2202.01036 (2022)] [Phys.Rev.D 99 (2019) 7, 074025]





Inhomogeneous regime
(Lifshitz point/regime)?

Inhomogeneous precondensation?
Pastor-Gutiérrez, Pawlowski, Sattler arXiv: 2602.11265 [hep-ph]

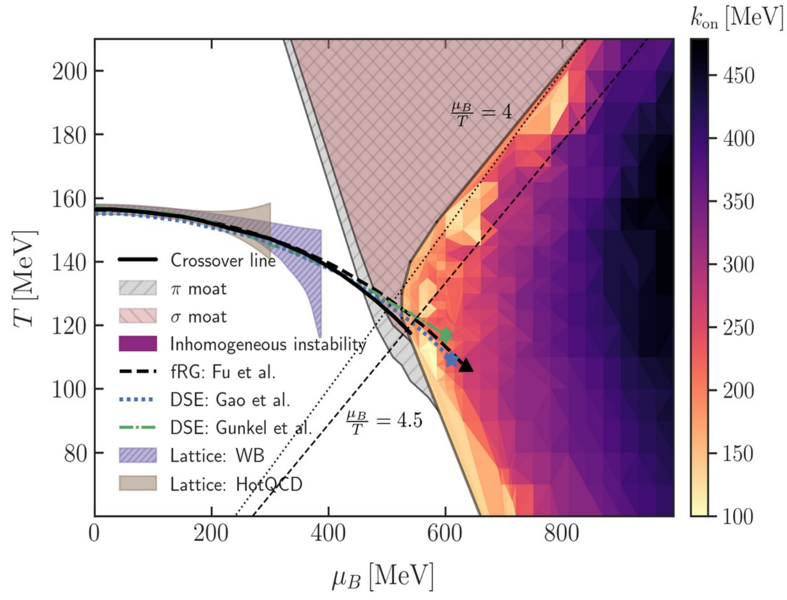
To Be Continued

Summary

- **Quantitative Results** up to μ/T < 4.5 in agreement with Lattice & other functional approaches.
- First self-consistent calculation of moat regime: **Inhomogeneous instability.**
- Possibly **inhomogeneous precondensation.**
- **Systematic error estimates** from LEGO® and regulator variation, checks for UV limit & gauge consistency.

Outlook

- *Self-consistent* treatment of **inhomogeneous condensation.** *(with Fu, Huang, **Kockler**, Pawlowski, Rennecke, Yin).*



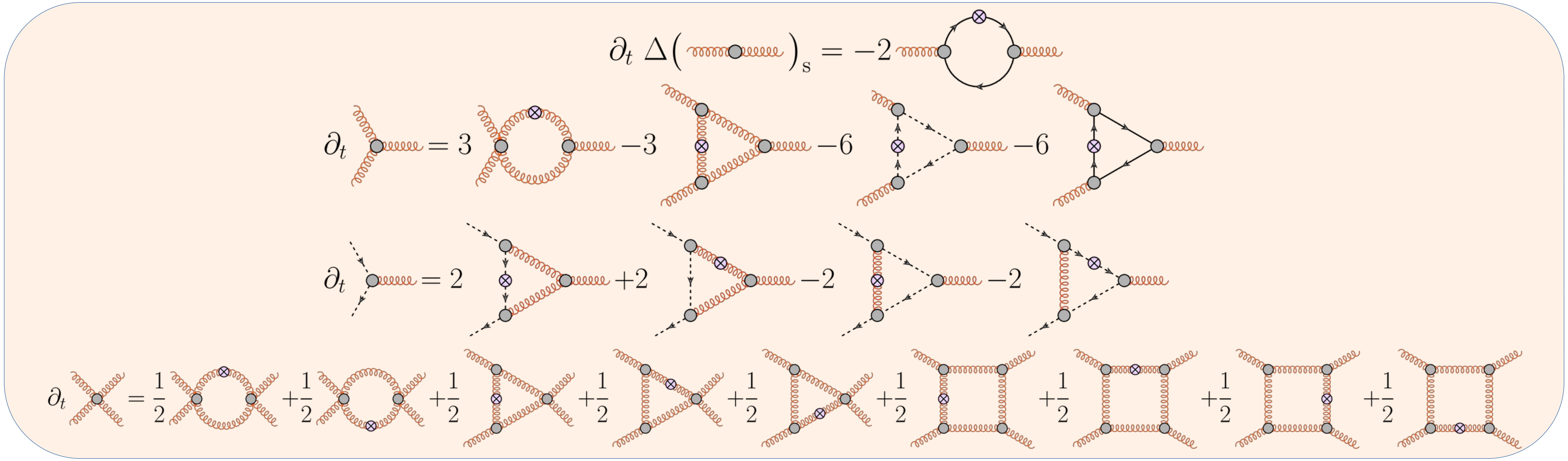
➔ **Understand existence of CEP/Lifshitz point.**

Thank you for your attention!

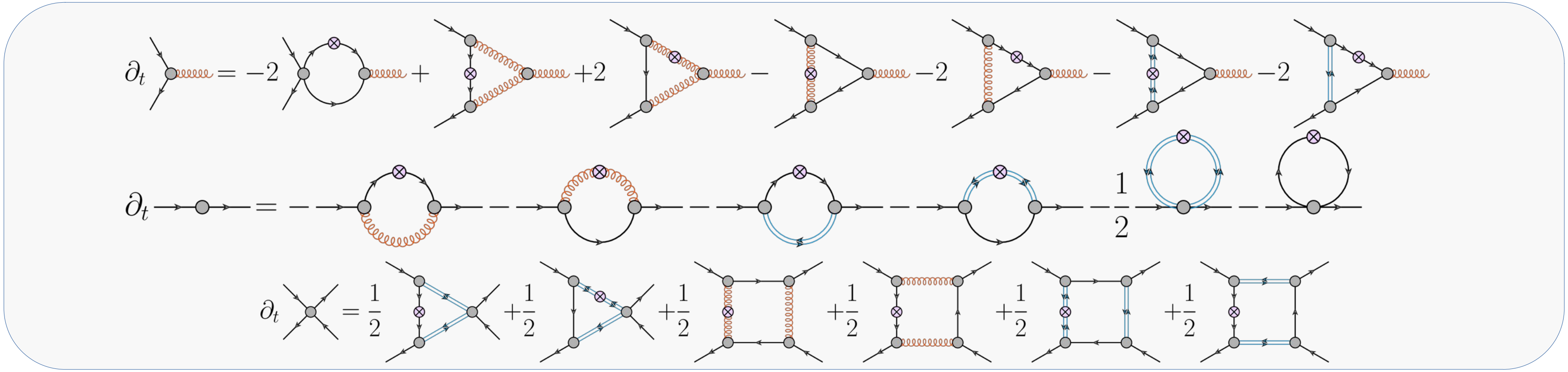
https://fqcd-collaboration.github.io/

Backup slides

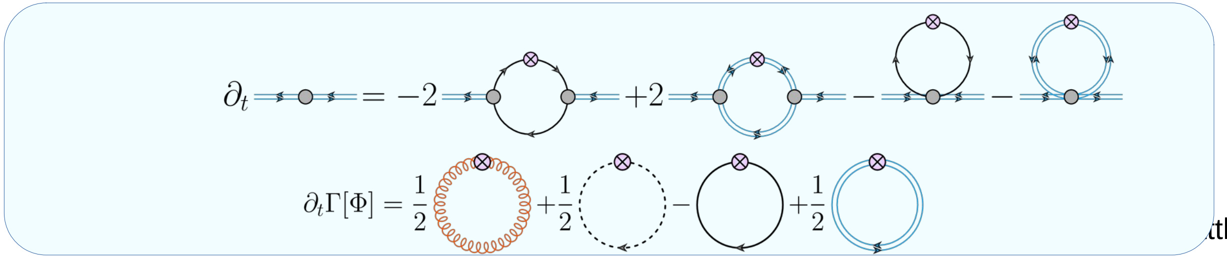
Gluons and Ghosts



Quarks



Mesons



First calculation with fully self-consistent moat regime!

Thermal precondensation in gauge-fermion theories

Pastor-Gutiérrez, Pawlowski, Sattler arXiv: 2602.11265 [hep-ph]

What is precondensation?

Condensate exists only on certain length-scales, often close to a 2nd order PT

Previously discussed, e.g. in

- Cold atoms: BCE-BCS crossover *Boettcher, Pawlowski, Diehl [1204.4394]*
- Condensed matter: Dirac semi-metals *Tolosa-Simeón, Classen, Scherer [2503.04911]*
- Gauge-fermion: diquark condensate in 2-colour QCD *Khan, Pawlowski, Rennecke, Scherer [1512.03673]*
- **Chiral QCD with** $N_f = 2, 3, 4$

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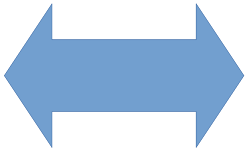

fluctuations

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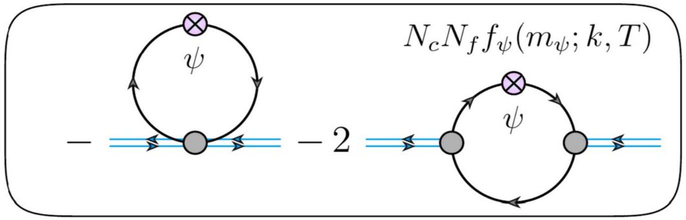

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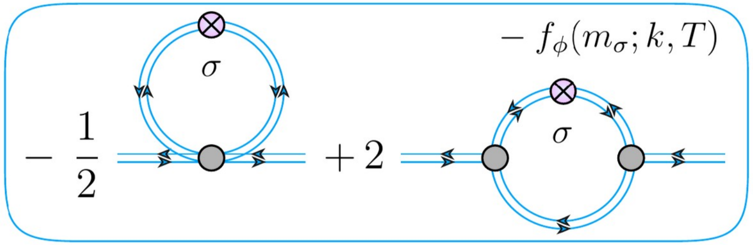

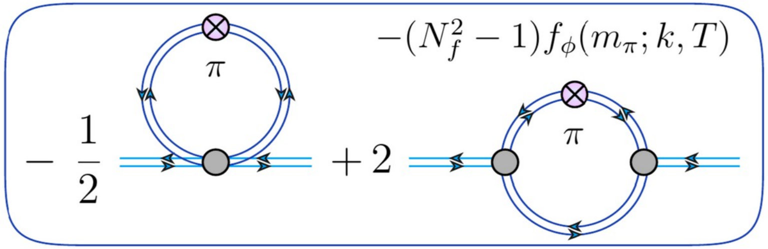

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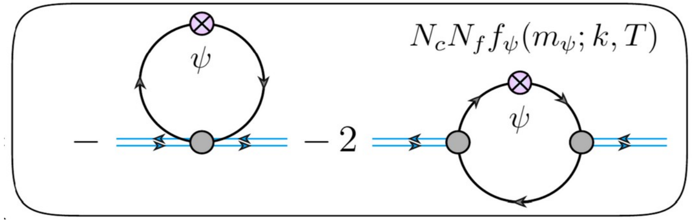
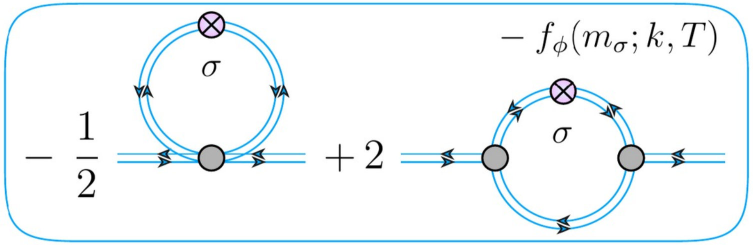


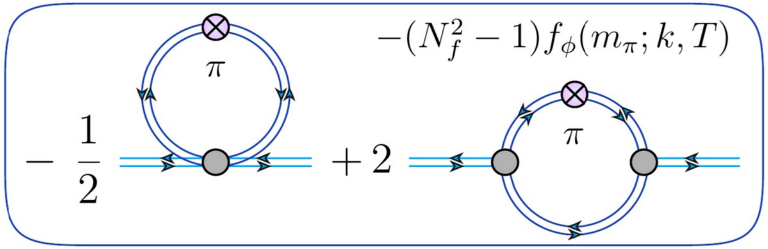
Competing

fluctuations

+

Different reaction to external parameter T





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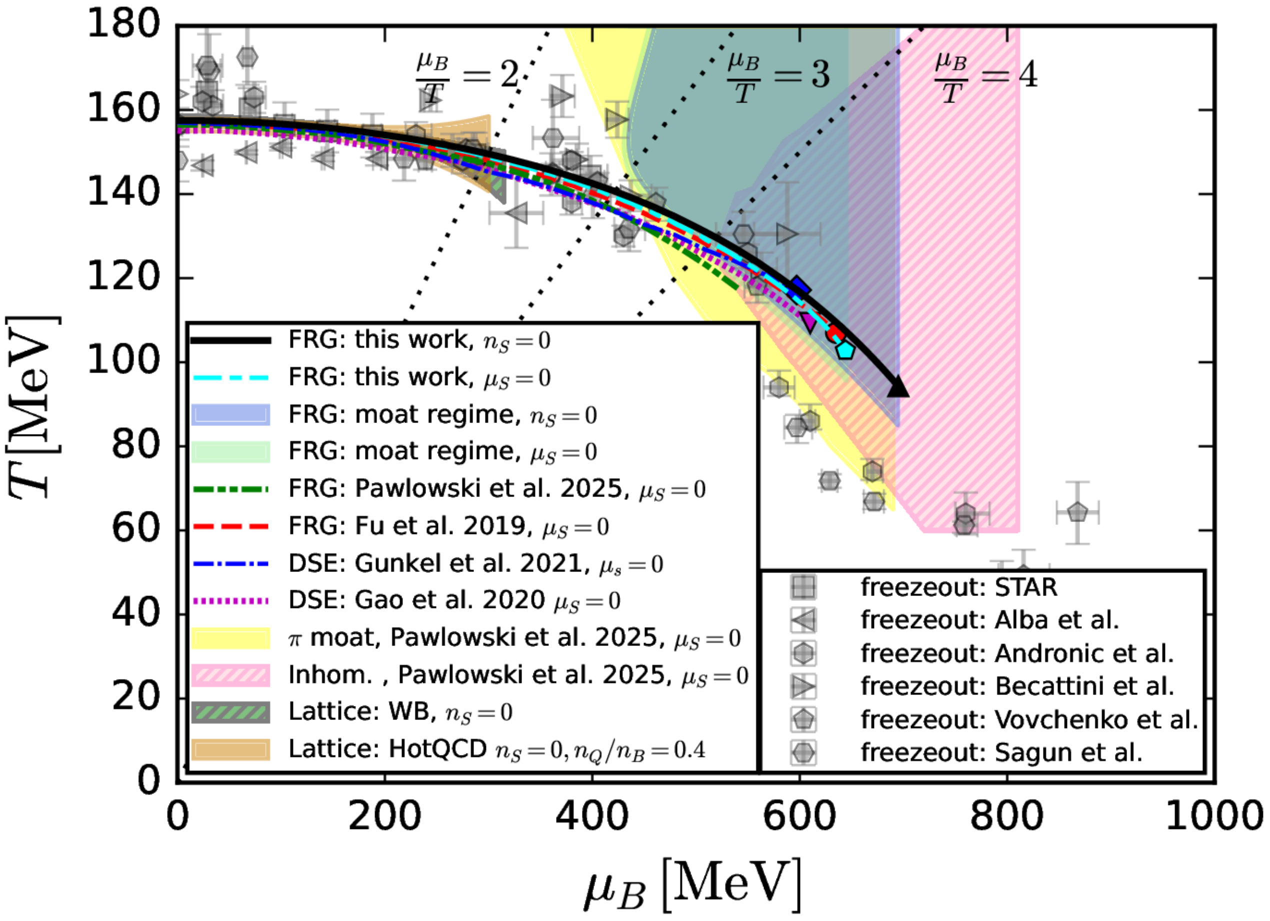
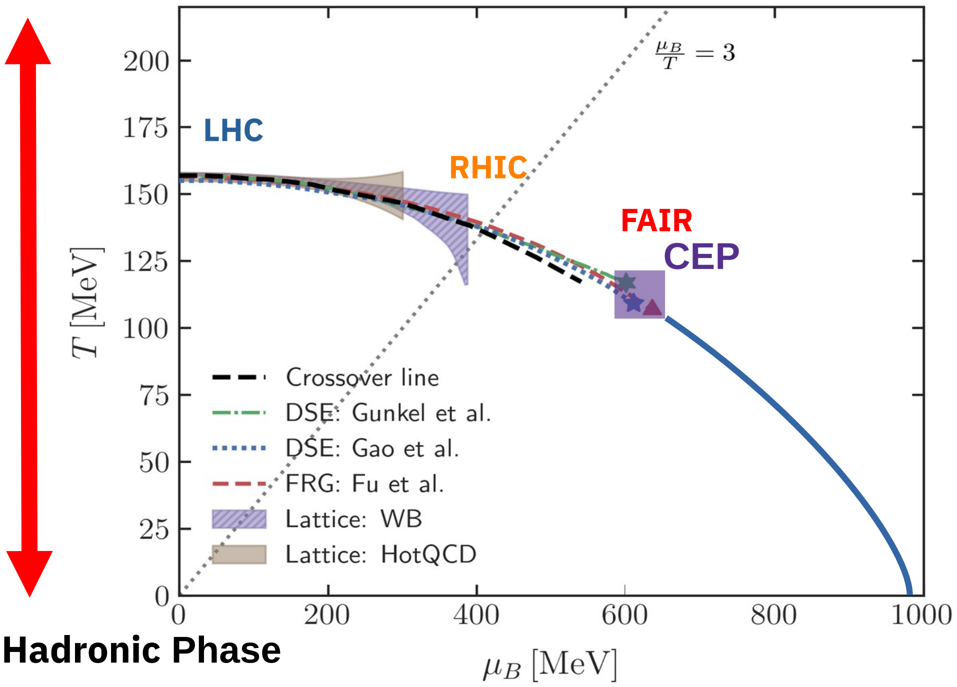
Overview with recent strangeness-neutral fRG results

(from Fu, Huang, Pawlowski, Rennecke, Wen 2603.13455 [hep-ph])



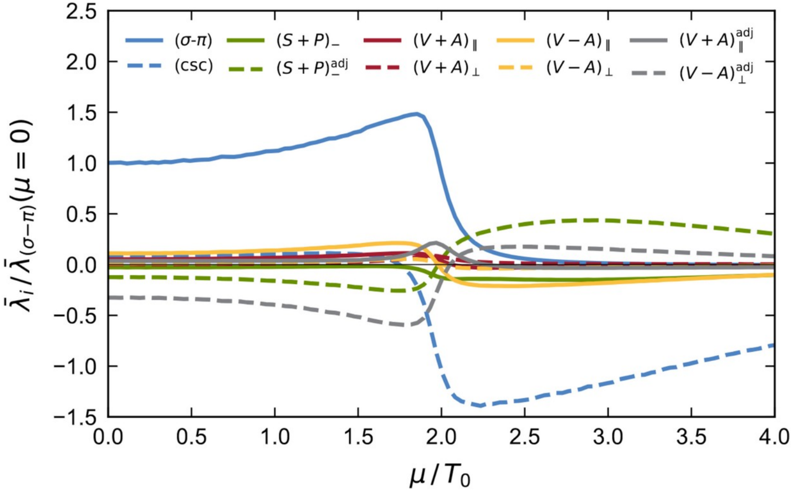
High-density QCD: some relevant channels and physics

Quark-Gluon Plasma

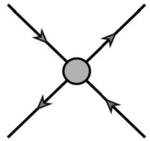


Hadronic Phase

Other four-fermi channels



Braun, Leonhardt, Pospiech [Phys.Rev.D 101 (2020) 3, 036004]



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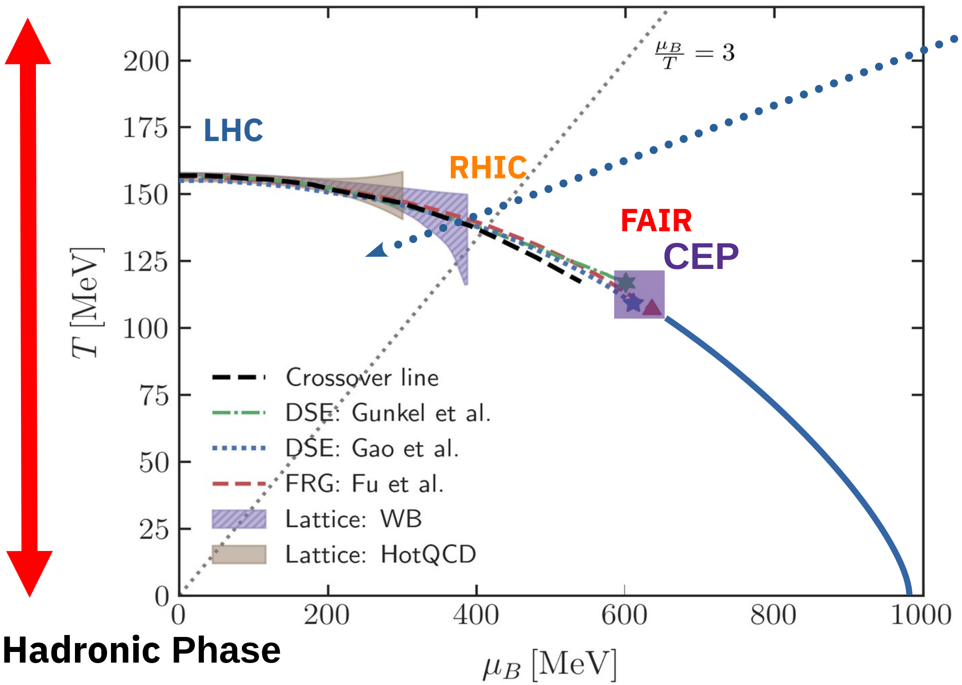

Hadronic Phase

$\langle \bar{q} q \rangle$

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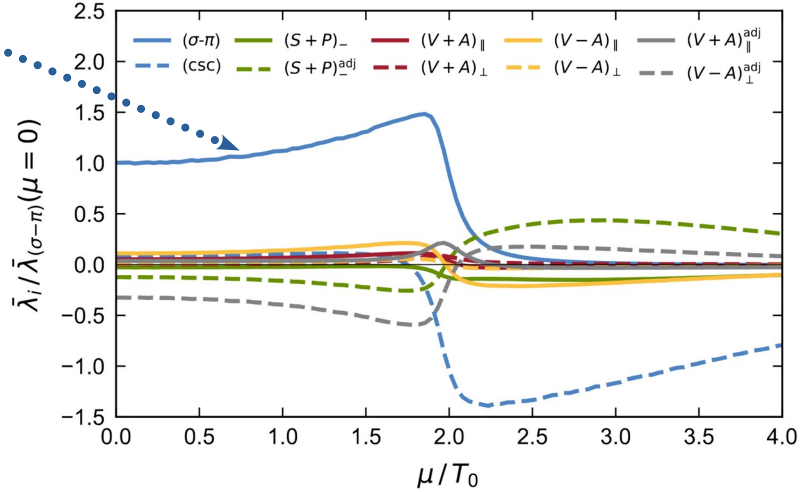

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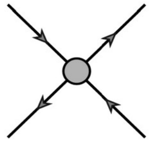

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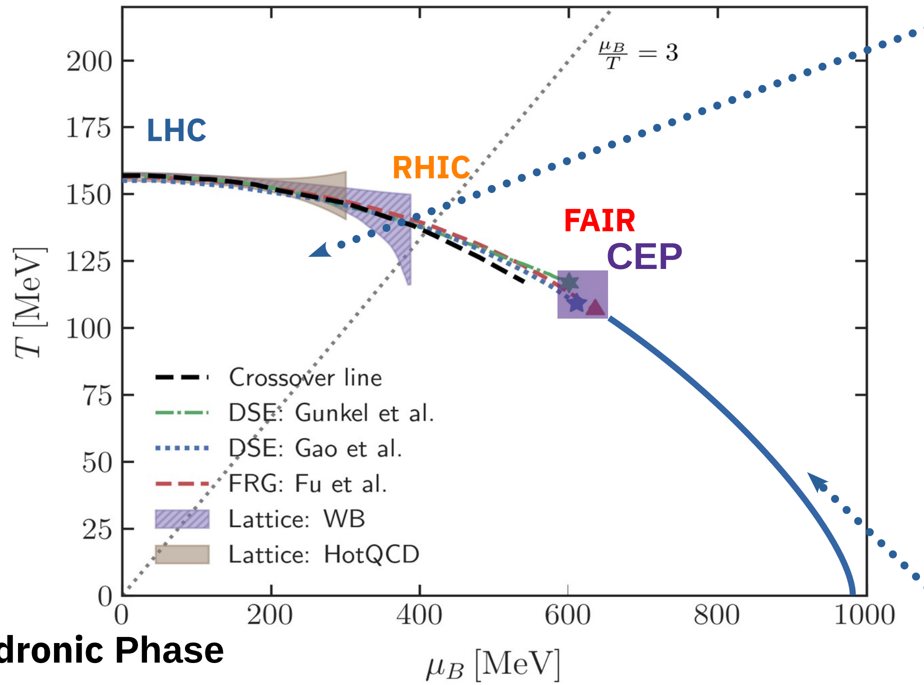

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$\langle d^\star d \rangle$

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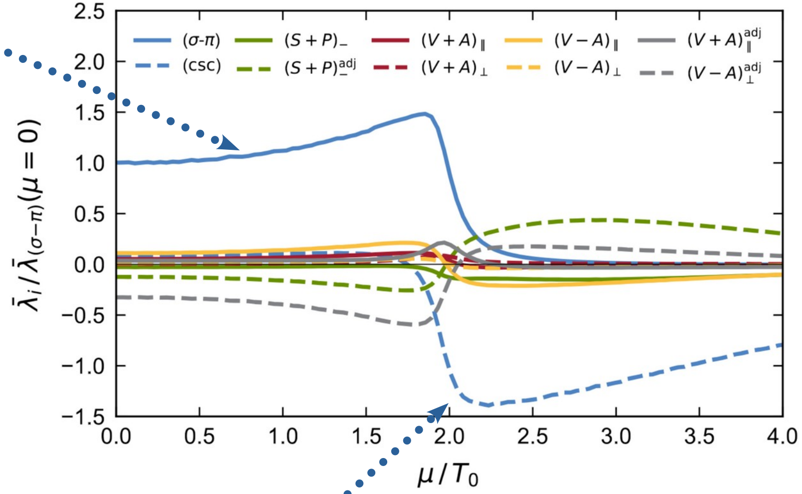

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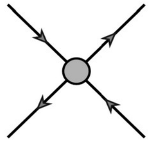

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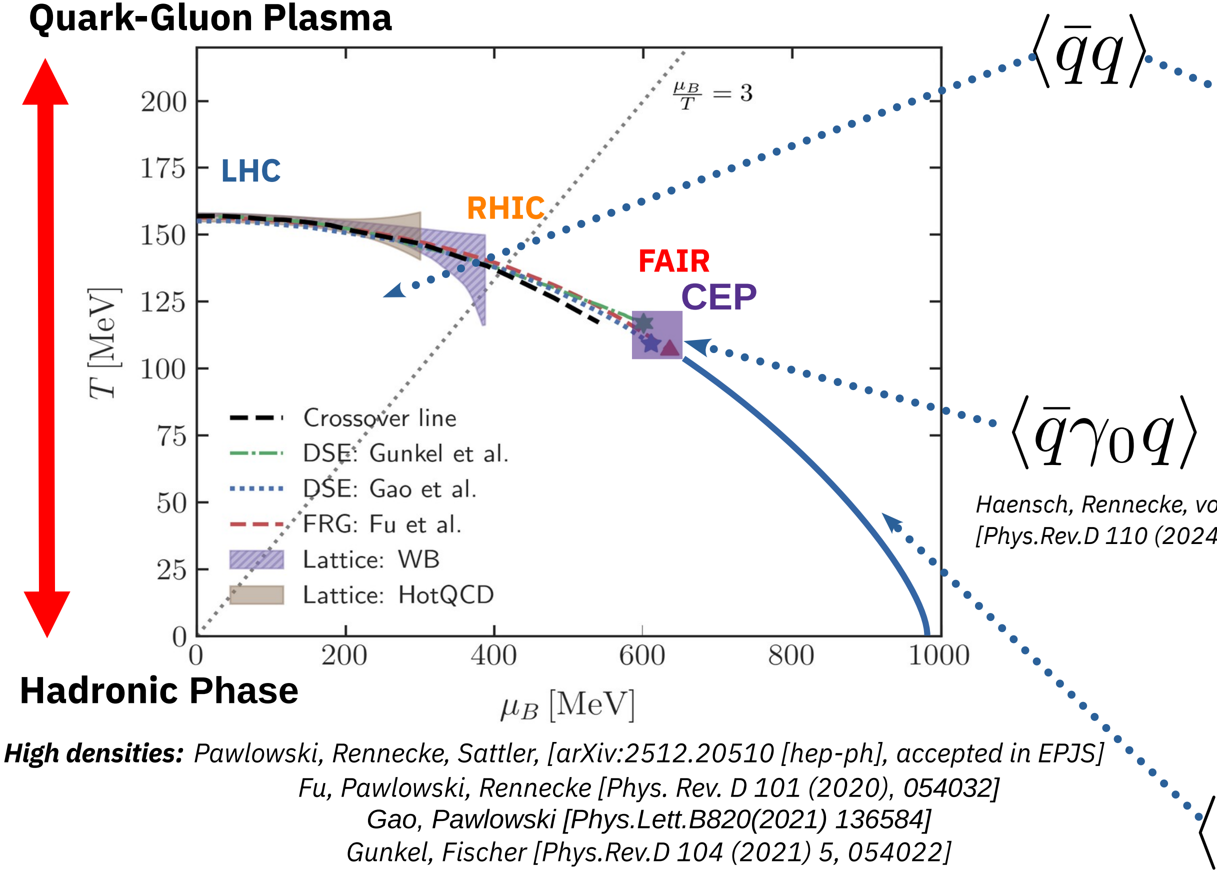

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Haensch, Rennecke, von Smekal [Phys.Rev.D 110 (2024) 3, 036018]

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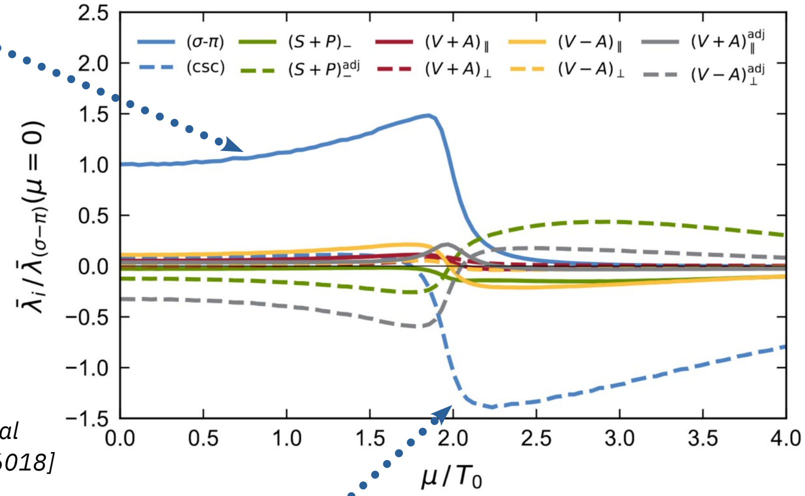

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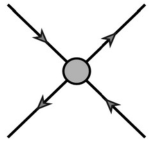

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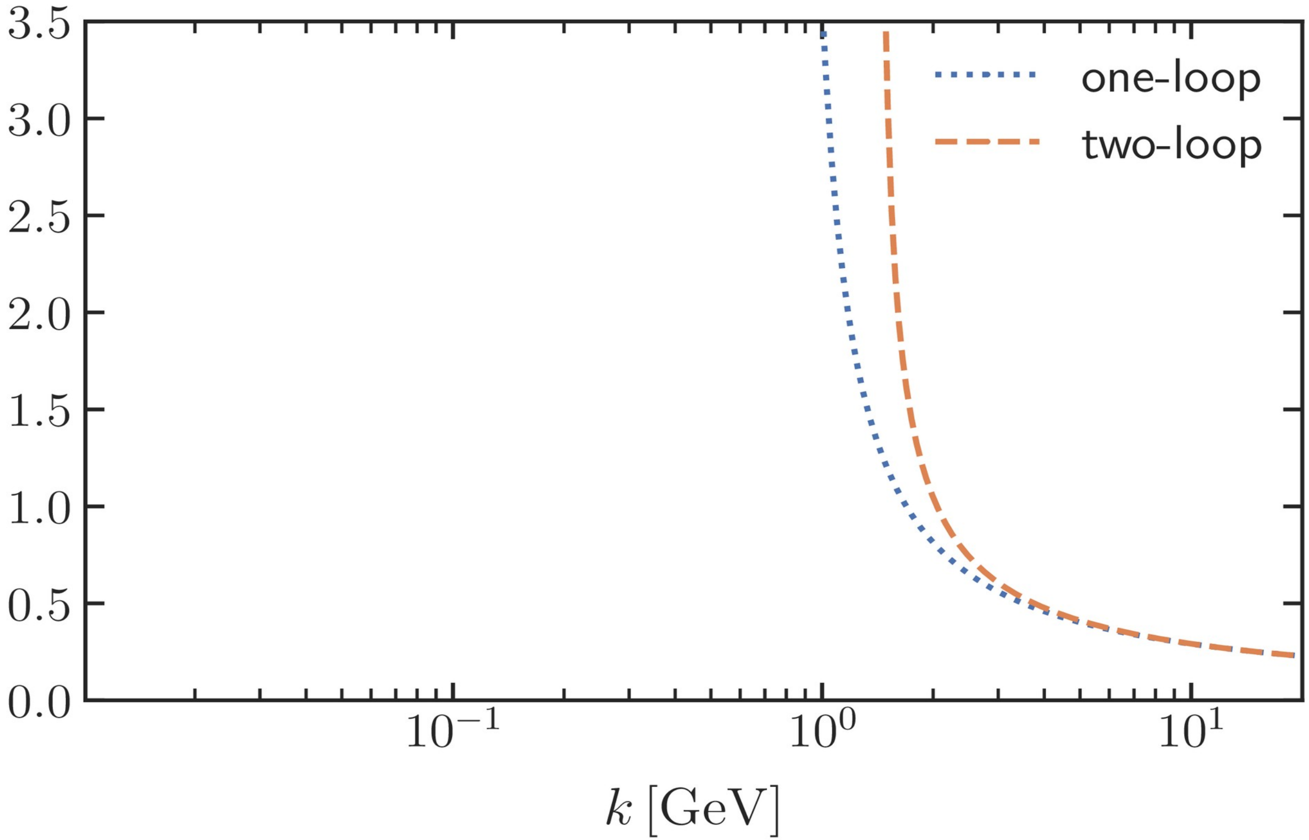
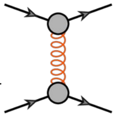
QCD from the fRG perspective



$p^2 Z_A$ 

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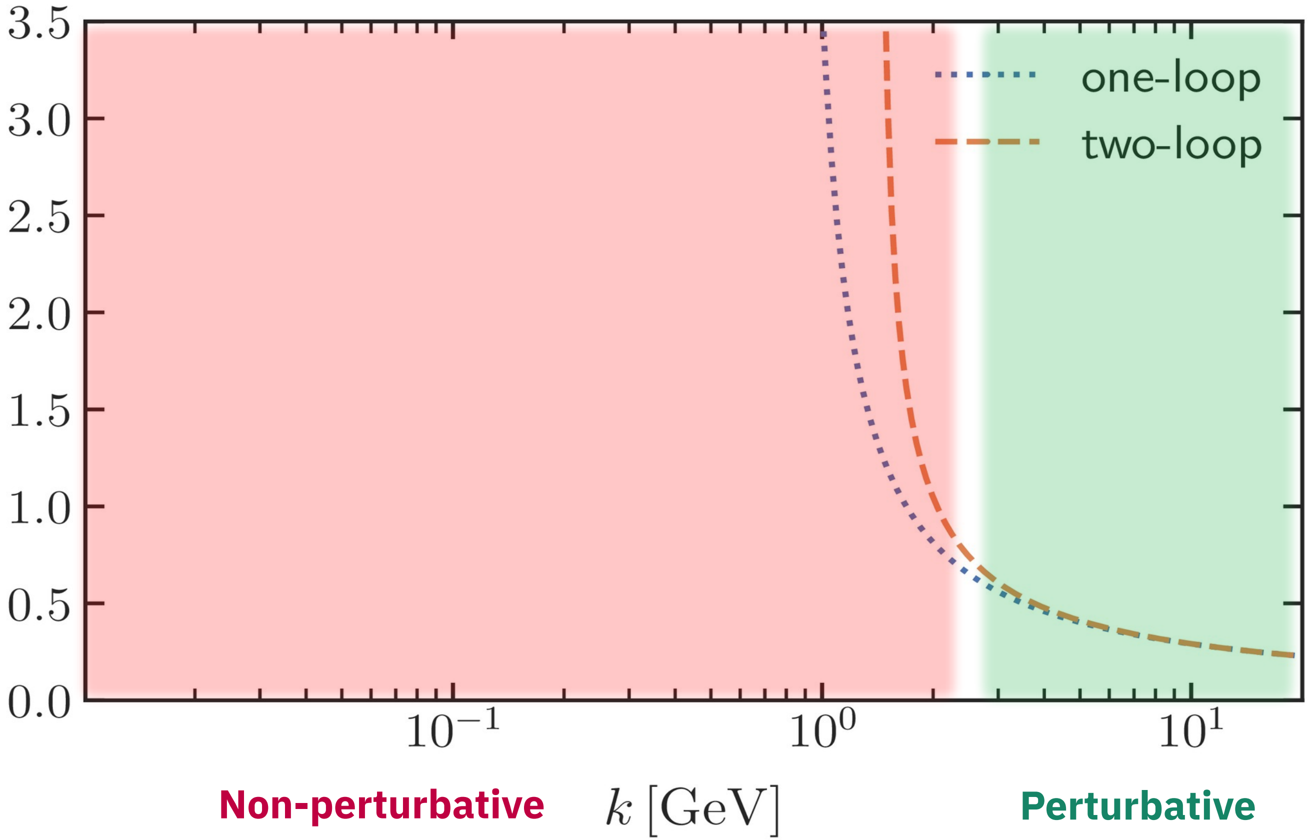

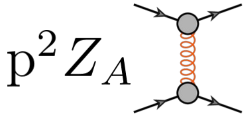

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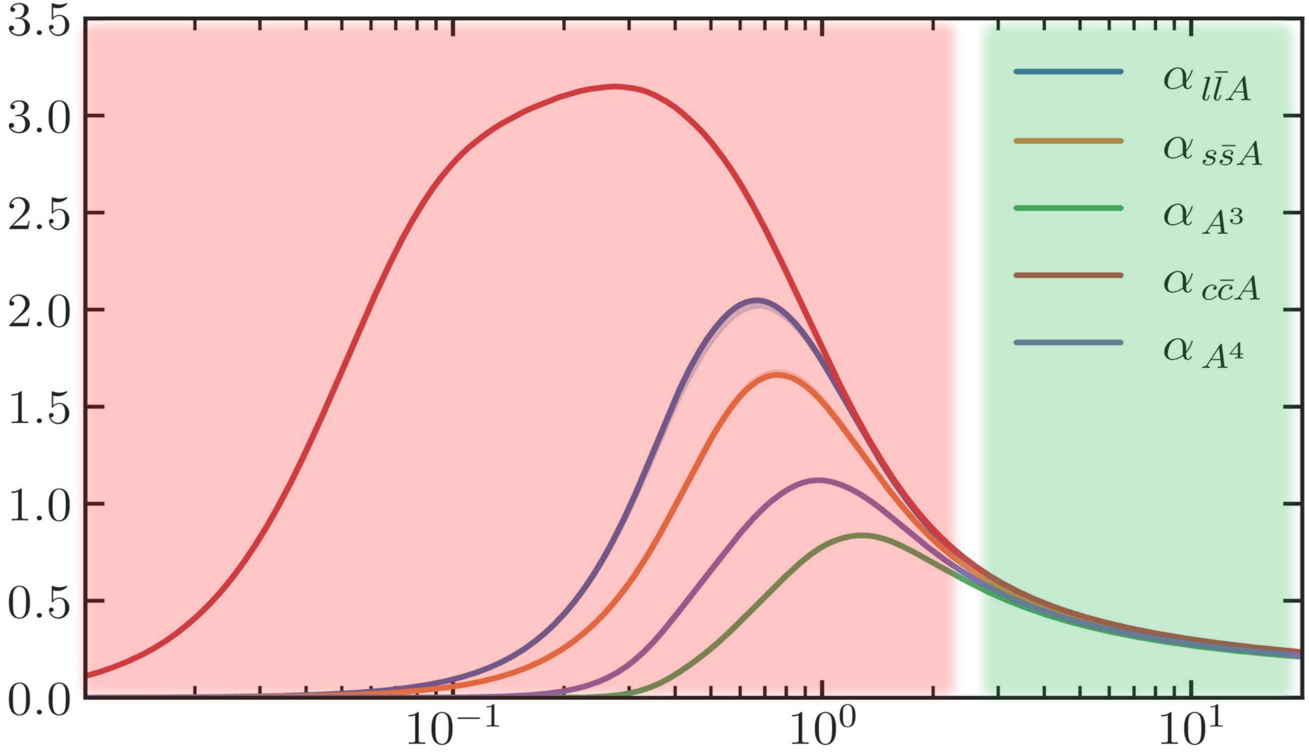

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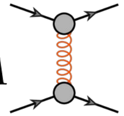
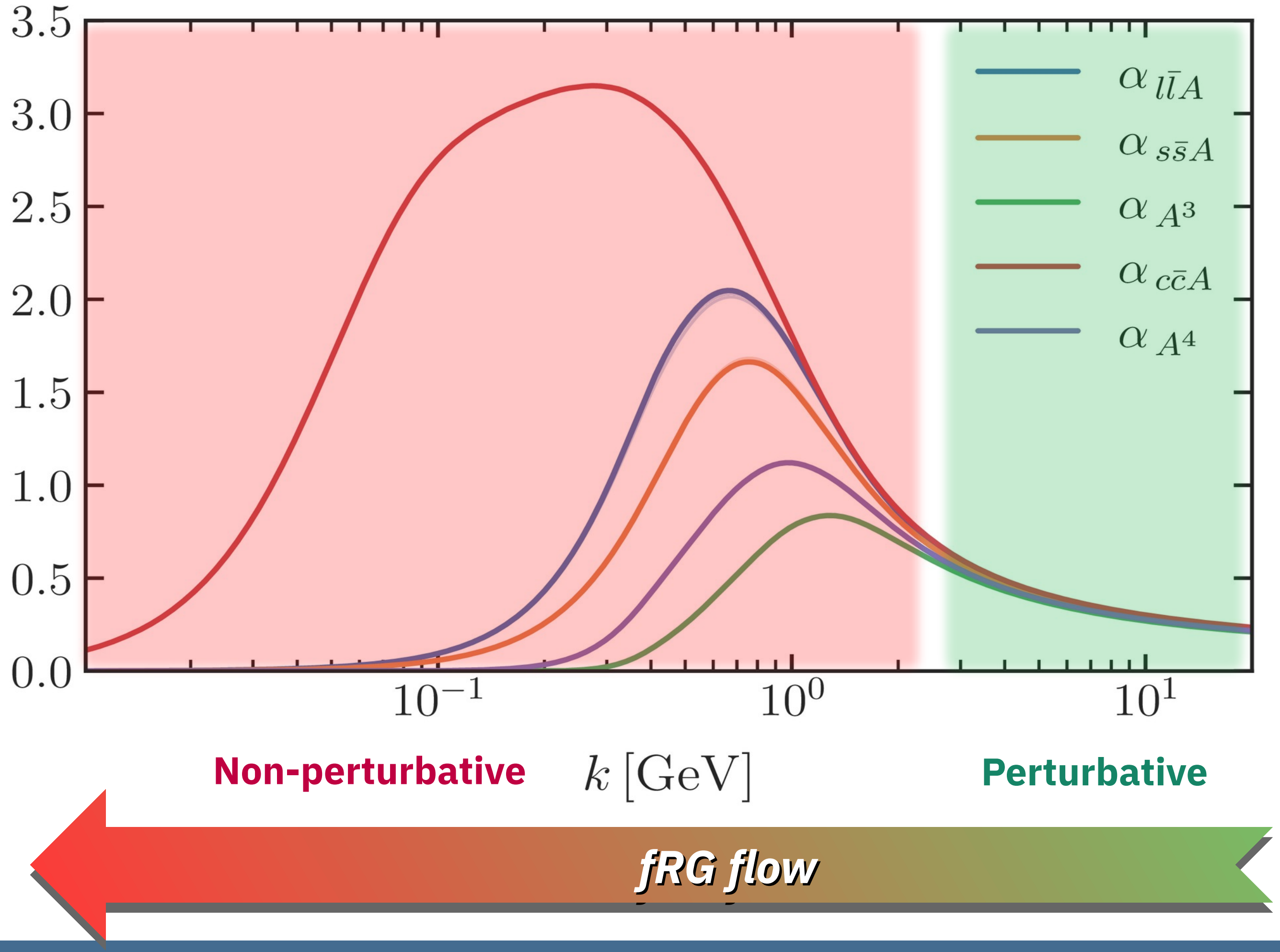


Non-perturbative $k\,[\mathrm{GeV}]$ **Perturbative**

QCD from the fRG perspective



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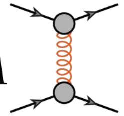

Macroscopic order

$$\sigma_0^2 = \lim_{\mathcal{V}\to\infty} \frac{1}{\mathcal{V}^2} \int d^3x \, d^3y \, \langle \hat{\sigma}(x) \hat{\sigma}(y) \rangle$$

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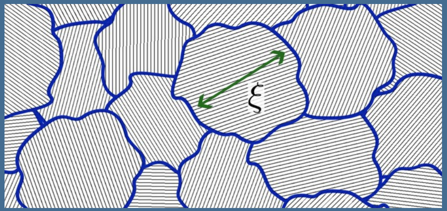

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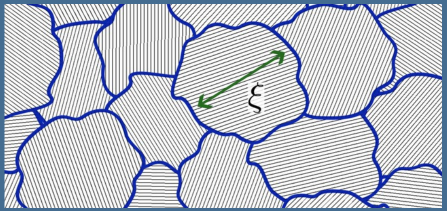

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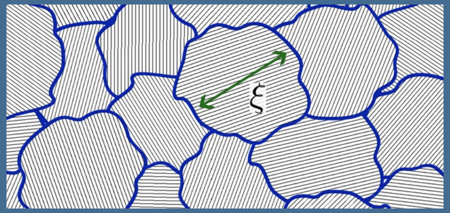
Precondensation

Condensate on a finite range of length-scales

$$\sigma_0^2(\boldsymbol{x}; r) = \begin{cases} 0, & r < r_{\mathrm{UV}} \,, \\ \neq 0, & r_{\mathrm{UV}} < r < \xi \,, \\ 0, & r > \xi \,. \end{cases}$$



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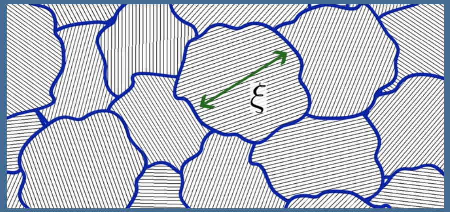


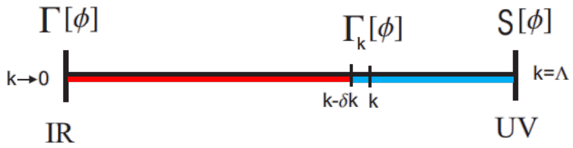
fRG

$r \sim 1/k$

$$Z[J] = \int [D\varphi] e^{-S[\varphi] + \int d^d x J(x)\varphi(x)}$$

Wilsonian approach:

Integrate out momentum shells



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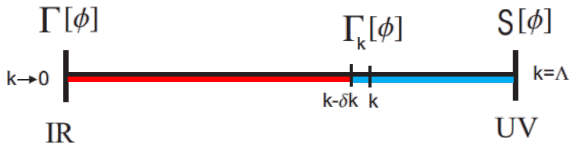

$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi] + \int d^d x J(x)\varphi(x)}$$

Wilsonian approach:

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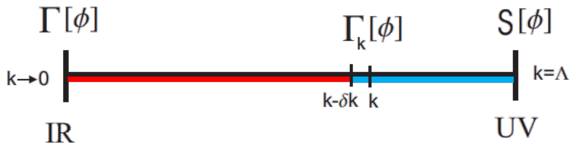

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Introduce mass-like **Regulator**

$$[D\varphi]_k = [D\varphi]_{\text{ren}} \, e^{-\frac{1}{2}\int d^d x \, \varphi(x) R_k(x) \varphi(x)}$$

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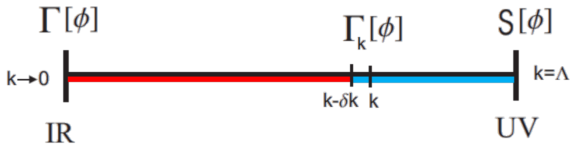

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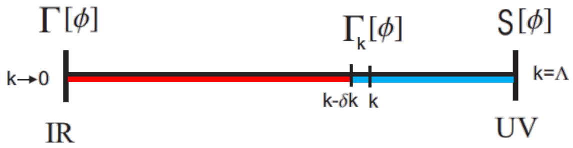
We can now take derivatives
with respect to k
Obtain differential equation!

$$Z_{\Lambda_{\text{UV}}}[J] \to Z[J]$$

Wilsonian approach:

Integrate out momentum shells



$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi] + \int d^dx J(x)\varphi(x)}$$

Simplify the notation a bit: $J_a\varphi^a := \int_x J_i(x)\varphi^i(x)$

Introduce mass-like **Regulator**

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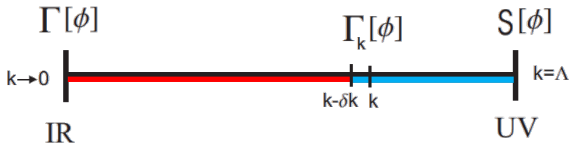

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Introduce mass-like **Regulator**

$$[D\varphi]_k = [D\varphi]_{\mathrm{ren}}\, e^{-\frac{1}{2}\varphi^a R_{k,ab} \varphi^b}$$

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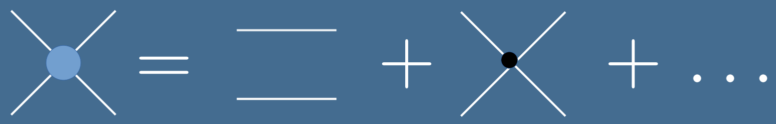
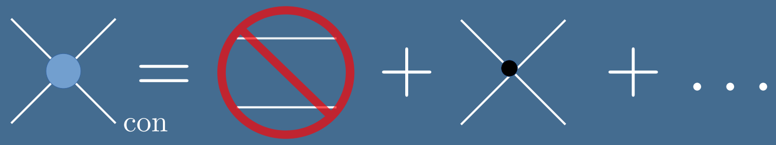
$$Z_k[J]$$



Schwinger functional

Connected diagrams

$$W_k[J] = \log Z_k[J]$$



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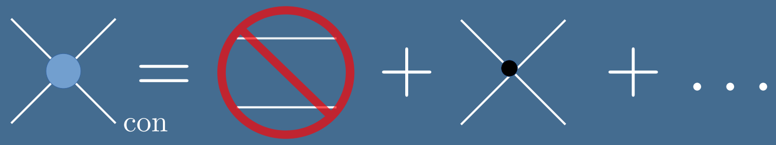


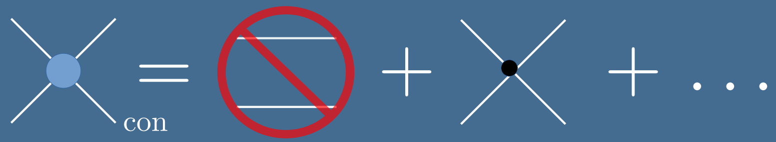
Polchinski equation

$$k\partial_k W_k[\Phi] = \frac{1}{2}\left(G_k^{ab} + \Phi^a \Phi^b\right) k\partial_k R_{k,ab}$$

Schwinger functional

Connected diagrams

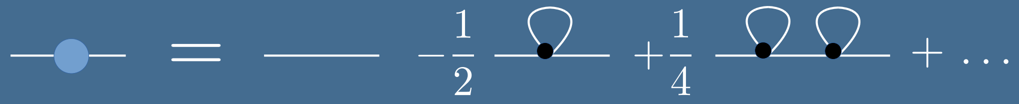
$$W_k[J] = \log Z_k[J]$$



Polchinski equation

$$k\partial_k W_k[\Phi] = \frac{1}{2}\Big(G_k^{ab} + \Phi^a\Phi^b\Big) k\partial_k R_{k,ab}$$

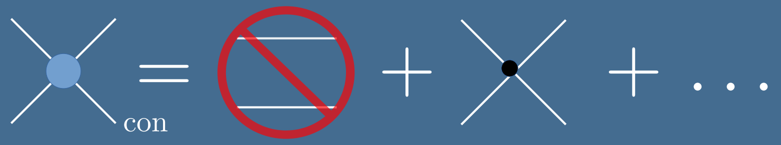
$G =$



Schwinger functional

Connected diagrams

$$W_k[J] = \log Z_k[J]$$



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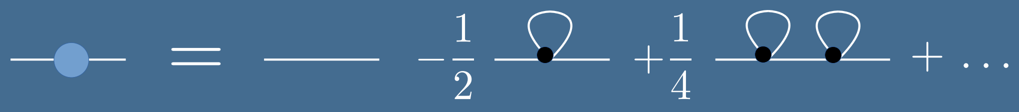

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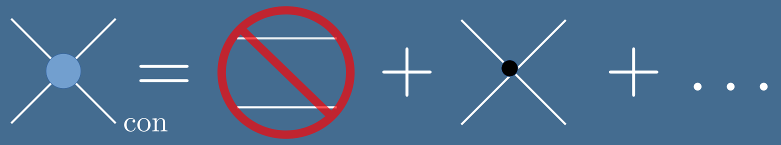

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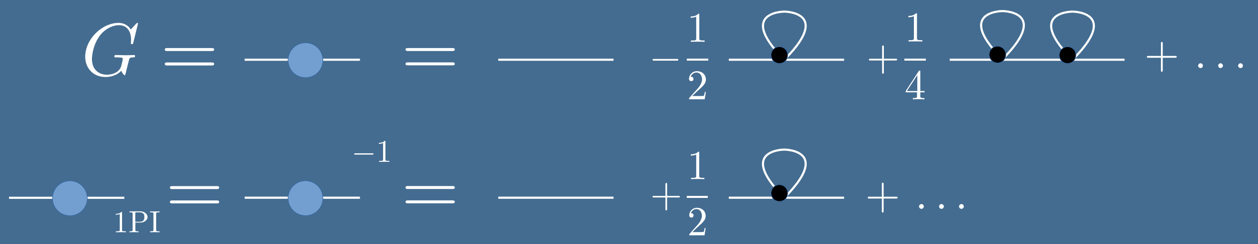
Legendre Transf.

$$\Phi = \langle\varphi\rangle_W$$



1PI effective action (Gibbs Free Energy)

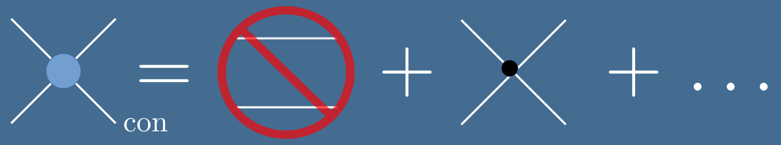
1PI diagrams

$$\Gamma_k[\Phi] = \max_J\left(J_a\,\Phi^a - W[J]\right) - \tfrac{1}{2}\Phi^a R_{k,ab}\Phi^b$$

Schwinger functional

Connected diagrams

$$W_k[J] = \log Z_k[J]$$



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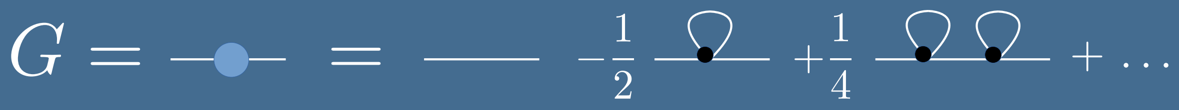

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1PI effective action (Gibbs Free Energy)

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Wetterich equation

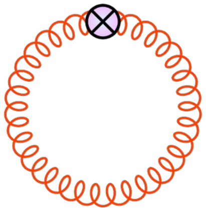
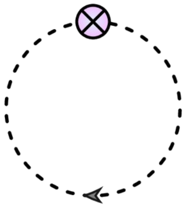
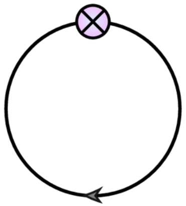
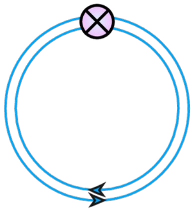
Wetterich [Phys.Lett.B 301 (1993) 90-94]

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} G^{ab}\partial_t R_{k,ab}$$

$$t = \ln\left(\frac{k}{\Lambda_{\mathrm{UV}}}\right)$$

Flow of 1PI effective action (Gibbs free energy):

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} G^{ab} \partial_t R_{k,ab}$$

$= \frac{1}{2}$  $+ \frac{1}{2}$  $-$  $+ \frac{1}{2}$ 

Flow of 1PI effective action (Gibbs free energy):

↓

Infinite tower of functional equations

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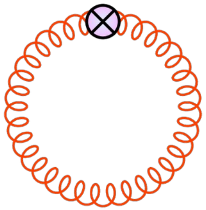
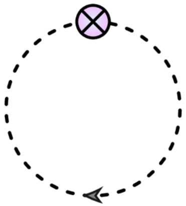
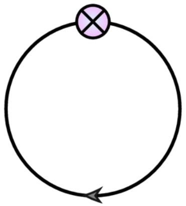
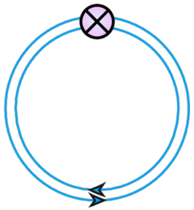
$= \frac{1}{2}$  $+\frac{1}{2}$  $-$  $+\frac{1}{2}$ 

$$\partial_t \Gamma[\bar\phi] = \frac{1}{2} \mathrm{Tr}\, G_k\, \partial_t R_k\,,$$

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$$\partial_t \Gamma^{(3)}[\bar\phi] = -\frac{1}{2} \mathrm{Tr} \left[\Gamma_k^{(5)} - 6\,\Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} + 6\,\Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\right] \left(G_k\, \partial_t R_k\, G_k\right),$$

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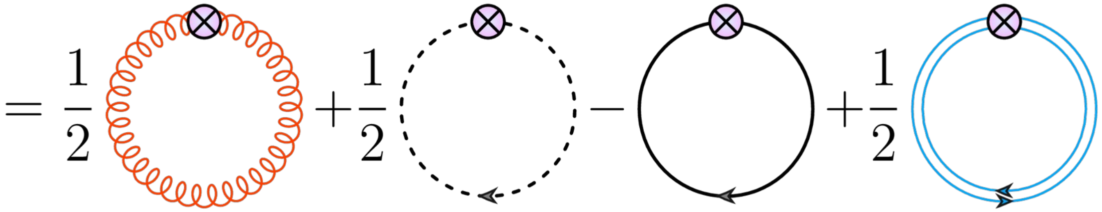
$\vdots \qquad\qquad \vdots$

Flow of 1PI effective action (Gibbs free energy):

↓

Infinite tower of functional equations

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} G^{ab} \partial_t R_{k,ab}$$



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Quantum Equation of Motion (EoM) $\left.\frac{\delta \Gamma}{\delta \Phi}\right|_{\mathrm{EoM}} = 0$

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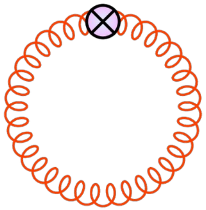
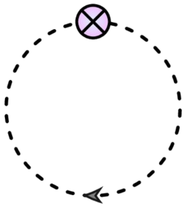
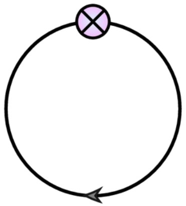
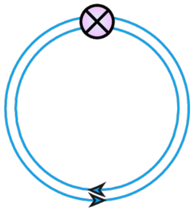
$= \frac{1}{2}$  $+\frac{1}{2}$  $-$  $+\frac{1}{2}$ 

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Quantum Equation of Motion (EoM) $\left.\frac{\delta \Gamma}{\delta \Phi}\right|_{\mathrm{EoM}} = 0$

Choice of truncation

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2} G^{ab} \partial_t R_{k,ab}$$

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$$\Big\downarrow$$

flowing field reparametrization

$$\dot{\Phi} = \mathcal{O}[\Phi]$$

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$$\dot{\Phi} = \mathcal{O}[\Phi]$$

Generalized Flow equation

Gies, Wetterich [Phys.Rev. D 65 (2002), 065001]
Pawlowski [Annals Phys. 322 (2007) 2831-2915]

$$\left(\partial_t + \dot{\Phi}^a \frac{\delta}{\delta \Phi^a} \right) \Gamma_k = \frac{1}{2} G^{ac} \left(\delta^b_c \partial_t + 2 \frac{\delta \dot{\Phi}^b}{\delta \Phi^c} \right) R_{k,ab}$$

TensorBases

Mathematica package

With J. Braun,
J. Pawlowski, A. Geißel,
N. Wink

FunKit

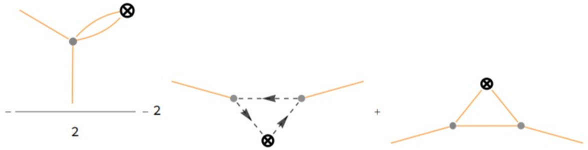
Mathematica package

DiFfRG framework

C++ library

- Library of tensor bases, extendable by everyone
- Automatically derived projectors

```
In[24]:= fRGAA = TakeDerivatives[WetterichEquation, {A[i1], A[i2]}] // FTruncate // FSimplify[#, "Symmetries" → symmetryP2] & // FPlot // FRoute // FPrint;
traceExprAA = F[TBGetProjector["AA", 1, {i1, i2} /. fRGAA["1-Loop"]["ExternalIndices"]], (fRGAA["1-Loop"]["Expression"] /. MakeDiagrammaticRules[])]
FlowAA = FormTrace[traceExprAA] // dressingRules // FORMSimplify // PropParam;
MakeCppFunction[FlowAA["1-Loop"], "Name" → "FlowAA", "Parameters" → {"l1", "p", "cos", "gS", "ZA", "Zc", "ZA3", "ZccbA"}]
```



$$\int_{l_1}(-2\,G_{\bar{c}^a c^b}(l_1,-l_1)\,\Gamma_{c^b\bar{c}^d A^m}(l_1,p_1-l_1,-p_1)\,G_{\bar{c}^d c^f}(l_1-p_1,p_1-l_1)\,\Gamma_{c^f\bar{c}^h A^n}(l_1-p_1,-l_1,p_1)\,G_{\bar{c}^h c^j}(l_1,-l_1)\,\partial_t R_{c^j\bar{c}^a}(l_1,-l_1))$$
$$+\int_{l_1} G_{A^aA^b}(-l_1,l_1)\,\Gamma_{A^cA^bA^m}(l_1+p_1,-l_1,-p_1)\,G_{A^cA^f}(-l_1-p_1,l_1+p_1)\,\Gamma_{A^gA^fA^n}(l_1,-l_1-p_1,p_1)\,G_{A^gA^j}(-l_1,l_1)\,\partial_t R_{A^jA^a}(-l_1,l_1)$$
$$+\int_{l_1}(-\frac{1}{2}G_{A^aA^b}(-l_1,l_1)\,\Gamma_{A^cA^bA^iA^j}(l_1,-l_1,-p_1,p_1)\,G_{A^cA^f}(-l_1,l_1)\,\partial_t R_{A^fA^a}(-l_1,l_1))$$

```
Out[27]= auto FlowAA(const auto &l1, const auto &p, const auto &cos, const auto &gS, const auto &ZA, const auto &Zc,
                     const auto &ZA3, const auto &ZccbA)
{
  return -4. *
         (dtZA(pow(1. + powr<6>(k), 0.16666666666666666667)) *
```

- Code generation for large fRG systems
- Hydrodynamic methods for full field dependences
- GPU accelerated

Our code is open Source, see github.com/satfra

Gauge consistency

Slavnov-**T**aylor **I**dentities (STIs) encode the *gauge consistency* in a gauge-fixed setting.

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In terms of BRST transformations $\mathfrak{s}A^a_\mu = D^{ab}_\mu c^b\,, \quad \mathfrak{s}q = -g c^a t^a q\,, \quad \mathfrak{s}c^a = -\frac{g}{2} f^{abc} c^b c^c\,, \quad \mathfrak{s}\bar{c}^a = \frac{1}{\xi}\mathcal{F}^a[A]\,.$

$$\int_x \langle J_A \cdot \mathfrak{s}A - J_c \cdot \mathfrak{s}c - J_{\bar{c}} \cdot \mathfrak{s}\bar{c} \rangle = 0\,.$$

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Adding a regulator leads to modifications ***at k > 0***.

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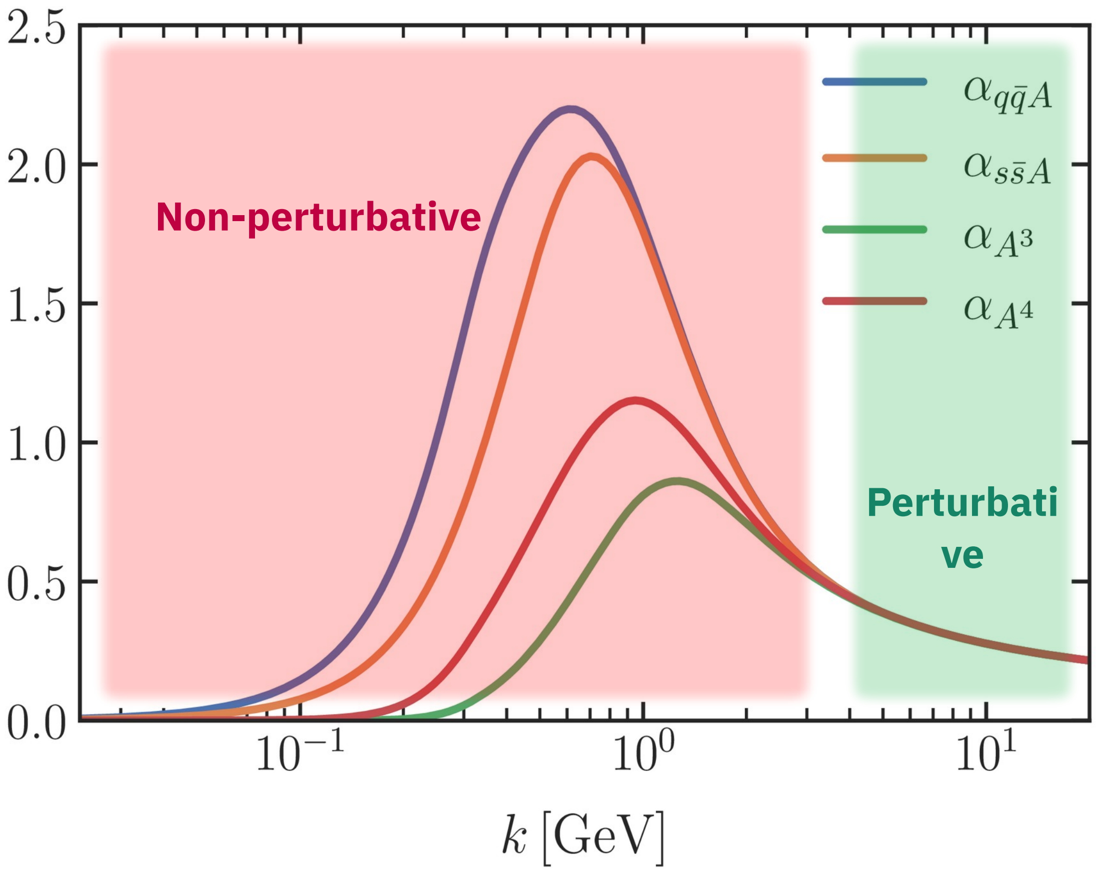
In the perturbative regime, the STIs lead at **k=0** to

$$\alpha_{q\bar{q}A}(\bar{p}) = \alpha_{c\bar{c}A}(\bar{p}) = \alpha_{A^3}(\bar{p}) = \alpha_{A^4}(\bar{p})\,.$$

Gauge consistency



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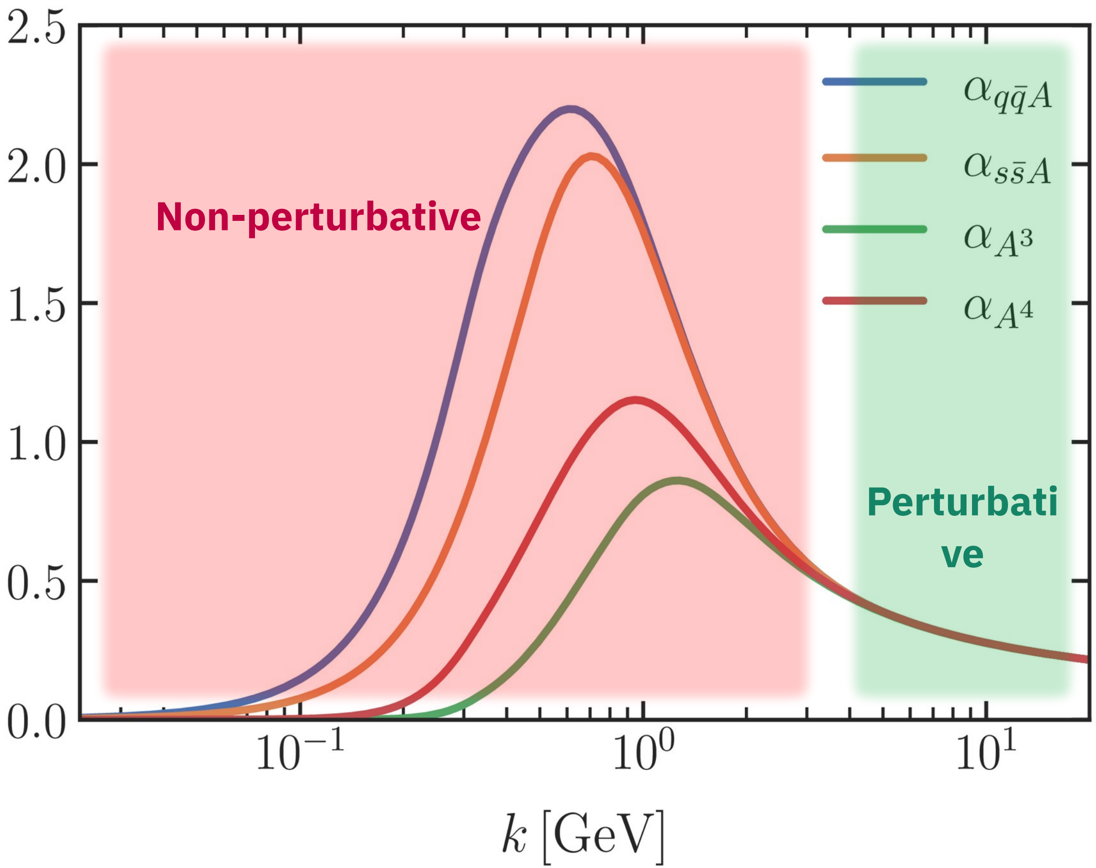
Gauge consistency

We make sure to fulfil perturbative STIs, which already leads to excellent results for the full STIs, see e.g.

Bonini, D'Attanasio, Marchesini, Nucl.Phys.B 437 (1995) 163-186
Bonini, D'Attanasio, Marchesini, Phys.Lett.B 346 (1995) 87-93
Ellwanger, Hirsch, Weber, Z.Phys.C 69 (1996) 687-698,
…….
Fischer, Maas, Pawlowski, Annals Phys. 324 (2009) 2408-2437
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, Phys.Rev.D 94 (2016) 5, 054005
Pawlowski, Schneider, Wink, arXiv:2202.11123 [hep-th]

Particularily important: *Gauge-consistency of the quark-propagator*

With Kockler, Pawlowski



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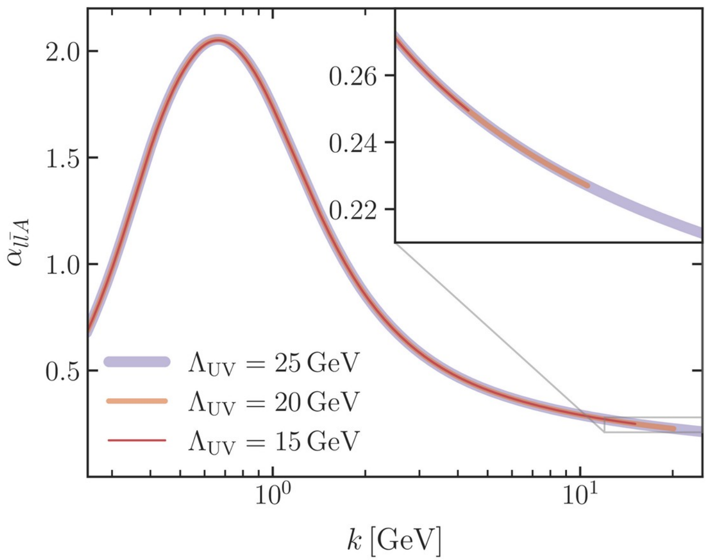
Do we depend on the UV cutoff?



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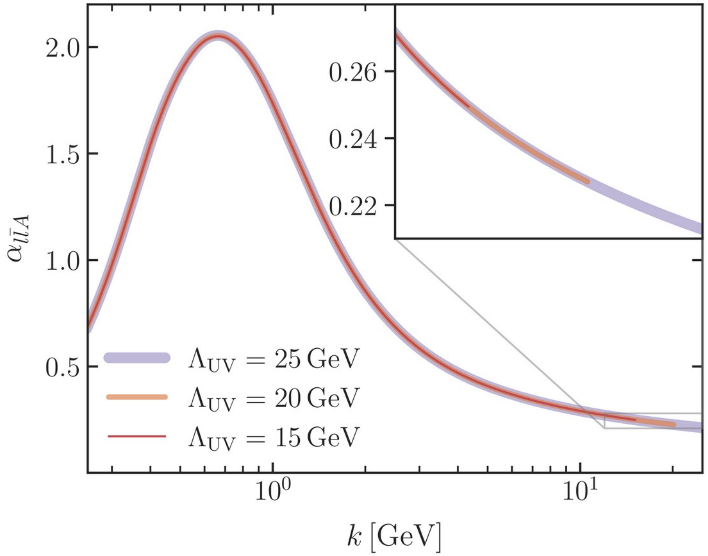

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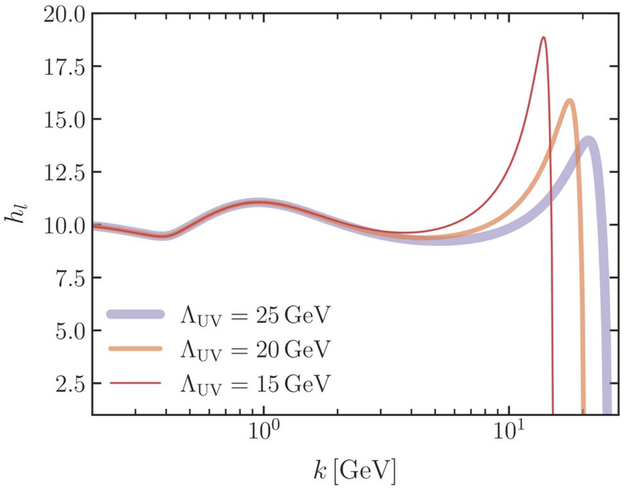

- Start of rebosonisation does not influence matter:
  fixed Point

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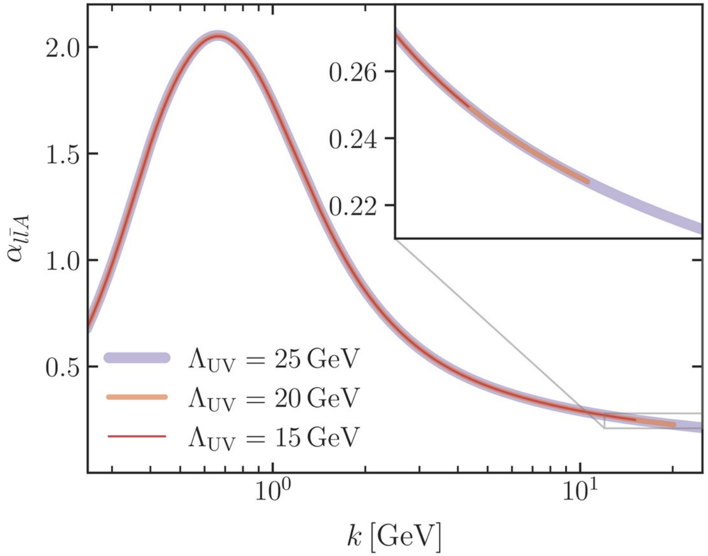

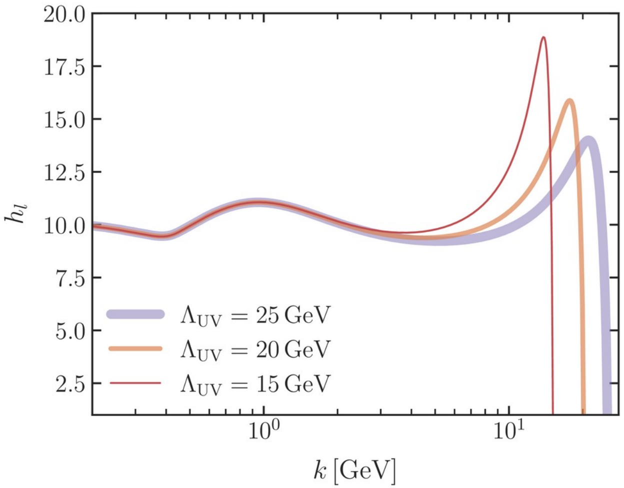

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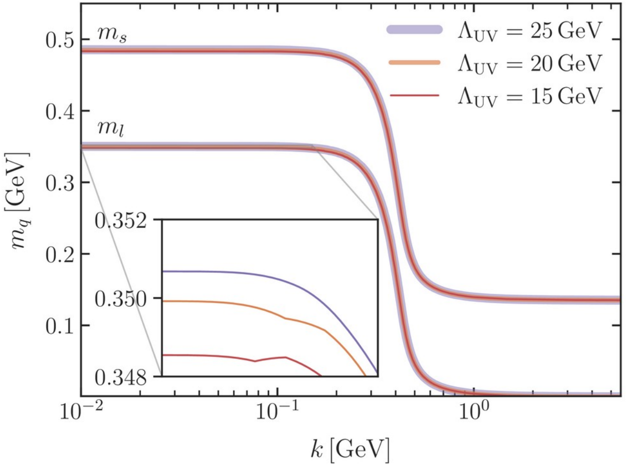
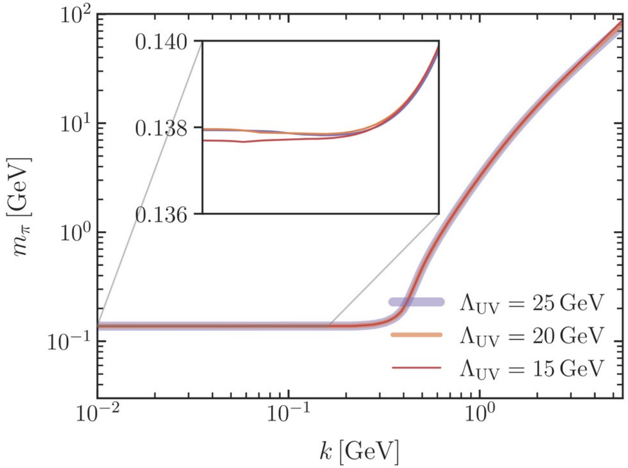


- Low-energy quantities:
  Error due to initial cutoff
  ~ 1MeV



Grossi, Wink [SciPost Phys.Core 6 (2023) 071]
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[Comput.Phys.Commun. 300 (2024) 109182]
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[Phys.Rev.D 111 (2025) 3, 036030]

- Flow equation does not depend on lower n-point functions

Hydrodynamic approach to fRG

$$k\partial_k \Gamma^{(n)} = \mathrm{Flow}[\Gamma^{(2)}, \ldots, \Gamma^{(n+2)}]$$

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$$\rho = \frac{\pi^2 + \sigma^2}{2}$$

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$$\delta^{(4)}(0)\, V_{\mathrm{eff}}(\rho) \sim \Gamma[\Phi]\big|_{p=0}$$

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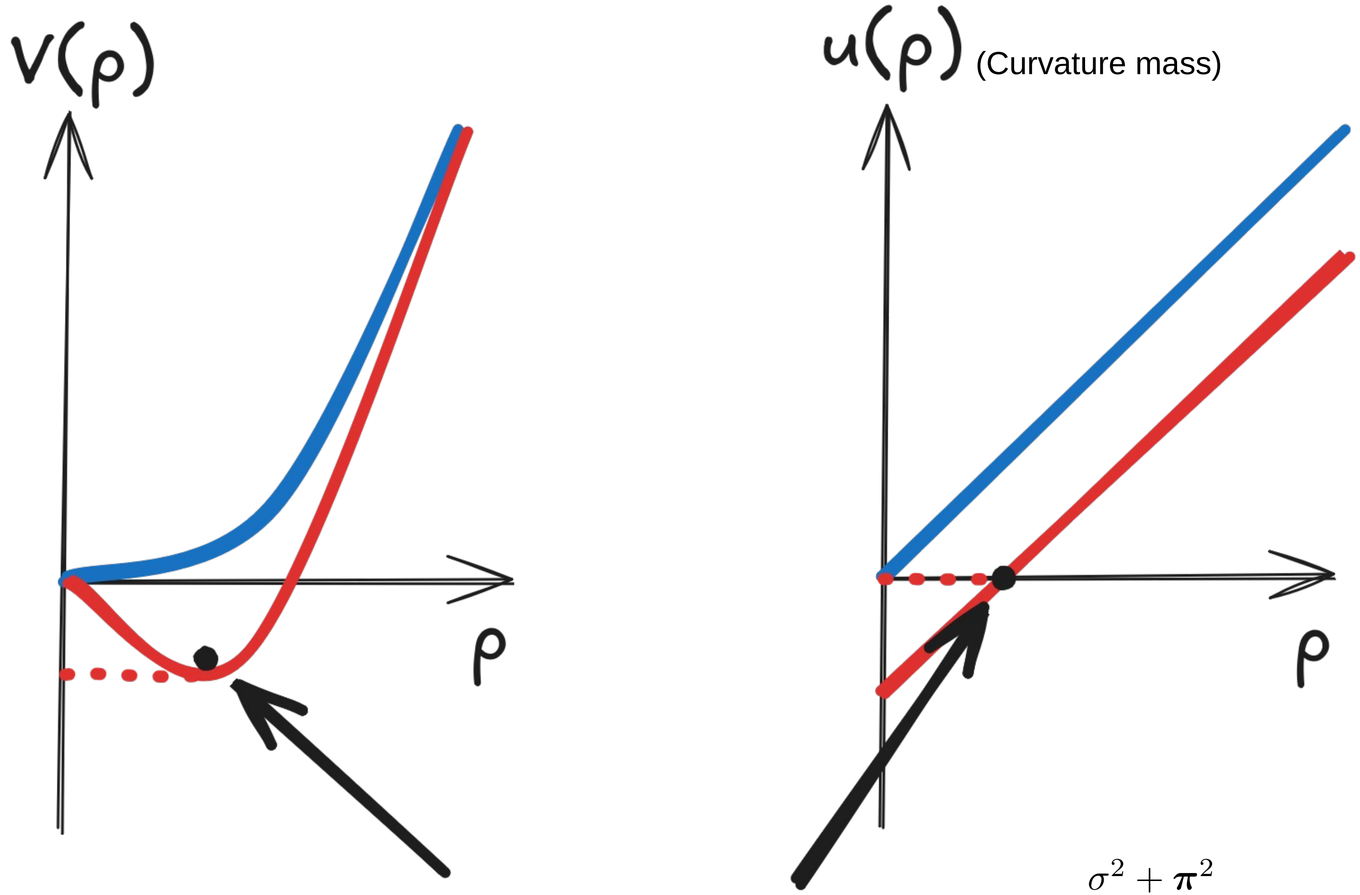
- Convection-Diffusion equation with sources(sinks)

$$k\partial_k u(\rho) = \partial_\rho F_k(u, \partial_\rho u, \rho)$$

- Ground state is at minimum of $V(\rho)$ equivalently, at $u(\rho) = 0$.



Ground state

$$\left.\frac{\delta \Gamma}{\delta \Phi}\right|_{\mathrm{EoM}} = 0$$

$$\rho := \frac{\sigma^2 + \boldsymbol{\pi}^2}{2}$$

$$u(\rho) := \partial_\rho V(\rho) = m^2_{\pi,\mathrm{curv}}(\rho)$$

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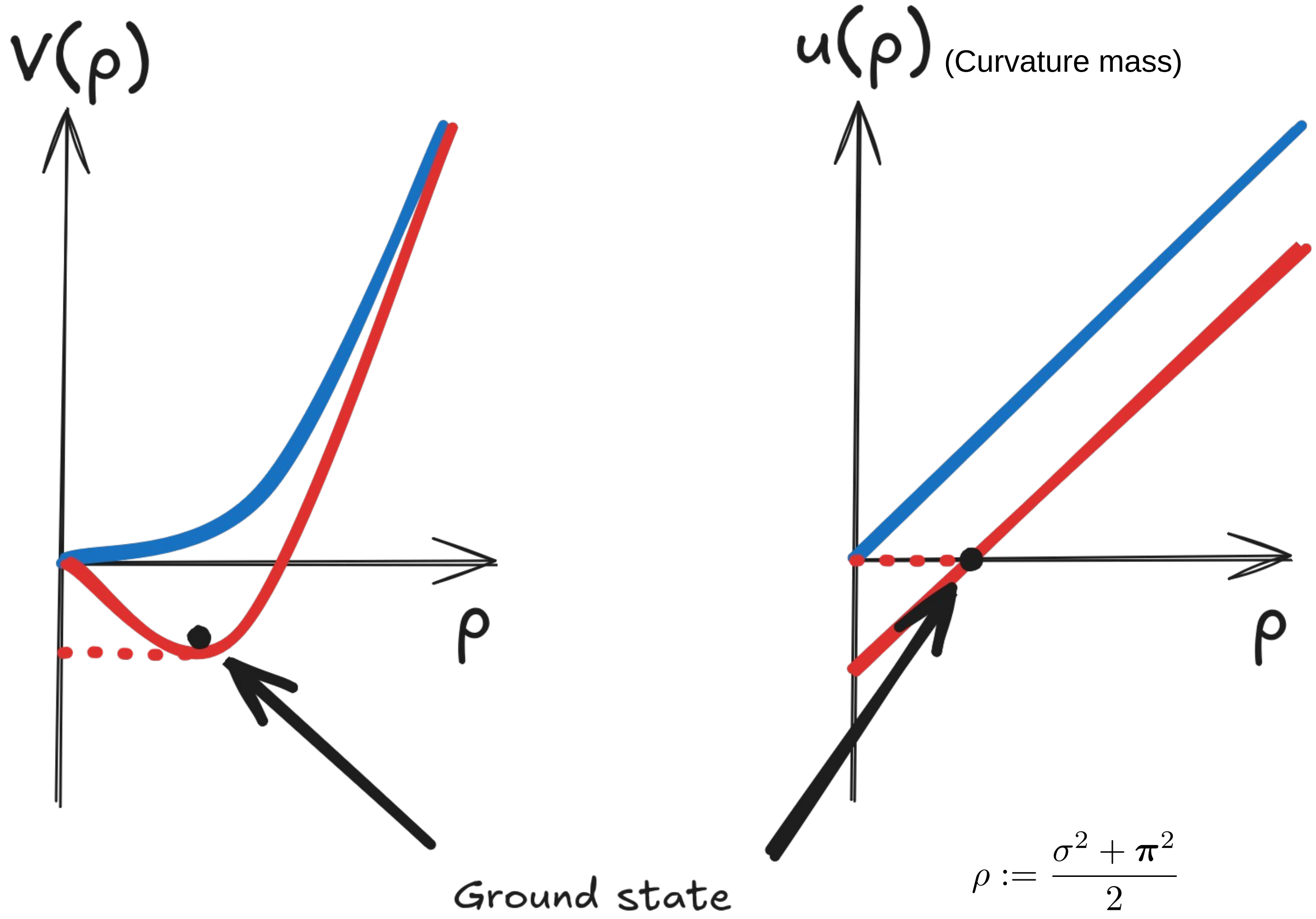
- Quarks act as sinks.



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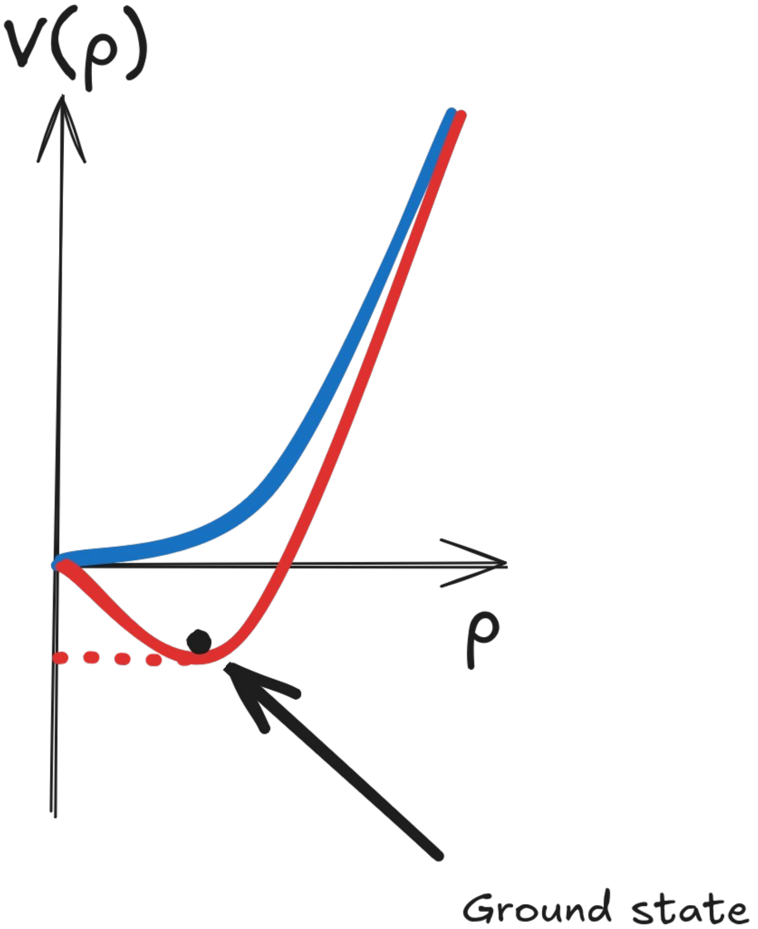
- Ground state is at minimum of $V(\rho)$ equivalently, at $u(\rho) = 0$.
- Quarks act as sinks.
- Mesons introduce advection and diffusion.



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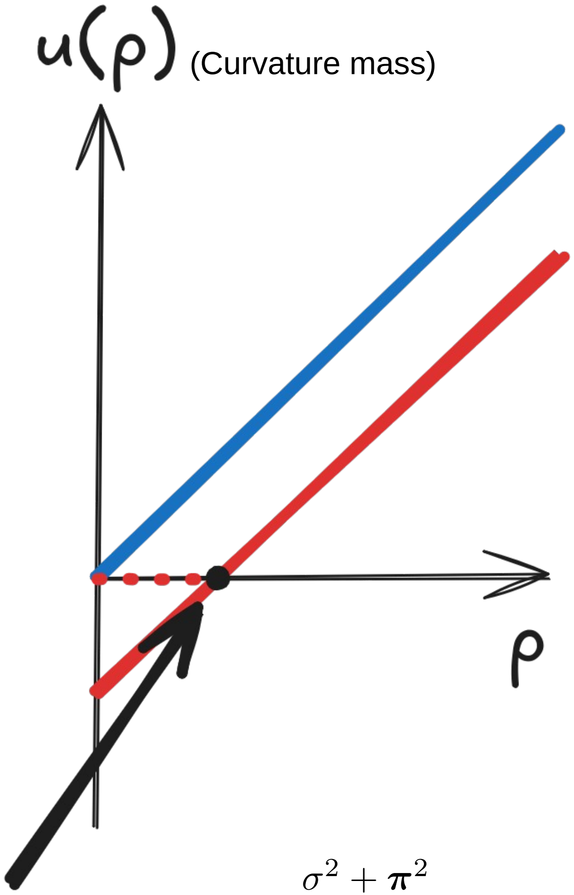

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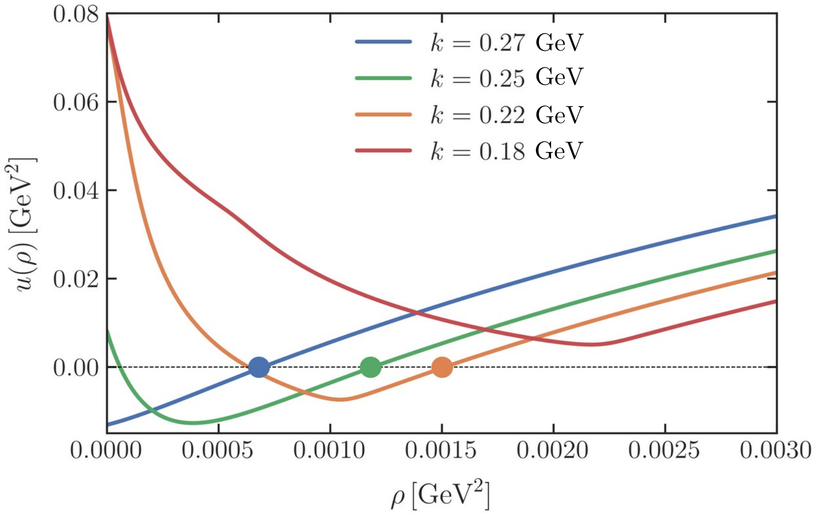
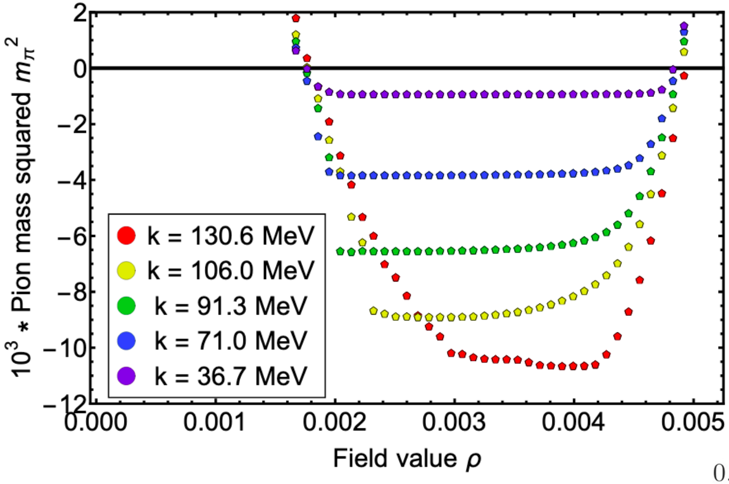
- First-order transitions



- Possible generation of shocks in $u(\rho)$



Grossi, Ihssen, Pawlowski, Wink
[Phys.Rev.D 104 (2021) 1, 016028]
Koenigstein, Steil, Wink, Grossi, Braun
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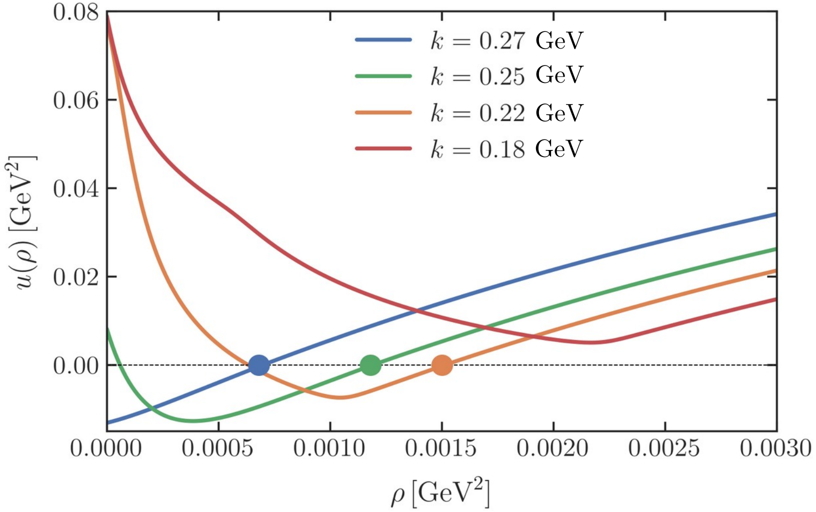

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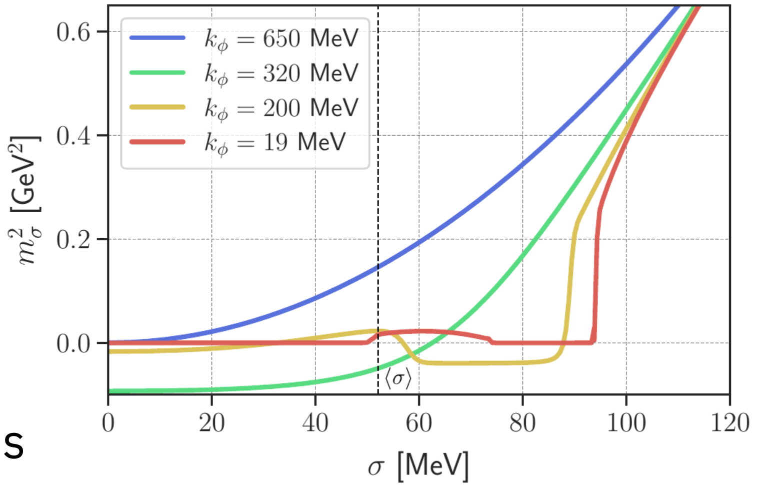

Ihssen, Pawlowski, Sattler, Wink
[Phys.Rev.D 111 (2025) 3, 036030]

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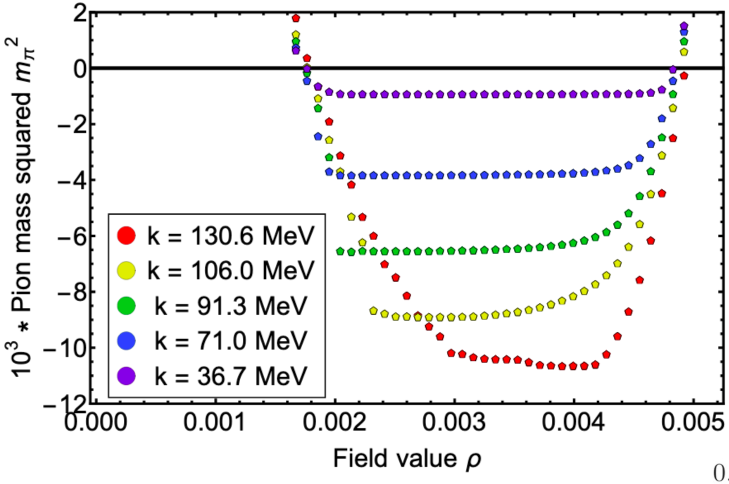

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Ihssen, Pawlowski, Sattler, Wink
[arXiv:2408.08413 (2024)]

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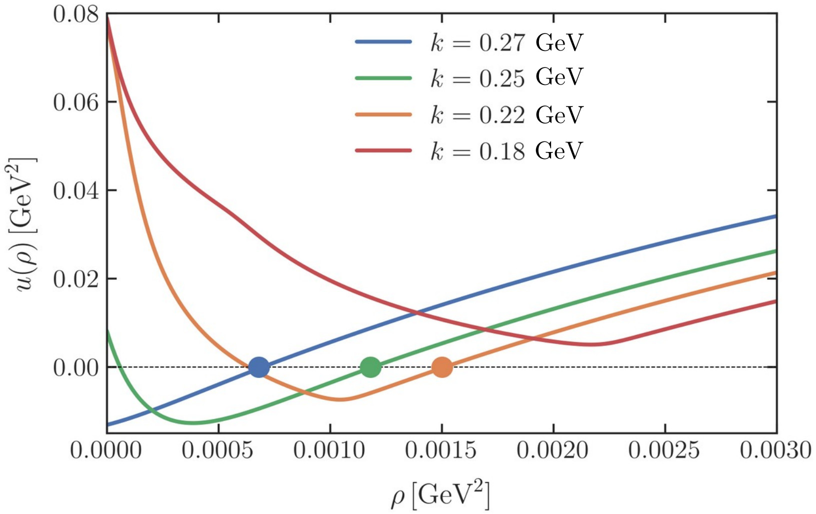

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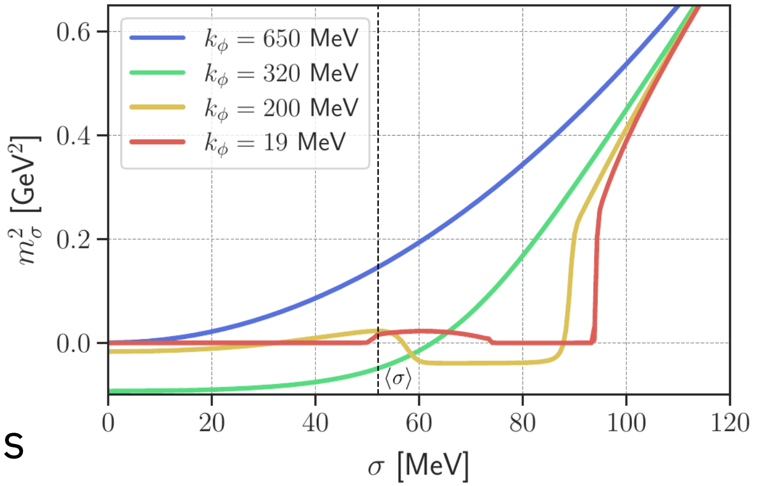

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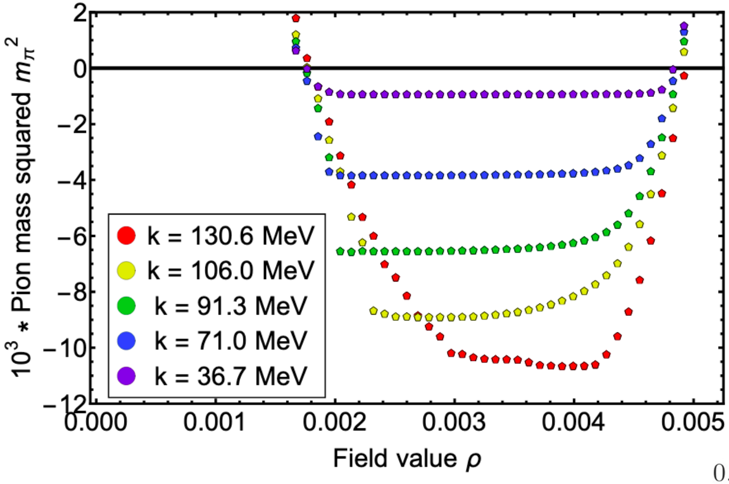

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FE/FV

Grossi, Wink [SciPost Phys.Core 6 (2023) 071]

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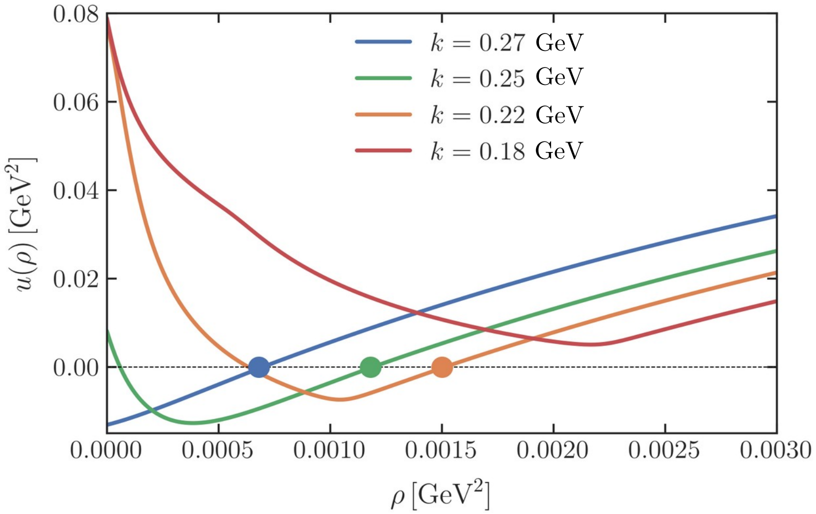

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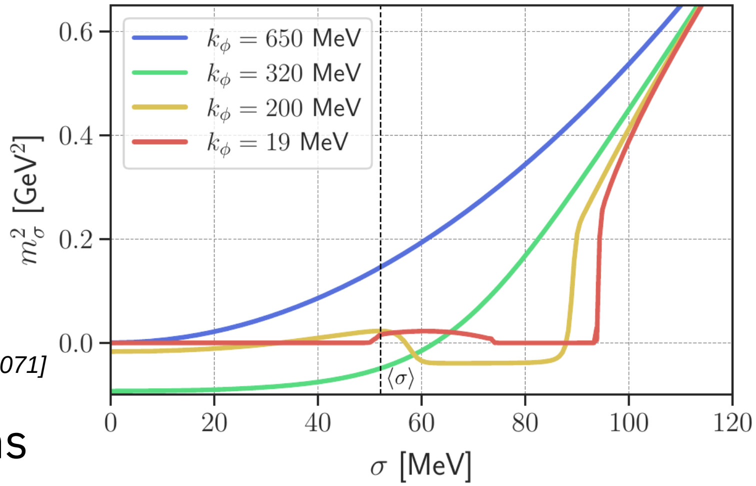

Ihssen, Pawlowski, Sattler, Wink
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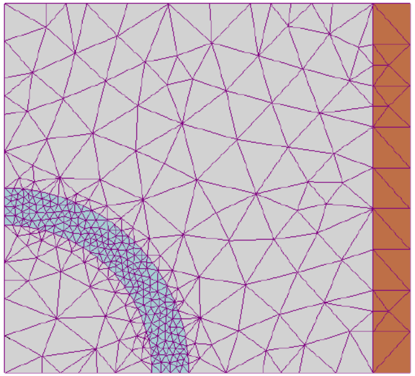
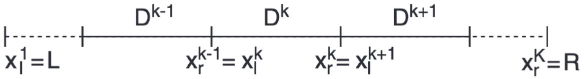
Finite element methods for couplings

1. Put it on a Mesh



or



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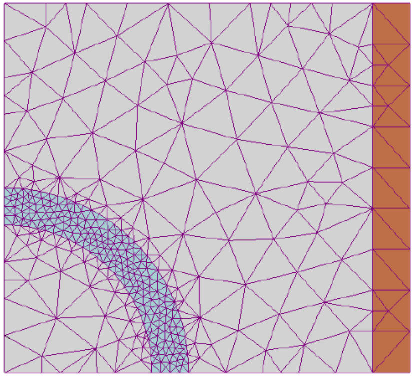

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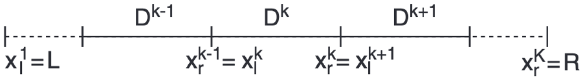


2. Choose basis functions

$$u_h(x,t)\Big|_{D_k} = \sum_{n}^{N} u_n(t)\psi_n(x)$$

+ sensible time integration

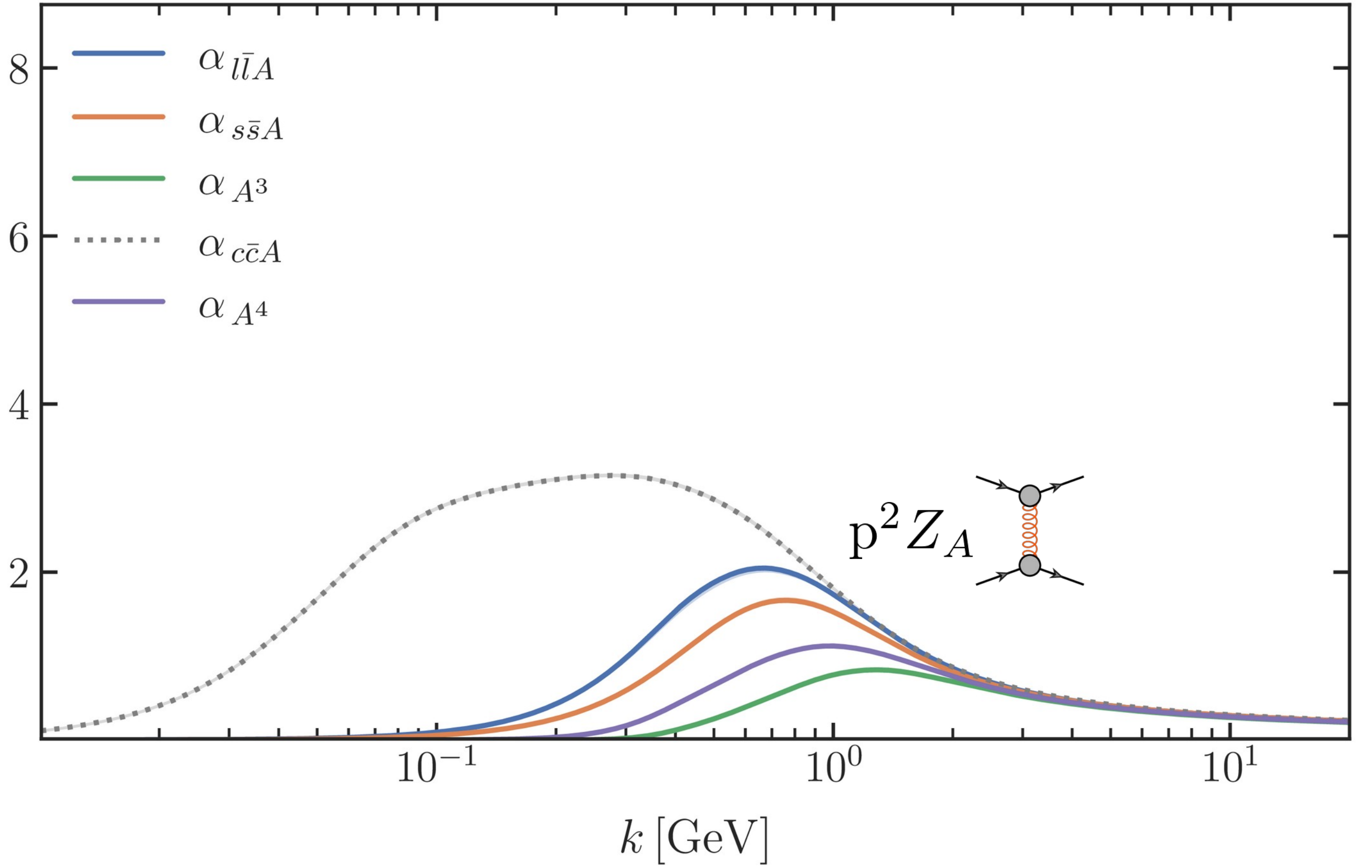
Sattler, Ihssen, Wink, [Phys.Rev.D 107 (2023) 11, 114009]

3. Discretise the PDE

$$\partial_t u + \partial_x F(u) = 0$$

$$\int_{D^k} \Big((\partial_t u_h^k)\phi_n - F(u_h^k)\partial_x \phi_n \Big)\, \mathrm{d}x = -\int_{\partial D^k} F^* \cdot \hat{n}\, \phi_n \, \mathrm{d}x$$

Systematic errors I: The LEGO® principle



Systematic errors I:
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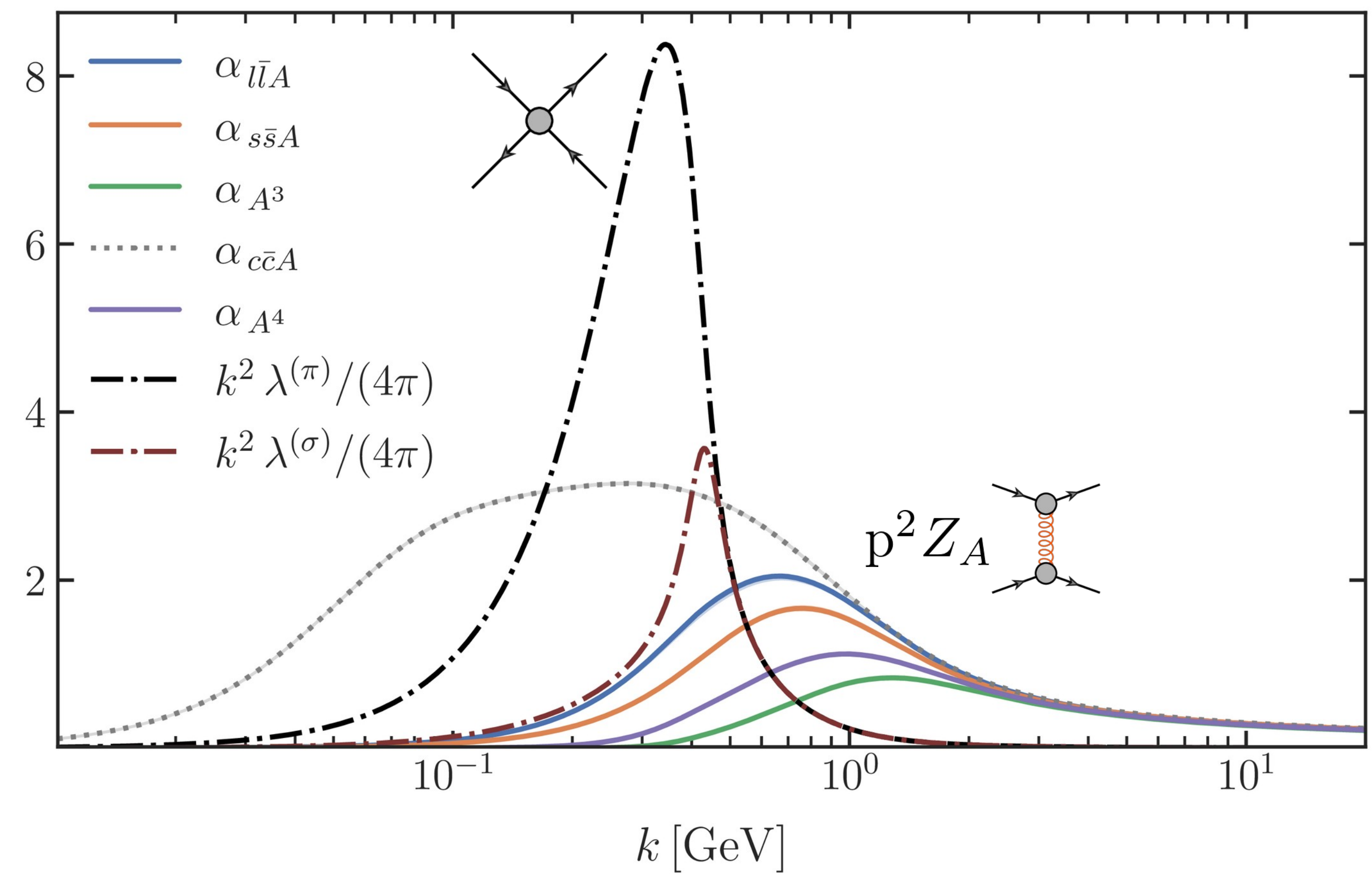

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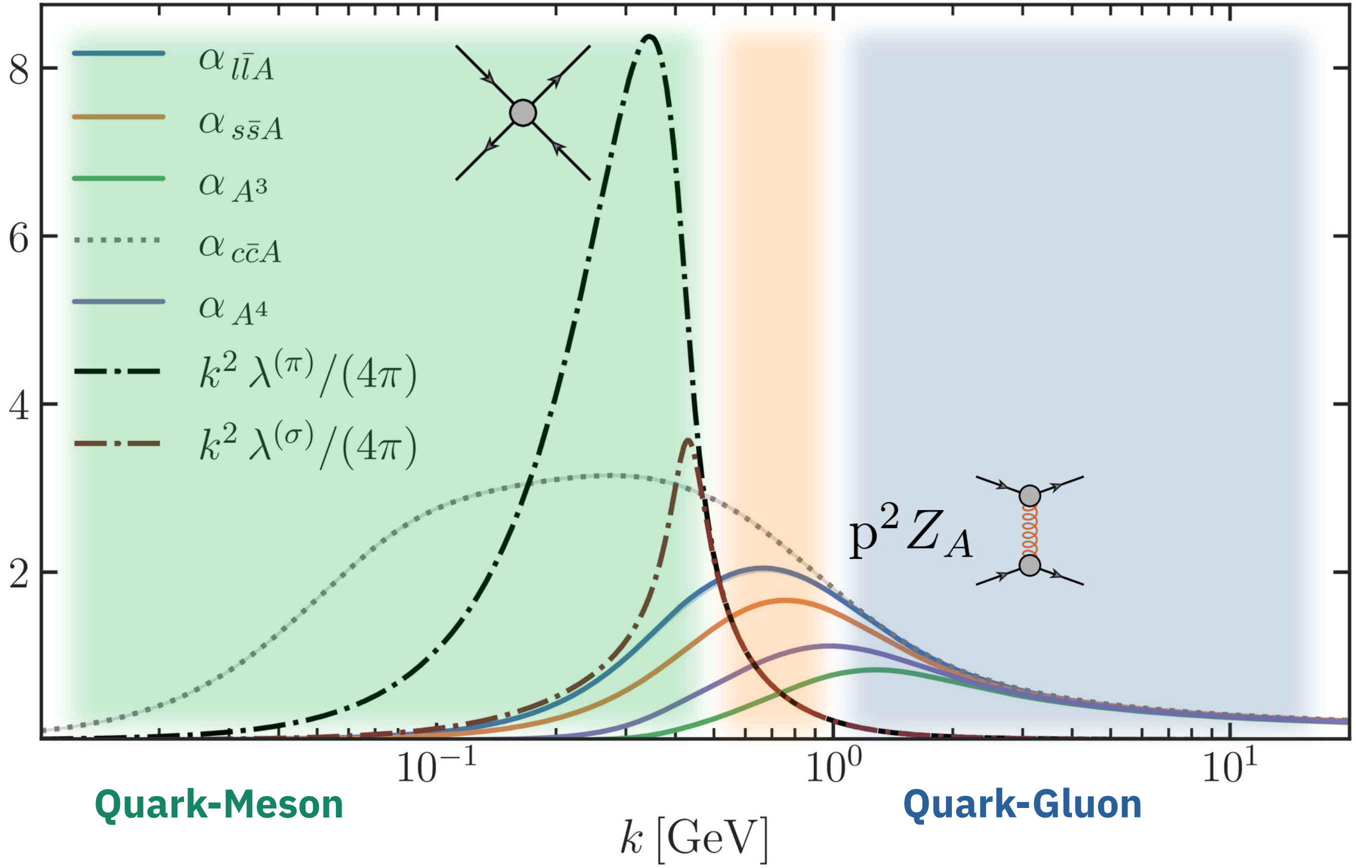

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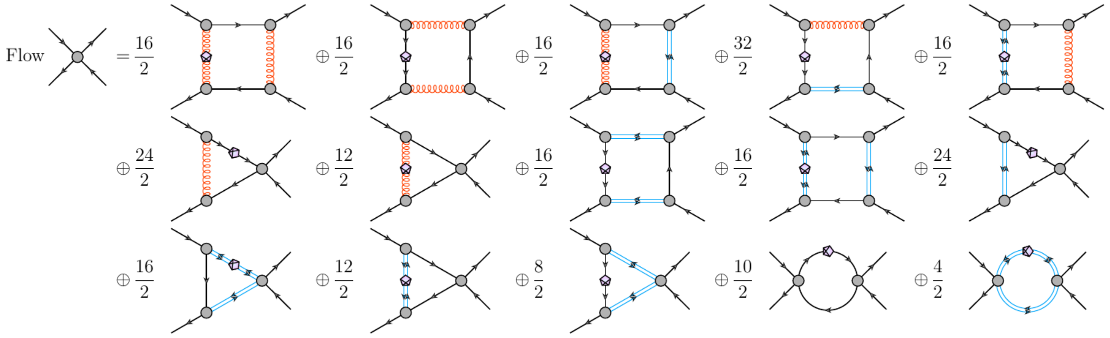
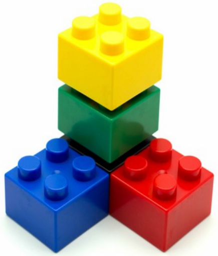
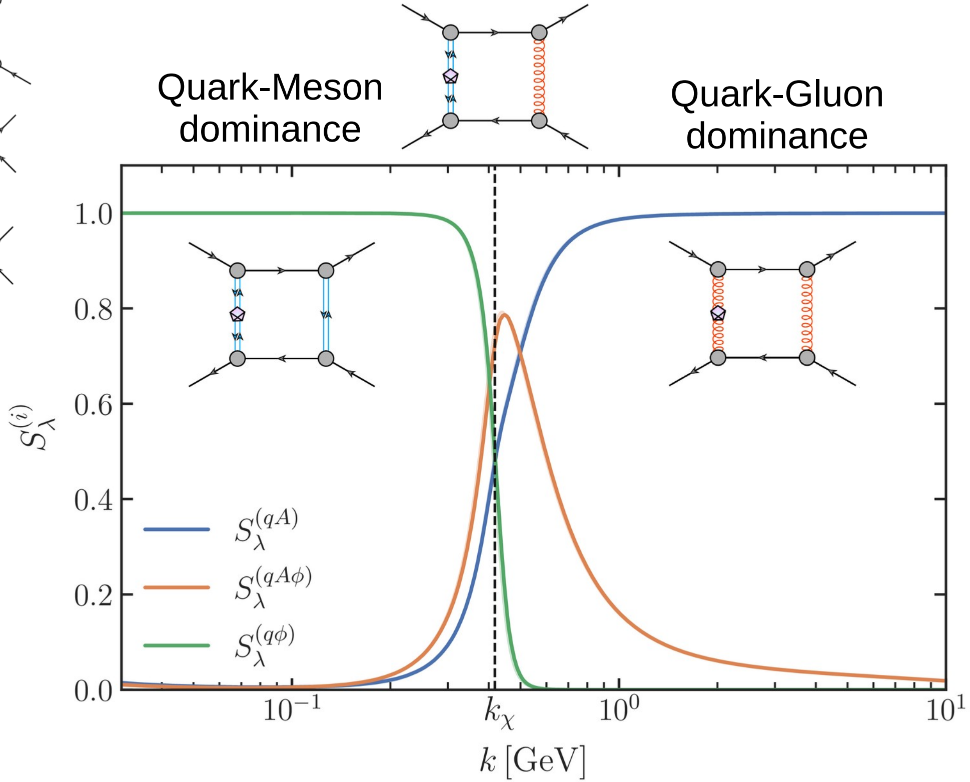


- ➔ Systematic error estimates from subsystems; preliminary estimate 10%.
- ➔ Low-energy effective theories.





Systematic errors I:
The LEGO® principle

Separate LEGO® blocks:

- Glue subsystem $\{\lambda_{\text{glue}}\} = \{\alpha_{A^3}\,,\, \alpha_{A^4}\,,\, \alpha_{c\bar{c}A}\}$
- Matter subsystem $\{\lambda_{\text{mat}}\} = \{h_\phi(\rho_0)\,,\, \lambda_{\phi,n}(\rho_0)\}$
- Interface blocks $\{\lambda_{\text{inter}}\} = \{\alpha_{l\bar{l}A}\}$

➔ Systematic error estimates from subsystems; preliminary estimate 10%.

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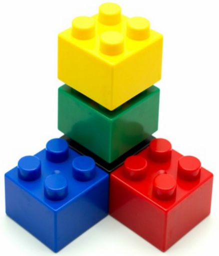
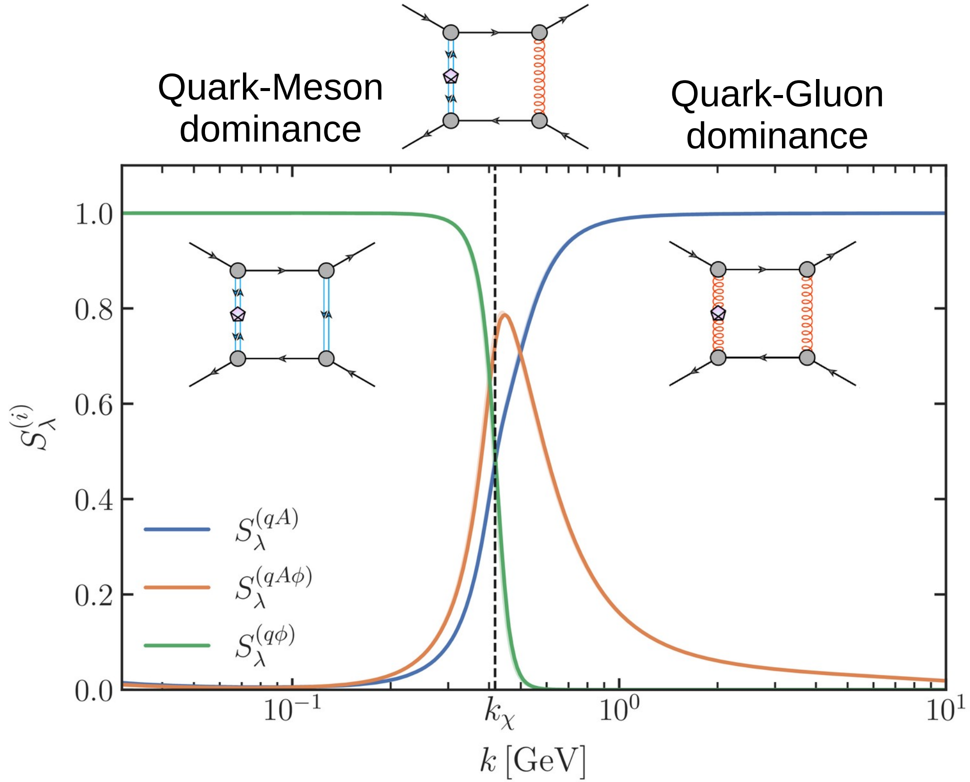
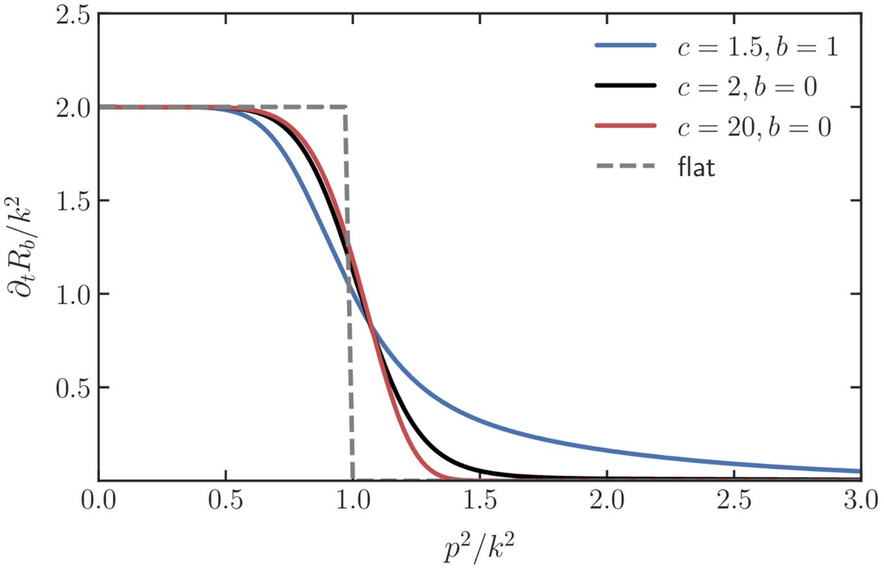




Systematic errors II: Regulator dependences

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{Tr}\, G_k \partial_t R_k$$

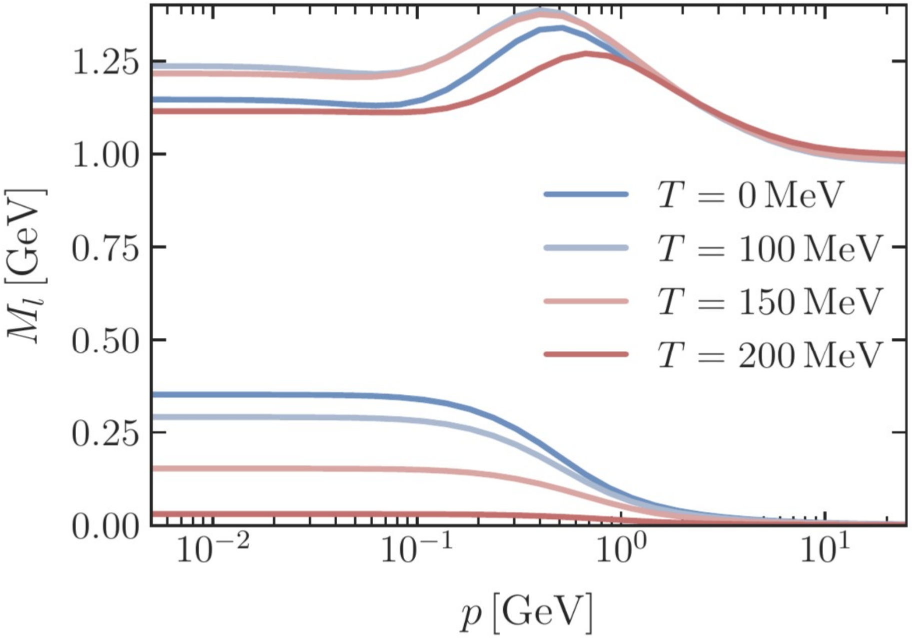
Easy regulator variation thanks to numerical framework:

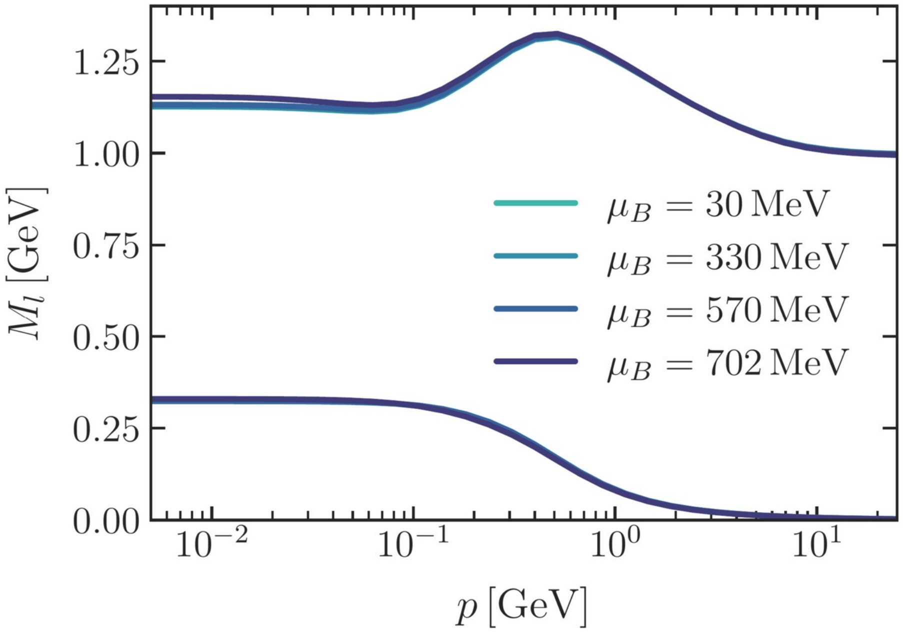


Observable	Value
$(f_K/f_\pi)_\chi$	$1.2168^{+0.0006}_{-0.0007}$
$f_{\pi,\chi}$ [MeV]	$93.2^{+3.5}_{-3.1}$
$m_{l,\chi}$ [MeV]	$311.6^{+0.3}_{-0.1}$
$m_{s,\chi}$ [MeV]	$446.7^{+0.3}_{-0.2}$
$m_{\sigma,\chi}$ [MeV]	$214.7^{+5.4}_{-9.3}$
$\sigma_{l,0,\chi}$ [MeV]	$67.1^{+1.2}_{-0.0}$

Chiral limit observables

Ihssen, Pawlowski, Sattler, Wink [arXiv:2408.08413]

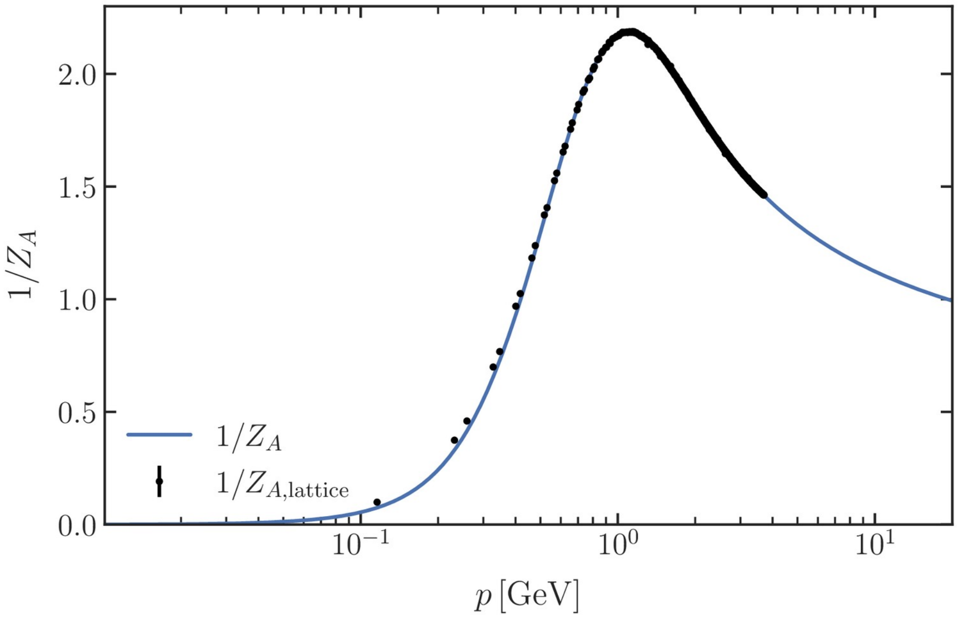




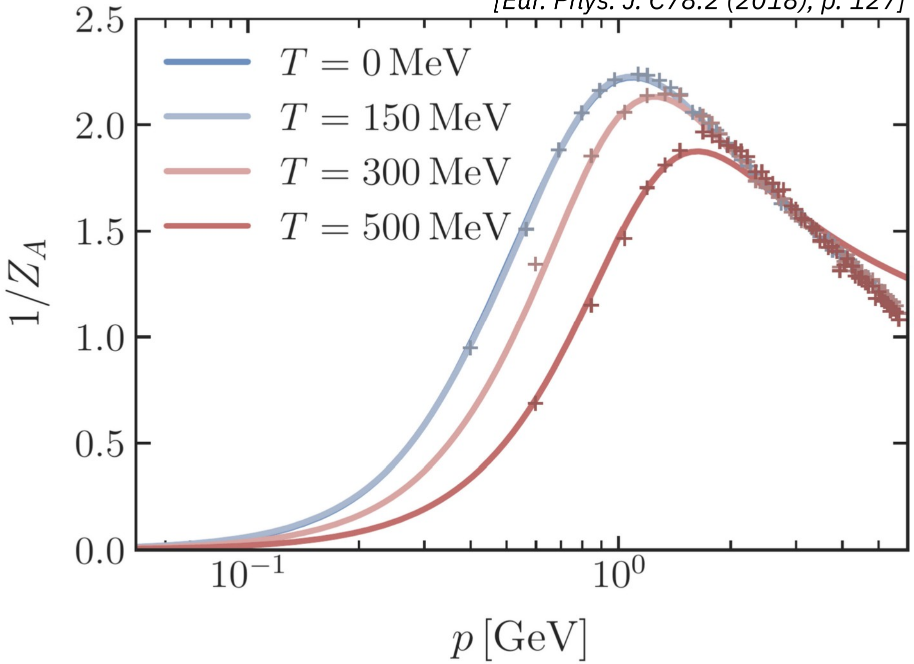
Momentum dependence of quark propagator

Silver Blaze symmetry

Lattice data from Boucaud et al.
[Phys.Rev.D 98 (2018) 11, 114515]



Lattice data from Ilgenfritz et al
[Eur. Phys. J. C78.2 (2018), p. 127]



Vacuum: fRG and lattice results fit except in deep infrared (solution branches)

Temperature dependence of Gluon propagator fits lattice calculations well.